1^{st} ORDER ORDINARY DIFFERENTIAL EQUATIONS

Question 1

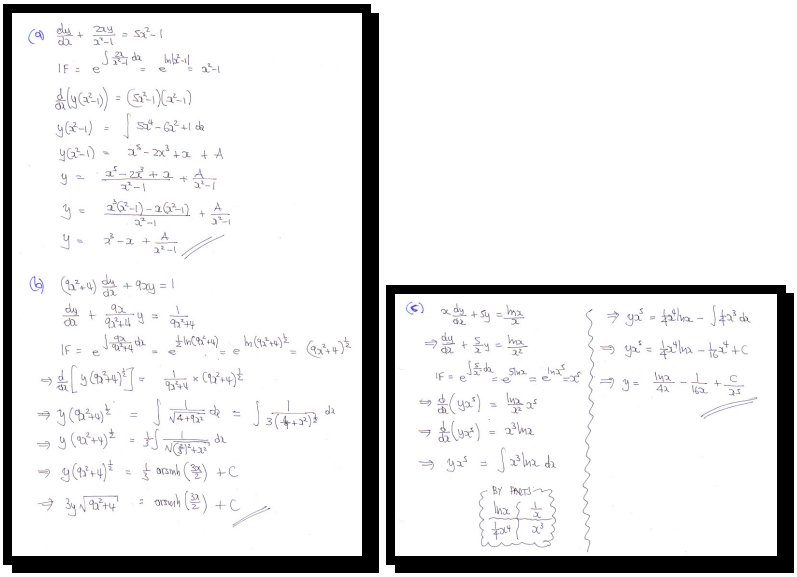
Find a general solution for each of the following differential equations.

a) $\dfrac{dy}{dx}+\dfrac{2xy}{x^2-1}=5x^2-1$

b) $\left(9x^2+4\right)\dfrac{dy}{dx}+9xy=1$

c) $x\dfrac{dy}{dx}+5y=\dfrac{\ln x}{x},\ \ x>0$

$$\boxed{y=x^3-x+\frac{A}{x^2-1}},\quad \boxed{3y\sqrt{9x^2+4}=\operatorname{arsinh}\left(\frac{3}{2}x\right)+C},\quad \boxed{y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{A}{x^5}}$$



Question 2

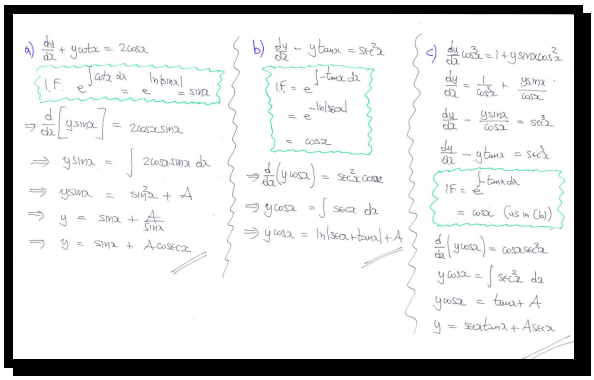
Find a general solution for each of the following differential equations.

a) $\dfrac{dy}{dx}+y\cot x=2\cos x$

b) $\dfrac{dy}{dx}-y\tan x=\sec^2 x$

c) $\dfrac{dy}{dx}\cos^3 x=1+y\sin x\cos^2 x$

$\boxed{y=\sin x+\operatorname{cosec} x}$, $\boxed{y=\sec x\left(\ln\left|\sec x+\tan x\right|+C\right)}$, $\boxed{y=\sec x\tan x+C\sec x}$



Question 3

Find a general solution for each of the following differential equations.

a) $\dfrac{dy}{dx} - y\tanh x = \sinh 2x$

b) $\dfrac{dy}{dx} - \dfrac{y}{x} = x\tanh x$

c) $x\dfrac{dy}{dx} + \dfrac{xy}{\coth x} = \operatorname{sech} x$

$\boxed{y = 2\cosh^2 x + A\cosh x}$, $\boxed{y = x\ln\left(\cosh x\right) + Cx}$, $\boxed{y = \ln|x| + C\operatorname{sech} x}$

Question 4

Find a solution for each of the following differential equations, subject to the boundary conditions given.

a) $\dfrac{dy}{dx}+\dfrac{y}{x}=x$, subject to $x=1,\ y=1$

b) $\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{\ln x}{x}$, subject to $x=1,\ y=0$

c) $\dfrac{dy}{dx}+\dfrac{y}{x}=\mathrm{e}^{x^2}$, subject to $x=1,\ y=\dfrac{1}{2}\mathrm{e}$

$$\boxed{y=\frac{1}{3}\left(x^2+\frac{2}{x}\right)},\quad \boxed{y=\frac{1}{x}-1+\ln x},\quad \boxed{y=\frac{\mathrm{e}^{x^2}}{2x}}$$

Question 5

Find a solution for each of the following differential equations, subject to the boundary conditions given.

a) $\dfrac{dy}{dx}-\dfrac{y}{x}=x^2$, subject to $x=1,\ y=1$

b) $\dfrac{dy}{dx}+\dfrac{3y}{x}=\left(x^4+3\right)^{\frac{1}{2}}$, subject to $x=1,\ y=\dfrac{1}{5}$

c) $\dfrac{dy}{dx}+2y=\mathrm{e}^{-3x}$, subject to $x=0,\ y=1$

$\boxed{y=\frac{1}{2}x\left(x+1\right)}$, $\boxed{10yx^3=\left(x^4+3\right)^{\frac{5}{2}}-30}$, $\boxed{y=2\mathrm{e}^{-2x}-\mathrm{e}^{-3x}}$

Question 6

Find a solution for each of the following differential equations, subject to the boundary conditions given.

a) $x\dfrac{dy}{dx}+2y=\dfrac{\sin 2x}{x}$, subject to $x=\dfrac{\pi}{4},\ y=\dfrac{8}{\pi^2}$

b) $\dfrac{dy}{dx}+y\tan x=\sin x$, subject to $x=0,\ y=0$

c) $\dfrac{dy}{dx}-y\tan x=4x^3\sec x$, subject to $x=0,\ y=1$

$\boxed{y=\dfrac{1}{2x^2}(1-\cos 2x)=\dfrac{\sin^2 x}{x^2}}$, $\boxed{y=\cos x\ln\left|\sec x\right|}$, $\boxed{y=\left(x^4+1\right)\sec x}$