MIXED SURD QUESTIONS

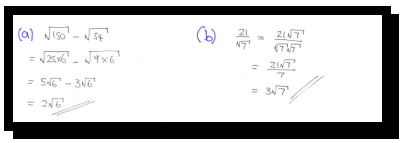
Question 1 (**)

Write each of the following expressions a single simplified surd.

a) $\sqrt{150}-\sqrt{54}$.

b) $\dfrac{21}{\sqrt{7}}$.

$\boxed{}$, $\boxed{2\sqrt{6}}$, $\boxed{3\sqrt{7}}$



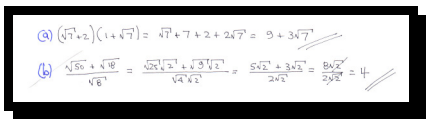
Question 2 (**)

Write each of the following surd expressions as simple as possible.

a) $\left(\sqrt{7}+2\right)\left(1+\sqrt{7}\right)$.

b) $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}$.

$\boxed{}$, $\boxed{9+3\sqrt{7}}$, $\boxed{4}$

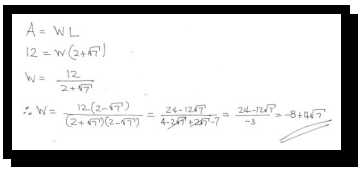


Question 3 (**)

A rectangle has area $12\,\text{cm}^2$ and length $2+\sqrt{7}$ cm.

Find its width in the form $a+b\sqrt{7}$, where a and b are integers.

$\boxed{-8+4\sqrt{7}}$



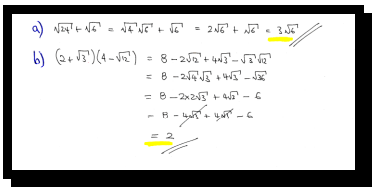
Question 4 (**)

Write each of the following surd expressions as simple as possible.

a) $\sqrt{24}+\sqrt{6}$.

b) $\left(2+\sqrt{3}\right)\left(4-\sqrt{12}\right)$.

$\boxed{}$, $\boxed{3\sqrt{6}}$, $\boxed{2}$



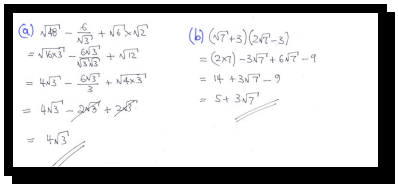
Question 5 (**)

Write each of the following surd expressions as simple as possible.

a) $\sqrt{48}-\dfrac{6}{\sqrt{3}}+\sqrt{6}\times\sqrt{2}$.

b) $\left(\sqrt{7}+3\right)\left(2\sqrt{7}-3\right)$.

$\boxed{4\sqrt{3}}$, $\boxed{5+3\sqrt{7}}$



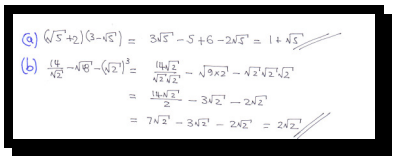
Question 6 (**)

Write each of the following surd expressions as simple as possible.

a) $\left(\sqrt{5}+2\right)\left(3-\sqrt{5}\right)$.

b) $\dfrac{14}{\sqrt{2}}-\sqrt{18}-\left(\sqrt{2}\right)^3$.

$\boxed{1+\sqrt{5}}$, $\boxed{2\sqrt{2}}$



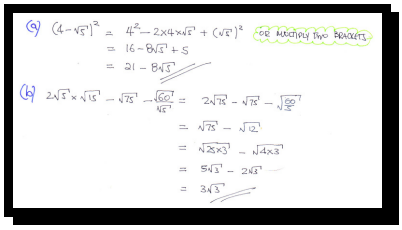
Question 7 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $\left(4-\sqrt{5}\right)^2$.

b) $2\sqrt{5}\times\sqrt{15}-\sqrt{75}-\dfrac{\sqrt{60}}{\sqrt{5}}$.

$\boxed{}$, $\boxed{21-8\sqrt{5}}$, $\boxed{3\sqrt{3}}$



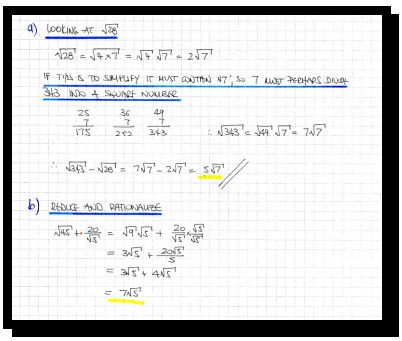
Question 8 **(**+)**

Write each of the following expressions a single simplified surd.

a) $\sqrt{343}-\sqrt{28}$.

b) $\sqrt{45}+\dfrac{20}{\sqrt{5}}$.

$\boxed{}$, $\boxed{5\sqrt{7}}$, $\boxed{7\sqrt{5}}$



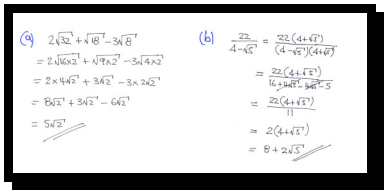
Question 9 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $2\sqrt{32}+\sqrt{18}-3\sqrt{8}$.

b) $\dfrac{22}{4-\sqrt{5}}$.

$\boxed{5\sqrt{2}}$, $\boxed{8+2\sqrt{5}}$



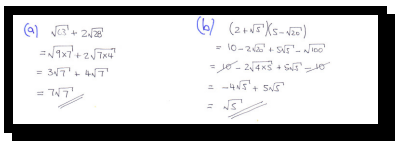
Question 10 **(**+)**

Write each of the following expressions a single simplified surd.

a) $\sqrt{63}+2\sqrt{28}$.

b) $\left(2+\sqrt{5}\right)\left(5-\sqrt{20}\right)$.

$\boxed{7\sqrt{7}}$, $\boxed{\sqrt{5}}$



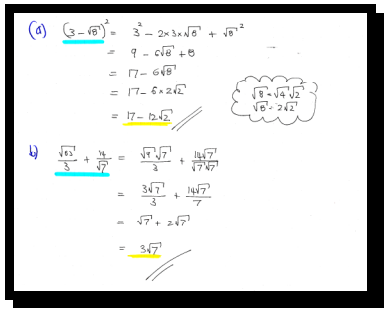
Question 11 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $\left(3-\sqrt{8}\right)^2$.

b) $\dfrac{\sqrt{63}}{3}+\dfrac{14}{\sqrt{7}}$.

$\boxed{}$, $\boxed{17-12\sqrt{2}}$, $\boxed{3\sqrt{7}}$



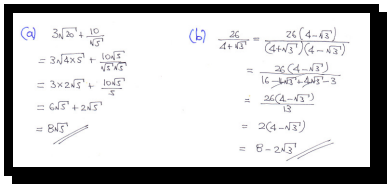
Question 12 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $3\sqrt{20}+\dfrac{10}{\sqrt{5}}$.

b) $\dfrac{26}{4+\sqrt{3}}$.

$\boxed{8\sqrt{5}}$, $\boxed{8-2\sqrt{3}}$

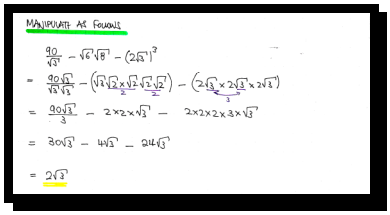


Question 13 **(**+)**

Write the following expression in the form $k\sqrt{3}$, where k is an integer.

$$\frac{90}{\sqrt{3}}-\sqrt{6}\times\sqrt{8}-\left(2\sqrt{3}\right)^3$$

$\boxed{}$, $\boxed{2\sqrt{3}}$



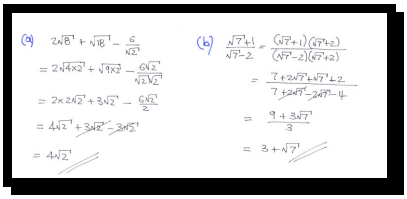
Question 14 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $2\sqrt{8}+\sqrt{18}-\dfrac{6}{\sqrt{2}}$.

b) $\dfrac{\sqrt{7}+1}{\sqrt{7}-2}$.

$\boxed{4\sqrt{2}}$, $\boxed{3+\sqrt{7}}$



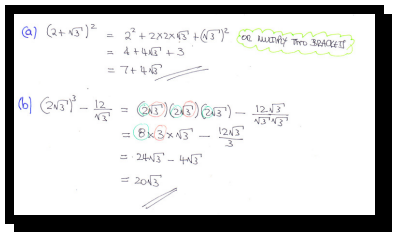
Question 15 (+)**

Write each of the following surd expressions as simple as possible.

a) $\left(2+\sqrt{3}\right)^2$.

b) $\left(2\sqrt{3}\right)^3-\dfrac{12}{\sqrt{3}}$.

$\boxed{7+4\sqrt{3}}$, $\boxed{20\sqrt{3}}$

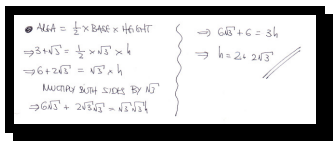


Question 16 (+)**

The area of a triangle is $\left(3+\sqrt{3}\right)\text{cm}^2$.

Given the base of the triangle is $\sqrt{3}$ cm, find in exact simplified surd form the height of the triangle.

$\boxed{h=2+2\sqrt{3}}$



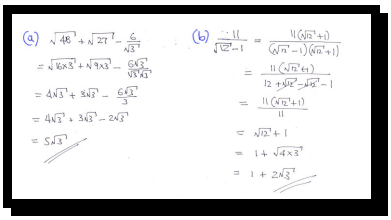
Question 17 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $\sqrt{48}+\sqrt{27}-\dfrac{6}{\sqrt{3}}$.

b) $\dfrac{11}{\sqrt{12}-1}$.

$\boxed{5\sqrt{3}}$, $\boxed{1+2\sqrt{3}}$



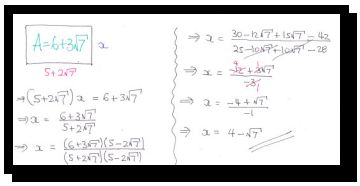
Question 18 **(**+)**

A rectangular room has an area of $6+3\sqrt{7}$ m^2.

The length of the room is $5+2\sqrt{7}$ m.

Find the width of the room, giving the answer as an exact surd in its simplest form.

$\boxed{}$, $\boxed{4-\sqrt{7}}$



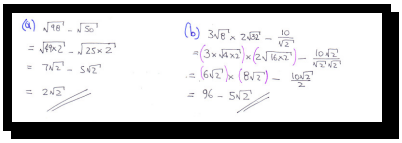
Question 19 **(**+)**

Write each of the following surd expressions as simple as possible.

a) $\sqrt{98}-\sqrt{50}$.

b) $3\sqrt{8}\times 2\sqrt{32}-\dfrac{10}{\sqrt{2}}$.

$\boxed{2\sqrt{2}}$, $\boxed{96-5\sqrt{2}}$

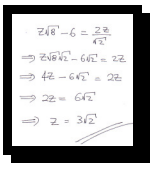


Question 20 **(**+)**

$$z\sqrt{8}-6=\frac{2z}{\sqrt{2}}.$$

Solve the above equation giving your answer in the form $k\sqrt{2}$, where k is an integer.

$\boxed{}$, $\boxed{z=3\sqrt{2}}$



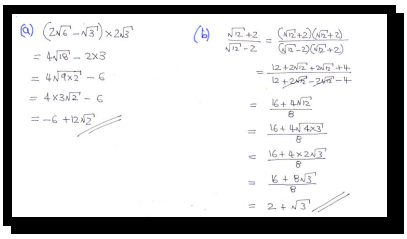
Question 21 (+)**

Write each of the following surd expressions as simple as possible.

a) $\left(2\sqrt{6}-\sqrt{3}\right)\times 2\sqrt{3}$.

b) $\dfrac{\sqrt{12}+2}{\sqrt{12}-2}$.

$\boxed{-6+12\sqrt{2}}$, $\boxed{2+\sqrt{3}}$

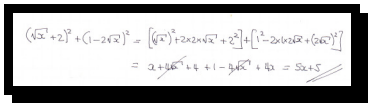


Question 22 (+)**

$$f(x)\equiv\left(\sqrt{x}+2\right)^2+\left(1-2\sqrt{x}\right)^2.$$

Express $f(x)$, $x\geq 0$ in the form $ax+b$.

$\boxed{5x+5}$



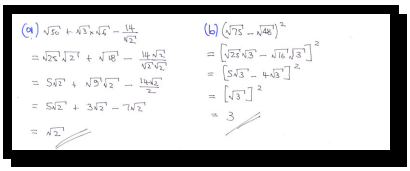
Question 23 (***)

Write each of the following surd expressions as simple as possible.

a) $\sqrt{50}+\sqrt{3}\times\sqrt{6}-\dfrac{14}{\sqrt{2}}$.

b) $\left(\sqrt{75}-\sqrt{48}\right)^2$.

$\boxed{\sqrt{2}}$, $\boxed{3}$



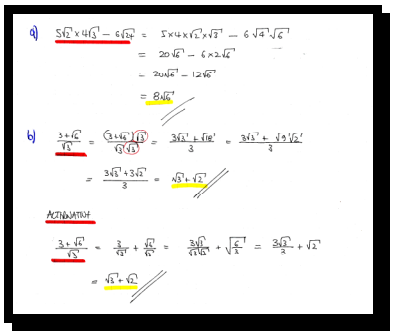
Question 24 (***)

Write each of the following surd expressions as simple as possible.

a) $5\sqrt{2}\times 4\sqrt{3}-6\sqrt{24}$.

b) $\dfrac{3+\sqrt{6}}{\sqrt{3}}$.

$\boxed{}$, $\boxed{8\sqrt{6}}$, $\boxed{\sqrt{3}+\sqrt{2}}$



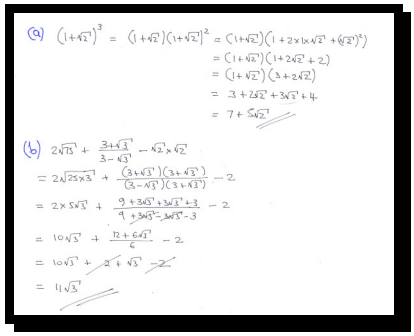
Question 25 (***)

Write each of the following surd expressions as simple as possible.

a) $\left(1+\sqrt{2}\right)^3$.

b) $2\sqrt{75}+\dfrac{3+\sqrt{3}}{3-\sqrt{3}}-\sqrt{2}\times\sqrt{2}$.

$\boxed{}$, $\boxed{7+5\sqrt{2}}$, $\boxed{11\sqrt{3}}$



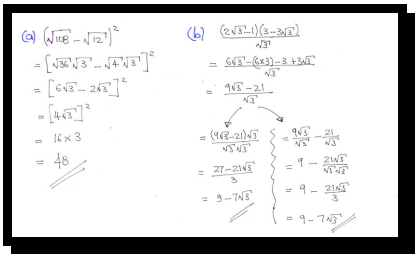
Question 26 (***)

Write each of the following surd expressions as simple as possible.

a) $\left(\sqrt{108}-\sqrt{12}\right)^2$.

b) $\dfrac{\left(2\sqrt{3}-1\right)\left(3-3\sqrt{3}\right)}{\sqrt{3}}$.

$\boxed{48}$, $\boxed{9-7\sqrt{3}}$



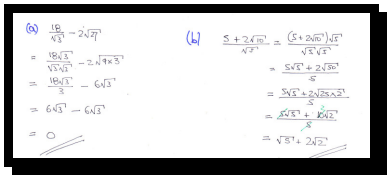
Question 27 **(***)**

Write each of the following surd expressions as simple as possible.

a) $\dfrac{18}{\sqrt{3}}-2\sqrt{27}$.

b) $\dfrac{5+2\sqrt{10}}{\sqrt{5}}$.

$\boxed{0}$, $\boxed{\sqrt{5}+2\sqrt{2}}$

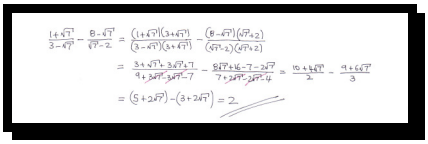


Question 28 **(***)**

$$\frac{1+\sqrt{7}}{3-\sqrt{7}}-\frac{8-\sqrt{7}}{\sqrt{7}-2}.$$

Show that the above surd expression simplifies to a positive integer, stating its value.

$\boxed{}$, $\boxed{2}$



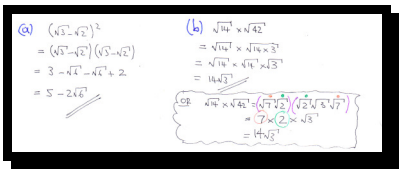
Question 29 (***)

Write each of the following surd expressions as simple as possible.

a) $\left(\sqrt{3}-\sqrt{2}\right)^2$.

b) $\sqrt{14}\times\sqrt{42}$.

$\square$, $\boxed{5-2\sqrt{6}}$, $\boxed{14\sqrt{3}}$



Question 30 (***)

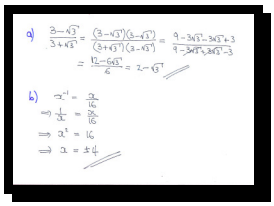
a) Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where a and b are integers

$$\frac{3-\sqrt{3}}{3+\sqrt{3}}.$$

b) Solve the equation

$$x^{-1}=\frac{x}{16}, \quad x\neq 0.$$

$\square$, $\boxed{2-\sqrt{3}}$, $\boxed{x=\pm 4}$

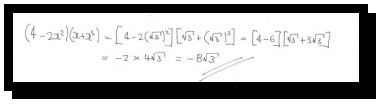


Question 31 (***)

If $x=\sqrt{3}$ show that

$$\left(4-2x^2\right)\left(x+x^3\right)=-8\sqrt{3}\,.$$

proof

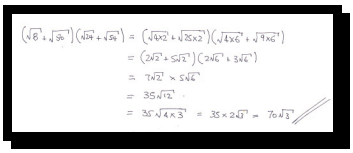


Question 32 (***)

Simplify fully

$$\left(\sqrt{8}+\sqrt{50}\right)\left(\sqrt{24}+\sqrt{54}\right).$$

$\square$, $70\sqrt{3}$



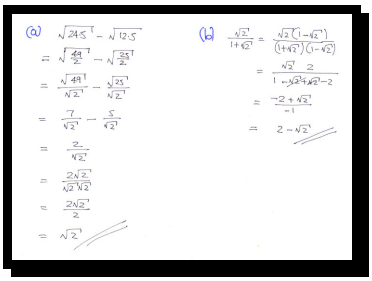
Question 33 (***)

Write each of the following surd expressions as simple as possible.

a) $\sqrt{24.5}-\sqrt{12.5}$.

b) $\dfrac{\sqrt{2}}{1+\sqrt{2}}$.

$\boxed{}$, $\boxed{\sqrt{2}}$, $\boxed{2-\sqrt{2}}$

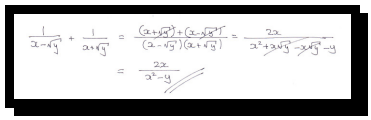


Question 34 (***)

$$\frac{1}{x-\sqrt{y}}+\frac{1}{x+\sqrt{y}}\,.$$

Write the above expression as a single fraction in its simplest form.

$$\boxed{\frac{2x}{x^2-y}}$$

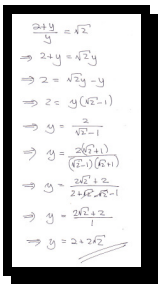


Question 35 (***)

$$\frac{2+y}{y}=\sqrt{2}\,.$$

Solve the above equation giving the answer in the form $a+b\sqrt{2}$, where a and b are integers.

$\square$, $\boxed{y=2+2\sqrt{2}}$



Question 36 (***)

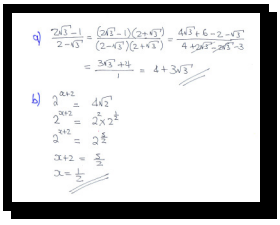
a) Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where a and b are integers

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}}.$$

b) Solve the equation

$$2^{x+2}=4\sqrt{2}\,.$$

$\square$, $\boxed{4+3\sqrt{3}}$, $\boxed{x=\frac{1}{2}}$



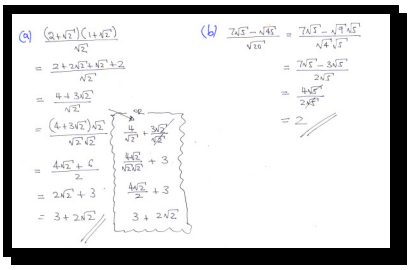
Question 37 (***)

Write each of the following surd expressions as simple as possible.

a) $\dfrac{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}\right)}{\sqrt{2}}$.

b) $\dfrac{5\sqrt{5}-\sqrt{45}}{\sqrt{20}}$.

$\boxed{3+2\sqrt{2}}$, $\boxed{2}$

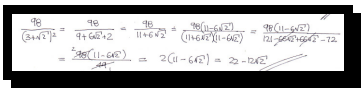


Question 38 (***)

$$\frac{98}{\left(3+\sqrt{2}\right)^2}.$$

Write the above surd expression in the form $a+b\sqrt{2}$, where a and b are integers.

$\boxed{}$, $\boxed{22-12\sqrt{2}}$



Question 39 (*+)**

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{3}$.

i. $\sqrt{108}+\sqrt{3}$.

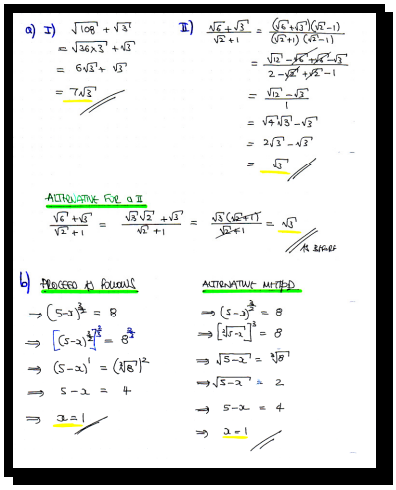
ii. $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1}$.

b) Solve the equation

$$(5-x)^{\frac{3}{2}}=8.$$

Detailed workings must be shown in this question.

$\boxed{}$, $\boxed{7\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{x=1}$



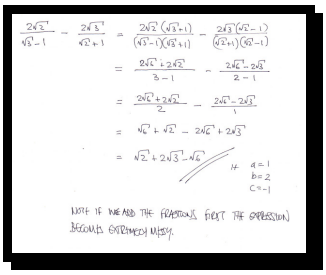
Question 40 (*+)**

It is given that that for some constants a, b and c

$$\frac{2\sqrt{2}}{\sqrt{3}-1}-\frac{2\sqrt{3}}{\sqrt{2}+1}\equiv a\sqrt{2}+b\sqrt{3}+c\sqrt{6}\,.$$

Find the value of a, the value of b and the value of c.

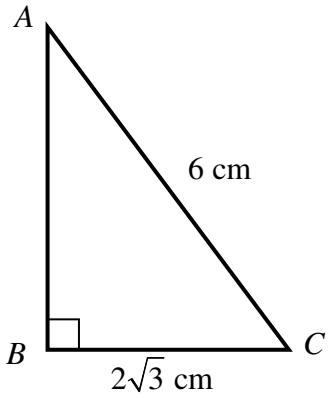
$\boxed{}$, $\boxed{a=1}$, $\boxed{b=2}$, $\boxed{c=-1}$



Question 41 (*+)**

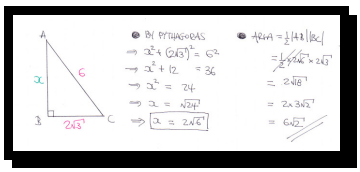
A right angled triangle ABC is shown in the figure below.

The lengths of AC and BC are 6 cm and $2\sqrt{3}$ cm, respectively.



Find the area of the triangle ABC in the form $k\sqrt{2}$, where k is an integer.

$$\boxed{k=6}$$



Question 42 (*+)**

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{2}$.

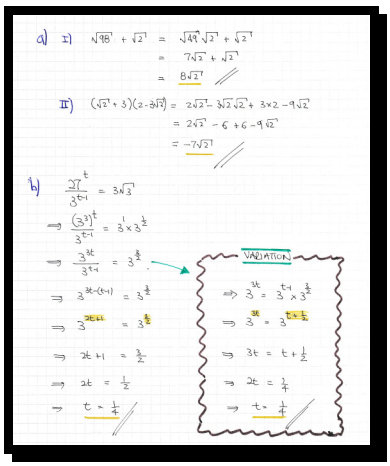
i. $\sqrt{98}+\sqrt{2}$.

ii. $\left(\sqrt{2}+3\right)\left(2-3\sqrt{2}\right)$.

b) Solve the equation

$$\frac{27^t}{3^{t-1}}=3\sqrt{3}.$$

$\boxed{}$, $\boxed{8\sqrt{2}}$, $\boxed{-7\sqrt{2}}$, $\boxed{t=\frac{1}{4}}$



Question 43 (*+)**

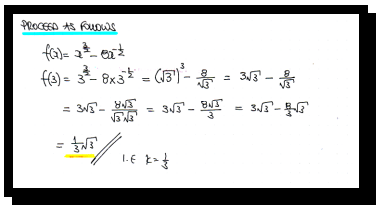
$$f(x) \equiv x^{\frac{3}{2}} - 8x^{-\frac{1}{2}}, \ x > 0$$

Show clearly that

$$f(3) = k\sqrt{3},$$

where k is a constant.

$\boxed{}$, proof



Question 44 **(***+)**

Write each of the following surd expressions as simple as possible.

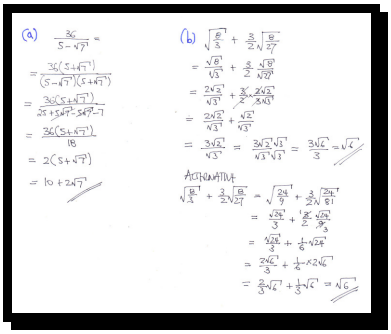
a) $\dfrac{36}{5-\sqrt{7}}$

Give the answer in the form $a+b\sqrt{7}$, where a and b are integers.

b) $\sqrt{\dfrac{8}{3}}+\dfrac{3}{2}\sqrt{\dfrac{8}{27}}$.

Give the answer in the form $\sqrt{k}$, where k is an integer.

$\boxed{}$, $\boxed{10+2\sqrt{7}}$, $\boxed{\sqrt{6}}$



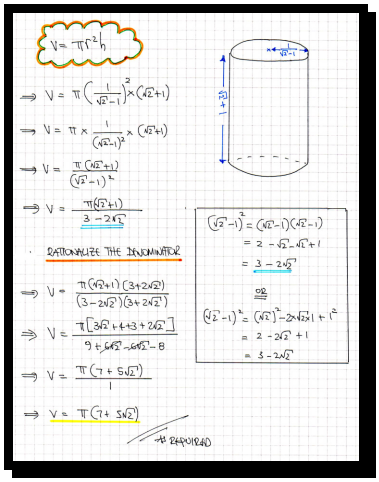
Question 45 (*+)**

A cylinder has a radius of $\left(\dfrac{1}{\sqrt{2}-1}\right)$ cm and a height of $\left(\sqrt{2}+1\right)$ cm.

Show, by detailed working, that the volume of this cylinder is exactly

$$\pi\left(7+5\sqrt{2}\right)\text{cm}^3 \ .$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 46 (*+)**

a) Solve the equation

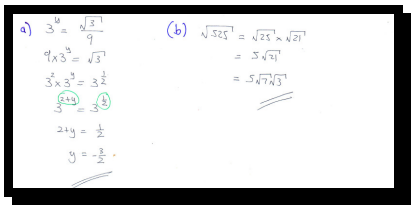
$$3^{y}=\frac{\sqrt{3}}{9}.$$

b) Express

$$\sqrt{525},$$

in the form $a\sqrt{b}\sqrt{c}$, where a, b and c are prime numbers.

$\square$, $\boxed{y=-\frac{3}{2}}$, $\boxed{5\sqrt{3}\sqrt{7}}$



Question 47 (*+)**

a) If x is a real number solve the following indicial equation

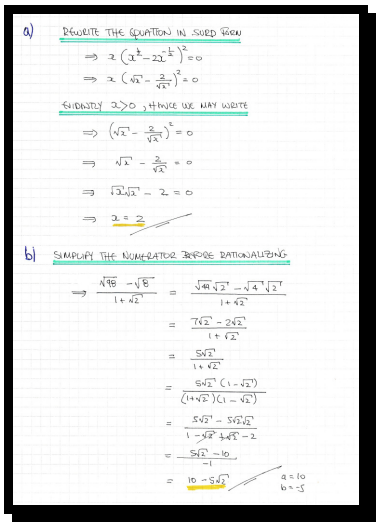
$$x\left(x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)^2=0\,.$$

b) Express

$$\frac{\sqrt{98}-\sqrt{8}}{1+\sqrt{2}}\,,$$

in the form $a+b\sqrt{2}$, where a and b are integers.

$\boxed{}$, $\boxed{x=2}$, $\boxed{10-5\sqrt{2}}$

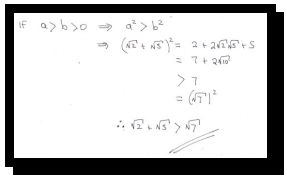


Question 48 (*+)**

Show clearly, without approximating and without using any calculating aid that

$$\sqrt{2}+\sqrt{5}>\sqrt{7}\,.$$

proof



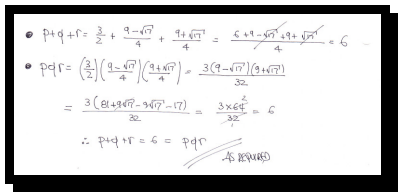
Question 49 (*+)**

$$p=\frac{3}{2},\ q=\frac{9-\sqrt{17}}{4}\quad\text{and}\quad r=\frac{9+\sqrt{17}}{4}.$$

Prove that

$$p+q+r=pqr\,.$$

, proof



Question 50 (*+)**

a) Simplify fully each of the following expressions, writing the final answer as a single simplified surd.

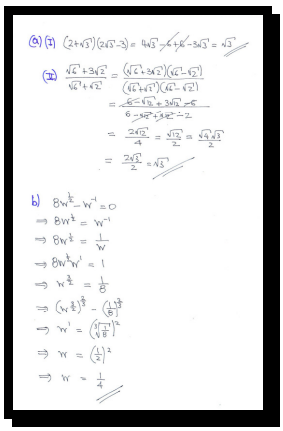
i. $\left(2+\sqrt{3}\right)\left(2\sqrt{3}-3\right)$.

ii. $\dfrac{\sqrt{6}+3\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.

b) Solve the equation

$$8w^{\frac{1}{2}}-w^{-1}=0, \quad w\neq 0.$$

$\square$, $\boxed{\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{w=\frac{1}{4}}$

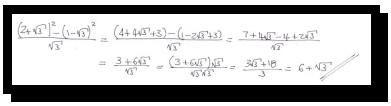


Question 51 (*+)**

$$\frac{\left(2+\sqrt{3}\right)^2-\left(1-\sqrt{3}\right)^2}{\sqrt{3}}.$$

Write the above surd expression in the form $a+b\sqrt{3}$, where a and b are integers.

$\square$, $\boxed{6+\sqrt{3}}$

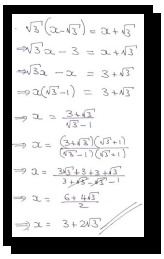


Question 52 (*+)**

$$\sqrt{3}\left(x-\sqrt{3}\right)=x+\sqrt{3}.$$

Solve the above equation giving the answer in the form $a+b\sqrt{3}$, where a and b are integers.

$\square$, $\boxed{x=3+2\sqrt{3}}$

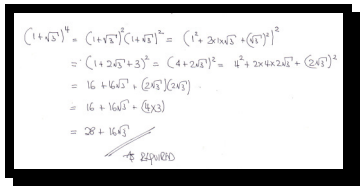


Question 53 (****)

Show clearly that

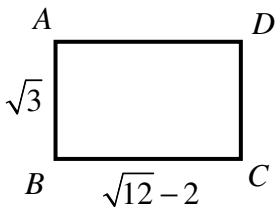
$$\left(1+\sqrt{3}\right)^4 = 28+16\sqrt{3}\,.$$

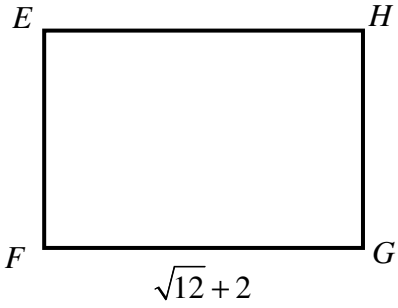
$\boxed{}$, $\boxed{\text{proof}}$



Question 54 (****)

The two rectangles shown in the figure below are similar.



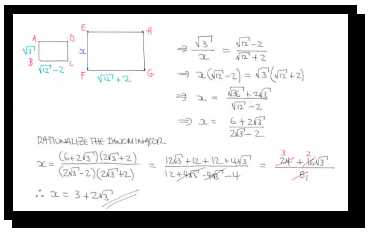


It is further given that in suitable units

$$|AB|=\sqrt{3}\,, \quad |BC|=\sqrt{12}-2 \quad \text{and} \quad |FG|=\sqrt{12}+2\,.$$

Find the exact length of EF.

$\square$, $\boxed{|EF|=3+2\sqrt{3}}$



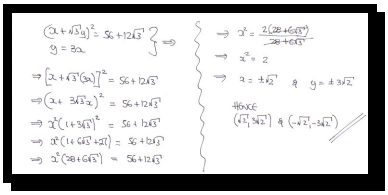
Question 55 (****)

Solve the following system of simultaneous equations

$$\left(x+y\sqrt{3}\right)^2=56+12\sqrt{3}$$

$$y=3x\,.$$

$\square$, $\boxed{\left(\sqrt{2},3\sqrt{2}\right),\left(-\sqrt{2},-3\sqrt{2}\right)}$



Question 56 (**+)**

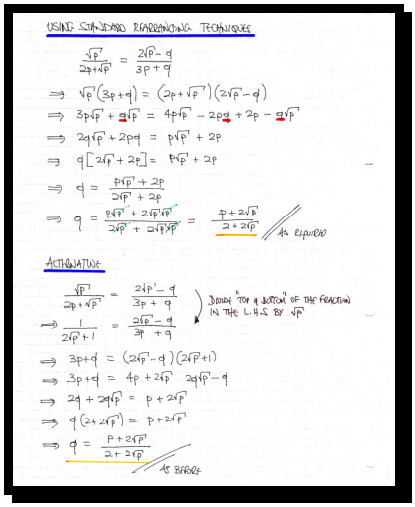
The positive constants p and q satisfy the following equation

$$\frac{\sqrt{p}}{2p+\sqrt{p}}=\frac{2\sqrt{p}-q}{3p+q}.$$

Show by a detailed method that

$$q=\frac{p+2\sqrt{p}}{2+2\sqrt{p}}.$$

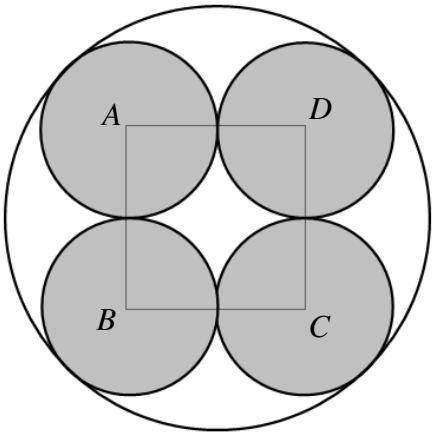
[ ], [proof]



Question 57 (**+)**

Four circles are touching in such a way so that their centres form the corners of a square $ABCD$. These four circles are circumscribed by a larger circle.

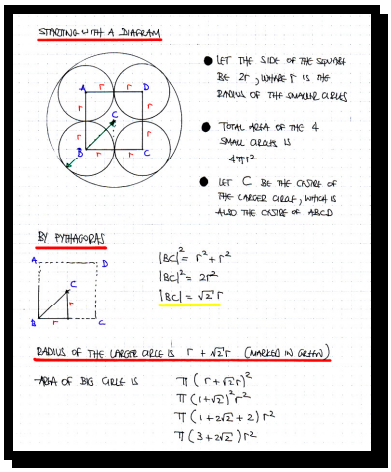
This is shown in the figure below.

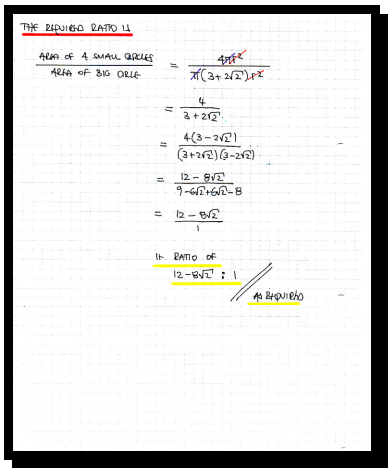


Show that the ratio of the total area of the four smaller circles to the area of the larger circle is given by

$$12-8\sqrt{2}:1$$

, proof



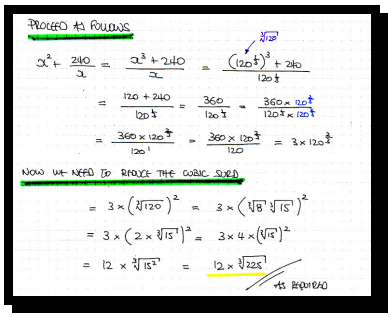


Question 58 (**+)**

If $x=\sqrt[3]{120}$, show clearly that

$$x^2+\frac{240}{x}=12\sqrt[3]{225}.$$

[ ], proof

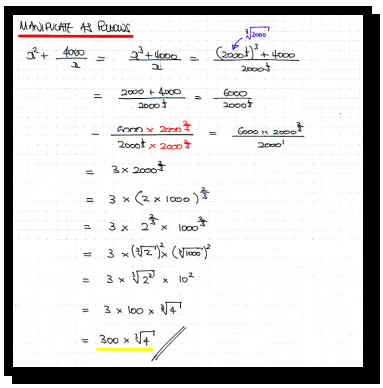


Question 59 (**+)**

If $x=\sqrt[3]{2000}$, show clearly that

$$x^2+\frac{4000}{x}=300\times\sqrt[3]{4}.$$

[ ], proof



Question 60 (**+)**

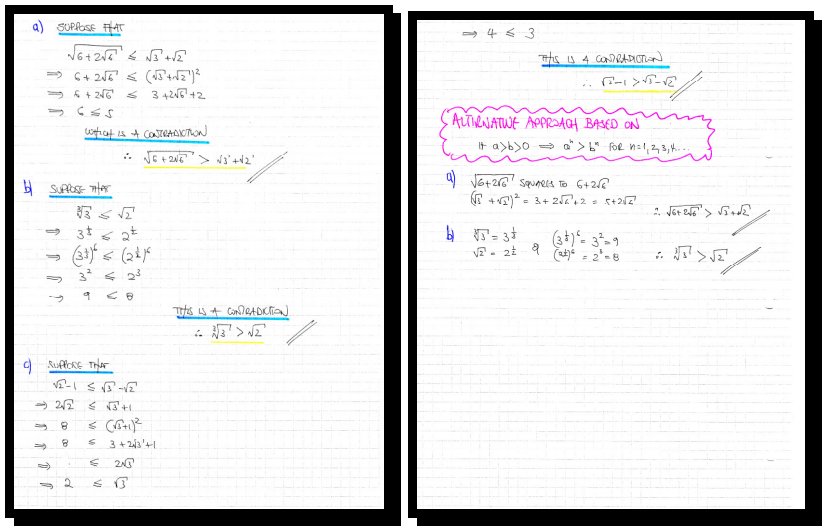
Show clearly, without approximating and without using any calculating aid, that

a) $\sqrt{6+2\sqrt{6}} > \sqrt{3}+\sqrt{2}$.

b) $\sqrt[3]{3} > \sqrt{2}$.

c) $\sqrt{2}-1 > \sqrt{3}-\sqrt{2}$.

$\boxed{}$, $\boxed{\text{proof}}$



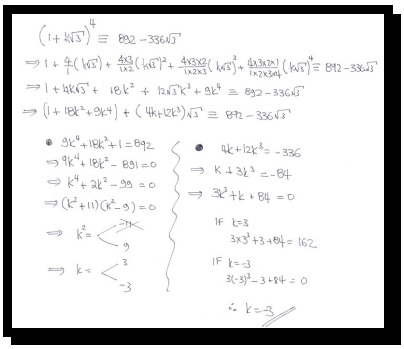
Question 61 (**+)**

It is given that if k is a non zero constant then

$$\left(1+k\sqrt{3}\right)^4 \equiv 892-336\sqrt{3}\,.$$

Determine the value of k.

$$\boxed{k=-3}$$



Question 62 (**+) non calculator**

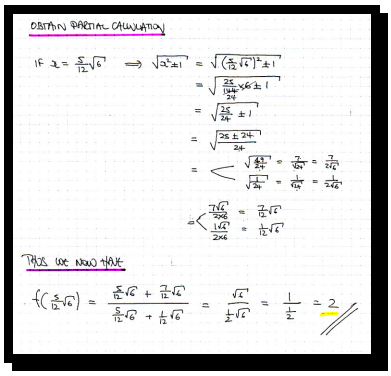
$$f(x)\equiv\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2-1}},\ x\in\mathbb{R},\ |x|\geq 1.$$

Find without the use of a calculator the value of

$$f\left(\frac{5}{12}\sqrt{6}\right).$$

Detailed workings must be shown in this question.

$\square$, $\boxed{f\left(\frac{5}{12}\sqrt{6}\right)=2}$

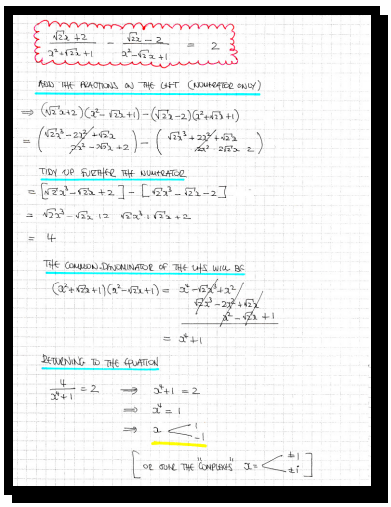


Question 63 **(****+)**

Solve the following equation.

$$\frac{2+\sqrt{2}\,x}{x^2+\sqrt{2}\,x+1}+\frac{2-\sqrt{2}\,x}{x^2-\sqrt{2}\,x+1}=2\,,\quad x\in\mathbb{R}\,.$$

$\boxed{}$, $\boxed{x=\pm 1}$



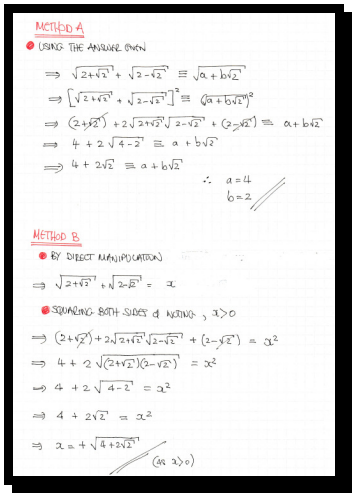
Question 64 (**+)**

Show that $\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}$ can be expressed in the form

$$\sqrt{a+b\sqrt{2}}\,,$$

where a and b are integers to be found.

$\square$, $\boxed{\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}=\sqrt{4+2\sqrt{2}}}$



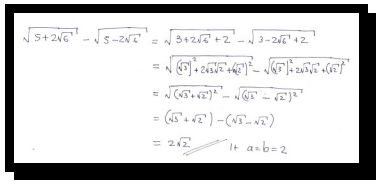
Question 65 (*****)

Show clearly without the use of any calculating aid that

$$\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}=a\sqrt{b}\,,$$

where a and b are integers to be found.

$\boxed{a=b=2}$



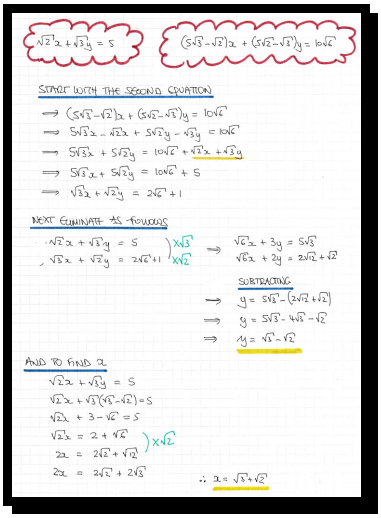
Question 66 (*****)

Determine, in exact simplified surd form, the solution pair (a,b) of the following simultaneous equations.

$$\sqrt{2}\,x+\sqrt{3}\,y=5 \qquad \text{and} \qquad \left(5\sqrt{3}-\sqrt{2}\right)x+\left(5\sqrt{2}-\sqrt{3}\right)y=10\sqrt{6}\,.$$

Detailed workings must be shown in this question.

$\boxed{}$, $\boxed{x=\sqrt{3}+\sqrt{2},\quad y=\sqrt{3}-\sqrt{2}}$



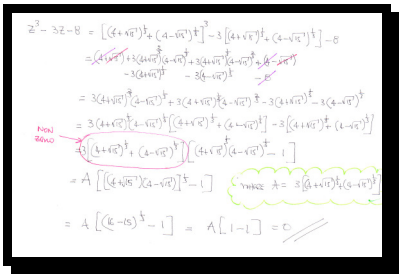
Question 67 **(*****)**

$$z=\sqrt[3]{4+\sqrt{15}}+\sqrt[3]{4-\sqrt{15}}\,.$$

Verify that z is a solution of the equation

$$z^3-3z-8=0\,.$$

proof

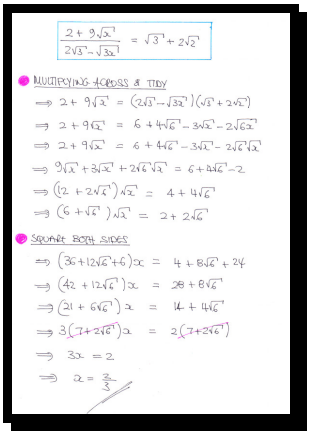


Question 68 **(*****)**

Find the rational solution of the following equation

$$\frac{2+9\sqrt{x}}{2\sqrt{3}-\sqrt{3x}}=\sqrt{3}+2\sqrt{2}\,,\ x\in\mathbb{Q}\,.$$

, $x=\frac{2}{3}$



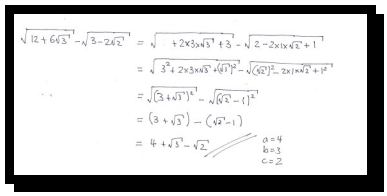
Question 69 (*****)

Show clearly without the use of any calculating aid that

$$\sqrt{12+6\sqrt{3}}-\sqrt{3-2\sqrt{2}}=a+\sqrt{b}-\sqrt{c}\,,$$

where a, b and c are integers to be found.

$$\boxed{a=4,\ \ b=3,\ \ c=2}$$



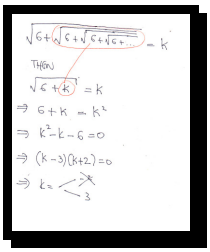
Question 70 (*****)

Show clearly without the use of any calculating aid that

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}=k\,,$$

where k is an integer to be found.

$$\boxed{k=3}$$



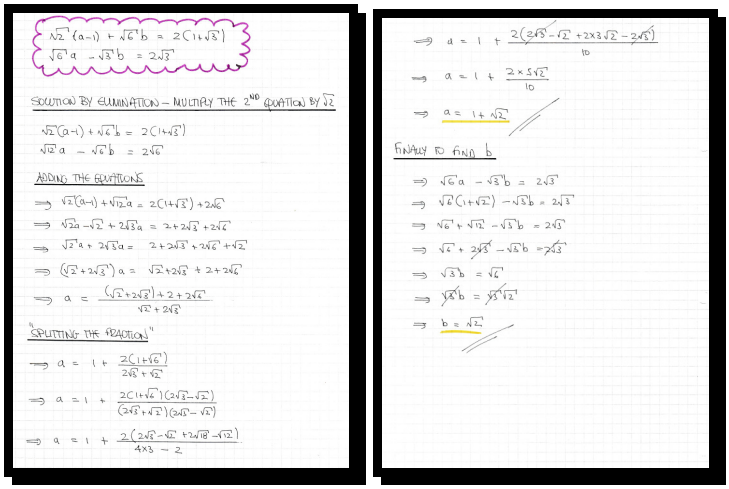
Question 71 (*****)

Determine, in exact simplified surd form, the solution pair (a,b) of the following simultaneous equations.

$$\sqrt{2}\,(a-1)+\sqrt{6}\,b=2\left(1+\sqrt{3}\right) \quad \text{and} \quad \sqrt{6}\,a-\sqrt{3}\,b=2\sqrt{3}\,.$$

Detailed workings must be shown in this question.

$\boxed{}$, $\boxed{a=1+\sqrt{2},\quad b=\sqrt{2}}$



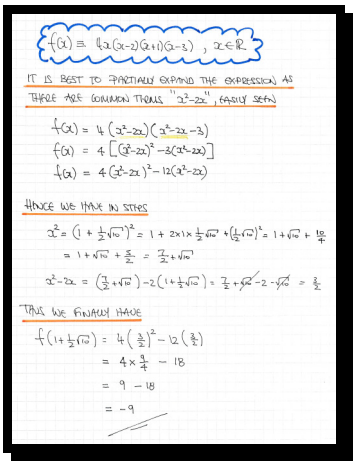
Question 72 (**+) non calculator**

$$f(x)\equiv 4x(x-2)(x+1)(x-3),\quad x\in\mathbb{R}.$$

Evaluate $f\left(1+\frac{1}{2}\sqrt{10}\right)$.

You must show detailed workings in this question.

$\boxed{}$, $\boxed{f\left(1+\frac{1}{2}\sqrt{10}\right)=-9}$



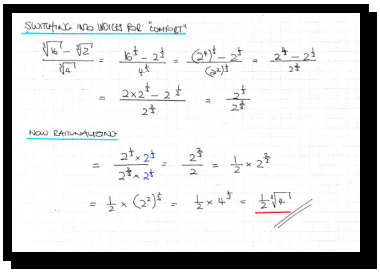
Question 73 (**+)**

Show with a detailed method that

$$\frac{\sqrt[3]{16}-\sqrt[3]{2}}{\sqrt[3]{4}} = k\sqrt[3]{4}$$

where k is a constant to be found.

$\boxed{\phantom{k=\frac{1}{2}}}$, $\boxed{k=\frac{1}{2}}$

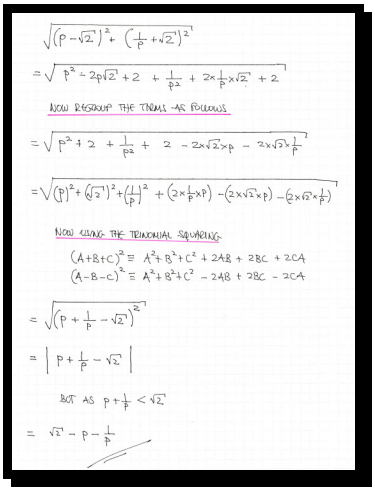


Question 74 (*****)

$$f(p)\equiv\left(p-\sqrt{2}\right)^2+\left(\frac{1}{p}-\sqrt{2}\right)^2,\ p\in\mathbb{R},\ p\neq 0.$$

Given that $p+\frac{1}{p}<\sqrt{2}$, find $\sqrt{f(p)}$ in its simplest form.

$\boxed{}$, $\boxed{\text{proof}}$



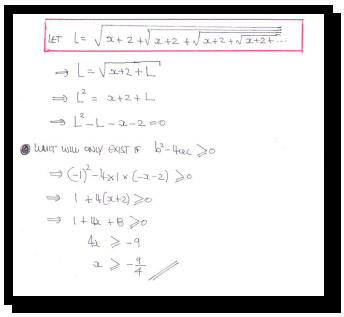
Question 75 **(*****)**

$$\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+...}}}}} \ ,$$

It is given that the above nested radical converges to a limit L, $L \in \mathbb{R}$.

Determine the range of possible values of x.

$\square$, $\boxed{x \geq -\frac{9}{4}}$

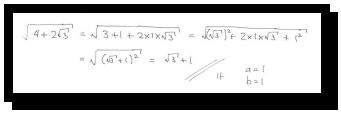


Question 76 **(*****)**

Show clearly that

$$\sqrt{4+2\sqrt{3}} = 1+\sqrt{3} \ .$$

$\boxed{\text{proof}}$



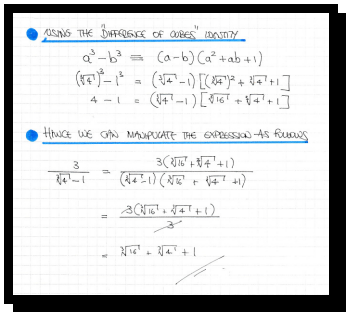
Question 77 **(*****)**

Show that

$$\frac{3}{\sqrt[3]{4}-1}$$

can be written in the form $\sqrt[3]{a}+\sqrt[3]{b}+1$, where a and b are integers to be found.

$\square$, $\boxed{\sqrt[3]{16}+\sqrt[3]{4}+1}$



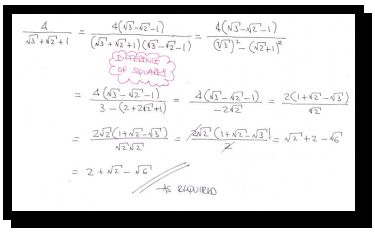
Question 78 **(*****)**

Show clearly that

$$\frac{4}{\sqrt{3}+\sqrt{2}+1}=2+\sqrt{2}-\sqrt{6}\,.$$

You may not use verification in this question

$\square$, proof



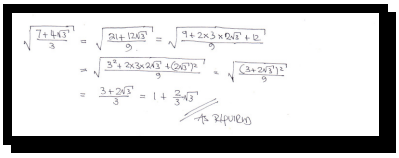
Question 79 **(*****)**

Show clearly that

$$\sqrt{\frac{1+4\sqrt{3}}{3}} = a + b\sqrt{3},$$

where a and b are constants to be found.

$$\boxed{a=1}, \quad \boxed{b=\frac{2}{3}}$$



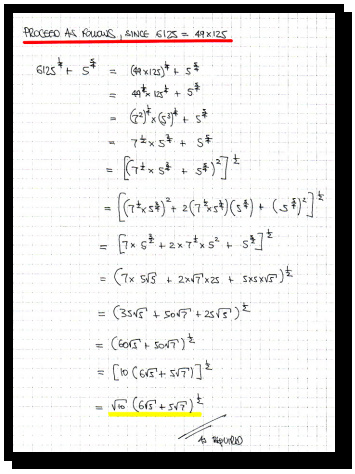
Question 80 **(*****)**

Show that

$$6125^{\frac{1}{4}} + 5^{\frac{5}{4}}$$

can be written in the form $\sqrt{10}\left(a\sqrt{5}+b\sqrt{7}\right)^{\frac{1}{2}}$, where a and b are positive integers to be found.

$$\boxed{}, \quad \boxed{\sqrt{10}\left(6\sqrt{5}+5\sqrt{7}\right)^{\frac{1}{2}}}$$

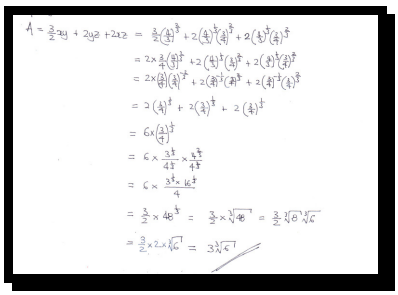


Question 81 (*****)

$$A=\frac{3}{2}xy+2yz+2xz\,.$$

Given that $x=\left(\frac{4}{3}\right)^{\frac{1}{3}}$, $y=\left(\frac{4}{3}\right)^{\frac{1}{3}}$ and $z=\left(\frac{3}{4}\right)^{\frac{2}{3}}$, show clearly that $A=3\sqrt[3]{6}$

$\boxed{}$, $\boxed{\text{proof}}$

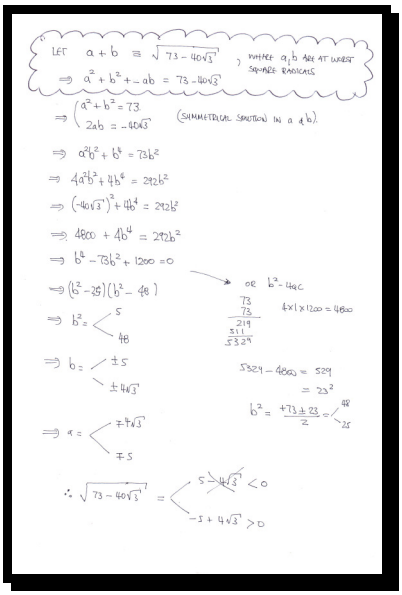


Question 82 (*****)

Find an exact simplified value for

$$\sqrt{73-40\sqrt{3}}\,.$$

$\boxed{-5+4\sqrt{3}}$



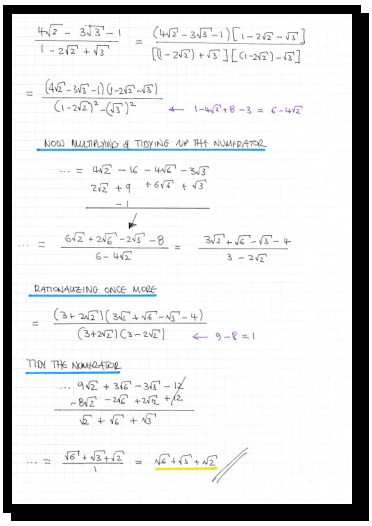
Question 83 **(*****)**

Rationalize the denominator of the following surd.

$$\frac{4}{\sqrt{3}+\sqrt{2}+1}=2+\sqrt{2}-\sqrt{6}\,.$$

Show detailed workings in this question.

, proof



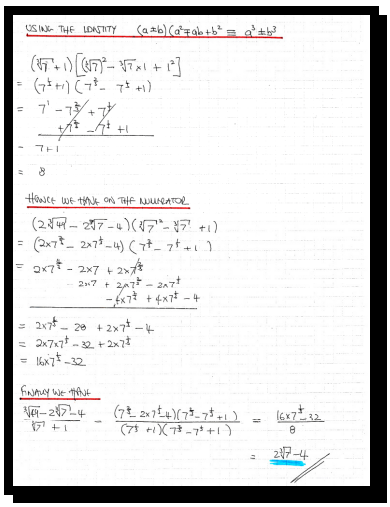
Question 84 **(*****)**

Show that

$$\frac{\sqrt[3]{49}-2\sqrt[3]{7}-4}{\sqrt[3]{7}+1}$$

can be written in the form $a\sqrt[3]{7}+b$, where a and b are integers to be found.

$\square$, $\boxed{2\sqrt[3]{7}-4}$



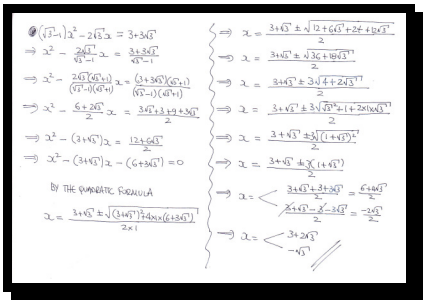
Question 85 (*****)

Solve the following quadratic equation

$$\left(\sqrt{3}-1\right)x^2-2\sqrt{3}x=3+3\sqrt{3}\,.$$

Give one of the roots in the form $p+q\sqrt{3}$ and the other root in the form $r\sqrt{3}$, where p, q and r are integers.

$\square$, $\boxed{x=-\sqrt{3},\quad x=3+2\sqrt{3}}$



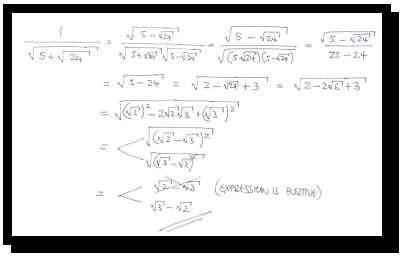
Question 86 (*****)

Express

$$\frac{1}{\sqrt{5+\sqrt{24}}},$$

in the form $\sqrt{p}-\sqrt{q}$, where p and q are integers.

$\boxed{\sqrt{3}-\sqrt{2}}$

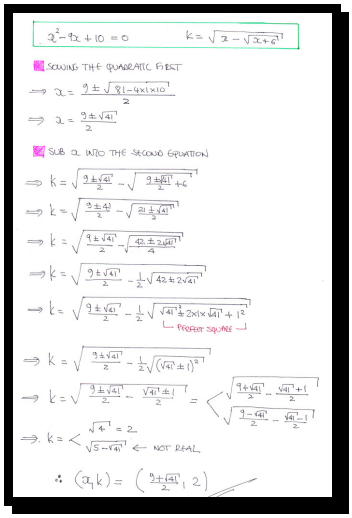


Question 87 (*****)

Solve the following simultaneous equations, to find in exact form where appropriate, the value or values of x and k.

$$x^2-9x+10=0 \quad \text{and} \quad k=\sqrt{x-\sqrt{x+6}}$$

$\square$, $\boxed{x=\frac{1}{2}\left(9+\sqrt{41}\right),\quad k=2}$

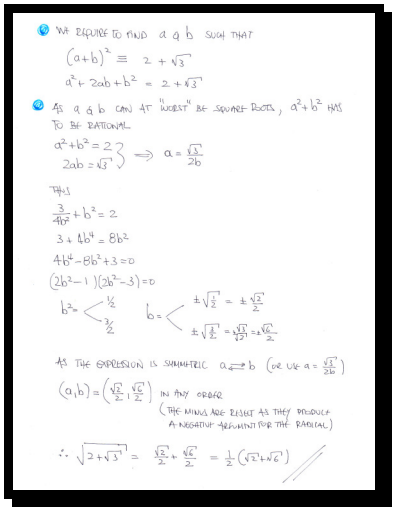


Question 88 (*****)

Find in exact simplified form

$$\sqrt{2+\sqrt{3}}\ .$$

$$\boxed{\frac{1}{2}\left(\sqrt{2}+\sqrt{6}\right)}$$



Question 89 **(*****)**

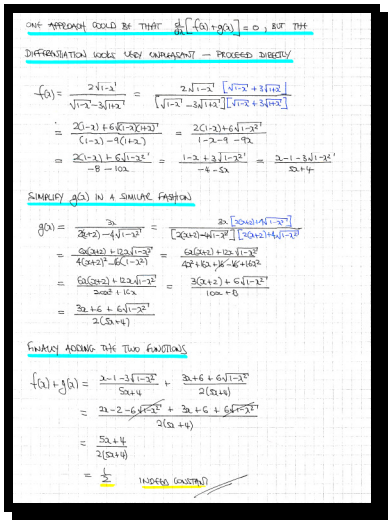
The functions f and f are defined as

$$f(x) \equiv \frac{2\sqrt{1-x}}{\sqrt{1-x}-3\sqrt{1+x}}, \quad -1 \le x \le 1.$$

$$g(x) \equiv \frac{3x}{2(x+2)-4\sqrt{1-x^2}}, \quad -1 \le x \le 1.$$

Show that $f(x)+g(x)$ is a **constant** function.

$\boxed{}$, $\boxed{\text{proof}}$

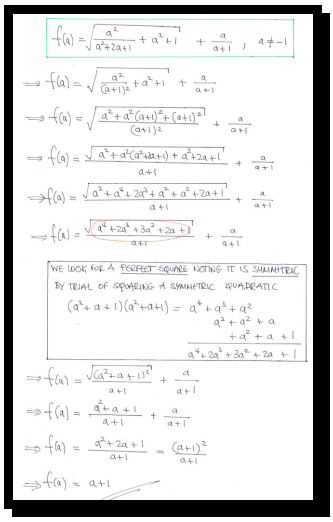


Question 90 (*****)

$$f(a)\equiv\frac{a}{a+1}+\sqrt{1+a^2+\frac{a^2}{a^2+2a+1}}\ ,\ a\in\mathbb{R}\ ,\ a\neq-1\ .$$

Show that $f(a)$ can be simplified to a linear polynomial in a.

$\boxed{}$, $\boxed{f(a)=a+1}$



Question 91 (*****)

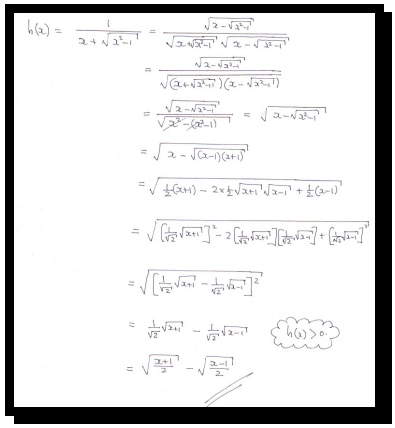
$$h(x)\equiv\frac{1}{\sqrt{x+\sqrt{x^2-1}}},\ x\in\mathbb{R},\ x\geq 1.$$

Show that $h(x)$ can be expressed in the form

$$\sqrt{f(x)}-\sqrt{g(x)},$$

where $f(x)$ and $g(x)$ are linear functions to be found.

$\square$, $\boxed{h(x)=\sqrt{\frac{x+1}{2}}-\sqrt{\frac{x-1}{2}}}$

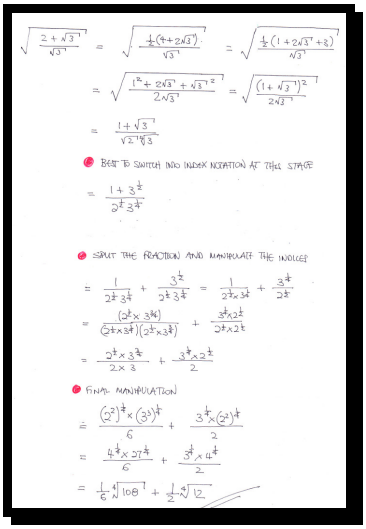


Question 92 (*****)

Using a detailed method show that

$$\sqrt{\frac{2+\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt[4]{108}}{6}+\frac{\sqrt[4]{12}}{2}.$$

[ ], [proof]

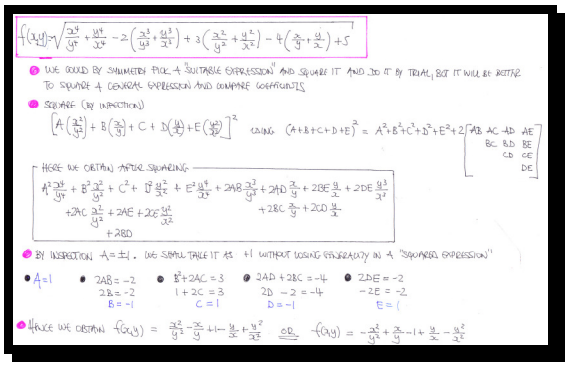


Question 93 (*****)

$$f(x,y)\equiv\sqrt{\frac{x^4}{y^4}+\frac{y^4}{x^4}-2\left(\frac{x^3}{y^3}+\frac{y^3}{x^3}\right)+3\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)-4\left(\frac{x}{y}+\frac{y}{x}\right)+5}\,.$$

Simplify $f(x,y)$ in a form not involving square roots.

$\square$, $\boxed{f(x,y)\equiv\pm\left(\frac{x^2}{y^2}-\frac{x}{y}+1-\frac{y}{x}+\frac{y^2}{x^2}\right)}$



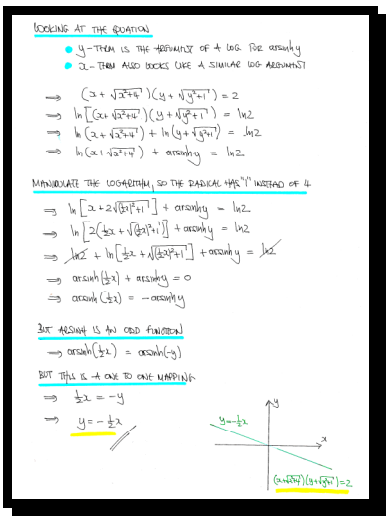
Question 94 **(*****)**

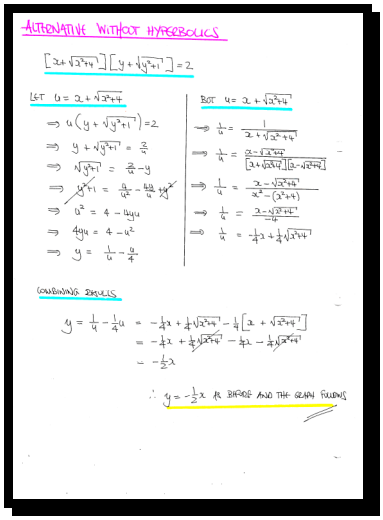
Sketch the graph of

$$\left[x+\sqrt{x^2+4}\right]\left[y+\sqrt{y^2+1}\right]=2, \quad x\in(-\infty,\infty),\ y\in(-\infty,\infty)$$

You must show a detailed method in this question

, proof



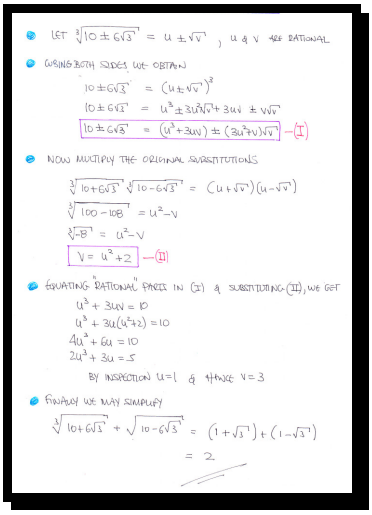


Question 95 (*****)

By using the substitution $\sqrt[3]{10\pm 6\sqrt{3}} = u \pm \sqrt{v}$, where $u \in \mathbb{Q}$, $v \in \mathbb{Q}$, simplify fully the following cubic radical expression.

$$\sqrt[3]{10+6\sqrt{3}} \ + \ \sqrt[3]{10-6\sqrt{3}} \ .$$

$\boxed{}$, $\boxed{2}$

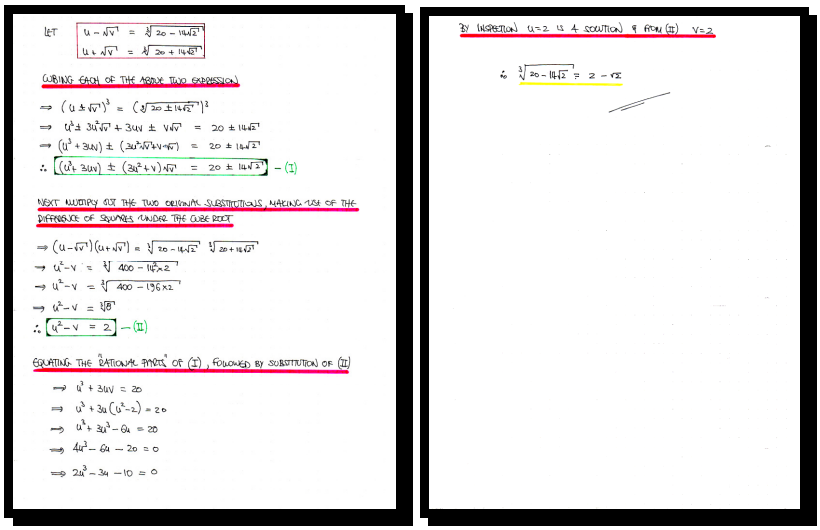


Question 96 (*****)

By using the substitution $\sqrt[3]{20\pm 14\sqrt{2}} = u \pm \sqrt{v}$, where $u \in \mathbb{Q}$, $v \in \mathbb{Q}$, simplify fully the following cubic radical expression.

$$\sqrt[3]{20+14\sqrt{2}} \, .$$

$\boxed{}$, $\boxed{2-\sqrt{2}}$



LET $u - \sqrt{v} = \sqrt[3]{20 - 14\sqrt{2}}$

$u + \sqrt{v} = \sqrt[3]{20 + 14\sqrt{2}}$

CUBING EACH OF THE ABOVE TWO EXPRESSIONS

$\Rightarrow (u \pm \sqrt{v})^3 = (\sqrt[3]{20 \pm 14\sqrt{2}})^3$

$\Rightarrow u^3 \pm 3u^2\sqrt{v} + 3uv \pm v\sqrt{v} = 20 \pm 14\sqrt{2}$

$\Rightarrow (u^3 + 3uv) \pm (3u^2\sqrt{v} + v\sqrt{v}) = 20 \pm 14\sqrt{2}$

$\therefore \boxed{(u^3 + 3uv) \pm (3u^2 + v)\sqrt{v} = 20 \pm 14\sqrt{2}}$ – (I)

NEXT MULTIPLY OUT THE TWO ORIGINAL SUBSTITUTIONS, MAKING USE OF THE DIFFERENCE OF SQUARES UNDER THE CUBE ROOT

$\Rightarrow (u - \sqrt{v})(u + \sqrt{v}) = \sqrt[3]{20 - 14\sqrt{2}} \sqrt[3]{20 + 14\sqrt{2}}$

$\rightarrow u^2 - v = \sqrt[3]{400 - 196 \times 2}$

$\rightarrow u^2 - v = \sqrt[3]{400 - 196 \times 2}$

$\Rightarrow u^2 - v = \sqrt[3]{8}$

$\therefore \boxed{u^2 - v = 2}$ – (II)

EQUATING THE RATIONAL PARTS OF (I), FOLLOWED BY SUBSTITUTION OF (II)

$\Rightarrow u^3 + 3uv = 20$

$\Rightarrow u^3 + 3u(u^2 - 2) = 20$

$\Rightarrow u^3 + 3u^3 - 6u = 20$

$\Rightarrow 4u^3 - 6u - 20 = 0$

$\Rightarrow 2u^3 - 3u - 10 = 0$

BY INSPECTION $u = 2$ IS A SOLUTION & FROM (II) $v = 2$

$\therefore \sqrt[3]{20 - 14\sqrt{2}} = 2 - \sqrt{2}$

Question 97 **(*****)**

$$f(x) \equiv x^2 + \frac{2x}{2+\sqrt{3}} - 1, \ x \in \mathbb{R}.$$

Factorize $f(x)$ into a product of 2 simple linear factors.

$\square$, $\boxed{\left(x+\sqrt{6}-\sqrt{3}-\sqrt{2}+2\right)\left(x-\sqrt{6}-\sqrt{3}+\sqrt{2}+2\right)}$

