2D VECTORS

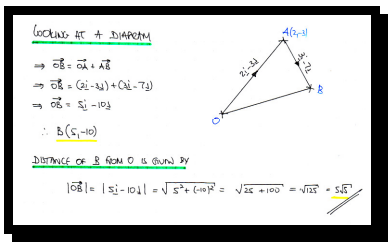
Question 1 (**)

Relative to a fixed origin O, the point A has coordinates $(2,-3)$.

The point B is such so that $\overrightarrow{AB}=3\mathbf{i}-7\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

Determine the distance of B from O.

$\boxed{}$, $\boxed{|OB|=5\sqrt{5}}$



Question 2 **(**+)**

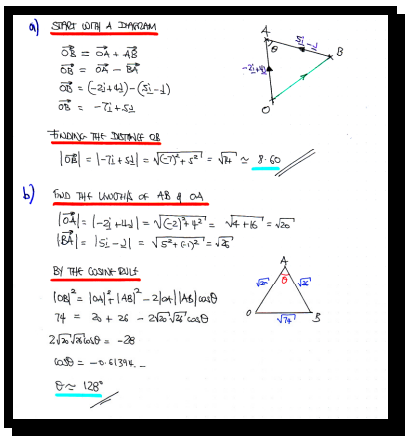
Relative to a fixed origin O, the point A has coordinates $(-2,4)$.

The point B is such so that $\overrightarrow{BA}=5\mathbf{i}-\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

a) Determine the distance of B from O.

b) Calculate the angle OAB.

$\boxed{}$, $\boxed{|OB|=\sqrt{74}}$, $\boxed{\measuredangle OAB\approx 128^\circ}$



Question 3 (***)

The points A, B and C lie on a plane so that

$$\overrightarrow{AB}=2\mathbf{i}+7\mathbf{j} \quad \text{and} \quad \overrightarrow{AC}=4\mathbf{i}-5\mathbf{j},$$

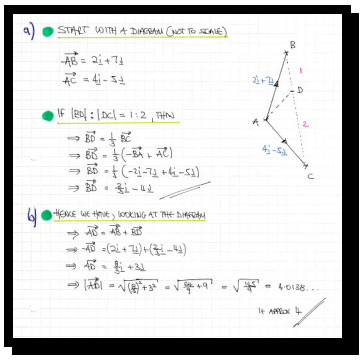
where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

The point D lies on the straight line segment BC, so that $|BD|:|DC|=1:2$.

a) Determine a simplified expression, in terms of $\mathbf{i}$ and $\mathbf{j}$, for $\overrightarrow{BD}$.

b) Show that the $|\overrightarrow{AD}|$ is approximately 4 units.

$\square$, $\boxed{\overrightarrow{BD}=\frac{2}{3}\mathbf{i}-4\mathbf{j}}$



Question 4 (***)

The following information is given for four points which lie on the same plane.

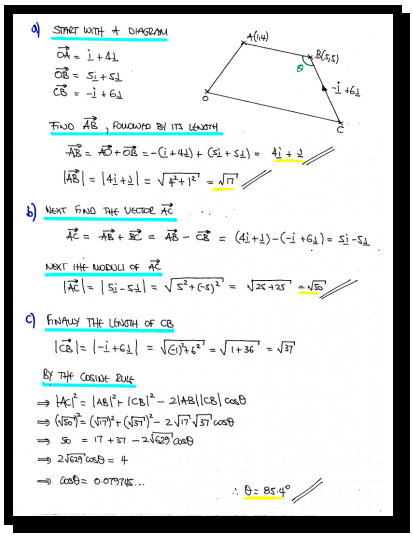
$$\overrightarrow{OA}=\mathbf{i}+4\mathbf{j}\,, \qquad \overrightarrow{OB}=5\mathbf{i}+5\mathbf{j} \qquad \text{and} \qquad \overrightarrow{CB}=-\mathbf{i}+6\mathbf{j}\,,$$

a) Find the vector $\overrightarrow{AB}$ and hence state its length

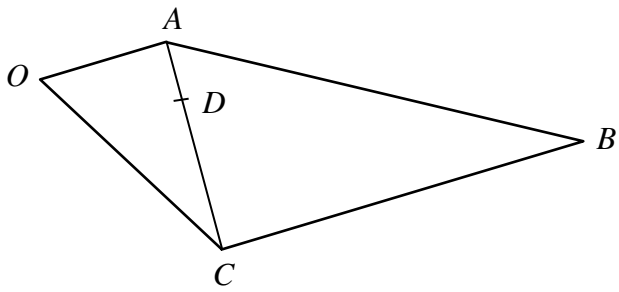
b) Determine the length of $\overrightarrow{AC}$.

c) Calculate the size of the angle ABC.

$$\boxed{}\,, \quad \boxed{\overrightarrow{AB}=4\mathbf{i}+\mathbf{j}}\,, \quad \boxed{\left|\overrightarrow{AB}\right|=\sqrt{17}}\,, \quad \boxed{\left|\overrightarrow{AC}\right|=\sqrt{50}}\,, \quad \boxed{\measuredangle ABC\approx 85.4^\circ}$$



Question 5 (***)



The figure above shows a trapezium $OABC$, where O is a fixed origin.

The position vectors of A and C are $12\mathbf{i}+4\mathbf{j}$ and $18\mathbf{i}-21\mathbf{j}$, respectively.

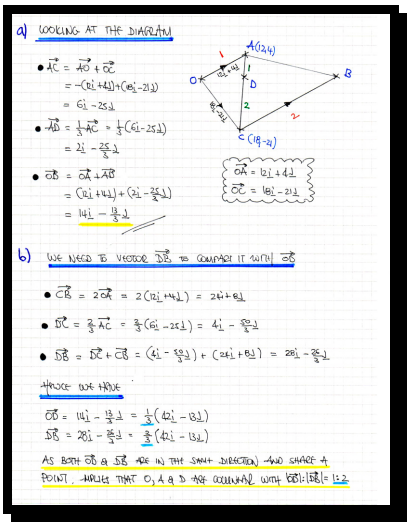
CB is parallel to OA, so that $\left|\overrightarrow{CB}\right|=2\left|\overrightarrow{OA}\right|$.

The point D lies on AC so that $AD:DC=1:2$.

a) Find a simplified expression, in terms of $\mathbf{i}$ and $\mathbf{j}$, for the position vector of D.

b) Show that that O, D and B are collinear and state the ratio of $OD:DB$.

$\boxed{}$, $\boxed{\overrightarrow{OD}=14\mathbf{i}-\frac{13}{3}\mathbf{j}}$, $\boxed{OD:DB=1:2}$



Question 6 **(***)**

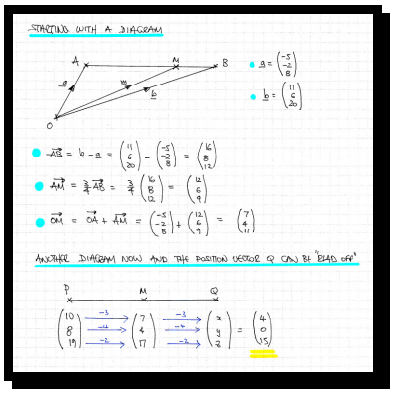
The points A and B have position vectors $\begin{pmatrix} -5 \\ -2 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ 6 \\ 20 \end{pmatrix}$, respectively.

The point M lies on AB so that $|AM| : |MB| = 3 : 1$

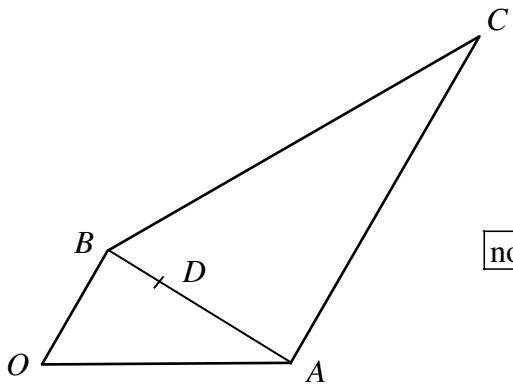
The point P has position vector $\begin{pmatrix} 10 \\ 8 \\ 19 \end{pmatrix}$.

Determine the position vector of the point Q, if M is the midpoint of PQ.

$\square$, $\mathbf{q} = \begin{pmatrix} 4 \\ 0 \\ 15 \end{pmatrix}$



Question 7 **(***+)**



not drawn to scale

The figure above shows a trapezium $OBCA$ where OB is parallel to AC.

The point D lies on BA so that $BD:DA=1:2$.

It is further given that $\overrightarrow{OA}=7\mathbf{i}-4\mathbf{j}$, $\overrightarrow{OB}=3\mathbf{i}+2\mathbf{j}$ and $\overrightarrow{AC}=2\,\overrightarrow{OB}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

a) Determine simplified expressions, in terms of $\mathbf{i}$ and $\mathbf{j}$, for each of the vectors $\overrightarrow{OC}$, $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OD}$.

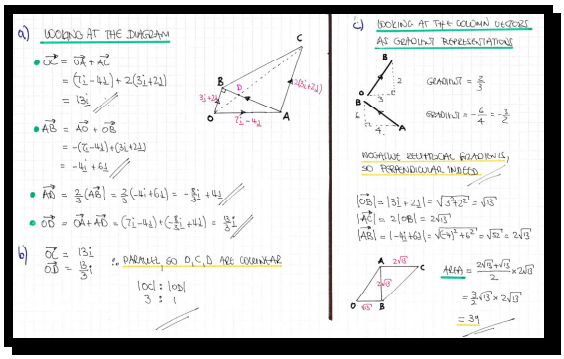
b) Deduce, showing your reasoning, that O, D and C are collinear and state the ratio of $OC:OD$.

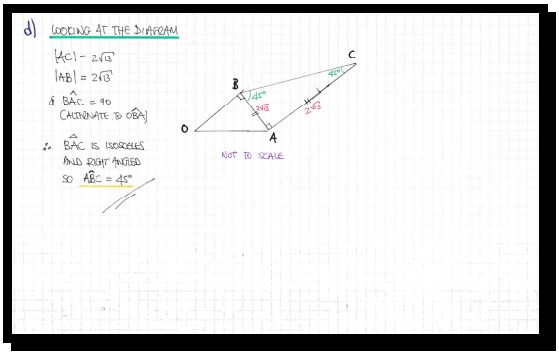
c) Show that $\angle OBA=90^\circ$ and hence find the area of the trapezium $OBCA$.

d) State the size of the angle $\angle ABC$.

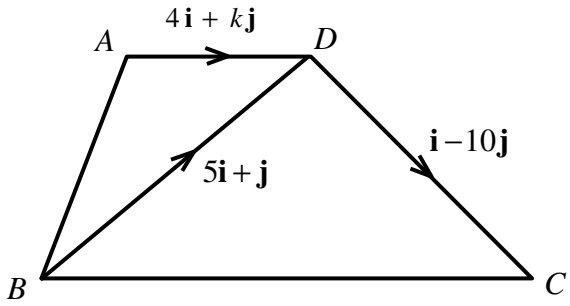
$\boxed{}$, $\boxed{\overrightarrow{OC}=13\mathbf{i}}$, $\boxed{\overrightarrow{AB}=-4\mathbf{i}+6\mathbf{j}}$, $\boxed{\overrightarrow{AD}=-\frac{8}{3}\mathbf{i}+4\mathbf{j}}$, $\boxed{\overrightarrow{OD}=\frac{13}{3}\mathbf{i}}$, $\boxed{OC:OD=3:1}$,

$\boxed{\text{area}=39}$, $\boxed{\angle ABC=45^\circ}$





Question 8 **(***+)**



The figure above shows a trapezium $ABCD$, where AD is parallel to BC.

The following information is given for this trapezium.

$$\overrightarrow{BD} = 5\mathbf{i} + \mathbf{j}, \qquad \overrightarrow{DC} = \mathbf{i} - 10\mathbf{j} \qquad \text{and} \qquad \overrightarrow{AD} = 4\mathbf{i} + k\mathbf{j},$$

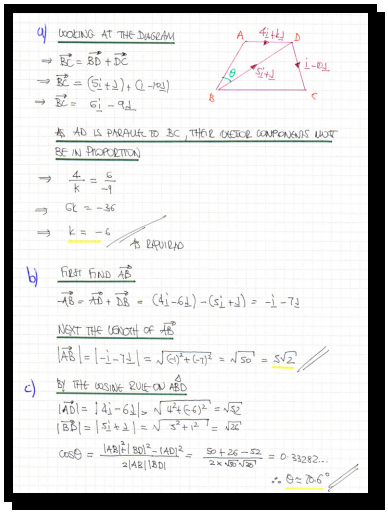
where k is an integer.

a) Use vector algebra to show that $k = -6$.

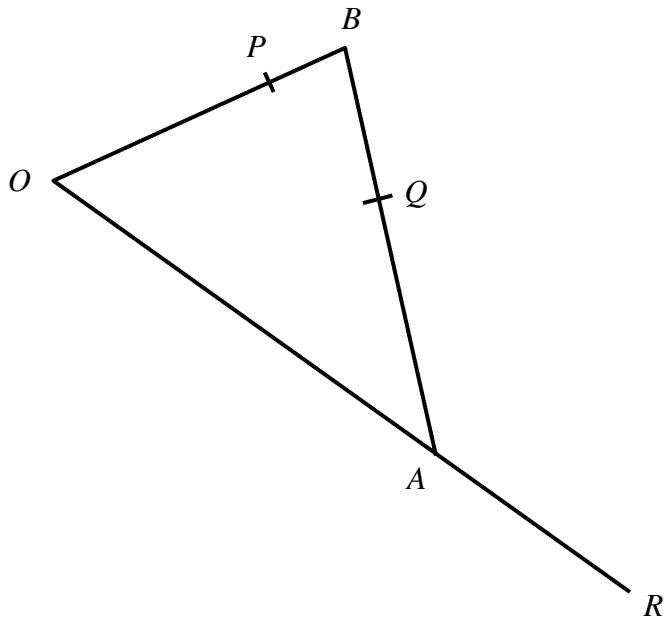
b) Find the length of $\overrightarrow{AB}$.

c) Calculate the size of the angle ABD.

$\boxed{}$, $\boxed{\left|\overrightarrow{AB}\right| = \sqrt{50} = 5\sqrt{2}}$, $\boxed{\angle ABD \approx 70.6^\circ}$



Question 9 **(***+)**



The figure above shows a triangle OAB, where O is a fixed origin.

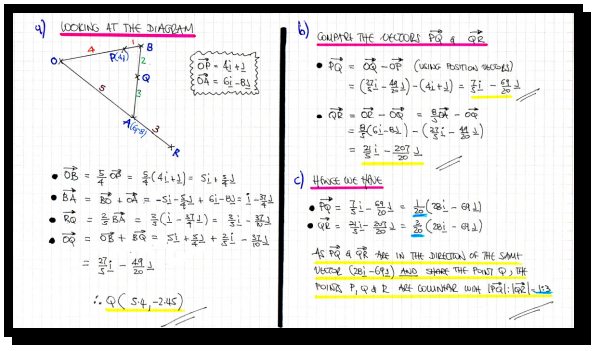
- The point A has coordinates $(6,-8)$.
- The point P, whose coordinates are $(4,1)$, lies on OB so that $OP:PB=4:1$.
- The point Q lies on AB so that $AQ:QB=3:2$
- The side OA is extended to the point R so that $OA:AR=5:3$.

a) Use vector methods to determine the coordinates of Q.

b) Determine expressions, in terms of $\mathbf{i}$ and $\mathbf{j}$, for the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

c) Deduce, showing your reasoning, that P, Q and R are collinear and state the ratio of $PQ:QR$.

$\boxed{}$, $\boxed{Q\left(\frac{27}{5},-\frac{49}{20}\right)}$, $\boxed{\overrightarrow{PQ}=\frac{7}{5}\mathbf{i}-\frac{69}{20}\mathbf{j}}$, $\boxed{\overrightarrow{QR}=\frac{21}{5}\mathbf{i}-\frac{207}{20}\mathbf{j}}$, $\boxed{PQ:QR=1:3}$



Question 10 (*+)**

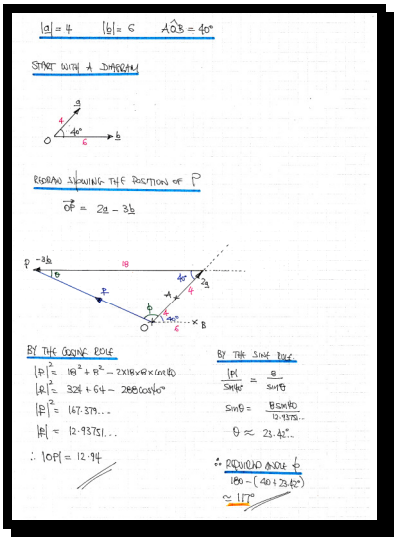
The points A, B and P lie on the x-y plane, where the point O is the origin.

It is further given that

$$|OA|=4\,,\quad |OB|=6\qquad\text{and}\qquad \sphericalangle AOB=40^\circ\,.$$

If $\overrightarrow{OP}=2\left(\overrightarrow{OA}\right)-3\left(\overrightarrow{OB}\right)$ determine the distance of P from the origin and the angle between $\overrightarrow{OP}$ and $\overrightarrow{OA}$.

$\boxed{}$, $\boxed{|OP|\approx 12.94}$, $\boxed{\approx 117^\circ}$



Question 11 **(****)**

The points $A(-1,4)$, $B(2,3)$ and $C(8,1)$ lie on the x-y plane, where O is the origin.

a) Show that A, B and C are collinear.

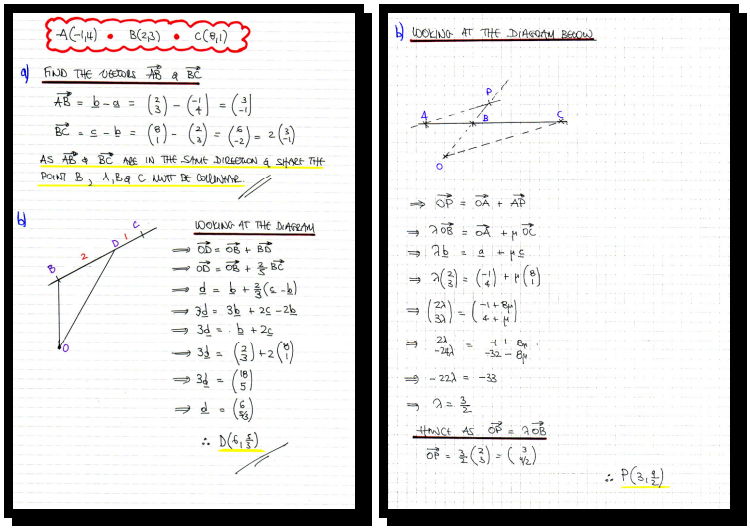
The point D lies on BC so that $\overrightarrow{BD}:\overrightarrow{BC}=2:3$.

b) Find the coordinates of D.

The straight line OB is extended to the point P, so that $\overrightarrow{AP}$ is parallel to $\overrightarrow{OC}$.

c) Determine the coordinates of P.

$\square$, $\boxed{D\left(6,\frac{5}{3}\right)}$, $\boxed{P\left(3,\frac{9}{2}\right)}$



Question 12 (**+)**

Relative to a fixed origin O on a horizontal plane, the points A and B have respective position vectors $3\mathbf{i}-2\mathbf{j}$ and $5\mathbf{i}+4\mathbf{j}$.

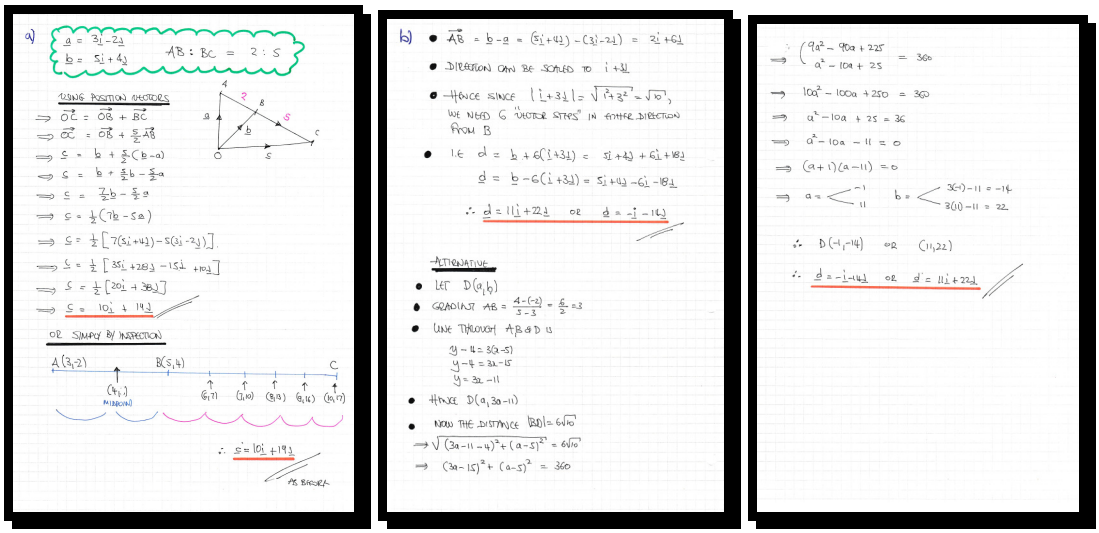
The point C lies on the same plane as A and B so that $\overrightarrow{AB}:\overrightarrow{BC}=2:5$.

a) Find the position vector of C.

The point D lies on the same plane as A and B so that A, B and D are collinear.

b) Given that $|BD|=6\sqrt{10}$, determine the possible position vectors of D.

$\boxed{}$, $\boxed{\mathbf{c}=10\mathbf{i}+19\mathbf{j}}$, $\boxed{\mathbf{d}=-\mathbf{i}-14\mathbf{j}\quad\cup\quad\mathbf{d}=11\mathbf{i}+22\mathbf{j}}$



Question 13 (**+)**

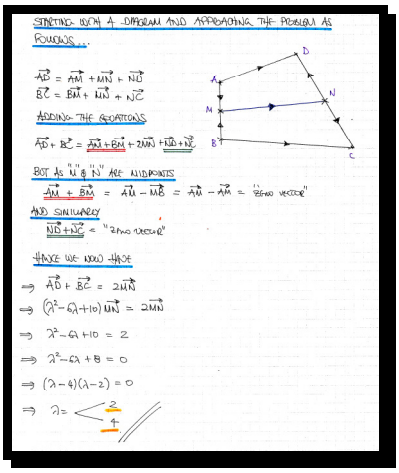
The four vertices of a quadrilateral $ABCD$ lie on the same plane.

The points M and N are the midpoints of AB and CD, respectively.

Determine the possible values of the scalar constant λ, given further that

$$\left(\lambda^2-6\lambda+10\right)\overrightarrow{MN} = \overrightarrow{AD}+\overrightarrow{BC}.$$

$\boxed{}$, $\boxed{\lambda=2 \quad \cup \quad \lambda=4}$



Question 14 (**+)**

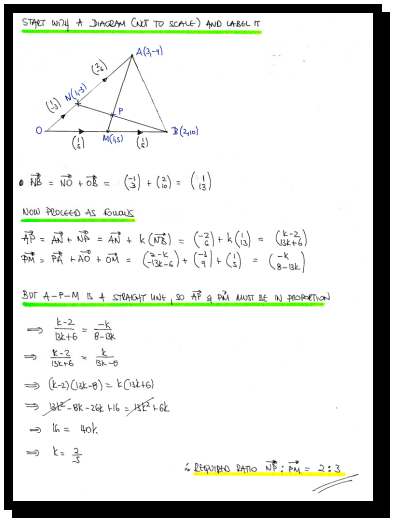
Relative to a fixed origin O, the points A and B have position vectors $3\mathbf{i}-9\mathbf{j}$ and $2\mathbf{i}+10\mathbf{j}$, respectively.

The point M is the midpoint of OB and the point N lies on OA so that $\overrightarrow{OA}=3\overrightarrow{ON}$.

The point P is the point of intersection of AM and BN.

Determine the ratio $\overrightarrow{NP}:\overrightarrow{PB}$.

$\boxed{}$, $\boxed{\overrightarrow{NP}:\overrightarrow{PB}=2:3}$



Question 15 (*****)

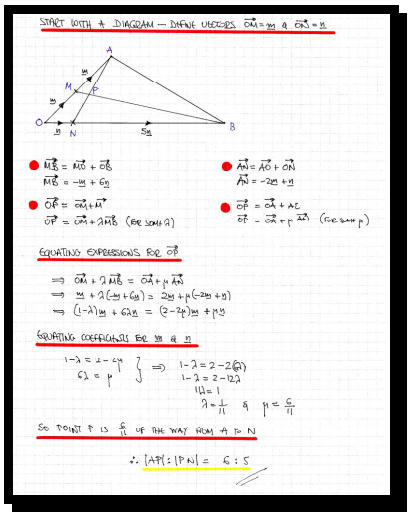
A triangle OAB is given.

The point M is the midpoint of OA.

The point N lies on OB so that $|ON| : |NB| = 1 : 5$

If the point P is the intersection of the straight lines AN and BM, use vector algebra to find the ratio of $|AP| : |EP|$.

$\boxed{}$, $\boxed{|AP| : |PN| = 6 : 5}$



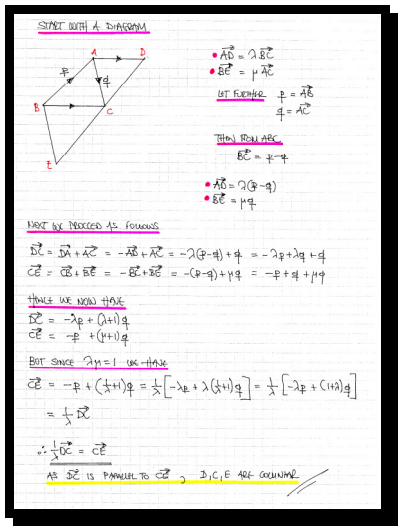
Question 16 (*****)

The triangle ABC is given.

The points D and E are such so that $\overrightarrow{AD}=\lambda\overrightarrow{BC}$ and $\overrightarrow{BE}=\mu\overrightarrow{AC}$, where λ and μ are positive scalar constants.

Given further that $\lambda\mu=1$, show that D, C and E are collinear.

, proof



Question 17 **(*****)**

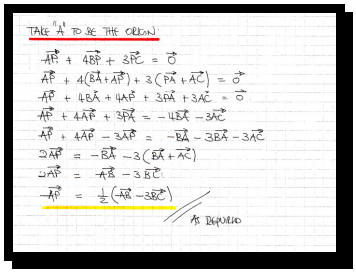
It is given that

$$\overrightarrow{AP} + 4\overrightarrow{BP} + 3\overrightarrow{PC} = \vec{0}.$$

Show that

$$\overrightarrow{AP} = \frac{1}{2}\left[\overrightarrow{AB} - 3\overrightarrow{BC}\right].$$

, proof

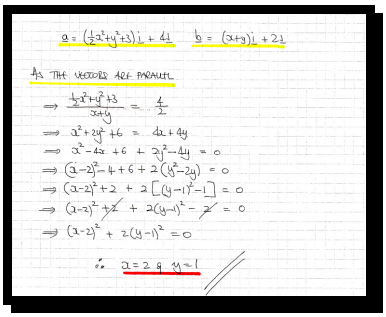


Question 18 **(*****)**

$$\mathbf{a} = \left(\frac{1}{2}x^2 + y^2 + 3\right)\mathbf{i} + 4\mathbf{j} \quad \text{and} \quad \mathbf{b} = (x+y)\mathbf{i} + 2\mathbf{j}.$$

Determine the value of x and the value of y given that $\mathbf{a}$ and $\mathbf{b}$ are parallel.

, $x = 2, \quad y = 1$



Introducing Elementary 3D Vectors

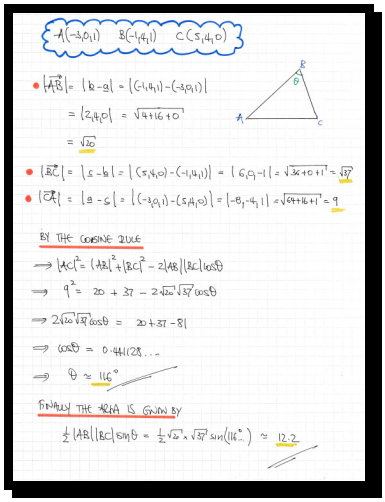
Question 1 (**)

Relative to a fixed origin O, the points A, B and C have respective position vectors

$$-3\mathbf{i}+\mathbf{k}, \qquad -\mathbf{i}+4\mathbf{j}+\mathbf{k} \qquad \text{and} \qquad 5\mathbf{i}+4\mathbf{j}.$$

Calculate the size of the angle ABC and hence find the area of the triangle ABC.

$\boxed{}$, $\boxed{\measuredangle ABC \approx 116^\circ}$, $\boxed{\text{area} \approx 12.2}$



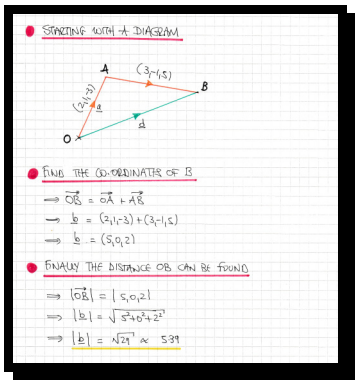
Question 2 **(**)**

Relative to a fixed origin O, the point A has coordinates $(2,1,-3)$.

The point B is such so that $\overrightarrow{AB}=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$.

Determine the distance of B from O.

$\boxed{}$, $\boxed{|OB|=\sqrt{29}}$



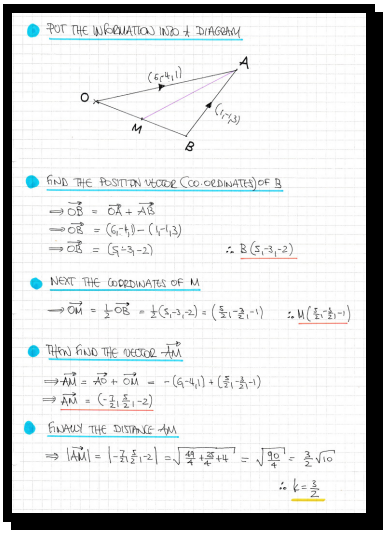
Question 3 (**)

Relative to a fixed origin O, the point A has coordinates $(6,-4,1)$.

The point B is such so that $\overrightarrow{BA}=\mathbf{i}-\mathbf{j}+3\mathbf{k}$.

If the point M is the midpoint of OB, show that $\left|\overrightarrow{AM}\right|=k\sqrt{10}$, where k is a rational constant to be found.

$\boxed{}$, $\boxed{k=\frac{3}{2}}$



Question 4 (+)**

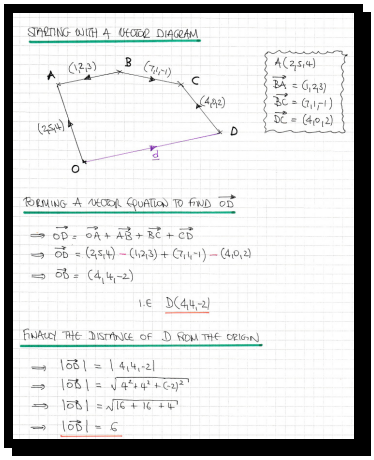
Relative to a fixed origin O, the point A has coordinates $(2,5,4)$.

The points B, C and D are such so that

$$\overrightarrow{BA}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}, \quad \overrightarrow{BC}=7\mathbf{i}+\mathbf{j}-\mathbf{k} \quad \text{and} \quad \overrightarrow{DC}=4\mathbf{i}+2\mathbf{k}.$$

Determine the distance of D from the origin.

$\boxed{}$, $\boxed{|OD|=6}$



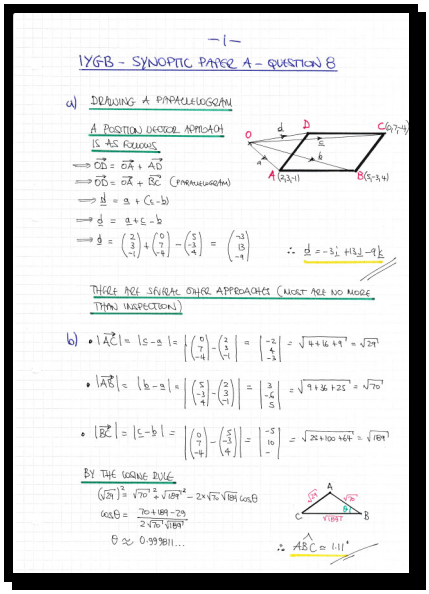
Question 5 (+)**

Relative to a fixed origin O, the points A, B and C have respective position vectors $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $5\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $7\mathbf{j}-4\mathbf{k}$.

a) Given that $ABCD$ is a parallelogram, determine the position vector of D.

b) Determine the distance AC and hence calculate the angle ABC.

$\boxed{}$, $\boxed{\mathbf{d}=-3\mathbf{i}+13\mathbf{j}-9\mathbf{k}}$, $\boxed{|AC|=\sqrt{29}}$, $\boxed{\angle ABC\approx 1.11^\circ}$



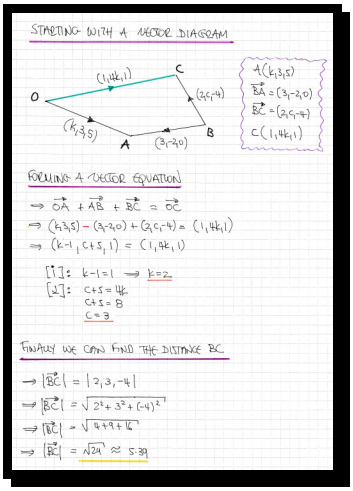
Question 6 (***)

Relative to a fixed origin O, the point A has coordinates $(k,3,5)$, where k is a scalar constant.

The points B and C are such so that $\overrightarrow{BA}=3\mathbf{i}-2\mathbf{j}$ and $\overrightarrow{BC}=2\mathbf{i}+c\mathbf{j}-4\mathbf{k}$, where c is a scalar constant.

If the coordinates of C are $(1,4k,1)$, determine the distance BC.

$\boxed{}$, $\boxed{|BC|=\sqrt{29}}$

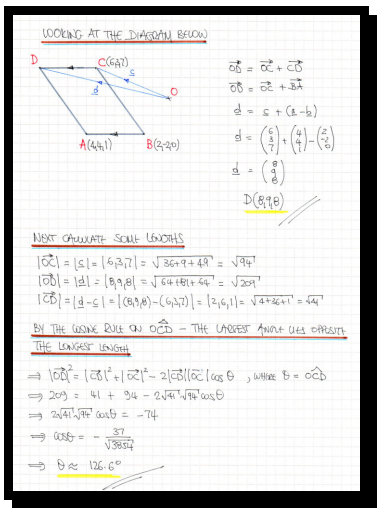


Question 7 (***)

The points $A(4,4,1)$, $B(2,-2,0)$ and $C(6,3,7)$ are referred relative to a fixed origin O.

If A, B, C and the point D form the parallelogram $ABCD$, use vector algebra to find the coordinates of D and hence calculate the angle OCD.

$\boxed{}$, $\boxed{D(8,9,8)}$, $\boxed{\angle OCD \approx 126.6^\circ}$



Question 8 (***)

$OABC$ is a square, where O is the origin, and the vertices A and C have respective position vectors $2\mathbf{i}+4\mathbf{j}+4\mathbf{k}$ and $4\mathbf{i}+2\mathbf{j}-4\mathbf{k}$.

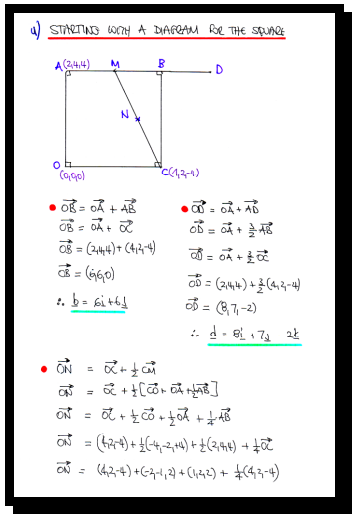
The point M is the midpoint of AB and the point N is the midpoint of MC.

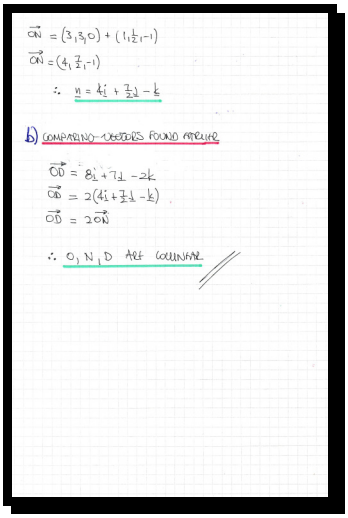
The point D is such so that $\overrightarrow{AD}=\frac{3}{2}\overrightarrow{AB}$.

a) Find the position vectors of the points B, D and N.

b) Deduce, showing your reasoning, that O, N and D are collinear.

$\boxed{}$, $\boxed{\overrightarrow{OB}=6\mathbf{i}+6\mathbf{j}}$, $\boxed{\overrightarrow{OD}=8\mathbf{i}+7\mathbf{j}-2\mathbf{k}}$, $\boxed{\overrightarrow{ON}=4\mathbf{i}+\frac{7}{2}\mathbf{j}-\mathbf{k}}$





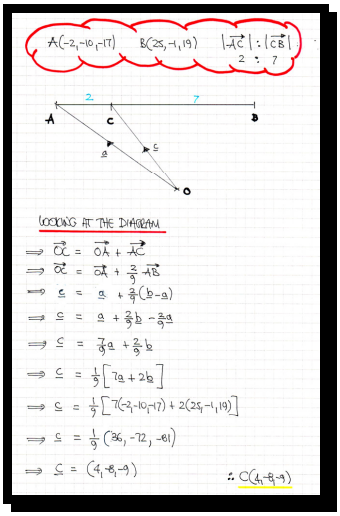
Question 9 **(***)**

The points $A(-2,-10,-17)$ and $B(25,-1,19)$ are referred relative to a fixed origin O.

The point C is such so that ACB forms a straight line.

Given further that $\dfrac{\left|\overrightarrow{AC}\right|}{\left|\overrightarrow{CB}\right|}=\dfrac{2}{7}$, determine the coordinates of C.

$\boxed{}$, $\boxed{C(4,-8,-9)}$



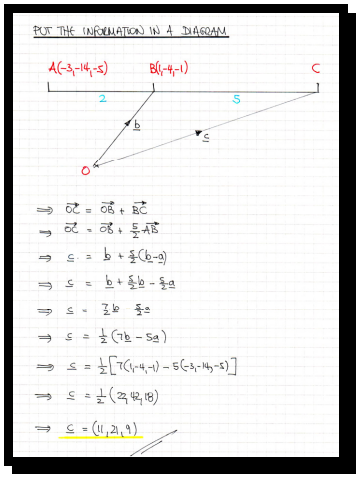
Question 10 (***)

The points $A(-3,-14,-5)$ and $B(1,-4,-1)$ are referred relative to a fixed origin O.

The point C is such so that ABC forms a straight line.

Given further that $\dfrac{\left|\overrightarrow{AB}\right|}{\left|\overrightarrow{BC}\right|}=\dfrac{2}{5}$, determine the coordinates of C.

$\boxed{}$, $\boxed{C(11,21,9)}$



Question 11 (***)

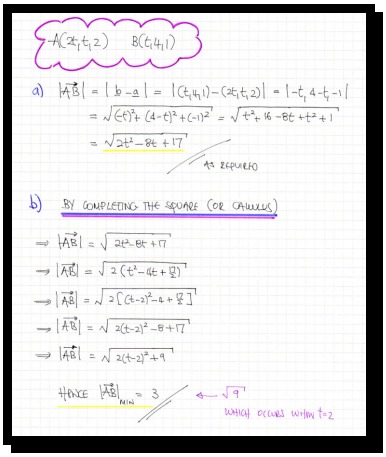
The variable points $A(2t,t,2)$ and $B(t,4,1)$, where t is a scalar variable, are referred relative to a fixed origin O.

a) Show that

$$\left|\overrightarrow{AB}\right| = \sqrt{2t^2-8t+17}\,.$$

b) Hence find the shortest distance between A and B, as t varies.

$\boxed{}$, $\boxed{\left|\overrightarrow{AB}\right|_{\min} = 3}$



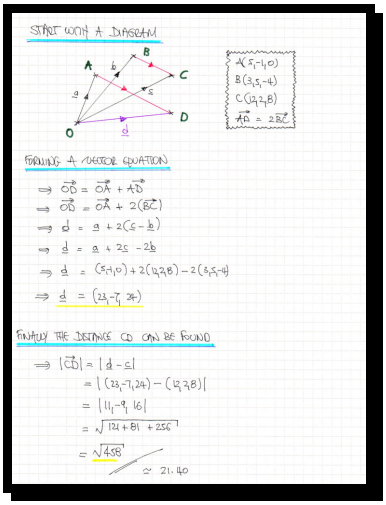
Question 12 (***)

The points $A(5,-1,0)$, $B(3,5,-4)$, $C(12,2,8)$ are referred relative to a fixed origin O.

The point D is such so that $\overrightarrow{AD}=2\overrightarrow{BC}$.

Determine the distance CD.

$\boxed{}$, $\boxed{|CD|=\sqrt{458}\approx 21.40}$

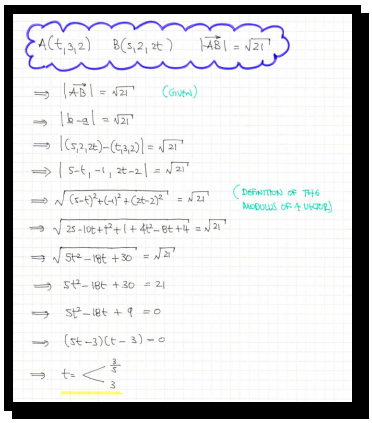


Question 13 (***)

The points $A(t,3,2)$ and $B(5,2,2t)$, where t is a scalar constant, are referred relative to a fixed origin O.

Given that $\left|\overrightarrow{AB}\right|=\sqrt{21}$, find the possible values of t.

$\square$, $\boxed{t=3,\ t=\frac{3}{5}}$

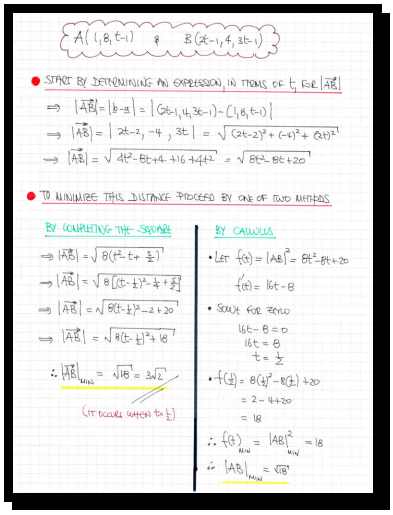


Question 14 (*+)**

The variable points $A(1,8,t-1)$ and $B(2t-1,4,3t-1)$, where t is a scalar variable, are referred relative to a fixed origin O.

Find the shortest distance between A and B, as t varies.

$\square$, $\left|\overrightarrow{AB}\right|_{\min}=\sqrt{18}$



Question 15 **(***+)**

The points $A(1,1,2)$, $B(2,1,5)$, $C(4,0,1)$ and D form the parallelogram $ABCD$, where the above coordinates are measured relative to a fixed origin.

a) Find the coordinates of D.

The points E, B and D are collinear, so that B is the midpoint of ED.

b) Determine the coordinates of E.

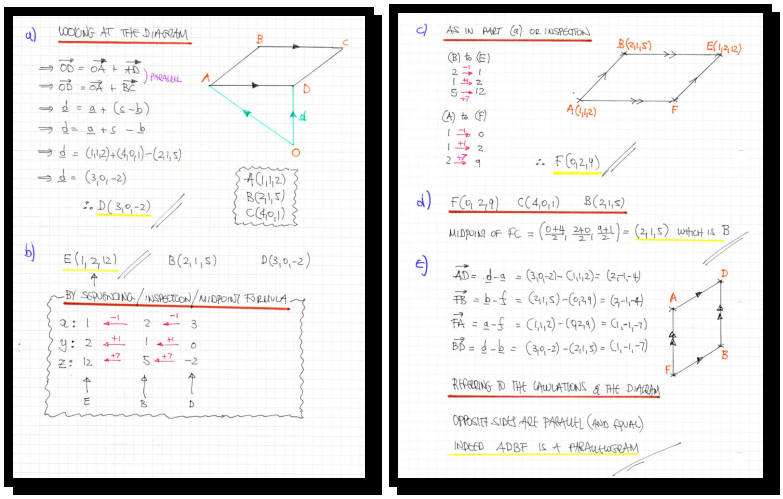
The point F is such so that $ABEF$ is also a parallelogram.

c) Find the coordinates of F.

d) Show that B is the midpoint of FC.

e) Prove that $ADBF$ is another parallelogram.

$\boxed{}$, $\boxed{D(3,0,-2)}$, $\boxed{E(1,2,12)}$, $\boxed{F(0,2,9)}$



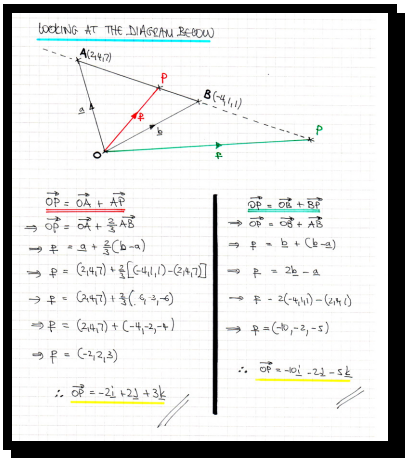
Question 16 (*+)**

With respect to a fixed origin, the points A and B have position vectors $2\mathbf{i}+4\mathbf{j}+7\mathbf{k}$ and $-4\mathbf{i}+\mathbf{j}+\mathbf{k}$, respectively.

The point P lies on the straight line through A and B.

Find the possible position vectors of P if $\left|\overrightarrow{AP}\right|=2\left|\overrightarrow{PB}\right|$.

$\boxed{}$, $\boxed{\overrightarrow{OP}=\mathbf{p}=-2\mathbf{i}+2\mathbf{j}+3\mathbf{k}}$, $\boxed{\overrightarrow{OP}=\mathbf{p}=-10\mathbf{i}-2\mathbf{j}-5\mathbf{k}}$



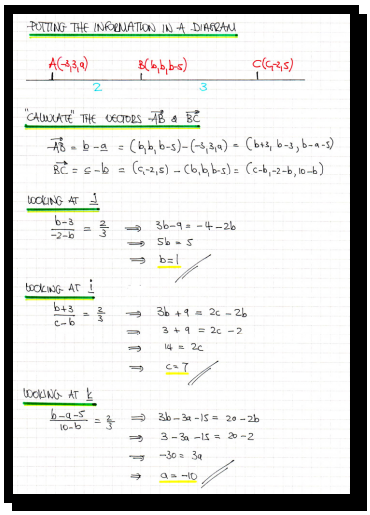
Question 17 (*+)**

The points $A(-3,3,a)$, $B(b,b,b-5)$ and $C(c,-2,5)$, where a, b and c are scalar constants, are referred relative to a fixed origin O.

Ii is further given that A, B and C are collinear and the ratio $\left|\overrightarrow{AB}\right| : \left|\overrightarrow{BC}\right| = 2:3$.

Use vector algebra to find the value of a, the value of b and the value of c.

$\boxed{}$, $\boxed{[a,b,c]=[-10,1,7]}$



Question 18 (*+)**

The points $A(7,4,3)$, B and $C(1,2,-1)$ form the parallelogram $OABC$, where the above coordinates are measured relative to a fixed origin O.

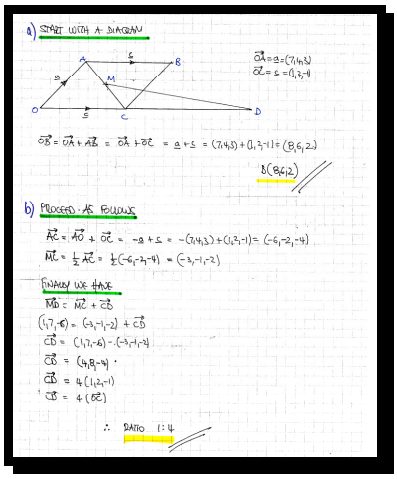
a) Find the coordinates of B.

The side OC is extended in the $\overrightarrow{OC}$ direction to a point D.

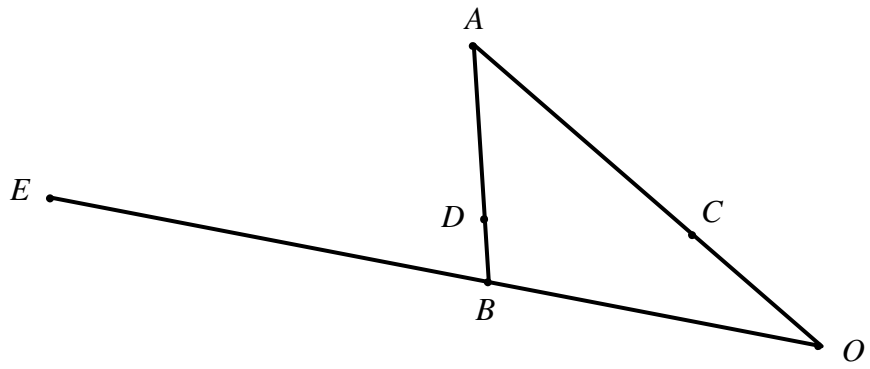
The point M is the midpoint of AC.

b) Given further that $\overrightarrow{MD}=\mathbf{i}+7\mathbf{j}-6\mathbf{k}$, determine $\left|\overrightarrow{OC}\right| : \left|\overrightarrow{CD}\right|$.

$\square$, $\boxed{B(8,6,2)}$, $\boxed{\left|\overrightarrow{OC}\right| : \left|\overrightarrow{CD}\right| = 1 : 4}$



Question 19 (*+)**



The figure above shows the triangle OAB, where O is the origin and the position vectors of A and B relative to O, are $-6\mathbf{i}+27\mathbf{j}-9\mathbf{k}$ and $4\mathbf{i}+6\mathbf{j}-6\mathbf{k}$, respectively.

The point E is such so that O, B and E are collinear with $OB:BE=1:2$

The point C is such so that O, C and A are collinear with $OC:CA=1:2$

The point D is such so that B, D and A are collinear with $BD:DA=1:3$

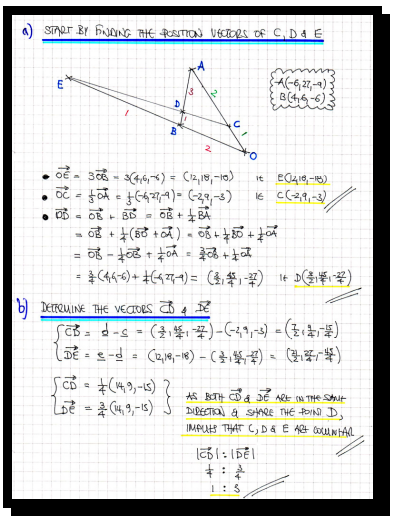
a) Determine the coordinates of C, D and E, relative to O.

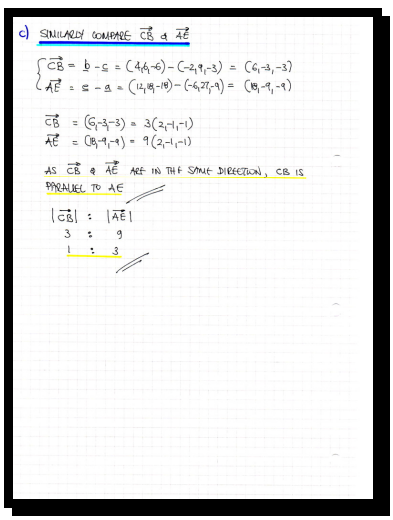
b) Show that the points C, D and E are collinear, and find the ratio $CD:DE$.

c) Show further that BC is parallel to EA, and find the ratio $BC:EA$.

$\boxed{}$, $\boxed{C(-2,9,-3)}$, $\boxed{D\left(\frac{3}{2},\frac{45}{4},-\frac{27}{4}\right)}$, $\boxed{E(12,18,-18)}$, $\boxed{CD:DE=1:3}$,

$\boxed{BC:EA=1:3}$





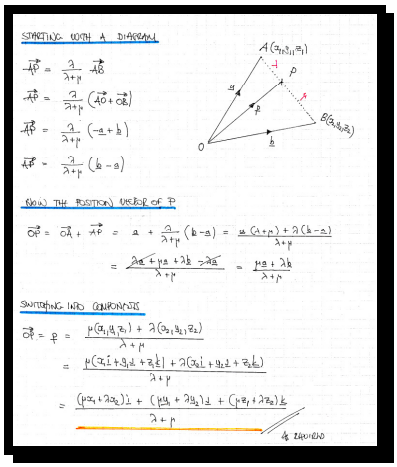
Question 20 (*+)**

The points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are referred relative to a fixed origin O.

If the point P is such so that $\overrightarrow{AP} : \overrightarrow{PB} = \lambda : \mu$, use vector algebra to show that

$$\overrightarrow{OP} = \frac{(\mu x_1 + \lambda x_2)\mathbf{i} + (\mu y_1 + \lambda y_2)\mathbf{j} + (\mu z_1 + \lambda z_2)\mathbf{k}}{\lambda + \mu}.$$

[ ], [proof]

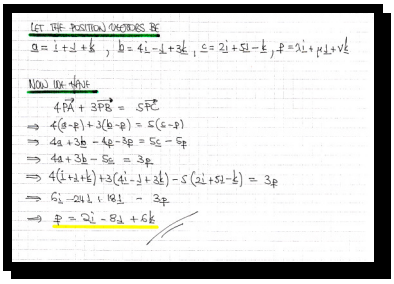


Question 21 (*+)**

Relative to a fixed origin, the coordinates of three points $A(1,1,1)$, $B(4,-1,3)$ and $C(2,5,-1)$, are given.

Find the position vector of the point P if $4\overrightarrow{PA} + 3\overrightarrow{PB} = 5\overrightarrow{PC}$.

$\square$, $\mathbf{p} = 2\mathbf{i} - 8\mathbf{j} + 6\mathbf{k}$



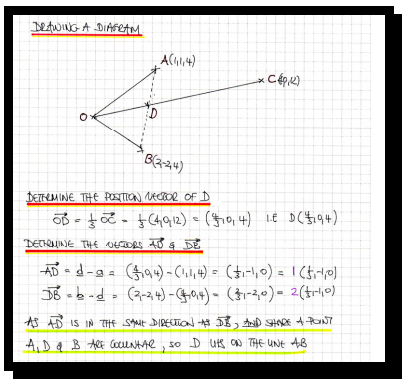
Question 22 (****)

Relative to a fixed origin O, the positions vectors of the points A, B and C are defined below.

$$\overrightarrow{OA}=\mathbf{i}+\mathbf{j}+4\mathbf{k}\,, \qquad \overrightarrow{OB}=2\mathbf{i}-2\mathbf{j}+4\mathbf{k}\,, \qquad \overrightarrow{OC}=4\mathbf{i}+12\mathbf{k}\,.$$

If $\overrightarrow{OD}=\frac{1}{3}\overrightarrow{OC}$ prove that the point D lies on the straight line AB.

[ ], [proof]



Question 23 (****)

Relative to a fixed origin O, the position vectors of three points A, B and C are

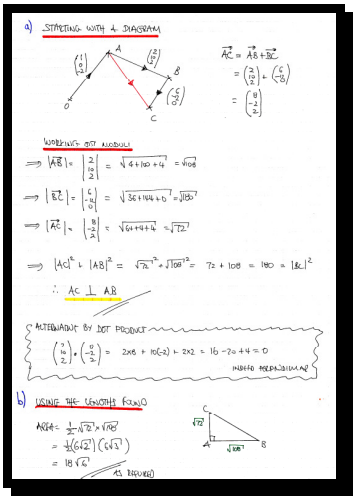
$$\overrightarrow{OA}=\mathbf{i}-2\mathbf{k}\,,\qquad \overrightarrow{AB}=2\mathbf{i}+10\mathbf{j}+2\mathbf{k}\qquad \text{and}\qquad \overrightarrow{BC}=6\mathbf{i}-12\mathbf{j}\,.$$

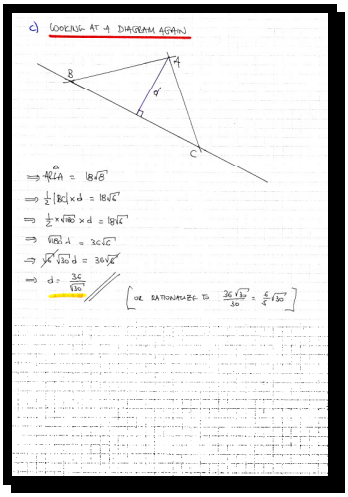
a) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{AB}$.

b) Show further that the area of the triangle ABC is $18\sqrt{6}$.

c) Hence, or otherwise, determine the shortest distance of A from the straight line through B and C.

$\boxed{}$, $\boxed{\text{distance}=\frac{6}{5}\sqrt{30}}$





Question 24 (****)

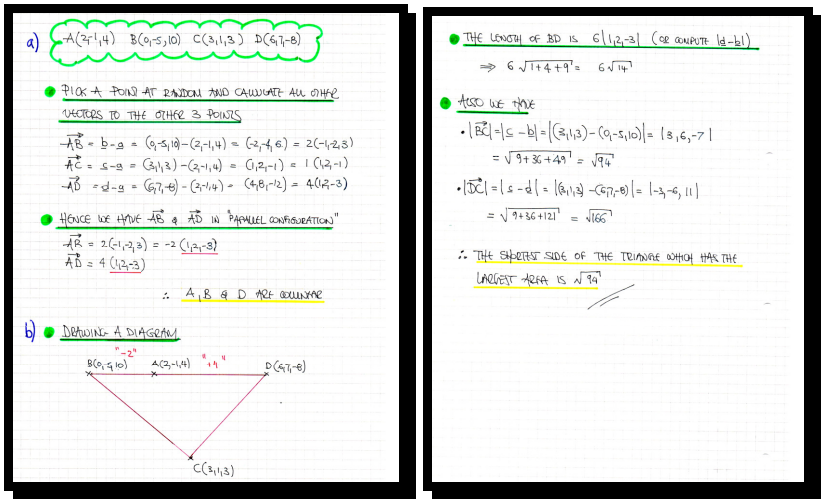
The points $A(2,-1,4)$, $B(0,-5,10)$, $C(3,1,3)$ and $D(6,7,-8)$ are referred relative to a fixed origin O.

a) Use vector algebra to show that three of the above four points are collinear.

A triangle is drawn using three of the above four points as its vertices.

b) Given further that the triangle has the largest possible area, determine, in exact surd form, the length of its shortest side.

$\square$, $\boxed{\sqrt{94}}$



Question 125 (****)

Relative to a fixed origin, the points P and Q have position vectors $9\mathbf{j}-2\mathbf{k}$ and $7\mathbf{i}-8\mathbf{j}+11\mathbf{k}$, respectively.

a) Determine the distance between the points P and Q.

b) Find the position vector of the point M, where M is the midpoint of PQ.

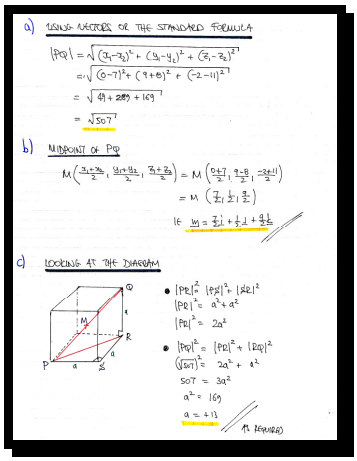
The points P and Q are vertices of a cube, so that PQ is one of the longest diagonals of the cube.

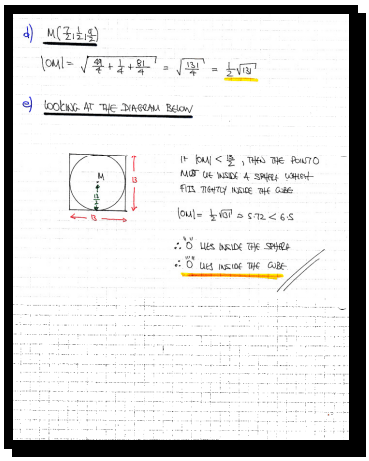
c) Show that the length of one of the sides of the cube is 13 units.

d) Calculate the distance of the point M from the origin O.

e) Show that the origin O lies inside the cube.

$\boxed{}$, $\boxed{|PQ|=\sqrt{507}}$, $\boxed{\overrightarrow{OM}=\frac{7}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{9}{2}\mathbf{k}}$, $\boxed{|OM|=\frac{1}{2}\sqrt{131}}$





Question 26 (**+)**

The points $A(3,2,14)$, $B(0,1,13)$ and $C(5,6,8)$ are defined with respect to a fixed origin O.

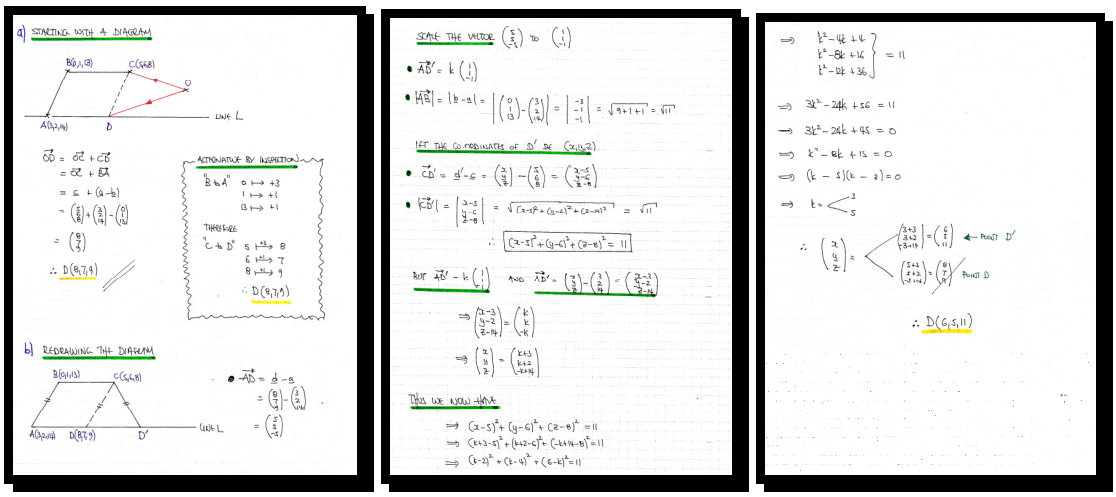
The straight line L passes through A and it is parallel to the vector $\overrightarrow{BC}$.

The point D lies on L so that $ABCD$ is a parallelogram.

a) Find the coordinates of D.

b) If instead $ABCD$ is an isosceles trapezium and the point D still lies on L, determine the new coordinates of D.

$\boxed{}$, $\boxed{D(8,7,9)}$, $\boxed{D(6,5,11)}$



Question 27 (*****)

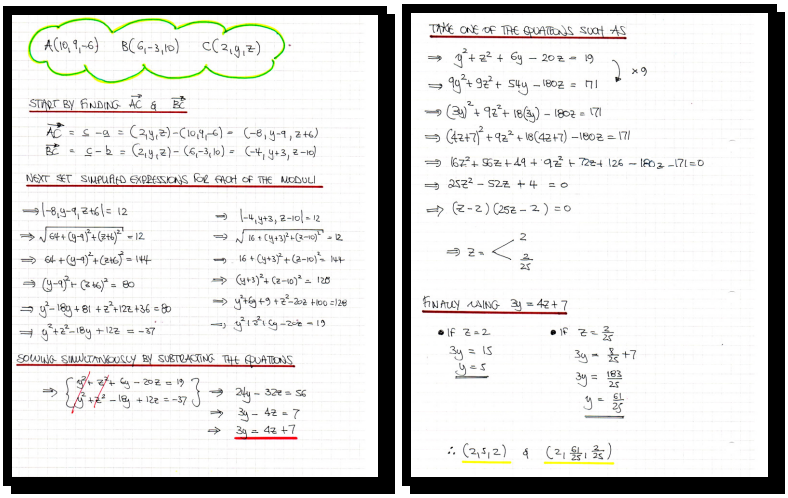
With respect to a fixed origin, the points A and B have position vectors $10\mathbf{i}+9\mathbf{j}-6\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+10\mathbf{k}$, respectively.

The position vector of the point C has $\mathbf{i}$ component equal to 2.

The distance of C from both A and B is 12 units.

Show that one of the two possible position vectors of C is $2\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and determine the other.

$\square$, $\boxed{\mathbf{c}=2\mathbf{i}+\frac{61}{25}\mathbf{j}+\frac{2}{25}\mathbf{k}}$



Question 28 (*****)

The vertices of the triangle OAB have coordinates $A(6,-18,-6)$, $B(7,-1,3)$, where O is a fixed origin.

The point N lies on OA so that $ON : NA = 1:2$.

The point M is the midpoint of OB.

The point P is the intersection of AM and BN.

By using vector methods, or otherwise, determine the coordinates of P.

$\square$, $\boxed{P(4,-4,0)}$

