RECURRENCE RELATIONS

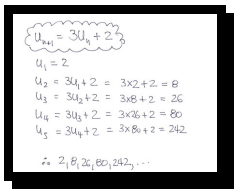
Question 1 (**)

A sequence u_1, u_2, u_3, u_4, … is given by the recurrence relation

$$u_{n+1} = 3u_n + 2, \quad u_1 = 2.$$

Find the value of u_2, u_3, u_4 and u_5.

$$\boxed{u_2 = 8}, \ \boxed{u_3 = 26}, \ \boxed{u_4 = 80}, \ \boxed{u_5 = 242}$$



Question 2 (**)

A sequence y_1, y_2, y_3, y_4, … is given by

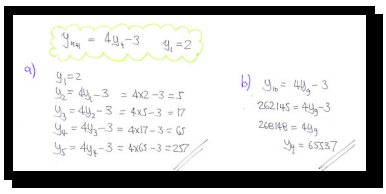
$$y_{n+1} = 4y_n - 3, \quad y_1 = 2.$$

a) Find the value of y_2, y_3, y_4 and y_5.

It is further given that $y_{10} = 262145$.

b) Calculate the value of y_9.

$$\boxed{y_2 = 5}, \ \boxed{y_3 = 17}, \ \boxed{y_4 = 65}, \ \boxed{y_5 = 257}, \ \boxed{y_9 = 65537}$$



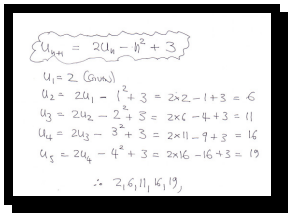
Question 3 (**)

A sequence u_1, u_2, u_3, u_4, … is given by the recurrence relation

$$u_{n+1} = 2u_n - n^2 + 3, \quad u_1 = 2.$$

Find the value of u_2, u_3, u_4 and u_5.

$$\boxed{u_2 = 6}, \ \boxed{u_3 = 11}, \ \boxed{u_4 = 16}, \ \boxed{u_5 = 19}$$



Question 4 (**)

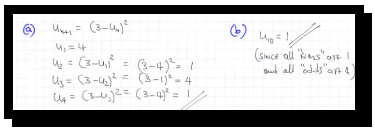
A sequence u_1, u_2, u_3, u_4, … is given by

$$u_{n+1} = (3 - u_n)^2, \quad u_1 = 4.$$

a) Find the value of u_2, u_3 and u_4.

b) State the value of u_{10}.

$$\boxed{}, \ \boxed{u_2 = 1}, \ \boxed{u_3 = 4}, \ \boxed{u_4 = 1}, \ \boxed{u_{10} = 1}$$



Question 5 (**)

A sequence b_1, b_2, b_3, b_4, … is given by

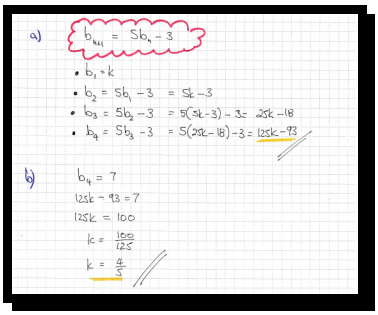
$$b_{n+1}=5b_n-3, \quad b_1=k,$$

where k is a non zero constant.

a) Find the value of b_4 in terms of k.

b) Given that $b_4=7$, determine the value of k.

$\square$, $\boxed{b_4=125k-93}$, $\boxed{k=\frac{4}{5}}$



Question 6 (**)

A recurrence relation is defined for $n \geq 1$ by

$$a_{n+1} = 3a_n + 4 , \quad a_1 = k ,$$

where k is a non zero constant.

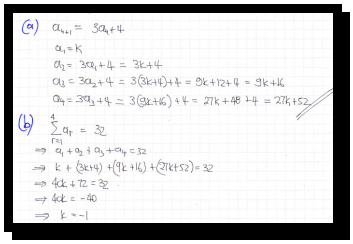
a) Find the value of a_4 in terms of k.

It is further given that

$$\sum_{r=1}^{4} a_r = 32 .$$

b) Determine the value of k.

$\boxed{a_4 = 27k + 52}$, $\boxed{k = -1}$



Question 7 (**)

A sequence u_1, u_2, u_3, u_4, … is given by

$$u_{n+1} = 3u_n - 9\,, \quad u_1 = k\,,$$

where k is a non zero constant.

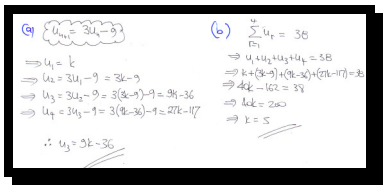
a) Find the value of u_3 in terms of k.

It is further given that

$$\sum_{r=1}^{4} u_r = 38\,.$$

b) Find the value of k.

$\boxed{u_3 = 9k - 36}$, $\boxed{k = 5}$



Question 8 (**)

A recurrence relation is defined for $n \geq 1$ by

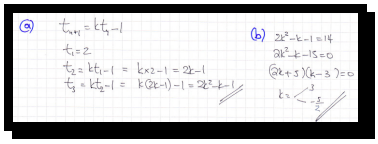
$$t_{n+1} = kt_n - 1, \quad t_1 = 2,$$

where k is a non zero constant.

a) Find the value of t_3 in terms of k.

b) Given that $t_3 = 14$ find the possible values of k.

$\boxed{t_3 = 2k^2 - k - 1}$, $\boxed{k = -\frac{5}{2}, 3}$



Question 9 (**)

A sequence u_1, u_2, u_3, u_4, … is given by

$$u_{n+1} = ku_n + 5, \quad u_1 = 2,$$

where k is a non zero constant.

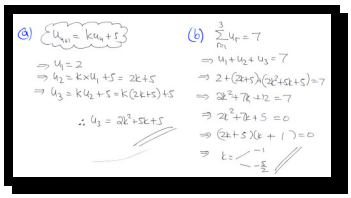
a) Find the value of u_3 in terms of k.

It is further given that

$$\sum_{r=1}^{3} u_r = 7.$$

b) Find the possible values of k.

$\boxed{u_3 = 2k^2 + 5k + 5}$, $\boxed{k = -1, -\frac{5}{2}}$



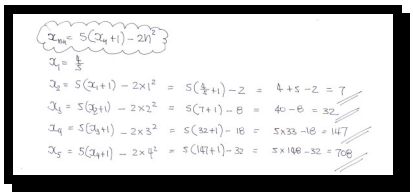
Question 10 (**)

A sequence $x_1,\ x_2,\ x_3,\ x_4,\ \ldots$ is given by the recurrence formula

$$x_{n+1}=5\left(x_n+1\right)-2n^2,\quad x_1=\frac{4}{5}.$$

Calculate the value of $x_2,\ x_3,\ x_4$, and x_5.

$$\boxed{x_2=7},\ \boxed{x_3=32},\ \boxed{x_4=147},\ \boxed{x_5=708}$$



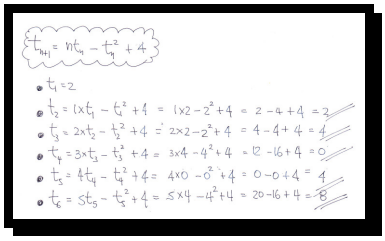
Question 11 (**)

A sequence $t_1,\ t_2,\ t_3,\ t_4,\ t_5,\ t_6\ldots$ is given by the recurrence formula

$$t_{n+1}=nt_n-\left(t_n\right)^2+4,\quad t_1=2.$$

Find the value of $t_2,\ t_3,\ t_4,\ t_5$ and t_6.

$$\boxed{t_2=2},\ \boxed{t_3=4},\ \boxed{t_4=0},\ \boxed{t_5=4},\ \boxed{t_6=8}$$



Question 12 (+)**

A recurrence relation is defined for $n \geq 1$ by

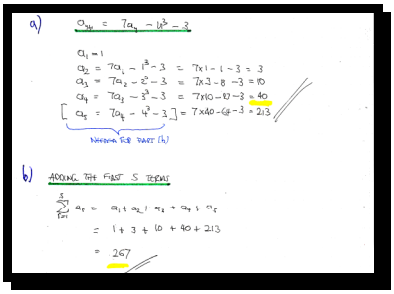
$$a_{n+1} = 7a_n - n^3 - 3, \quad a_1 = 1.$$

a) Find the value of a_4.

b) Evaluate the sum

$$\sum_{r=1}^{5} a_r .$$

$\boxed{}$, $\boxed{a_4 = 40}$, $\boxed{267}$



Question 13 (+)**

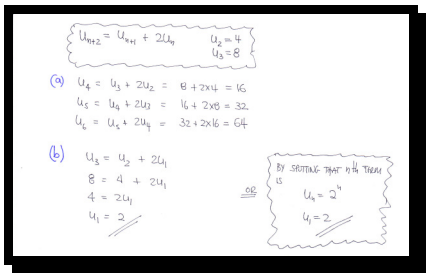
A sequence $u_1, u_2, u_3, u_4, u_5, u_6, \ldots$ is given by

$$u_{n+2} = u_{n+1} + 2u_n, \quad u_2 = 4, \quad u_3 = 8.$$

a) Find the value of u_4, u_5 and u_6.

b) Determine the value of u_1.

$$\boxed{u_4 = 16}, \ \boxed{u_5 = 32}, \ \boxed{u_6 = 64}, \ \boxed{u_1 = 2}$$



Question 14 (+)**

A sequence of numbers is given by the recurrence relation

$$u_{n+1}=\frac{1}{1-u_n},\quad n\geq 1,\quad u_1=2.$$

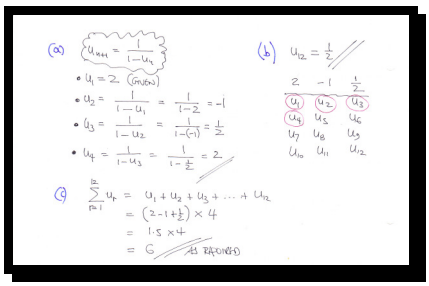
a) Find the value of u_2, u_3 and u_4.

b) State the value of u_{12}.

c) Show clearly that

$$\sum_{r=1}^{12} u_r = 6.$$

$\square$, $\boxed{u_2=-1,\ u_3=\frac{1}{2},\ u_4=2}$, $\boxed{u_{12}=\frac{1}{2}}$



Question 15 (+)**

A sequence of numbers is given by the recurrence relation

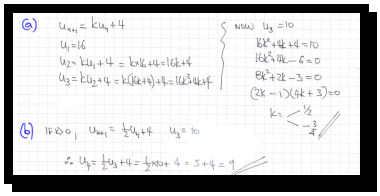
$$u_{n+1} = ku_n + 4, \quad n \geq 1, \quad u_1 = 16,$$

where k is a non zero constant.

a) If $u_3 = 10$, find the possible values of k.

b) Determine the value of u_4, given that $k > 0$.

$$\boxed{k = \frac{1}{2}, -\frac{3}{4}}, \quad \boxed{u_4 = 9}$$



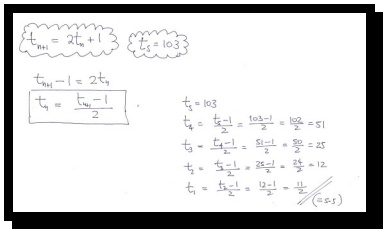
Question 16 (+)**

A sequence $t_1, t_2, t_3, t_4, t_5, \ldots$ is given by

$$t_{n+1} = 2t_n + 1, \quad t_5 = 103.$$

Find the value of t_1.

$$\boxed{t_1 = \frac{11}{2}}$$



Question 17 (+)**

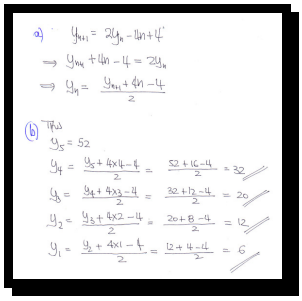
A sequence $y_1,\ y_2,\ y_3,\ y_4,\ \ldots$ is given by the recurrence formula

$$y_{n+1}=2y_n-4n+4,\quad y_5=52.$$

a) Make y_n the subject of the above recurrence formula.

b) Hence determine the value of $y_4,\ y_3,\ y_2$ and y_1.

$$\boxed{y_n=\frac{y_{n+1}+4n-4}{2}},\ \boxed{y_4=32},\ \boxed{y_3=20},\ \boxed{y_2=12},\ \boxed{y_1=6}$$



Question 18 (***)

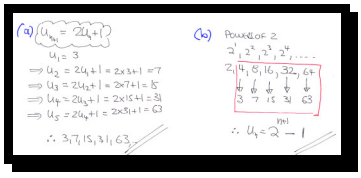
A sequence is defined for $n\geq1$ by the recurrence relation

$$u_{n+1}=2u_n+1,\quad u_1=3.$$

a) Find the first five terms of the sequence.

b) By considering the first few powers of 2, write down an expression for the n^{th} term of the sequence.

$$\boxed{3,\ 7,\ 15,\ 31,\ 63,\ldots},\ \boxed{u_n=2^{n+1}-1}$$



Question 19 (***)

A sequence of numbers is given by the recurrence relation

$$a_{n+1}=5-\frac{18}{4+a_n},\quad n\geq 1,\quad a_2=0.$$

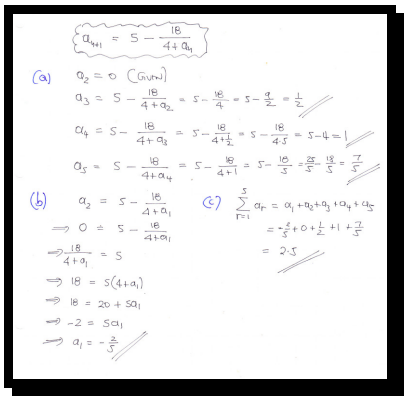
a) Find the value of a_3, a_4 and a_5.

b) Determine the value of a_1.

c) Calculate the value of

$$\sum_{r=1}^{5} a_r .$$

$\square$, $\boxed{u_3=\frac{1}{2},\quad u_3=1,\quad u_5=\frac{7}{5}}$, $\boxed{u_1=-\frac{2}{5}}$, $\boxed{\sum_{r=1}^{5} u_r=2.5}$



Question 20 (***)

A sequence $x_1,\ x_2,\ x_3,\ x_4,\ \ldots$ is given by

$$x_{n+1}=\frac{k-5x_n}{x_n},\quad x_1=1,\quad k>5,$$

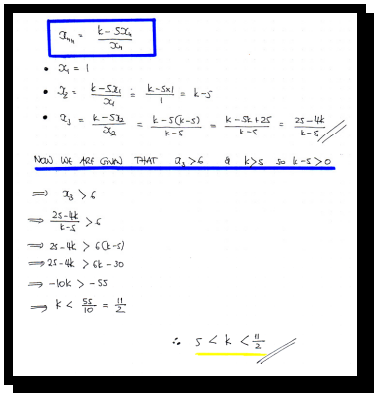
where k is a non zero constant.

a) Determine the value of x_3 in terms of k, giving the final answer as a single simplified fraction.

It is further given that $x_3>6$.

b) Find the range of the values of k.

$\square$, $\boxed{x_3=\dfrac{25-4k}{k-5}}$, $\boxed{5<k<\dfrac{11}{2}}$



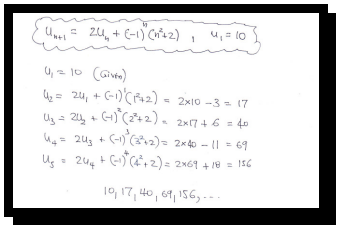
Question 21 (***)

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by the recurrence relation

$$u_{n+1}=2u_n+(-1)^n\left(n^2+2\right),\quad u_1=10.$$

Find the value of $u_2,\ u_3,\ u_4$ and u_5.

$$\boxed{u_2=17},\ \boxed{u_3=40},\ \boxed{u_4=69},\ \boxed{u_5=156}$$



Question 22 (*+)**

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is defined by the recursive relation

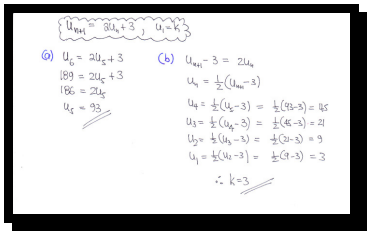
$$u_{n+1}=2u_n+3,\quad u_1=k,$$

where k is a non zero constant.

a) Given that $u_6=189$, find the value of u_5.

b) Determine the value of k.

$$\boxed{u_5=93},\ \boxed{k=3}$$



Question 23 (*+)**

A sequence a_1, a_2, a_3, a_4, … is given by

$$a_{n+1} = p + qa_n,$$

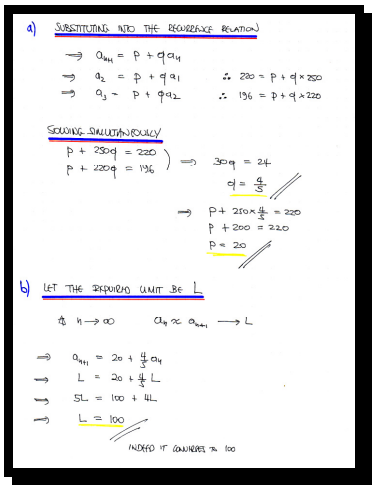
where p and q are non zero constants.

It is given that $a_1 = 250$, $a_2 = 220$ and $a_3 = 196$.

a) Determine the value of p and the value of q.

b) Show clearly that the sequence converges to 100.

$\boxed{}$, $\boxed{p = 20}$, $\boxed{q = \frac{4}{5}}$



Question 24 (*+)**

A sequence $x_1,\ x_2,\ x_3,\ x_4,\ldots$ is given by

$$x_{n+1}=\frac{a+2x_n}{x_n},\quad x_1=2,$$

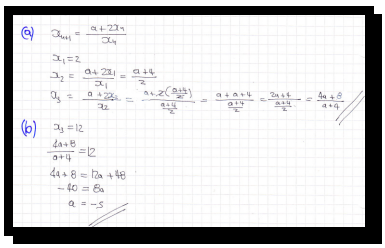
where a is a non zero constant.

a) Find a simplified expression for x_3 in terms of a.

It is given that $x_3=12$.

b) Determine the value of a.

$\square$, $\boxed{x_3=\dfrac{4a+8}{a+4}}$, $\boxed{a=-5}$



Question 25 (*+)**

A recurrence relation is defined for $n \geq 1$ by

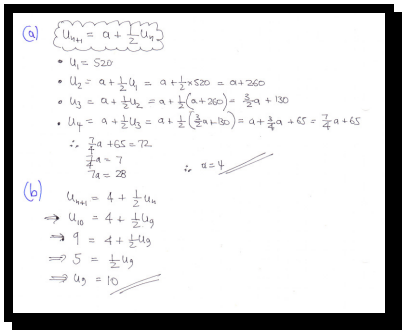
$$u_{n+1} = a + \frac{1}{2}u_n, \quad u_1 = 520,$$

where a is a non zero constant.

a) Given that $u_4 = 72$, find the value of a.

b) Given further that $u_{10} = 9$, find the value of u_9.

$\square$, $\boxed{a = 4}$, $\boxed{u_9 = 10}$



Question 26 (*+)**

A sequence of positive numbers is given by the recurrence relation for $n \geq 1$ by

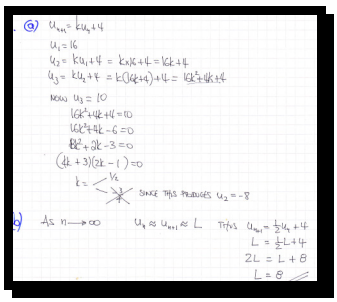
$$u_{n+1} = ku_n + 4, \quad u_1 = 16,$$

where k is a non zero constant.

a) Given that $u_3 = 10$, find the value of k.

b) Given further that the sequence converges to a limit L, use an algebraic method to determine the value of L.

$$\boxed{k = \frac{1}{2}}, \quad \boxed{L = 8}$$



Question 27 (*+)**

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

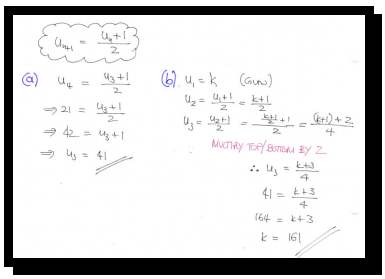
$$u_{n+1}=\frac{u_n+1}{2},\quad u_1=k,$$

where k is a non zero constant.

a) Given that $u_4=21$ find the value of u_3.

b) Determine the value of k.

$\boxed{}$, $\boxed{u_3=41}$, $\boxed{k=161}$



Question 28 (*+)**

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

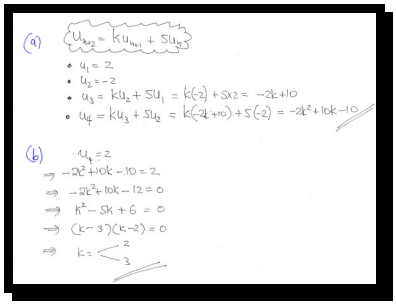
$$u_{n+2}=ku_{n+1}+5u_n,\qquad u_1=2,\qquad u_2=-2,$$

where k is a non zero constant.

a) Find the value of u_4 in terms of k.

b) Given that $u_4=2$, find the possible values of k.

$\boxed{u_4=-2k^2+10k-10}$, $\boxed{k=2,\ 3}$



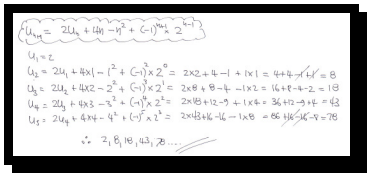
Question 29 (*+)**

A sequence is defined for $n\geq 1$ by the recurrence relation

$$u_{n+1}=2u_n+4n-n^2+(-1)^{n+1}2^{n-1},\quad u_1=2.$$

Find the first five terms of the sequence.

$\boxed{2,\ 8,\ 18,\ 43,\ 78,\ \ldots}$



Question 30 (*+)**

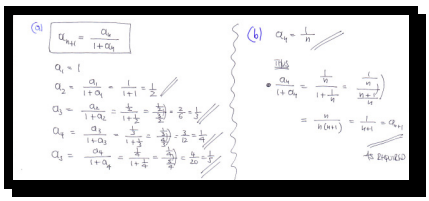
A sequence a_1, a_2, a_3, a_4, … is given by the recurrence formula

$$a_{n+1}=\frac{a_n}{1+a_n},\ a_1=1$$

a) Determine the value of a_2, a_3, a_4 and a_5.

b) State an expression for the n^{th} term of the sequence and verify that it satisfies the above recurrence formula.

$$\boxed{a_2=\frac{1}{2}},\ \boxed{a_3=\frac{1}{3}},\ \boxed{a_4=\frac{1}{4}},\ \boxed{a_5=\frac{1}{5}},\ \boxed{a_n=\frac{1}{n}}$$



Question 31 (*+)**

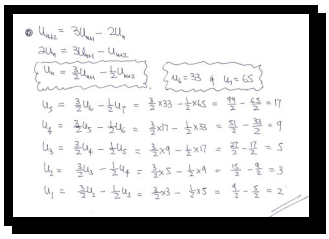
A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is defined by the recursive relation

$$u_{n+2}=3u_{n+1}-2u_n,\quad u_1=k,$$

where k is a non zero constant.

Given that $u_6=33$ and $u_7=65$, determine the value of k.

$$\boxed{k=2}$$



Question 32 (*+)**

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

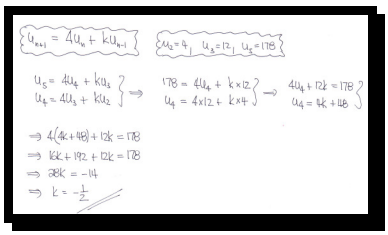
$$u_{n+1}=4u_n+ku_{n-1},$$

where k is a non zero constant.

It is further given that $u_2=4$, $u_3=12$ and $u_5=178$.

Determine the value of k.

$$\boxed{k=-\frac{1}{2}}$$



Question 33 (*+)**

A recurrence relation is defined for $n \geq 1$ by

$$t_{n+1} = at_n + b\,, \quad t_1 = 2\,,$$

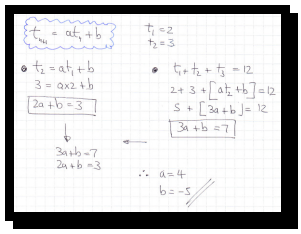
where a and b are non zero constants.

Given further that

$$t_2 = 3 \quad \text{and} \quad \sum_{r=1}^{3} t_r = 12\,,$$

find the possible value of a and the possible value of b.

$\boxed{}$, $\boxed{a = 4}$, $\boxed{b = -5}$



Question 34 (*+)**

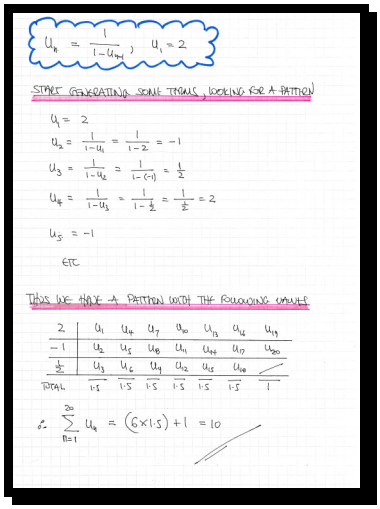
A sequence of numbers, u_1, u_2, u_3, u_4, ... , is defined by

$$u_n = \frac{1}{1-u_{n-1}}, \quad u_1 = 2.$$

Determine the value of

$$\sum_{n=1}^{20} u_n .$$

$\square$, $\displaystyle\sum_{n=1}^{20} u_n = 10$



Question 35 (****)

A recurrence relation is defined for $n \geq 1$ by

$$u_{n+1} = k + (-1)^n u_n, \quad u_1 = 4,$$

where k is a non zero constant.

a) Show clearly that $u_5 = 4$.

b) State, in terms of k, the value of u_{26}.

c) Given further that

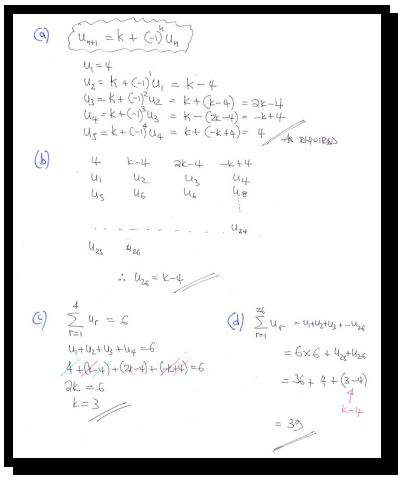
$$\sum_{r=1}^{4} u_r = 6 \ ,$$

find the value of k.

d) Evaluate the sum

$$\sum_{r=1}^{26} u_r \ .$$

$\boxed{}$, $\boxed{u_{26} = k - 4}$, $\boxed{k = 3}$, $\boxed{S_{26} = 39}$



Question 36 (****)

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ u_5,\ \ldots$ is given by

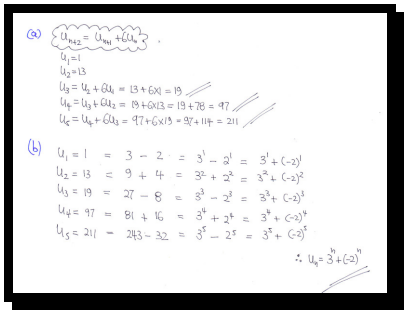
$$u_{n+2}=u_{n+1}+6u_n,\quad u_1=1,\quad u_2=13.$$

a) Find the value of u_3, the value of u_4 and the value of u_5.

b) Find a simplified expression for the n^{th} term of the above sequence by considering the first few terms of the sequence shown below

$$3-2,\ 9+4,\ 27-8,\ 81+16,\ 243-32,\ \ldots$$

$\square$, $\boxed{u_3=19}$, $\boxed{u_4=97}$, $\boxed{u_5=211}$, $\boxed{u_n=3^n+(-2)^n}$



Question 37 (****)

A sequence t_1, t_2, t_3, t_4, … is given by

$$t_{n+1} = a + bt_n,$$

where a and b are non zero constants.

It is given that $t_3 = 320$, $t_4 = 240$ and $t_5 = 200$.

a) Determine the value of a and the value of b.

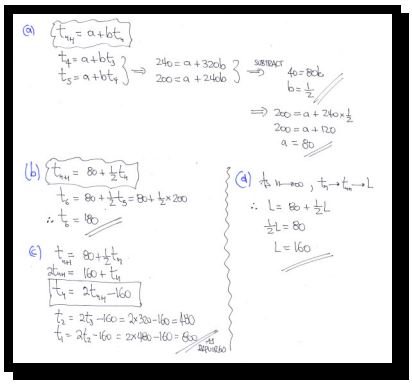
b) Find the value of t_6.

c) Show clearly that $t_1 = 800$.

The sequence converges to a limit L.

d) Determine the value of L.

$$\boxed{a = 80}, \quad \boxed{b = \frac{1}{2}}, \quad \boxed{t_6 = 180}, \quad \boxed{L = 160}$$



Question 38 (****)

A recurrence relation is defined for $n \geq 1$ by

$$a_{n+1} = (a_n)^2 - 4, \quad a_1 = k,$$

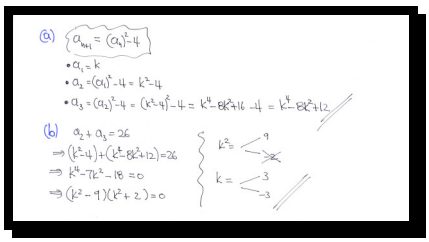
where k is a non zero constant.

a) Find the value of a_3 in terms of k.

It is given that $a_2 + a_3 = 26$.

b) Find the possible values of k.

$\square$, $\boxed{a_3 = k^4 - 8k^2 + 12}$, $\boxed{k = \pm 3}$



Question 39 (****)

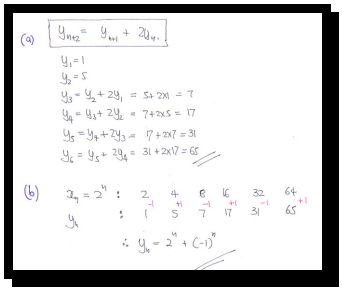
A sequence $y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6\ \ldots$ is given by

$$y_{n+2}=y_{n+1}+2y_n,\quad y_1=1,\quad y_2=5.$$

a) Find the value of $y_3,\ y_4\ ,\ y_5$ and y_6.

b) Find a simplified expression for the n^{th} term of the sequence, by considering the first few powers of 2.

$\square$, $\boxed{y_3=7}$, $\boxed{y_4=17}$, $\boxed{y_5=31}$, $\boxed{y_6=65}$ $\boxed{y_n=2^n+(-1)^n}$



Question 40 (****)

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

$$u_{n+1}=k-\frac{12}{u_n},\quad u_1=1,$$

where k is a non zero constant.

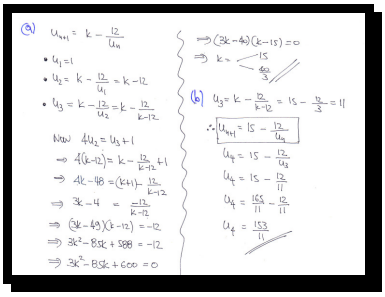
It is further given that

$$4u_2=u_3+1.$$

a) Show that one of the possible values of k is 15 and find the other.

b) If $k=15$ find the exact value of u_4.

$$\boxed{k=\frac{40}{3}},\ \boxed{u_4=\frac{153}{11}}$$



Question 41 (****)

A sequence of numbers is defined by the recurrence relation for $n \geq 1$

$$u_{n+1} = ku_n + 6, \quad u_1 = 4,$$

where k is a non zero constant.

a) Find, in terms of k, the value of u_2 and the value of u_3

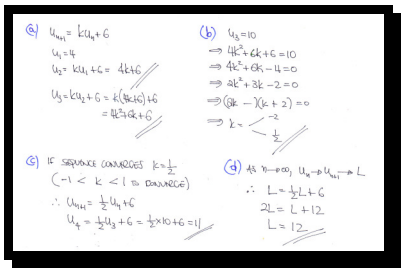
b) Given that $u_3 = 10$ find the possible values of k.

The sequence tends to a limit L.

c) Find the value of u_4.

d) Determine the value of L.

$$\boxed{u_2 = 4k + 6}, \quad \boxed{u_3 = 4k^2 + 6k + 6}, \quad \boxed{k = -2, \frac{1}{2}}, \quad \boxed{u_4 = 11}, \quad \boxed{L = 12}$$



Question 42 (****)

A recurrence relation is defined for $n \geq 1$ by

$$U_{n+1} = aU_n + b, \quad U_1 = k,$$

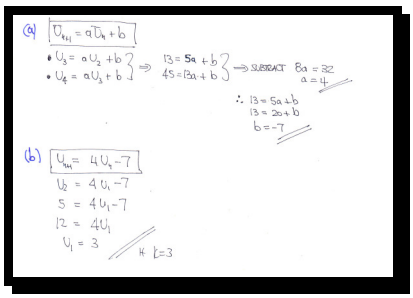
where a, b and k are non zero constants.

It is given that $U_2 = 5$, $U_3 = 13$ and $U_4 = 45$.

a) Find the value of a and the value of b.

b) Determine the value of k.

$$\boxed{a = 4}, \quad \boxed{b = -7}, \quad \boxed{U_1 = k = 3}$$



Question 43 (****)

A sequence of numbers is given by the recurrence relation

$$t_{n+1} = At_n + B\,, \quad n \geq 1,$$

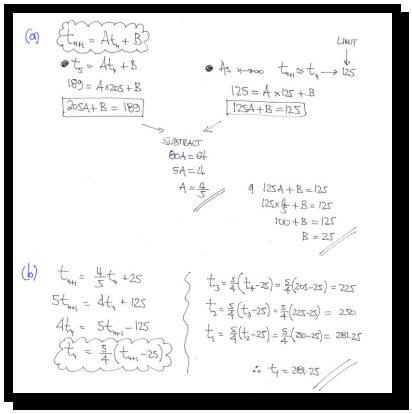
where A and B are non zero constants.

It is given that $t_4 = 205$ and $t_5 = 189$, and the sequence converges to 125.

a) By forming and solving two equations show that $A = \frac{4}{5}$ and $B = 25$.

b) Find the value of t_1.

$$\boxed{t_1 = 281.25}$$



Question 44 (****)

A sequence of numbers is given by the recurrence relation

$$u_{n+1} = pu_n + q\,, \quad n \geq 1,$$

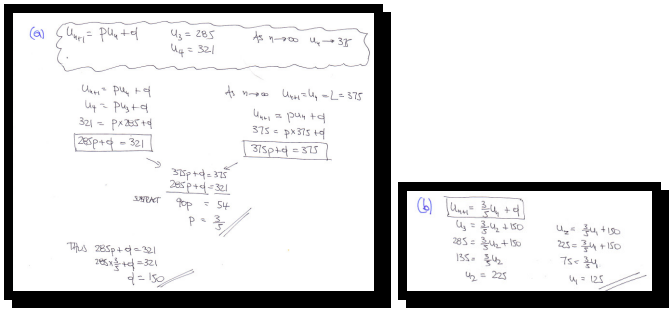
where p and q are non zero constants.

It is given that $u_3 = 285$ and $u_4 = 321$, and the sequence converges to 375.

a) Find the value of p and the value of q

b) Determine the value of u_1.

$$\boxed{p = \frac{3}{5}}\,, \quad \boxed{q = 150}\,, \quad \boxed{u_1 = 125}$$



Question 45 (****)

$$P_{n+1} = A + BP_n\,,\ t > 1\,.$$

The relationship above gives the amount of money Adrian pays into a pension scheme each year P_n, where n is the pension contribution in the n^{th} year.

Adrian's annual contributions in the 2^{nd}, 3^{rd} and 4^{th} years were £1625, £2425 and £3065, respectively.

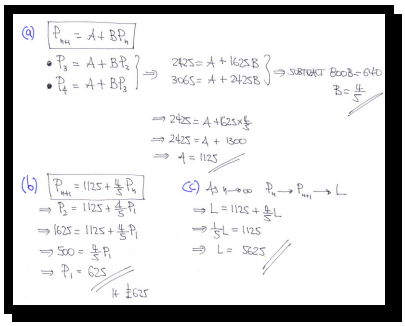
a) Find the value of A and the value of B.

b) Determine Adrian's annual contributions in the first year.

Adrian's annual contributions cannot exceed a certain amount L.

c) Find the value of L.

$\boxed{A = 1125}$, $\boxed{B = 0.8}$, $\boxed{P_1 = 625}$, $\boxed{L = 5625}$



Question 46 (****)

In a clinical trial the concentration C, of a certain blood agent, is measured at one hour intervals since a trial drug was first administered to a patient.

The following readings were obtained

$$C_3 = 88\,,\ C_4 = 76 \text{ and } C_5 = 70\,,$$

where C_t denotes the reading t hours after the drug was first administered.

It is thought that C satisfies the relationship

$$C_{t+1} = a + bC_t\,,\ t \geq 0\,.$$

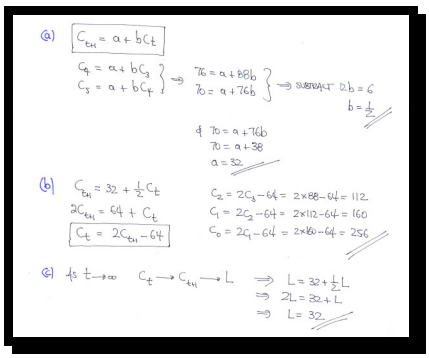
a) Find the value of a and the value of b.

b) Determine the **initial** concentration of the blood agent, when the drug was first administered.

The value of C converges to a limit L.

c) Find the value of L.

$\boxed{}$, $\boxed{a = 32}$, $\boxed{b = \frac{1}{2}}$, $\boxed{C_0 = 256}$, $\boxed{L = 64}$



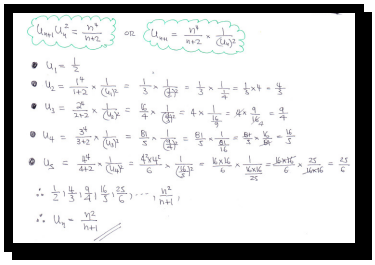
Question 47 (****)

A sequence u_1, u_2, u_3, u_4, … is given by

$$(u_{n+1})(u_n)^2 = \frac{n^4}{n+2}, \quad u_1 = \frac{1}{2}.$$

Calculate the value of u_2, u_3, u_4 and u_5, and hence write an expression for the n^{th} term of the sequence.

$$\boxed{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \ldots}, \quad \boxed{u_n = \frac{n^2}{n+1}}$$



Question 48 (****)

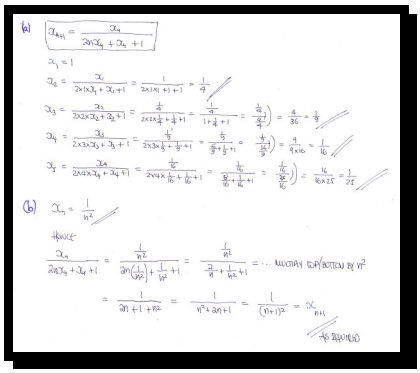
A sequence $x_1,\ x_2,\ x_3,\ x_4,\ \ldots$ is given by the recurrence formula

$$x_{n+1}=\frac{x_n}{2nx_n+x_n+1},\ x_1=1$$

a) Determine the value of $x_2,\ x_3,\ x_4$ and x_5.

b) State an expression for the n^{th} term of the sequence and verify that it satisfies the above recurrence formula.

$$\boxed{x_2=\frac{1}{4}},\ \boxed{x_3=\frac{1}{9}},\ \boxed{x_4=\frac{1}{16}},\ \boxed{x_5=\frac{1}{25}},\ \boxed{x_n=\frac{1}{n^2}}$$



Question 49 (****)

A recurrence relation obeys the relationship

$$x_{n+1}=\sqrt{x_n+12}\ ,\ x_1=k\ ,$$

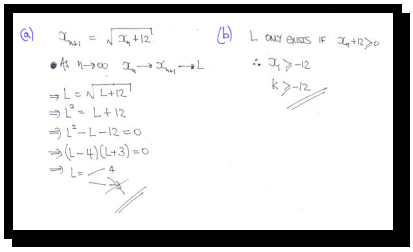
where k is a non zero constant.

This recurrence relation converges to a limit L, for a suitable range of values of k.

a) Find the value of L.

b) Determine the **range** of values of k, so L exists.

$$\boxed{L=4}\ ,\ \boxed{k\geq -12}$$



Question 50 (****)

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

$$u_{n+1}=\frac{1}{1-u_n},\quad u_1=k,$$

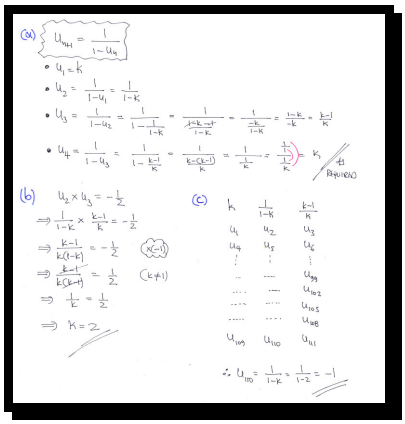
where k is a non zero constant.

a) Show clearly that $u_4=k$.

b) Given that $u_2\times u_3=-\frac{1}{2}$, determine the value of k.

c) State the value of u_{110}.

$\boxed{k=2}$, $\boxed{u_{110}=-1}$



Question 51 (****)

A sequence u_1, u_2, u_3, u_4, u_5 ... satisfies

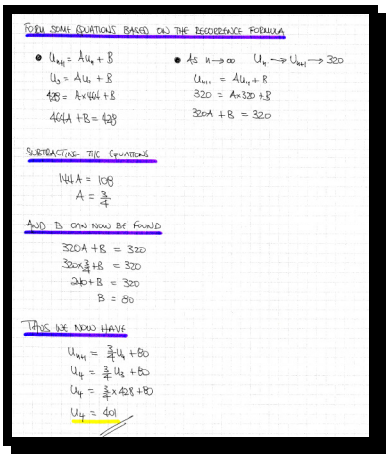
$$u_{n+1} = Au_n + B,$$

where A and B are non zero constants.

The second and third term of this sequence are 464 and 428, respectively.

Given further that the sequence converges to 320, find the value of the fourth term of this sequence.

$\boxed{}$, $\boxed{u_4 = 401}$



Question 52 (**+)**

A sequence of numbers is given by the recurrence relation

$$u_{n+1}=\frac{Au_n+2}{4+Bu_n},\quad n\geq 1,\quad u_1=\frac{1}{2},$$

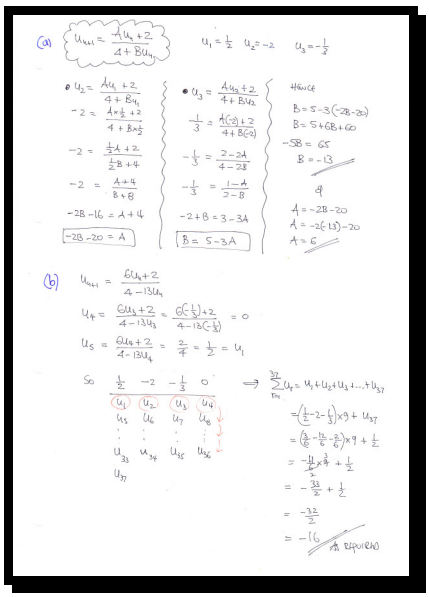
where A and B are non zero constants.

a) If $u_2=-2$ and $u_3=-\frac{1}{3}$, find the value of A and the value of B.

b) Show clearly that

$$\sum_{r=1}^{37}u_r=-16.$$

$\boxed{}$, $\boxed{A=6}$, $\boxed{B=-13}$



Question 53 (**+)**

A sequence u_1, u_2, u_3, u_4, … is given by

$$u_{n+1}=\frac{u_n}{a}+\frac{a}{u_n},\quad u_1=2,$$

where a is a non zero constant.

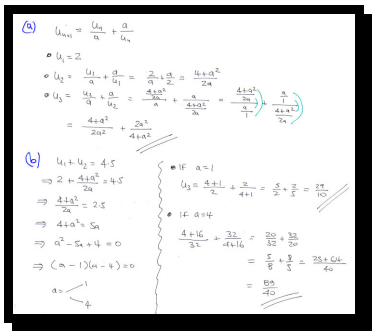
a) Find the value of u_3 in terms of a.

It is further given that

$$u_1+u_2=4.5.$$

b) Find the possible values of u_3.

$$\boxed{u_3=\frac{4+a^2}{2a^2}+\frac{2a^2}{4+a^2}},\quad \boxed{u_3=\frac{29}{10},\frac{89}{40}}$$



Question 54 (**+)**

A sequence $t_1, t_2, t_3, t_4, t_5, \ldots$ is given by

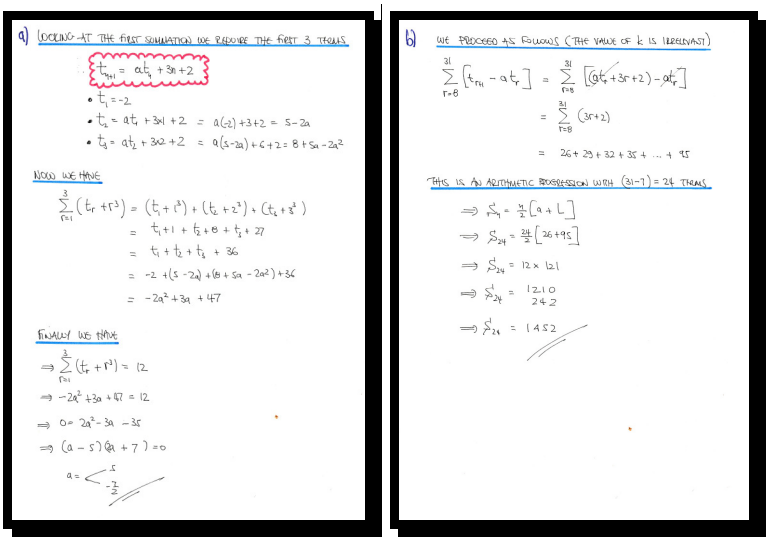
$$t_{n+1} = at_n + 3n + 2, \quad t \in \mathbb{N}, \quad t_1 = -2,$$

where a is a non zero constant.

a) Given that $\sum_{r=1}^{3}\left(r^3 + t_r\right) = 12$, determine the possible values of a.

b) Evaluate $\sum_{r=8}^{31}\left(t_{r+1} - at_r\right)$.

$\square$, $a = 5, \ a = -\frac{7}{2}$, 1452



Question 55 (**+)**

A sequence u_1, u_2, u_3, u_4, … is given by

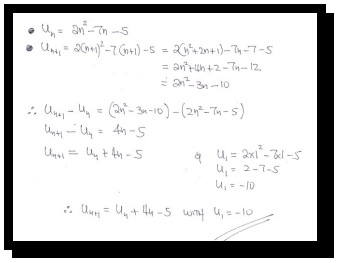
$$u_n = 2n^2 - 7n - 5 .$$

Find an expression for u_{n+1} as a recurrence relation of the form

$$u_{n+1} = Au_n + Bn + C , \ u_1 = D ,$$

where A, B, C and D are constants to be found.

$$\boxed{u_{n+1} = u_n + 4n - 5, \quad u_1 = -10}$$



Question 56 (**+)**

A sequence u_1, u_2, u_3, u_4, … is given by

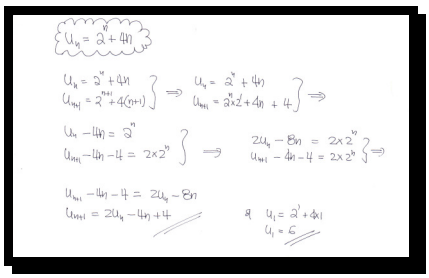
$$u_n = 2^n + 4n .$$

Find an expression for u_{n+1} as a recurrence relation of the form

$$u_{n+1} = Au_n + Bn + C , \ u_1 = D ,$$

where A, B, C and D are constants to be found.

$\boxed{}$, $\boxed{u_{n+1} = 2u_n - 4n + 4, \quad u_1 = 6}$



Question 57 (**+)**

A recurrence relation is defined for $n \geq 1$ by

$$t_{n+1} = at_n + b,$$

where a and b are non zero constants.

It is given that $t_2 = 176$, $t_3 = 248$ and $t_4 = 284$.

a) Find the value of a and the value of b.

b) Determine the value of t_1.

The sequence converges to a limit l.

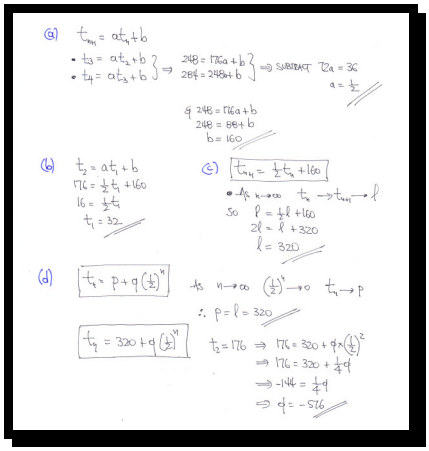
c) Find the value of l.

The n^{th} term of the sequence is given by

$$t_n = p + q\left(\frac{1}{2}\right)^n, \quad \text{where } p \text{ and } q \text{ are constants.}$$

d) Find the value of p and the value of q.

$$\boxed{a = \frac{1}{2}}, \ \boxed{b = 160}, \ \boxed{t_1 = 32}, \ \boxed{l = 320}, \ \boxed{p = 320}, \ \boxed{q = -576}$$



Question 58 (**+)**

A sequence u_1, u_2, u_3, u_4, … is given by

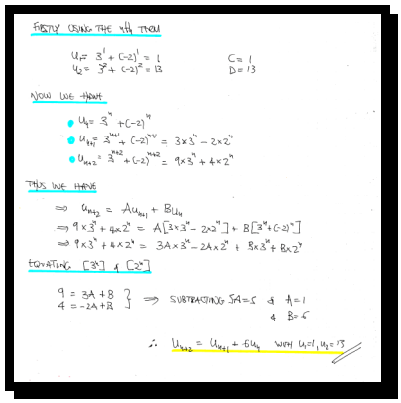
$$u_n = 3^n + (-2)^n .$$

Find an expression for u_{n+2}, as a recurrence relation of the form,

$$u_{n+2} = Au_{n+1} + Bu_n ,\ u_1 = C ,\ u_2 = D$$

where A, B, C and D are constants to be found.

$\boxed{}$, $\boxed{u_{n+2} = u_{n+1} + 6u_n ,\quad u_1 = 1,\ \ u_2 = 13}$



Question 59 (**+)**

A sequence $u_1,\ u_2,\ u_3,\ u_4,\ \ldots$ is given by

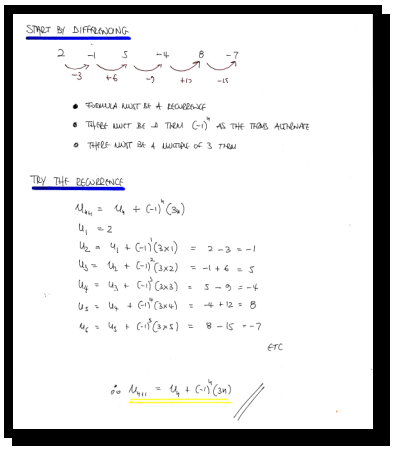
$$u_{n+1}=f\left(n,u_n\right).$$

The first few terms of the sequence are

$$2,\ -1,\ 5,\ -4,\ 8,\ -7,...$$

Find an expression for u_{n+1}, in the form $u_{n+1}=f\left(n,u_n\right)$.

$\boxed{}$, $\boxed{u_{n+1}=u_n+(-1)^n(3n)}$



Question 60 (**+)**

The n^{th} term of the sequence is given by

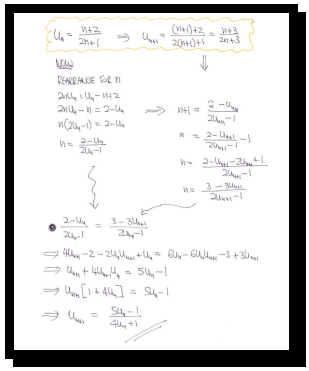
$$u_n = \frac{n+2}{2n+1},\ n \in \mathbb{N},\ n \geq 1.$$

Show that the same sequence can be generated by the recurrence relation

$$u_{n+1} = \frac{Au_n - 1}{Bu_n + 1},\ u_1 = 1,\ n \in \mathbb{N},\ n \geq 1,$$

where A and B are integers to be found.

$\boxed{}$, $\boxed{A=5}$, $\boxed{B=4}$



Question 61 (**+)**

A sequence is defined for $n \geq 1$ by the recurrence relation

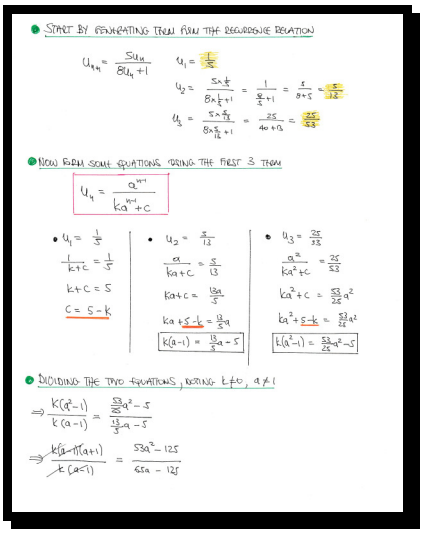
$$u_{n+1} = \frac{5u_n}{1+8u_n}, \quad u_1 = \frac{1}{5}.$$

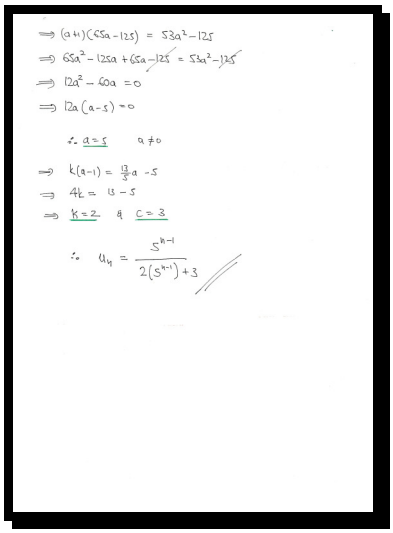
Determine an expression for u_n, given that it is of the form

$$u_n = \frac{a^{n-1}}{c + ka^{n-1}},$$

where a, c and k are constants to be found.

$\square$, $\boxed{u_n = \dfrac{5^{n-1}}{3+2\left(5^{n-1}\right)}}$





Question 62 (*****)

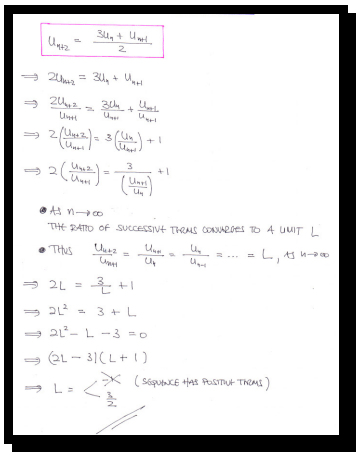
A sequence $u_1,\ u_2,\ u_3,\ u_4,\ u_5\ \ldots$ is given by the recurrence formula

$$u_{n+2}=\frac{3u_n+u_{n+1}}{2},\quad u_1=1,\ u_2=1.$$

It is further given that in this sequence **the ratio of consecutive terms** converges to a limit L.

Determine the value of L.

$\square$, $\boxed{\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right)=L=\frac{3}{2}}$



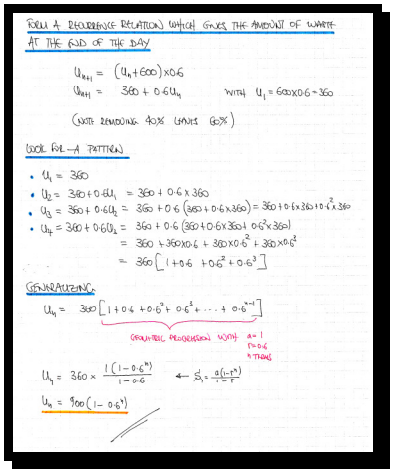
Question 63 (*****)

A factory gets permission to dispose, at the start of every day, 600 kg of waste into a stream of water.

The running stream removes 40% of the any waste present, by the end of the day.

Determine a simplified expression for the amount of waste present in the stream at the end of the n^{th} day.

$\square$, $\boxed{u_n = 900\left[1-\left(\frac{3}{5}\right)^n\right]}$



Question 64 **(*****)**

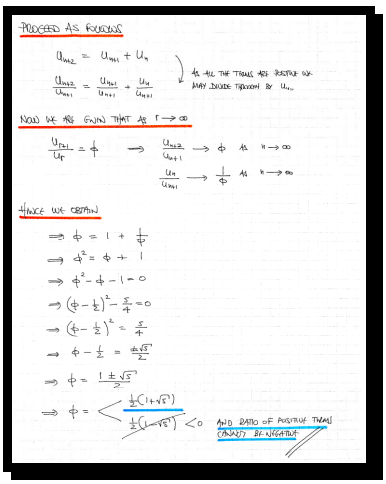
The Fibonacci sequence is given by the recurrence formula

$$u_{n+2}=u_{n+1}+u_n,\quad u_1=1,\ u_2=1.$$

It is further given that in this sequence **the ratio of consecutive terms** converges to a limit ϕ, known as the *Golden Ratio*.

Show, by using the above recurrence formula, that $\phi=\frac{1}{2}\left(1+\sqrt{5}\right)$.

$\boxed{}$, proof



Question 65 (*****)

The n^{th} term of a sequence is given by

$$u_n = 1 + \left(\frac{1}{3}\right)^n, \quad \text{where } n \geq 1.$$

a) By expressing u_{n+1} in terms of u_n, or otherwise, define the terms of the sequence as a recurrence relation.

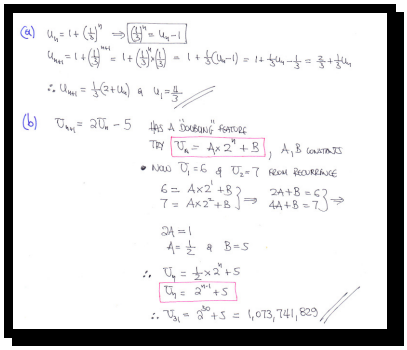
A recurrence relation is defined for $n \geq 1$ by

$$U_{n+1} = 2U_n - 5, \; U_1 = 6.$$

b) By finding the n^{th} term of the sequence, or otherwise, show that

$$u_{31} = 1,073,741,829.$$

$\square$, $\boxed{u_{n+1} = \frac{2+u_n}{3}, \; u_1 = \frac{4}{3}}$, $\boxed{U_n = 2^{n-1} + 5}$



Question 66 (*****)

A sequence is defined by the recurrence relation

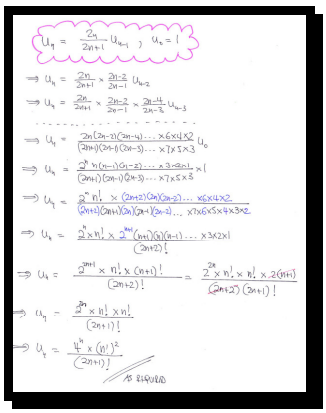
$$u_n = \frac{2n}{2n+1}u_{n-1}\,,\ n\in\mathbb{N}\quad u_0 = 1\,.$$

Show, by direct manipulation, that

$$u_n = \frac{4^n\times(n!)^2}{(2n+1)!}\,.$$

[*you may not use proof by induction*]

proof



Question 67 (*****)

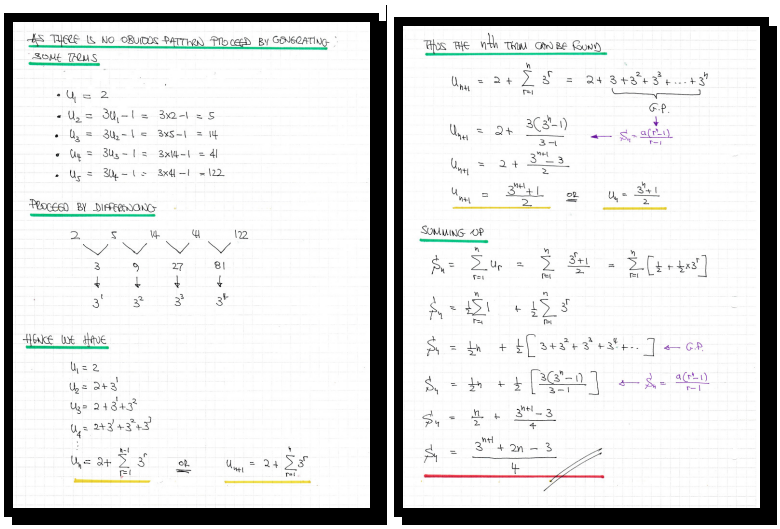
A sequence of numbers, u_1, u_2, u_3, u_4, ... , is defined by

$$u_{n+1}=3u_n-1, \quad u_1=2.$$

Determine, in terms of n, a simplified expression for

$$\sum_{r=1}^{n} u_r .$$

$\square$, $\boxed{S_n=\frac{1}{4}\left[3^{n+1}+2n-3\right]}$



Question 68 **(*****)**

It is given that

$$\sum_{r=1}^{n} u_r = 6^{n+1} - 10 \times 2^n + 4,$$

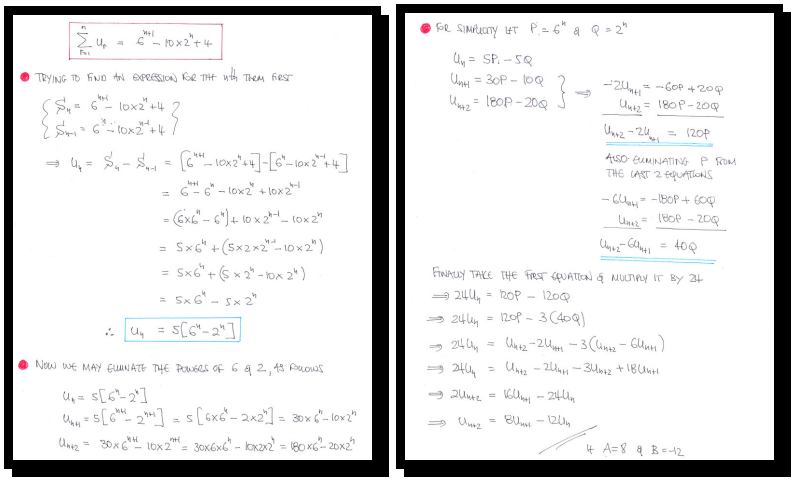
where u_n is the n^{th} term of a sequence.

Show clearly that

$$u_{n+2} = Au_{n+1} + Bu_n,$$

where A and B are integers to be found.

$\square$, $\boxed{u_{n+2} = 8u_{n+1} - 12u_n}$



Question 69 (*****)

The function f satisfies the following three relationships

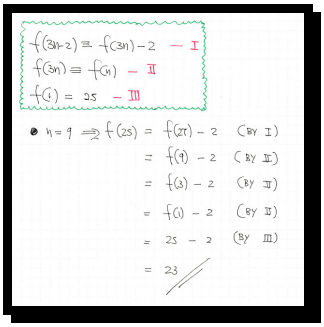
i. $f(3n-2) \equiv f(3n)-2,\ n \in \mathbb{N}$.

ii. $f(3n) \equiv f(n),\ n \in \mathbb{N}$.

iii. $f(1)=25$.

Determine the value of $f(25)$.

$\square$, $\boxed{f(25)=23}$



Question 70 (*****)

A sequence is defined by the recurrence relation

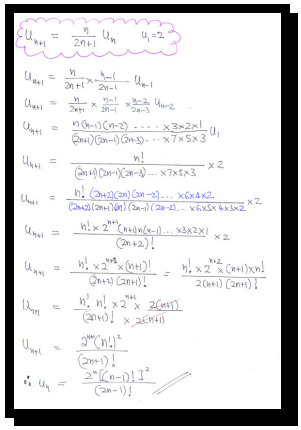
$$u_{n+1}=\frac{n}{2n+1}u_n\,,\ n\in\mathbb{N}\quad u_1=2\,.$$

Show, by direct manipulation, that

$$u_n=\frac{2^n\times\left[(n-1)!\right]^2}{(2n-1)!}\,.$$

[*You may not use proof by induction in this question*]

proof



Question 71 (*****)

Consider the following sequence.

$$\frac{1}{7},\ \frac{1}{2},\ \frac{7}{9},\ 1,\ \frac{13}{11},\ \frac{4}{3},\ \frac{19}{13},\ \frac{11}{7},\ \ldots,\quad x\in\mathbb{R},\quad x<2\,.$$

a) Determine the n^{th} of this sequence and hence find a recurrence relation formula for this sequence.

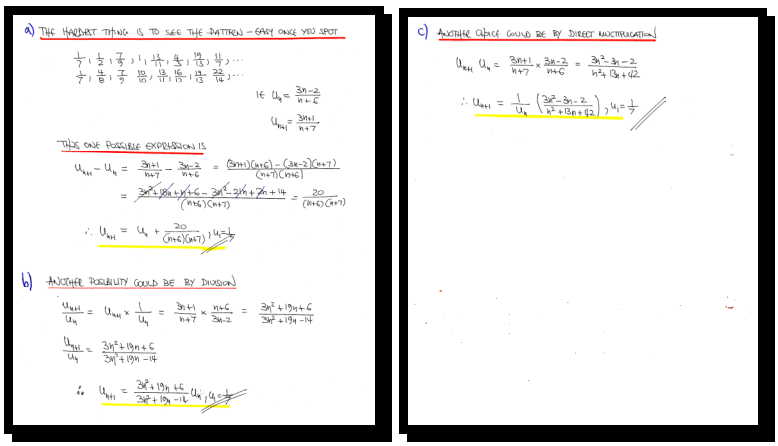
b) Find a **different**, to that given in part **(a)**, recurrence relation formula for the same sequence.

c) Determine a **third** recurrence relation formula for this sequence.

The recurrence relations in this question must be in the form $F\left(u_{n+1},\ u_n,\ n\right)$

$$\square,\quad \boxed{u_{n+1}=u_n+\frac{20}{(n+6)(n+7)}},\quad \boxed{u_{n+1}=\frac{3n^2+19n+6}{3n^2+19n-14}u_n},$$

$$\boxed{u_{n+1}=\left(\frac{1}{u_n}\right)\frac{3n^2-3n-2}{n^2+13n+42}}$$



Question 72 (*****)

The n^{th} term of a series is given recursively by

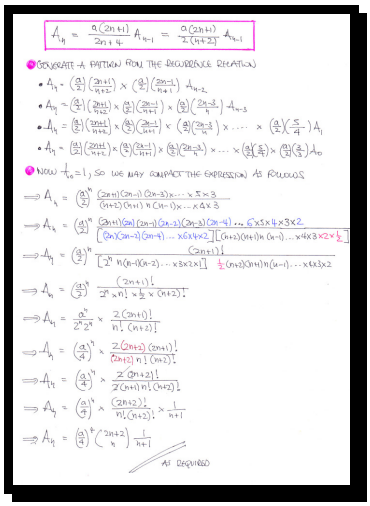
$$A_n = \frac{a(2n+1)}{2n+4} A_{n-1}, \ n \in \mathbb{N}, \ n \geq 1,$$

where a is a positive constant.

Given further that $A_0 = 1$, show that

$$A_n = \left(\frac{a}{4}\right)^n \binom{2n+2}{n} \frac{1}{n+1}.$$

proof



Question 73 (*****)

A sequence is defined as

$$u_{r+1}=u_r+\frac{2r}{r^4+r^2+1}, \qquad u_1=0, \quad r\in\mathbb{N}.$$

Determine the exact value of u_{61}.

$$\boxed{u_{61}=\frac{3660}{3661}}$$

