2^{nd} ORDER O.D.E.s

24 EXAM QUESTIONS

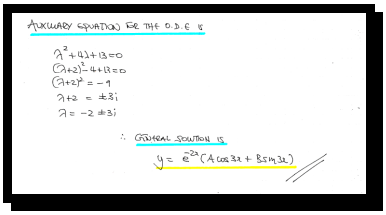
7 BASIC QUESTIONS

Question 1 (**)

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=0\,.$$

$\boxed{}$, $\boxed{y=\mathrm{e}^{-2x}\left[A\cos 3x+B\sin 3x\right]}$

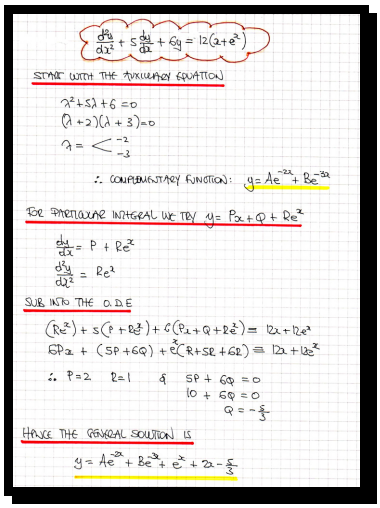


Question 2 (**)

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=12\left(x+\mathrm{e}^x\right).$$

$\square$, $\boxed{y=A\mathrm{e}^{-3x}+B\mathrm{e}^{-2x}+\mathrm{e}^x+2x-\frac{5}{3}}$

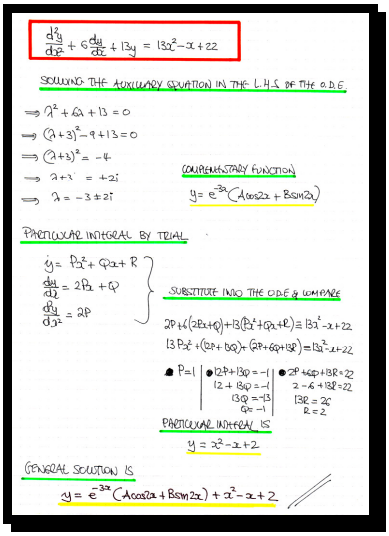


Question 3 (**)

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=13x^2-x+22\,.$$

$\boxed{}$, $\boxed{y=\mathrm{e}^{-3x}\left(A\cos 2x+B\sin 2x\right)+x^2-x+2}$



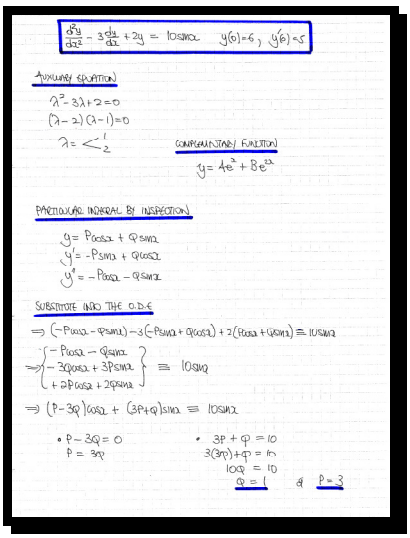
Question 4 (***)

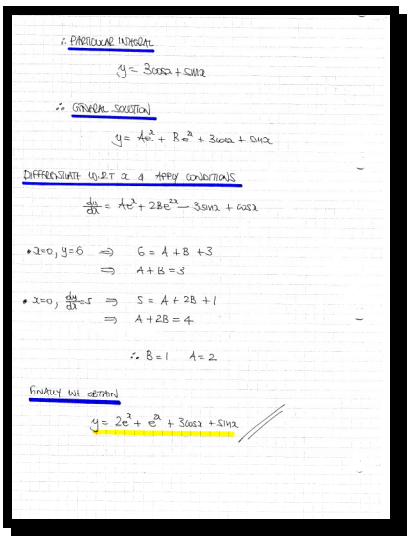
Find a solution of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=10\sin x,$$

subject to the boundary conditions $y=6$ and $\frac{dy}{dx}=5$ at $x=0$.

$\boxed{}$, $\boxed{y=2\mathrm{e}^{x}+\mathrm{e}^{2x}+3\cos x+\sin x}$





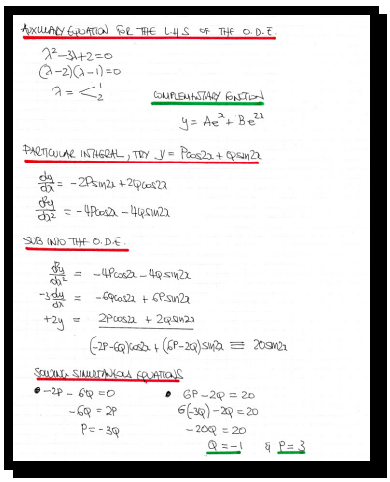
Question 5 (***)

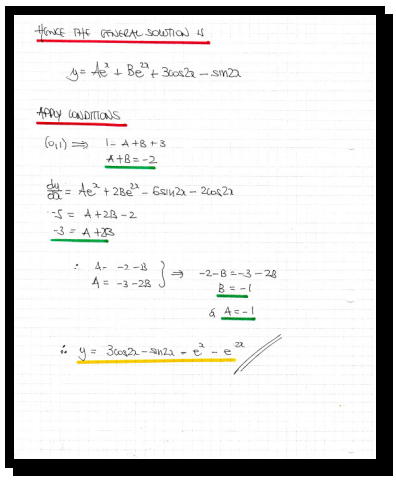
Find a solution of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=20\sin 2x\,,$$

subject to the boundary conditions $y=1$ and $\dfrac{dy}{dx}=-5$ at $x=0$.

$\boxed{}$, $\boxed{y=3\cos 2x-\sin 2x-\mathrm{e}^{2x}-\mathrm{e}^{x}}$



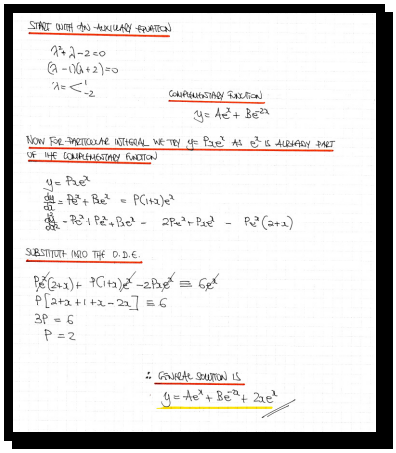


Question 6 (***)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=6\mathrm{e}^{x}.$$

$\boxed{}$, $\boxed{y=(A+2x)\mathrm{e}^{x}+B\mathrm{e}^{-2x}}$

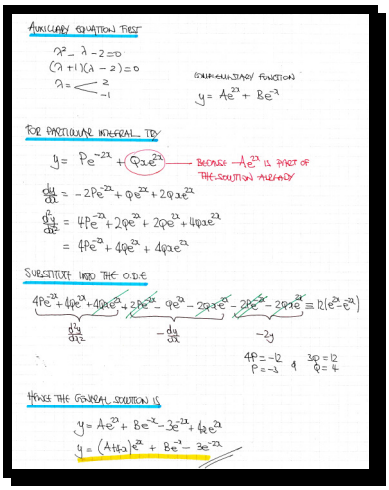


Question 7 (***)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=12\left(\mathrm{e}^{2x}-\mathrm{e}^{-2x}\right).$$

$\boxed{}$, $\boxed{y=\left(A+4x\right)\mathrm{e}^{2x}+B\mathrm{e}^{-x}-3\mathrm{e}^{-2x}}$



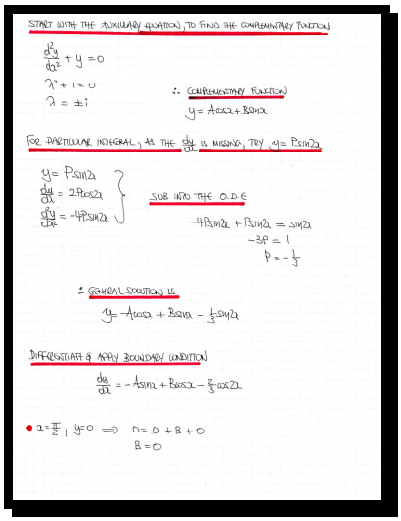
Question 8 (***)

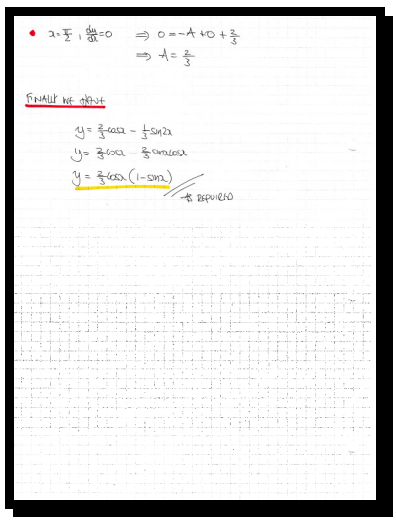
$$\frac{d^2y}{dx^2}+y=\sin 2x,\ \text{with}\ y=0\ ,\ \frac{dy}{dx}=0\ \ \text{at}\ x=\frac{\pi}{2}.$$

Show that a solution of the above differential equation is

$$y=\frac{2}{3}\cos x\left(1-\sin x\right).$$

$\boxed{}$, $\boxed{\text{proof}}$





16 STANDARD QUESTIONS

Question 1 **(***+)**

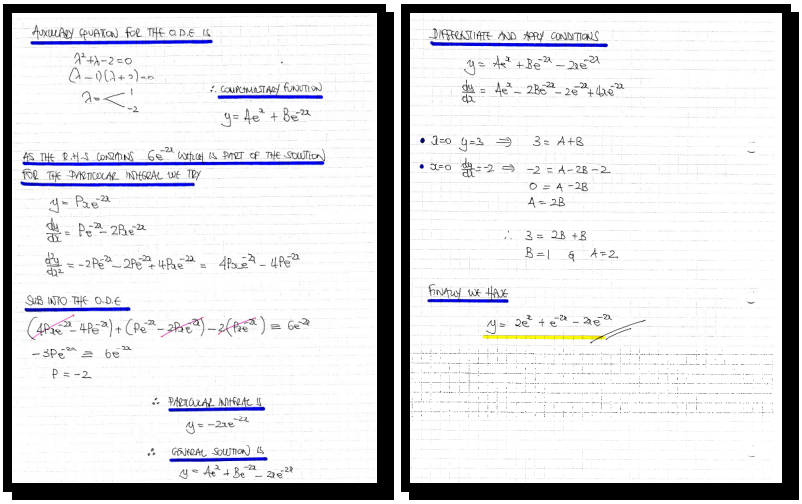
$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=6\mathrm{e}^{-2x},$$

with $y=3$ and $\frac{dy}{dx}=-2$ at $x=0$.

Show that the solution of the above differential equation is

$$y=2\mathrm{e}^{x}+(1-2x)\mathrm{e}^{-2x}.$$

[ ], [proof]

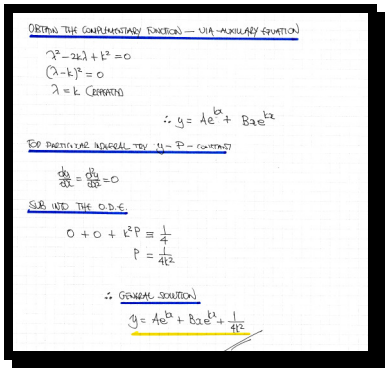


Question 2 (*+)**

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}-2k\frac{dy}{dx}+k^2y=\frac{1}{4},\ k>0.$$

$\boxed{}$, $\boxed{y=A\mathrm{e}^{kx}+Bx\mathrm{e}^{kx}+\frac{1}{4k^2}}$



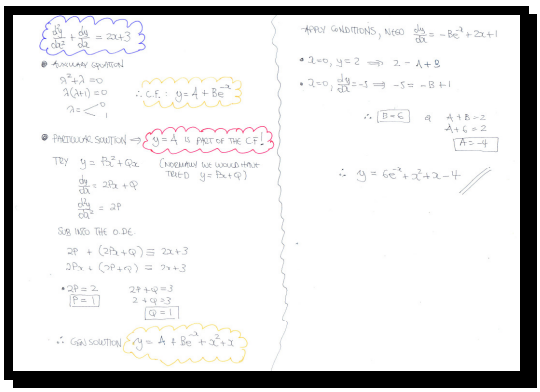
Question 3 (*+)**

Find the solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=2x+3\,,$$

subject to the conditions $y=2$, $\frac{dy}{dx}=-5$ at $x=0$.

$$\boxed{y=x^2+x-4+6\mathrm{e}^{-x}}$$



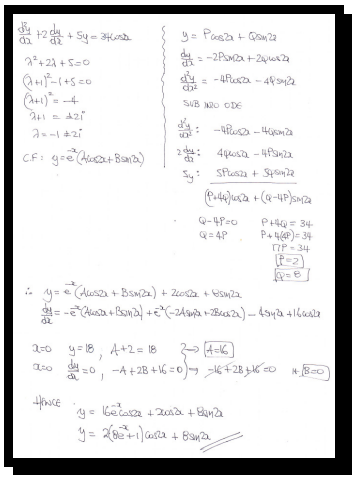
Question 4 (*+)**

Find a solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=34\cos 2x,$$

subject to the boundary conditions $y=18$ and $\frac{dy}{dx}=0$ at $x=0$.

$$\boxed{y=2\left(8\mathrm{e}^{-x}+1\right)\cos 2x+8\sin 2x}$$



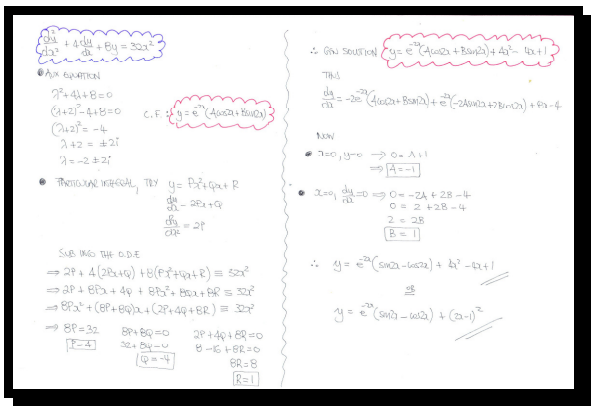
Question 5 (*+)**

The curve C has a local minimum at the origin and satisfies the differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+8y=32x^2 .$$

Find an equation for C.

$$\boxed{y=\mathrm{e}^{x}\left(\sin 2x+\cos 2x\right)+\left(2x-1\right)^2}$$



Question 6 **(***+)**

$$\frac{d^2x}{dt^2}+9x+12\sin 3t=0,\ t\geq 0,$$

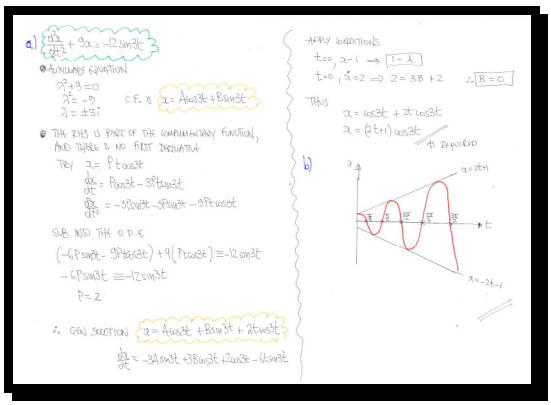
with $x=1$, $\frac{dx}{dt}=2$ at $t=0$.

a) Show that a solution of the differential equation is

$$x=(2t+1)\cos 3t.$$

b) Sketch the graph of x.

proof



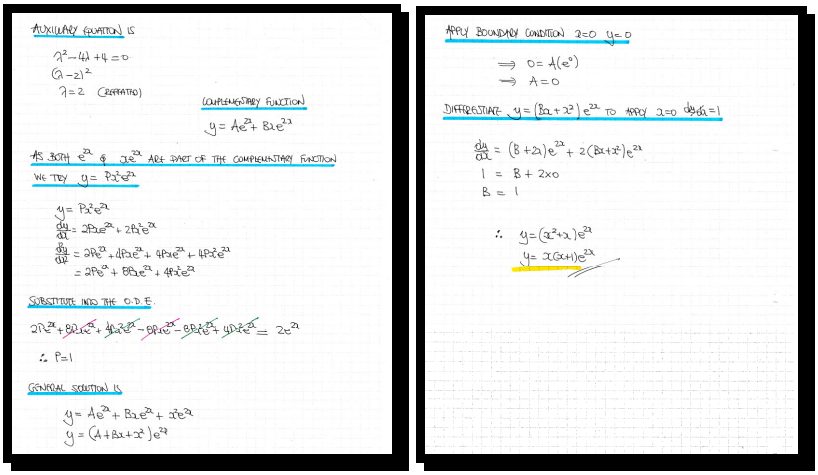
Question 7 **(***+)**

Solve the following differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=2\mathrm{e}^{2x},$$

subject to the boundary conditions $y=0$, $\frac{dy}{dx}=1$ at $x=0$.

$\square$, $\boxed{y=x(x+1)\mathrm{e}^{2x}}$



Question 8 **(***+)**

It is given that the functions of x, f and g, satisfy the following coupled first order differential equations.

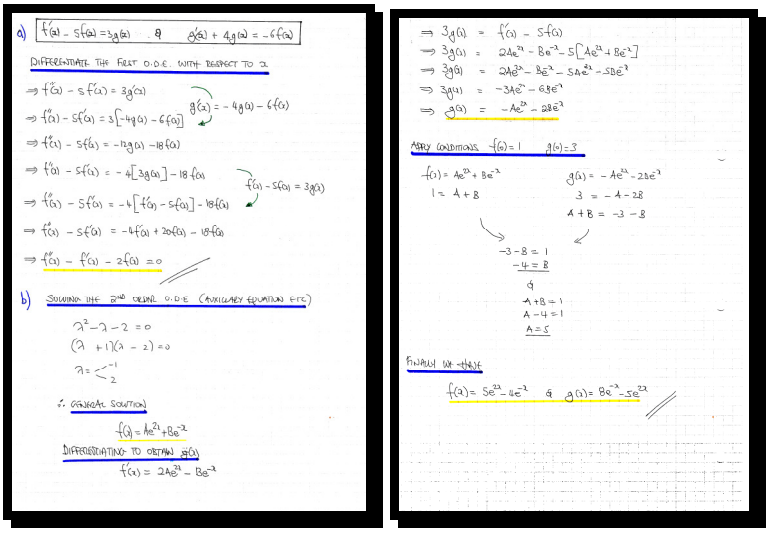
$$f'(x)-5f(x)=3g(x) \qquad \text{and} \qquad g'(x)+4g(x)=-6f(x).$$

a) Show that

$$f''(x)-f'(x)-2f(x)=0.$$

b) Given further that $f(0)=1$ and $g(0)=3$, solve the differential equation of part **(a)** to obtain simplified expressions for $f(x)$ and $g(x)$.

$$\boxed{}, \quad \boxed{\left[f(x),\ g(x)\right]=\left[5\mathrm{e}^{2x}-4\mathrm{e}^{-x},\ 8\mathrm{e}^{-x}-5\mathrm{e}^{2x}\right]}$$



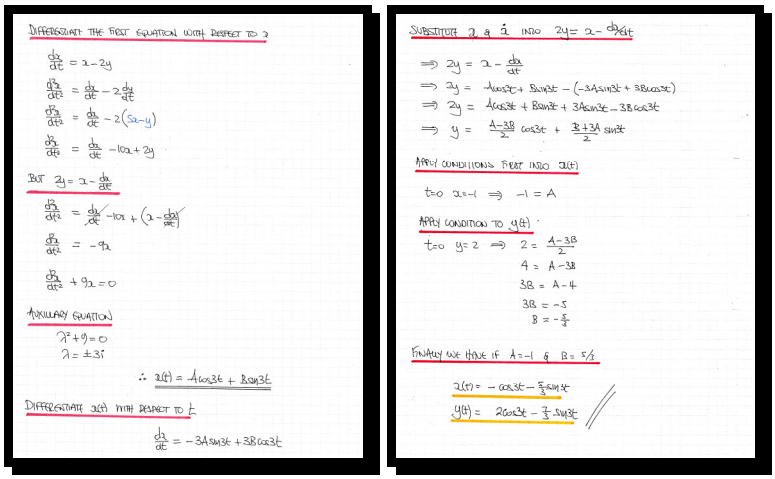
Question 9 (*+)**

The variables x and y satisfy the following coupled first order differential equations.

$$\frac{dx}{dt}=x-2y \qquad \text{and} \qquad \frac{dy}{dt}=5x-y\,.$$

Given further that $x=-1$, $y=2$ at $t=0$, solve the differential equations to obtain simplified expressions for x and y.

$\square$, $\boxed{x=-\cos 3t-\frac{5}{3}\sin 3t,\quad y=2\cos 3t-\frac{7}{3}\sin 3t}$



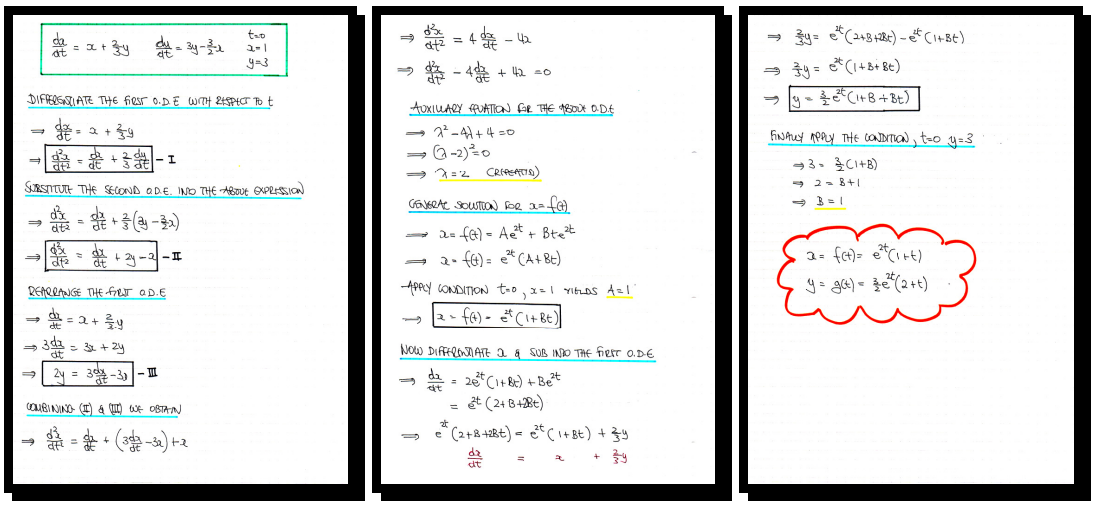
Question 10 (*+)**

It is given that the variables $x = f(t)$ and $y = g(t)$ satisfy the following coupled first order differential equations.

$$\frac{dx}{dt} = x + \frac{2}{3}y \qquad \text{and} \qquad \frac{dy}{dt} = 3y - \frac{3}{2}x\,.$$

Given further that $x = 1$, $y = 3$ at $t = 0$, solve the differential equations to obtain simplified expressions for $f(t)$ and $g(t)$.

$$\boxed{}\,, \qquad \boxed{\left[f(t),\ g(x)\right] = \left[\mathrm{e}^{2t} + t\mathrm{e}^{2t},\ \ 3\mathrm{e}^{2t} + \frac{3}{2}t\mathrm{e}^{2t}\right]}$$

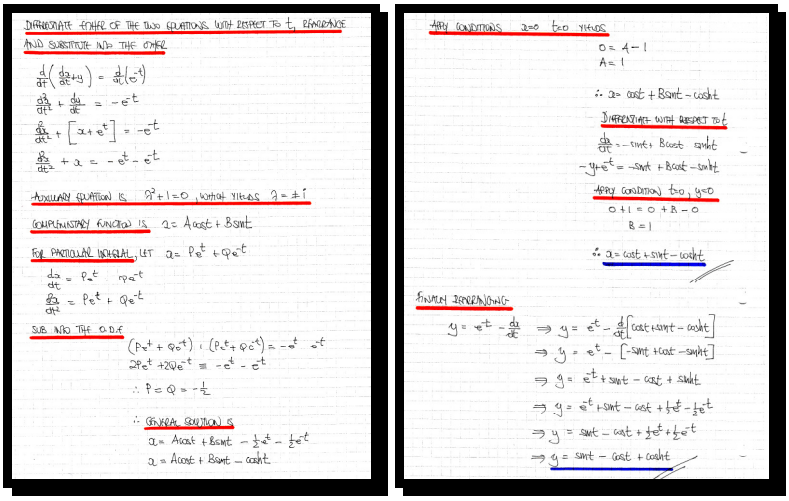


Question 11 (****)

$$\frac{dx}{dt}+y=\mathrm{e}^{-t} \qquad \text{and} \qquad \frac{dy}{dt}-x=\mathrm{e}^{t}\,.$$

Given that $x=0$, $y=0$ at $t=0$, solve the differential equations to obtain simplified expressions for $x=f(t)$ and $y=g(t)$.

$\boxed{}$, $\boxed{x=-\cosh t+\sin t+\cos t,\quad y=\cosh t+\sin t-\cos t}$



Question 12 (****)

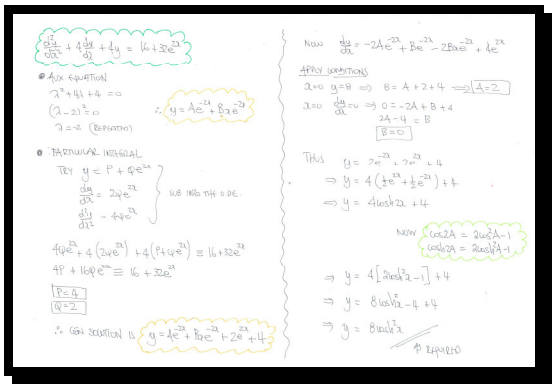
$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=16+32\mathrm{e}^{2x},$$

with $y=8$ and $\frac{dy}{dx}=0$ at $x=0$.

Show that the solution of the above differential equation is

$$y=8\cosh^2 x.$$

proof



Question 13 **(****)**

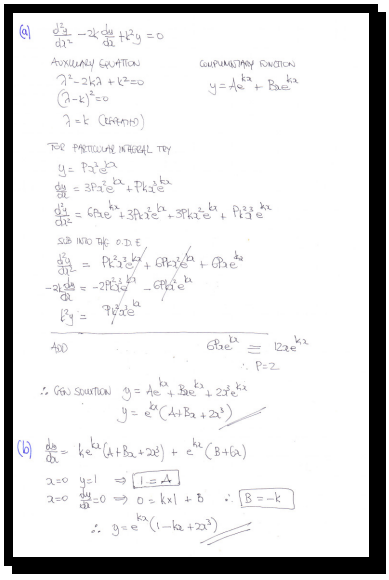
$$\frac{d^2y}{dx^2}-2k\frac{dy}{dx}+k^2y=12x\mathrm{e}^{kx},\ k>0$$

a) Find a general solution of the differential equation given that $y=Px^3\mathrm{e}^{kx}$, where P is a constant, is part of the solution.

b) Given further that $y=1$, $\frac{dy}{dx}=0$ at $x=0$ show that

$$y=\mathrm{e}^{kx}\left(2x^3-kx+1\right).$$

$$\boxed{y=\mathrm{e}^{kx}\left(2x^3+Ax+B\right)}$$



Question 14 (****)

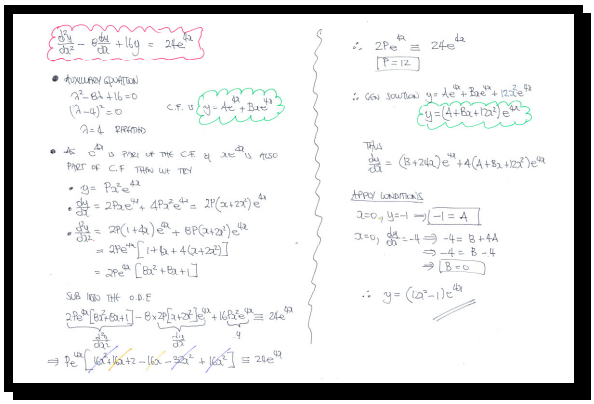
Show that the solution of the differential equation

$$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=24\mathrm{e}^{4x},$$

subject to the boundary conditions $y=-1$, $\frac{dy}{dx}=-4$ at $x=0$, can be written as

$$y=\left(12x^2-1\right)\mathrm{e}^{4x}.$$

proof



Question 15 (****)

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=4\mathrm{e}^{3x}.$$

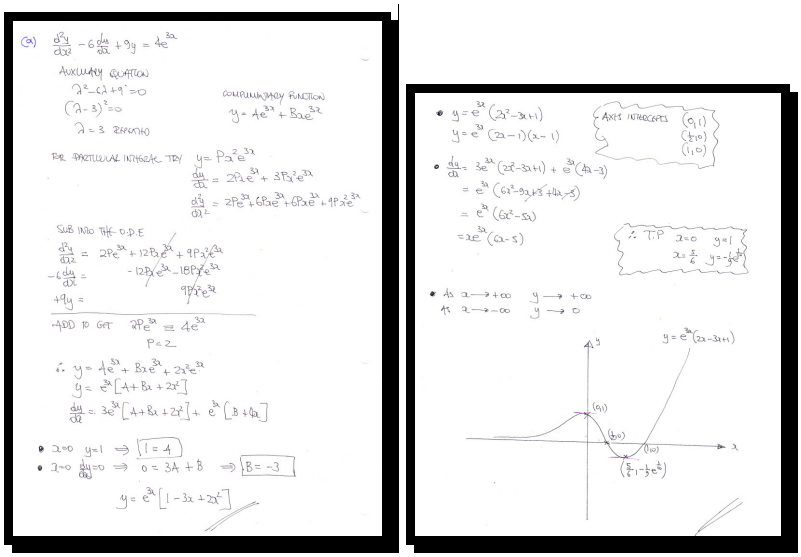
a) Find a solution of the differential equation given that $y=1$, $\frac{dy}{dx}=0$ at $x=0$.

b) Sketch the graph of y.

The sketch must include …

- the coordinates of any points where the graph meets the coordinate axes.
- the coordinates of any stationary points of the curve.
- clear indications of how the graph looks for large positive or negative values of x.

$$\boxed{y=\mathrm{e}^{3x}\left(2x^2-3x+1\right)}$$



Question 16 (****)

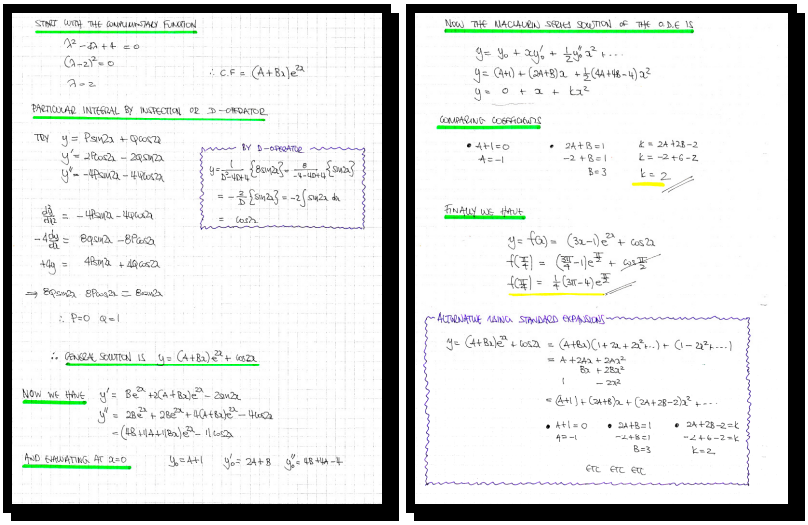
The curve with equation $y = f(x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8\sin 2x\,.$$

The first two non zero terms in Maclaurin series expansion of $f(x)$ are $x + kx^2$, where k is a constant.

Determine in any order the value of k and the exact value of $f\left(\frac{1}{4}\pi\right)$.

$\boxed{}$, $\boxed{k = 2}$, $\boxed{f\left(\frac{1}{4}\pi\right) = \frac{1}{4}(3\pi - 4)\mathrm{e}^{\frac{1}{2}\pi}}$



1 HARD QUESTION

Question 1 (**+)**

The function $y = f(x)$ satisfies the following differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 2e^{-x}(\sin 2x - 2\cos 2x),$$

subject to the boundary conditions $y = 0$, $\frac{dy}{dx} = 2$ at $x = 0$.

Solve the differential equation to show that

$$y = \cosh x \sin 2x .$$

No credit will be given for verification methods.

$\boxed{}$, $\boxed{\text{proof}}$

