1^{st} ORDER O.D.E.
EXAM QUESTIONS

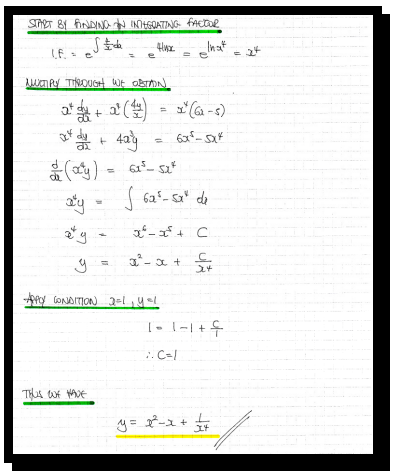
Question 1 (**)

$$\frac{dy}{dx}+\frac{4y}{x}=6x-5\,,\ x>0\,.$$

Determine the solution of the above differential equation subject to the boundary condition is $y=1$ at $x=1$.

Give the answer in the form $y=f\left(x\right)$.

$\boxed{}$, $\boxed{y=x^2-x+\frac{1}{x^4}}$



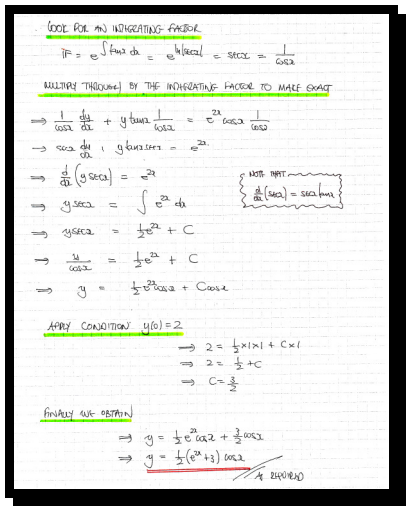
Question 2 (+)**

$$\frac{dy}{dx}+y\tan x=\mathrm{e}^{2x}\cos x\,,\ \ y(0)=2\,.$$

Show that the solution of the above differential equation is

$$y=\frac{1}{2}\left(\mathrm{e}^{2x}+3\right)\cos x\,.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 3 (+)**

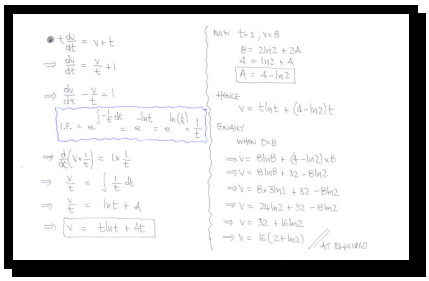
The velocity of a particle v ms^{-1} at time t s satisfies the differential equation

$$t\frac{dv}{dt}=v+t\,,\ t>0\,.$$

Given that when $t=2$, $v=8$, show that when $t=8$

$$v=16\left(2+\ln 2\right).$$

proof



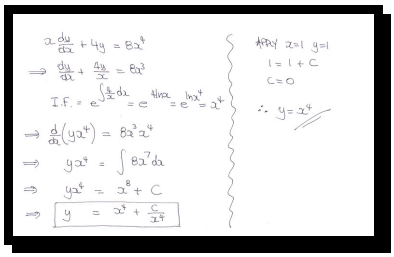
Question 4 (+)**

$$x\frac{dy}{dx}+4y=8x^4\text{, subject to } y=1 \text{ at } x=1\,.$$

Show that the solution of the above differential equation is

$$y=x^4\,.$$

proof

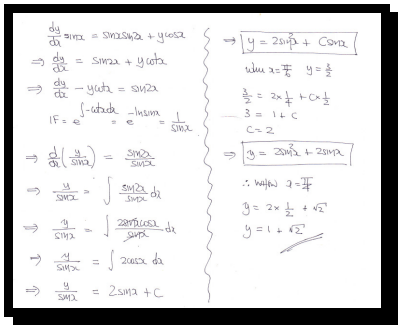


Question 5 (***)

$$\frac{dy}{dx}\sin x = \sin x \sin 2x + y\cos x\,.$$

Given that $y=\frac{3}{2}$ at $x=\frac{\pi}{6}$, find the exact value of y at $x=\frac{\pi}{4}$.

$\boxed{1+\sqrt{2}}$



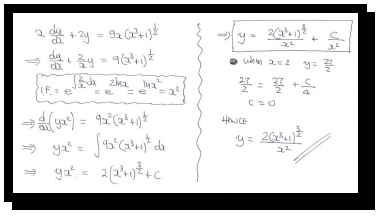
Question 6 (***)

$$x\frac{dy}{dx}+2y=9x\left(x^3+1\right)^{\frac{1}{2}}\ \text{, with } y=\frac{27}{2} \text{ at } x=2\,.$$

Show that the solution of the above differential equation is

$$y=\frac{2}{x^2}\left(x^3+1\right)^{\frac{3}{2}}\,.$$

$\boxed{\text{proof}}$



Question 7 (***)

A trigonometric curve C satisfies the differential equation

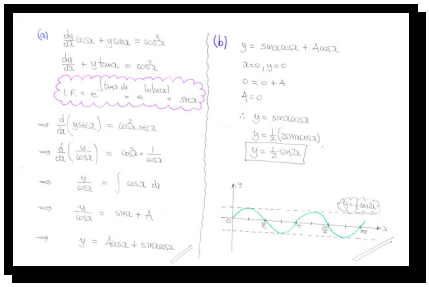
$$\frac{dy}{dx}\cos x + y\sin x = \cos^3 x\,.$$

a) Find a general solution of the above differential equation.

b) Given further that the curve passes through the Cartesian origin O, sketch the graph of C for $0 \le x \le 2\pi$.

The sketch must show clearly the coordinates of the points where the graph of C meets the x axis.

$$\boxed{y = \sin x\cos x + A\cos x}$$



Question 8 (***)

20 grams of salt are dissolved into a beaker containing 1 litre of a certain chemical.

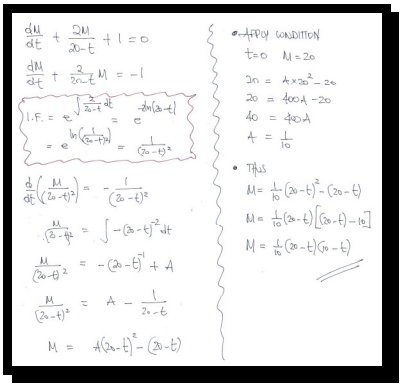
The mass of salt, M grams, which remains undissolved t seconds later, is modelled by the differential equation

$$\frac{dM}{dt}+\frac{2M}{20-t}+1=0\,,\ t\geq 0\,.$$

Show clearly that

$$M=\frac{1}{10}(10-t)(20-t)\,.$$

proof



Question 9 (*+)**

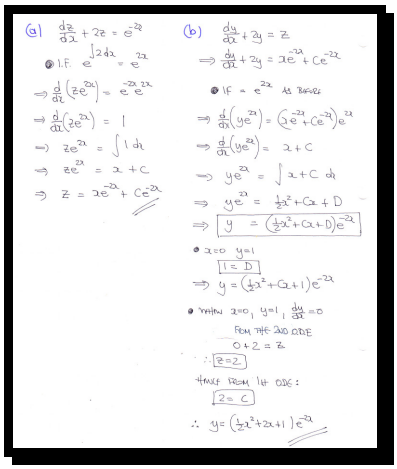
Given that $z=f(x)$ and $y=g(x)$ satisfy the following differential equations

$$\frac{dz}{dx}+2z=\mathrm{e}^{-2x} \quad \text{and} \quad \frac{dy}{dx}+2y=z\,,$$

a) Find z in the form $z=f(x)$

b) Express y in the form $y=g(x)$, given further that at $x=0$, $y=1$, $\dfrac{dy}{dx}=0$

$$\boxed{z=(x+C)\mathrm{e}^{-2x}}\,,\quad \boxed{y=\left(\frac{1}{2}x^2+2x+1\right)\mathrm{e}^{-2x}}$$



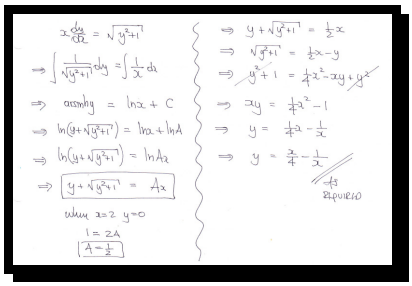
Question 10 (*+)**

$$x\frac{dy}{dx}=\sqrt{y^2+1}\ ,\ x>0\text{, with }\ y=0\ \text{ at }\ x=2\,.$$

Show that the solution of the above differential equation is

$$y=\frac{x}{4}-\frac{1}{x}\,.$$

proof



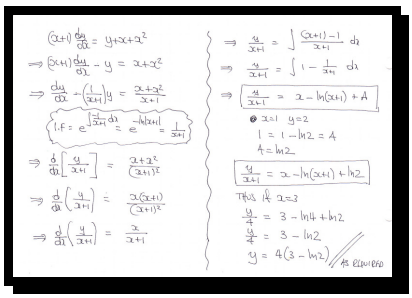
Question 11 (*+)**

$$(x+1)\frac{dy}{dx}=y+x+x^2\,,\ x>-1\,.$$

Given that $y=2$ at $x=1$, solve the above differential equation to show that

$$y=4\left(3-\ln 2\right)\ \text{ at }\ x=3\,.$$

proof

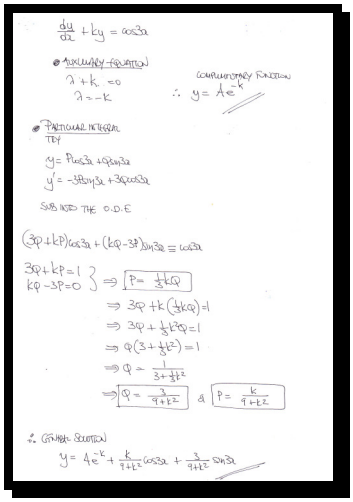


Question 12 (*+)**

$$\frac{dy}{dx}+ky=\cos 3x\,,\ k \text{ is a non zero constant.}$$

By finding a complimentary function and a particular integral, or otherwise, find the general of the above differential equation.

$$\boxed{y=A\mathrm{e}^{-x}+\frac{k}{9+k^2}\cos 3x+\frac{3}{9+k^2}\sin 3x}$$

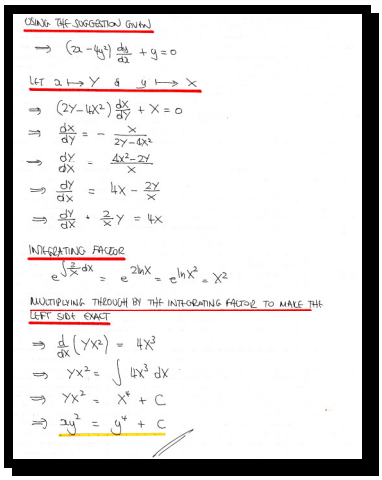


Question 13 **(***+)**

$$\left(2x-4y^2\right)\frac{dy}{dx}+y=0\,.$$

By reversing the role of x and y in the above differential equation, or otherwise, find its general solution.

$\square$, $\boxed{xy^2=y^4+C}$



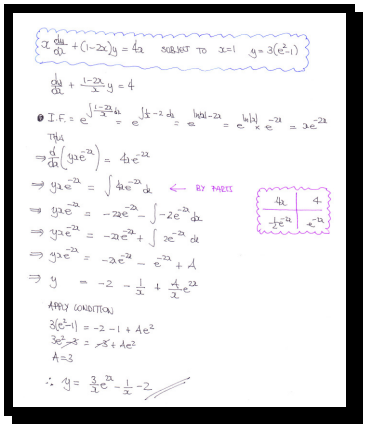
Question 14 (****)

The curve with equation $y=f\left(x\right)$ satisfies

$$x\frac{dy}{dx}+\left(1-2x\right)y=4x\,,\ \ x>0\,,\ \ f\left(1\right)=3\left(\mathrm{e}^2-1\right).$$

Determine an equation for $y=f\left(x\right)$.

$$\boxed{y=\frac{3}{x}\mathrm{e}^x-\frac{1}{x}-2}$$



Question 15 (****)

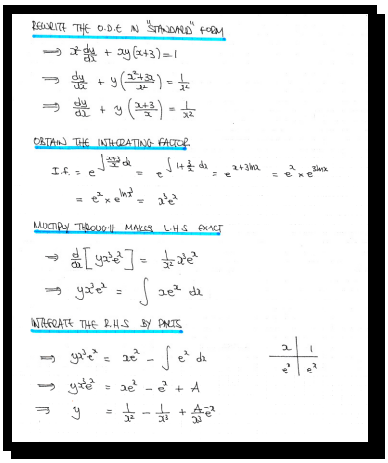
A curve C, with equation $y=f(x)$, passes through the points with coordinates $(1,1)$ and $(2,k)$, where k is a constant.

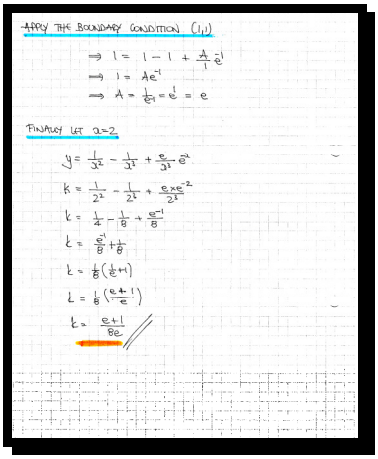
Given further that the equation of C satisfies the differential equation

$$x^2\frac{dy}{dx}+xy(x+3)=1,$$

determine the exact value of k.

$\square$, $\boxed{k=\dfrac{e+1}{8e}}$





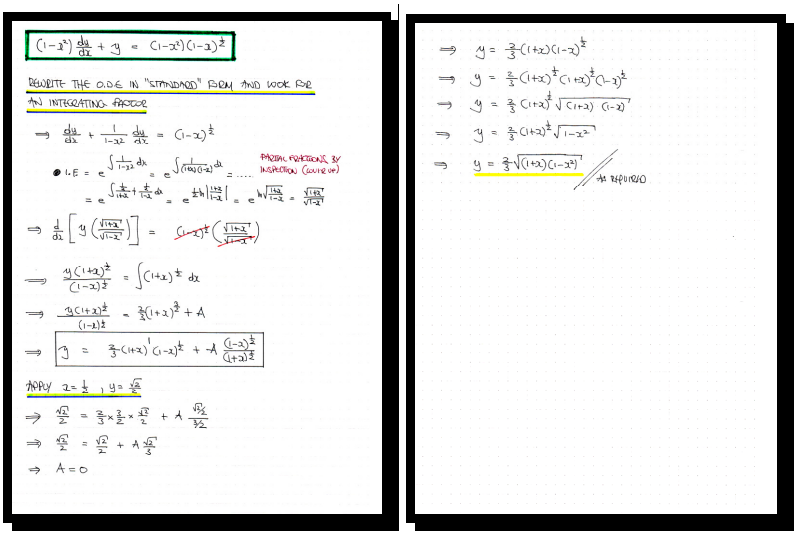
Question 16 **(****)**

$$\left(1-x^2\right)\frac{dy}{dx}+y=\left(1-x^2\right)\left(1-x\right)^{\frac{1}{2}}\ ,\ -1<x<1\,.$$

Given that $y=\dfrac{\sqrt{2}}{2}$ at $x=\dfrac{1}{2}$, show that the solution of the above differential equation can be written as

$$y=\frac{2}{3}\sqrt{\left(1-x^2\right)\left(1+x\right)}\,.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 17 **(****)**

A curve C, with equation $y = f(x)$, meets the y axis the point with coordinates $(0,1)$.

It is further given that the equation of C satisfies the differential equation

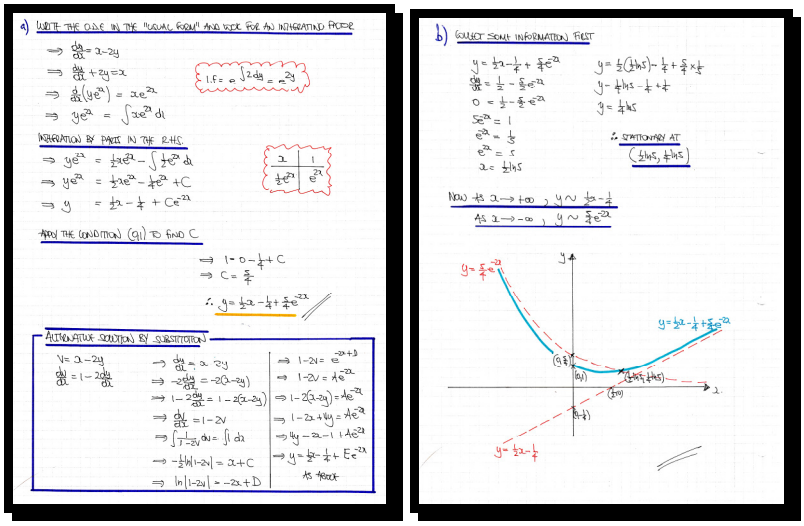
$$\frac{dy}{dx} = x - 2y .$$

a) Determine an equation of C.

b) Sketch the graph of C.

The graph must include in exact simplified form the coordinates of the stationary point of the curve and the equation of its asymptote.

$$\boxed{}, \quad \boxed{y = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}}$$



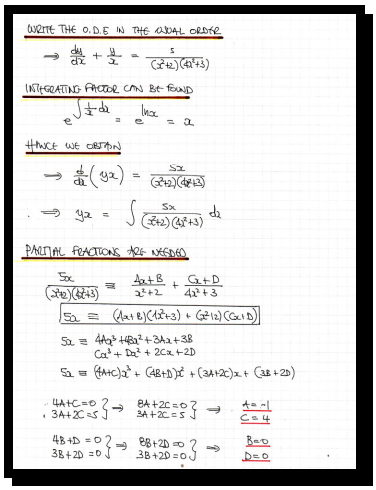
Question 18 (****)

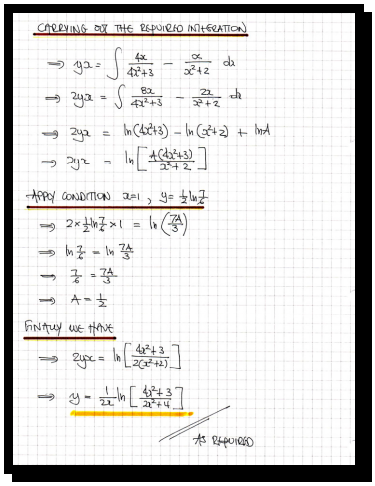
$$\frac{dy}{dx}+\frac{y}{x}=\frac{5}{\left(x^2+2\right)\left(4x^2+3\right)},\ x>0\,.$$

Given that $y=\frac{1}{2}\ln\frac{7}{6}$ at $x=1$, show that the solution of the above differential equation can be written as

$$y=\frac{1}{2x}\ln\left(\frac{4x^2+3}{2x^2+4}\right).$$

, proof





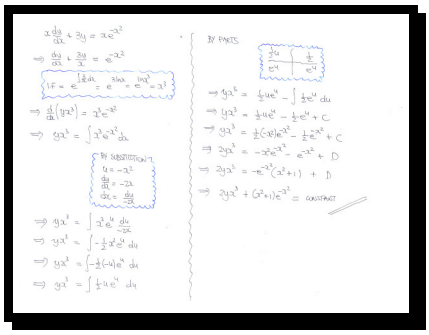
Question 19 (****)

$$x\frac{dy}{dx}+3y=x\mathrm{e}^{-x^2}\ ,\ x>0\,.$$

Show clearly that the general solution of the above differential equation can be written in the form

$$2yx^3+\left(x^2+1\right)\mathrm{e}^{-x^2}=\text{constant}\,.$$

proof



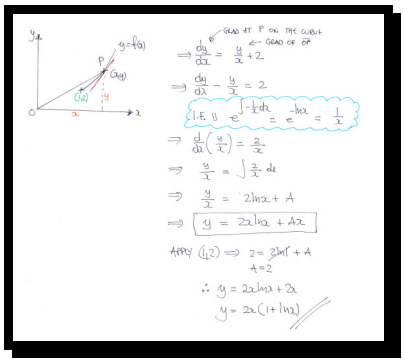
Question 20 (****)

The general point P lies on the curve with equation $y = f(x)$.

The gradient of the curve at P is 2 more than the gradient of the straight line segment OP.

Given further that the curve passes through $Q(1,2)$, express y in terms of x.

$$\boxed{y = 2x(1+\ln x)}$$



Question 21 (**+)**

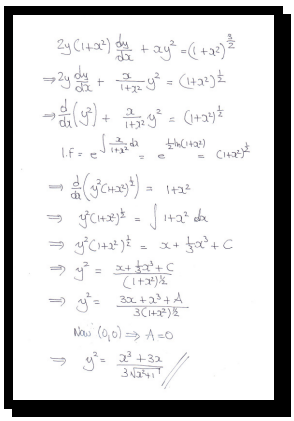
A curve with equation $y = f(x)$ passes through the origin and satisfies the differential equation

$$2y\left(1+x^2\right)\frac{dy}{dx}+xy^2=\left(1+x^2\right)^{\frac{3}{2}}.$$

By finding a suitable integrating factor, or otherwise, show clearly that

$$y^2=\frac{x^3+3x}{3\sqrt{x^2+1}}.$$

$\boxed{}$, $\boxed{\text{proof}}$



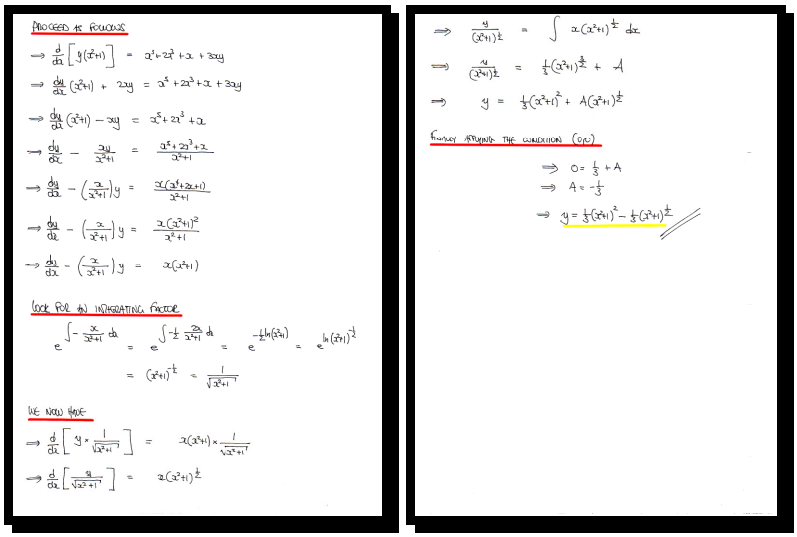
Question 22 (**+)**

The curve with equation $y=f\left(x\right)$ passes through the origin, and satisfies the relationship

$$\frac{d}{dx}\left[y\left(x^2+1\right)\right]=x^5+2x^3+x+3xy\,.$$

Determine a simplified expression for the equation of the curve.

$\square$, $\boxed{y=\frac{1}{3}\left(x^2+1\right)^2-\frac{1}{3}\left(x^2+1\right)^{\frac{1}{2}}}$



Question 23 (**+)**

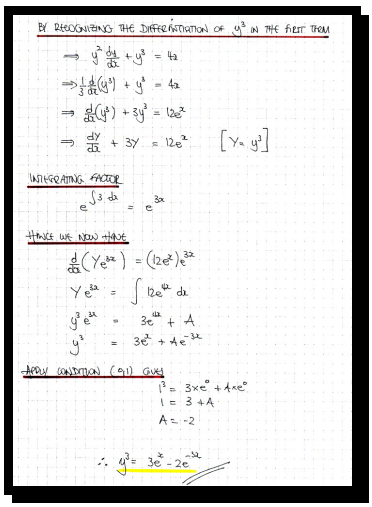
A curve with equation $y=f(x)$ passes through the point with coordinates $(0,1)$ and satisfies the differential equation

$$y^2\frac{dy}{dx}+y^3=4\mathrm{e}^x.$$

By finding a suitable integrating factor, solve the differential equation to show that

$$y^3=3\mathrm{e}^x-2\mathrm{e}^{-3x}.$$

$\boxed{}$, $\boxed{\text{proof}}$



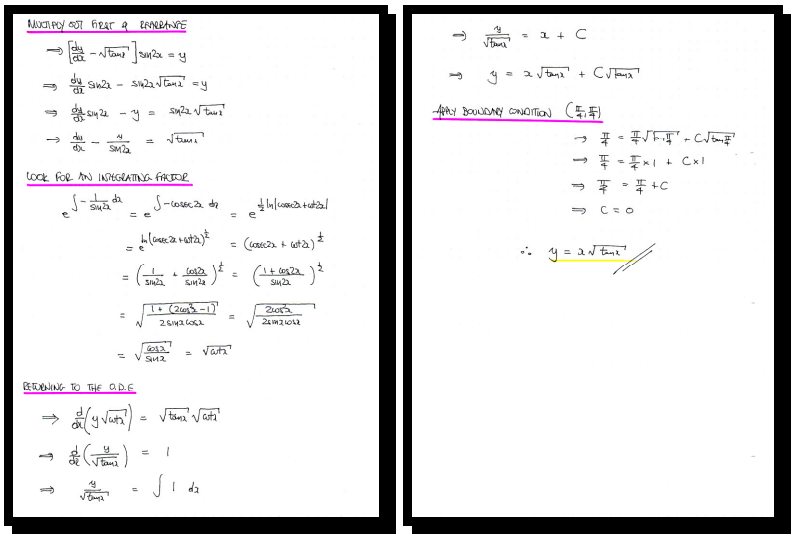
Question 24 (**+)**

It is given that a curve with equation $y = f(x)$ passes through the point $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ and satisfies the differential equation

$$\left(\frac{dy}{dx} - \sqrt{\tan x}\right)\sin 2x = y\,.$$

Find an equation for the curve in the form $y = f(x)$.

$\boxed{}$, $\boxed{y = x\sqrt{\tan x}}$



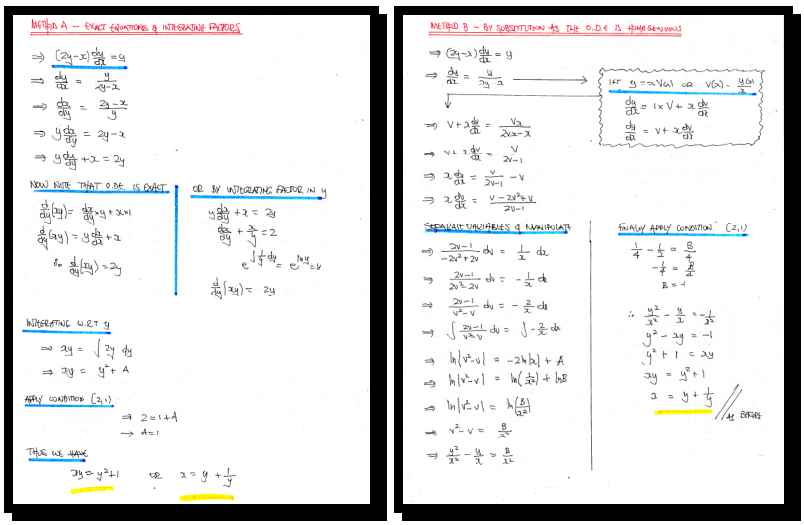
Question 25

The variables x and y satisfy

$$(2y-x)\frac{dy}{dx}=y\,,\quad y>0\,,\ x>0\,.$$

If $y=1$ at $x=2$, show that $x=y+\dfrac{1}{y}$.

V , [ ] , proof



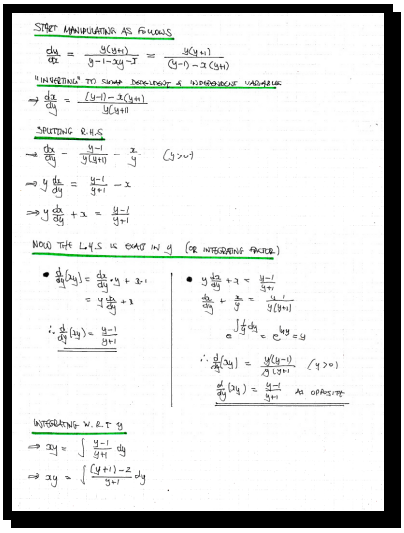
Question 26

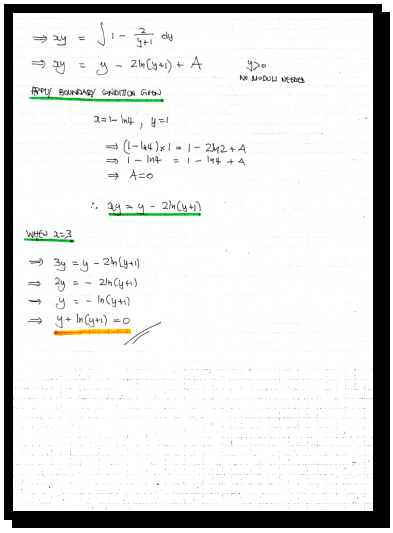
The variables x and y satisfy

$$\frac{dy}{dx}=\frac{y(y+1)}{y-x-xy-1},\quad y>0.$$

If $y=1$ at $x=1-\ln 4$, show that $y+\ln(y+1)=0$ at $x=3$.

V, [ ], proof





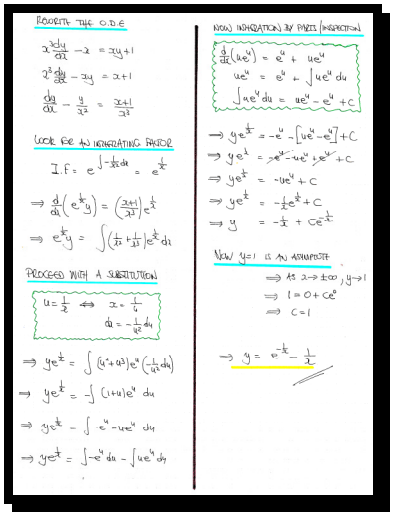
Question 27 (*****)

The curve with equation $y = f(x)$ has the line $y = 1$ as an asymptote and satisfies the differential equation

$$x^3 \frac{dy}{dx} - x = xy + 1, \ x \neq 0 .$$

Solve the above differential equation, giving the solution in the form $y = f(x)$.

$\square$, $\boxed{y = \mathrm{e}^{-\frac{1}{x}} - \frac{1}{x}}$



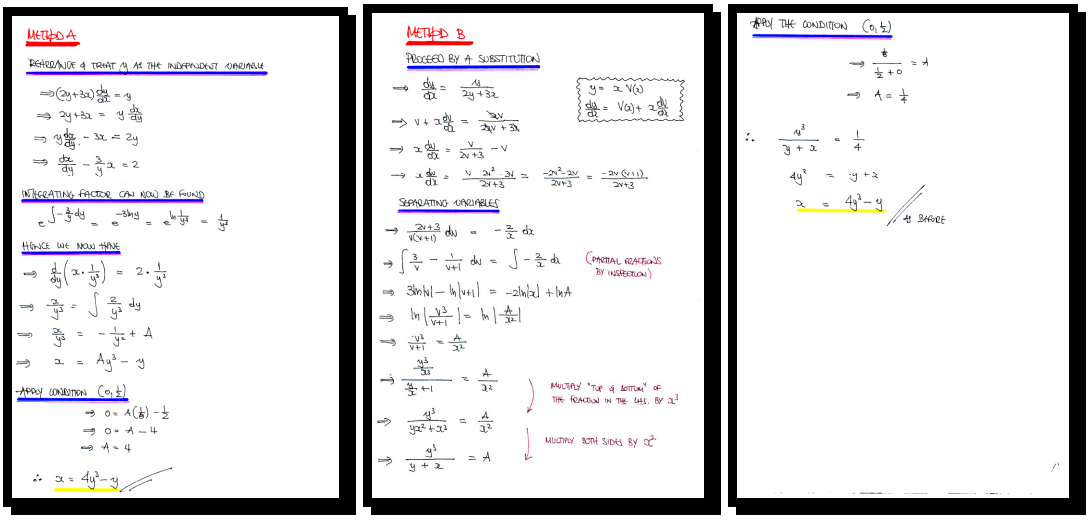
Question 28 **(*****)**

It is given that a curve with equation $x = f(y)$ passes through the point $\left(0, \frac{1}{2}\right)$ and satisfies the differential equation

$$(2y+3x)\frac{dy}{dx} = y\,.$$

Find an equation for the curve in the form $x = f(y)$.

$\boxed{}$, $\boxed{x = 4y^3 - y}$



Question 29 (*****)

Use suitable manipulations to solve this **exact** differential equation.

$$4x\frac{dy}{dx}+\sin 2y=4\cos^2 y\,,\quad y\left(\frac{1}{4}\right)=0\,.$$

Given the answer in the form $y=f\left(x\right)$.

$\boxed{}$, $\boxed{y=\arctan\left[2-\frac{1}{\sqrt{x}}\right]}$

