SIMPLE RECIPROCAL FUNCTIONS

Question 1 (+)**

The curves C_1 and C_2 have respective equations

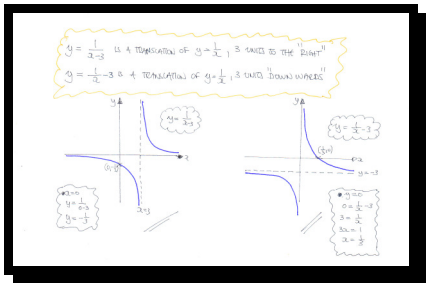
$$C_1: \ y=\frac{1}{x-3}, \ x\neq 3$$

$$C_2: \ y=\frac{1}{x}-3, \ x\neq 0$$

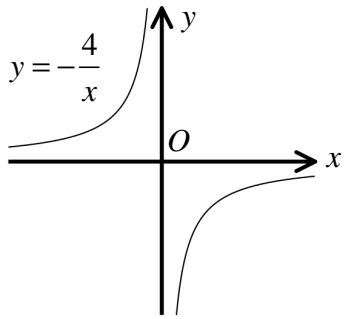
Sketch on separate diagrams the graph of C_1 and the graph of C_2.

Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.

, graph



Question 2 **(**+)**



The figure above shows the graph of the curve with equation

$$y=-\frac{4}{x},\ x\neq 0.$$

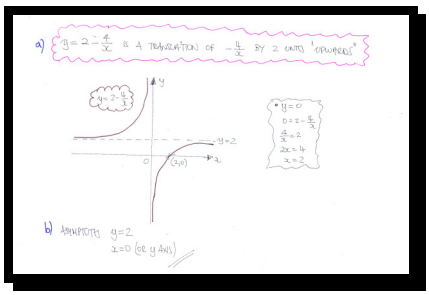
a) Sketch the graph of the curve C with equation

$$y=2-\frac{4}{x},\ x\neq 0.$$

Indicate clearly the coordinates of any points of intersection between C and the coordinate axes.

b) State the equations of the two asymptotes of C.

$\boxed{}$, $\boxed{x=0,\ y=2}$



Question 3 (***)

The curves C_1 and C_2 have respective equations

$$C_1: \ y=-\frac{1}{x}, \ x\neq 0$$

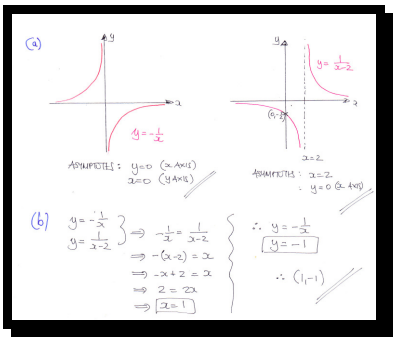
$$C_2: \ y=\frac{1}{x-2}, \ x\neq 2$$

a) Sketch on separate diagrams the graph of C_1 and the graph of C_2.

Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.

b) Find the coordinates of the point of intersection between C_1 and C_2.

$\boxed{}$, $\boxed{(1,-1)}$



Question 4 (***)

The curves C_1 and C_2 have respective equations

$$C_1: \ y=\frac{1}{x}+2, \ x\neq 0$$

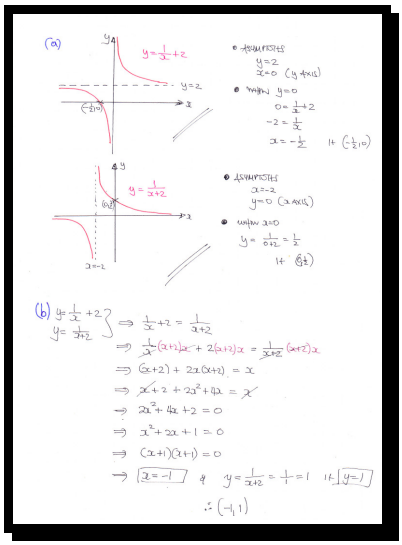
$$C_2: \ y=\frac{1}{x+2}, \ x\neq -2$$

a) Sketch on separate diagrams the graph of C_1 and the graph of C_2.

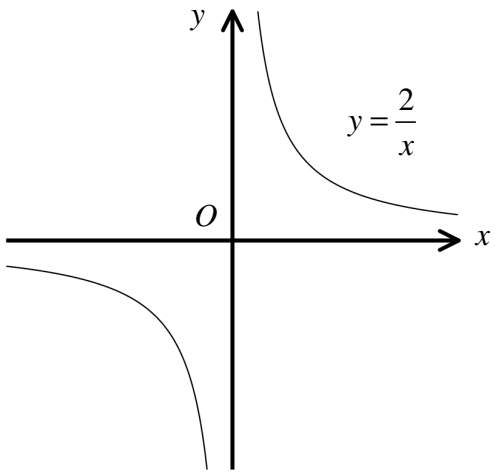
Indicate clearly in each graph any asymptotes and the coordinates of any intersections with the coordinate axes.

b) Find the coordinates of the point of intersection between C_1 and C_2.

$\boxed{}$, $\boxed{(-1,1)}$



Question 5 (***)



The figure above shows the graph of the curve C with equation

$$y=\frac{2}{x},\ x\neq 0.$$

a) Describe the geometric transformation which maps the graph of C onto the graph with equation

$$y=\frac{2}{x-2},\ x\neq 0.$$

b) Sketch the graph of the curve with equation

$$y=\frac{2}{x}+2,\ x\neq 0.$$

Indicate clearly the coordinates of any points of intersections between the curve and the coordinate axes. State the equations of the two asymptotes of the curve.

[continues overleaf]

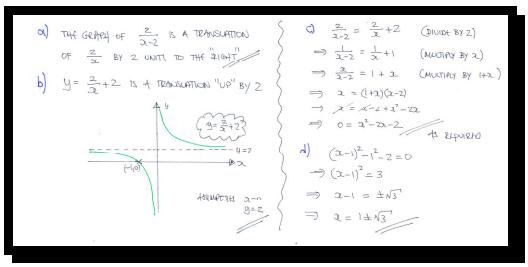
[continued from overleaf]

c) Show that the x coordinates of the points of intersection between the graph of $y=\dfrac{2}{x-2}$ and the graph of $y=\dfrac{2}{x}+2$ are the roots of the quadratic equation

$$x^2-2x-2=0\,.$$

d) Hence find, in exact surd form, the x coordinates of the points of intersection between the graph of $y=\dfrac{2}{x-2}$ and the graph of $y=\dfrac{2}{x}+2$.

$\boxed{}$, $\boxed{\text{translation, 2 units to the "right"}}$, $\boxed{x=1\pm\sqrt{3}}$



Question 6 (***)

A curve C has equation

$$y=\frac{1}{x^2},\ x\in\mathbb{R},\ x\neq 0.$$

a) Sketch the graph of C.

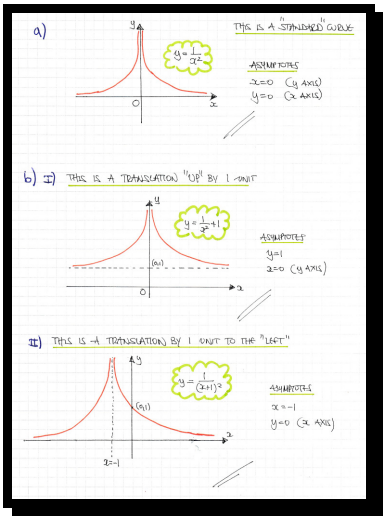
b) Sketch on separate set of axes the graph of ...

i. ... $y=\frac{1}{x^2}+1,\ x\in\mathbb{R},\ x\neq 0.$

ii. ... $y=\frac{1}{(x+1)^2},\ x\in\mathbb{R},\ x\neq -1.$

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.

[ ], graph



Question 7 (***)

A curve C has equation

$$f(x)=-\frac{1}{x^2},\ x\in\mathbb{R},\ x\neq 0.$$

a) Sketch the graph of C.

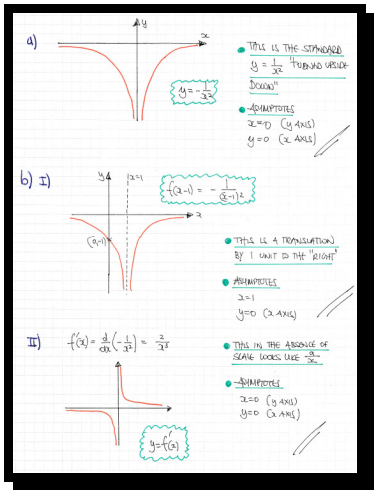
b) Sketch on separate set of axes the graph of ...

i. ... $f(x-1)$.

ii. ... $f'(x)$.

Mark clearly in each sketch the equations of any asymptotes to these curves and the coordinates of any intersections with the coordinate axes.

[ ], [graph]



Question 8 (*+)**

$$f(x)=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

$$g(x)=\frac{1}{x-1}+1,\ x\in\mathbb{R},\ x\neq 1.$$

a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include …

- … the coordinates of any points where $g(x)$ meet the coordinate axes.
- … the equations of any asymptotes of $g(x)$.

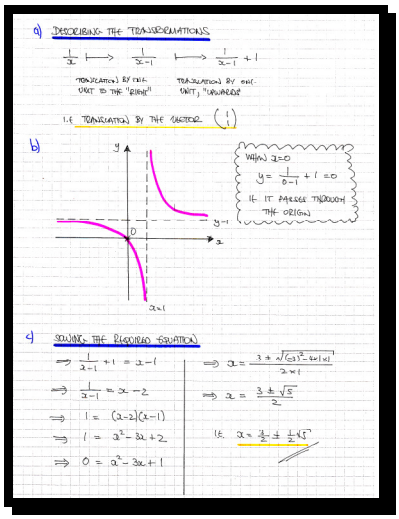
c) Solve the equation

$$g(x)=x-1,$$

giving the answers in the form $a+b\sqrt{5}$, where a and b are constants.

translation "right" by 1 unit, followed by translation "upwards" by 1 unit,

$\square$, $x=\frac{3}{2}\pm\frac{1}{2}\sqrt{5}$



Question 9 **(***+)**

A curve has equation $y=f(x)$ given by

$$f(x)=2+\frac{1}{2x-1},\ x\neq\frac{1}{2}.$$

a) Express $f(x)$ as a single simplified fraction.

Consider the following sequence of transformations T_1, T_2 and T_3.

$$\frac{1}{x}\xrightarrow{T_1}\frac{1}{x-1}\xrightarrow{T_2}\frac{1}{2x-1}\xrightarrow{T_3}2+\frac{1}{2x-1}.$$

b) Describe geometrically the transformations T_1, T_2 and T_3.

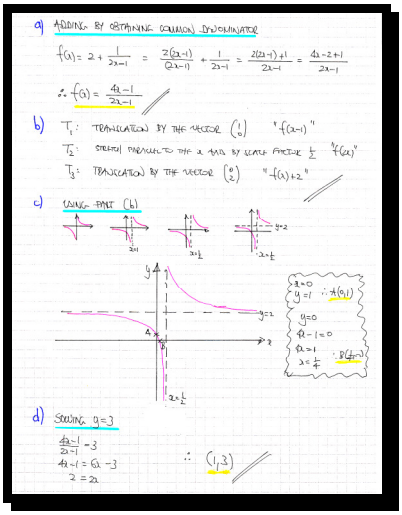
c) Hence sketch the graph of $f(x)$.

Indicate clearly any asymptotes and the coordinates of any intersections with the coordinate axes.

d) Find the coordinates of the point of intersection of $f(x)$ and the line $y=3$.

$\boxed{}$, $\boxed{T_1=\text{translation, "right", 1 unit}}$, $\boxed{T_2=\text{horizontal stretch by scale factor }\frac{1}{2}}$,

$\boxed{T_3=\text{translation, "upwards", 2 units}}$, $\boxed{(1,3)}$



Question 10 **(***+)**

Consider the following sequence of transformations T_1, T_2 and T_3.

$$\frac{1}{x} \xrightarrow{T_1} -\frac{1}{x} \xrightarrow{T_2} -\frac{1}{x+1} \xrightarrow{T_3} 2-\frac{1}{x+1}$$

a) Describe geometrically the transformations T_1, T_2 and T_3.

b) Hence sketch the graph of

$$y=2-\frac{1}{x+1}, \ x\neq -1.$$

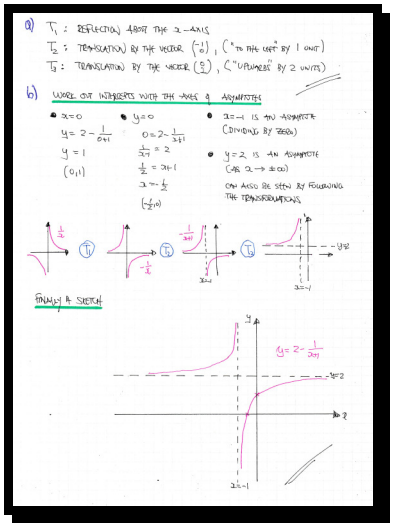
Indicate clearly any asymptotes and the coordinates of any intersections with the coordinate axes.

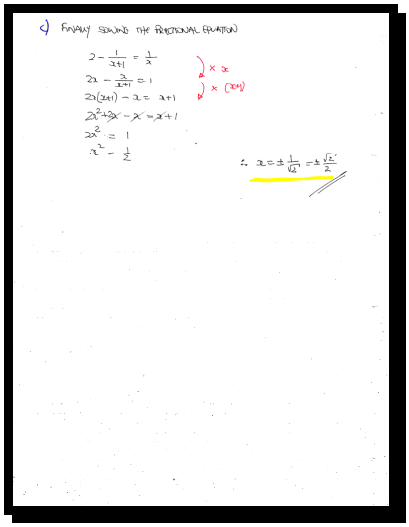
c) Solve the equation

$$2-\frac{1}{x+1}=\frac{1}{x}.$$

$\boxed{}$, $\boxed{T_1 = \text{reflection in the } x\text{-axis}}$, $\boxed{T_2 = \text{translation, "left", 1 unit}}$,

$\boxed{T_3 = \text{translation, "upwards", 2 units}}$, $\boxed{x=\pm\frac{\sqrt{2}}{2}}$





Question 11 **(***+)**

Consider a sequence of geometric transformations T_1, T_2 and T_3 which map the graph of the curve with equation $y_1 = \dfrac{1}{x}$ onto the graph of y_2.

T_1 : reflection in the x axis.

T_2 : translation in the negative x direction by 2 units.

T_3 : translation in the positive y direction by 2 units.

a) Show that the equation of y_2 is given by

$$y_2 = \frac{2x+3}{x+2}, \quad x \neq -2.$$

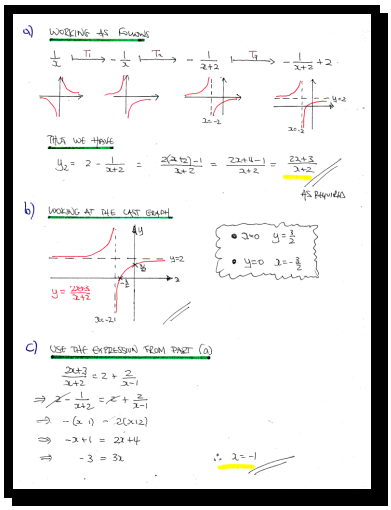
b) Sketch the graph of y_2.

Indicate clearly any asymptotes and coordinates of intersections with the axes.

c) Solve the equation

$$\frac{2x+3}{x+2} = 2 + \frac{2}{x-1}.$$

$\boxed{}$, $\boxed{x=-1}$



Question 12 **(***+)**

The curve C_1 has equation

$$y=-\frac{2}{x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Sketch the graph of C_1.

The curve C_2 has equation

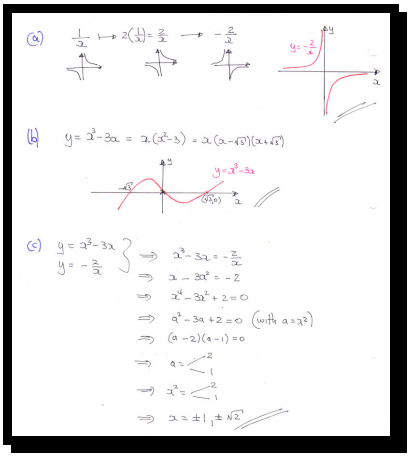
$$y=x^3-3x,\ x\in\mathbb{R}.$$

b) Sketch the graph of C_2.

The sketch must include the coordinates, in exact surd form where appropriate, of all the points where the curve meets the coordinate axes.

c) Find the x coordinates of the points of intersection between C_1 and C_2.

$\square$, $\boxed{x=\pm 1,\ \pm\sqrt{2}}$



Question 13 **(***+)**

$$f(x)=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

$$g(x)=2-\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

$$h(x)=\frac{6}{x+2},\ x\in\mathbb{R},\ x\neq -2.$$

b) Sketch in the same diagram the graphs of $g(x)$ and $h(x)$.

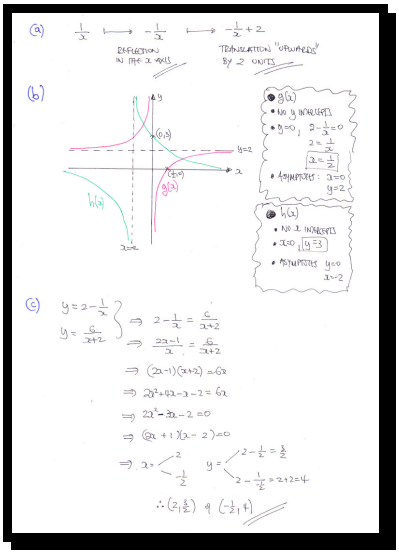
The sketch must include the coordinates of …

- … all the points where the curves meet the coordinate axes.
- … the equations of any asymptotes of the curves.

c) Solve the equation $g(x)=h(x)$.

$\boxed{}$, $\boxed{\text{reflection in the } x \text{ axis, followed by translation "upwards" by 1 unit}}$,

$\boxed{x=-\frac{1}{2},2}$



Question 14 (*+)**

$$y=\frac{2}{x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Describe mathematically the transformation that maps the graph of $y=\frac{1}{x}$ onto the graph of $y=\frac{2}{x}$.

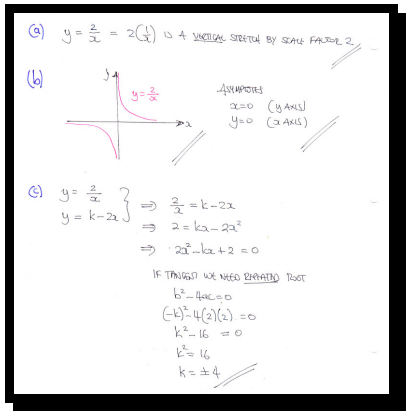
b) Sketch the graph of $y=\frac{2}{x}$.

Write down the equations of the asymptotes of the curve.

The straight line with equation $y=k-2x$, where k is a constant, is a tangent to the curve with equation $y=\frac{2}{x}$.

c) Determine the possible values of k.

$\boxed{}$, $\boxed{\text{stretch, vertically, by scale factor of } 2}$, $\boxed{k=\pm 4}$



Question 15 **(***+)**

$$f(x)=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

$$g(x)=\frac{1}{x+2}+2,\ x\in\mathbb{R},\ x\neq -2.$$

a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the …

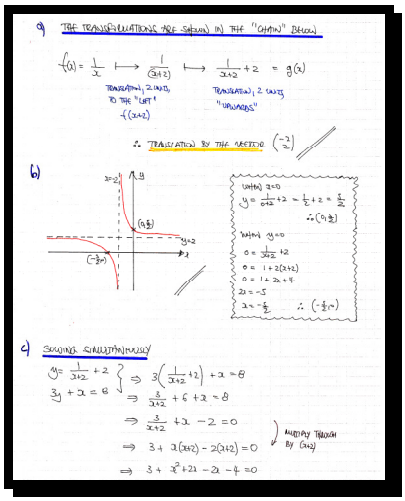
- … coordinates of all the points where the curve meet the coordinate axes.
- … equations of any asymptotes of the curve.

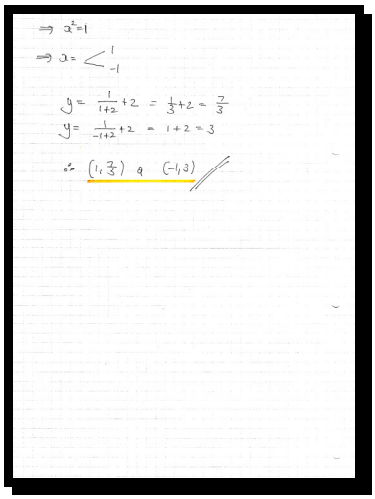
c) Find the coordinates of the points of intersection of $g(x)$ and the line with equation

$$3y+x=8.$$

$\boxed{}$, $\boxed{\text{translation "left" by 2 units, followed by translation "upwards" by 2 units}}$,

$\boxed{\left(1,\frac{7}{3}\right),(-1,3)}$





Question 16 **(***+)**

$$f(x)=\frac{4x-13}{x-3},\ x\in\mathbb{R},\ x\neq 3.$$

a) Show that the equation of $f(x)$ can be written as

$$f(x)=4-\frac{1}{x-3},\ x\in\mathbb{R},\ x\neq 3.$$

b) Sketch the graph of $f(x)$.

The sketch must include …

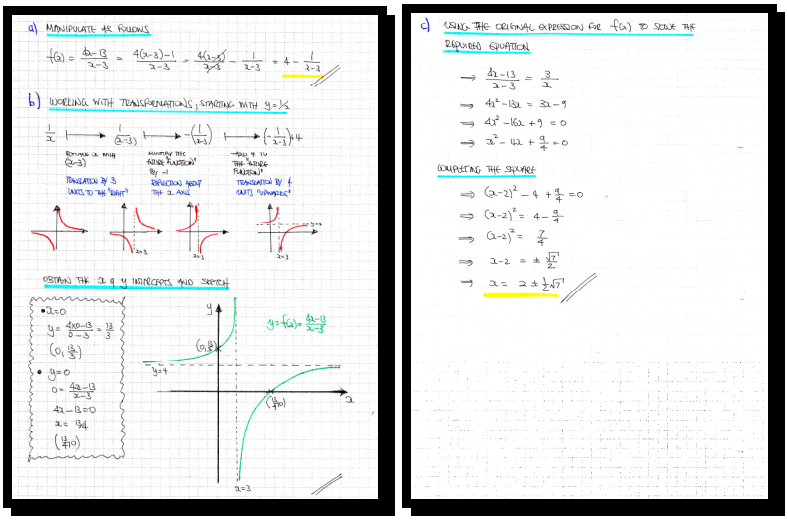
- … the coordinates of the points where $f(x)$ meets the coordinate axes.
- … the equations of any asymptotes of the curve.

c) Solve the equation

$$f(x)=\frac{3}{x},$$

giving the answers in the form $a+b\sqrt{7}$, where a and b are constants.

$\square$, $\boxed{x=2\pm\frac{1}{2}\sqrt{7}}$



Question 17 **(***+)**

$$f(x)=a-\frac{1}{b-x},\ x\in\mathbb{R},\ x\neq b,$$

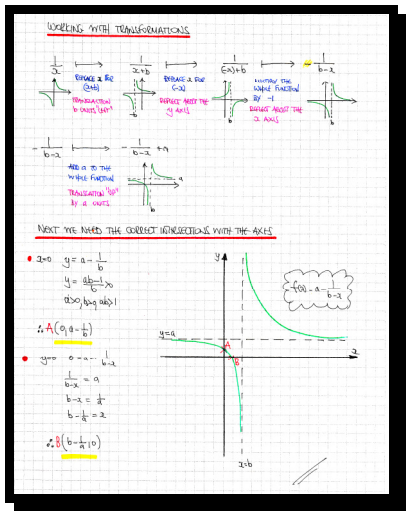
where a and b are positive constants such that $ab>1$.

Sketch the graph of $f(x)$.

The sketch must include, in terms of a and b, …

- … the coordinates of the points where $f(x)$ meets the coordinate axes.
- … the equations of any asymptotes of the curve.

[ ], [graph]



Question 18 **(****)**

The curve C_1 has equation

$$y=\frac{a}{x},\ x\neq 0,$$

where a is a positive constant.

a) Describe geometrically the transformation that maps the graph of C_1 onto the graph of C_2 whose equation is $y=\frac{a}{x}+1$.

b) Sketch the graph of C_2.

The sketch must include the coordinates of …

- … all the points where the curves meet the coordinate axes.
- … the equations of any asymptotes of the curves.

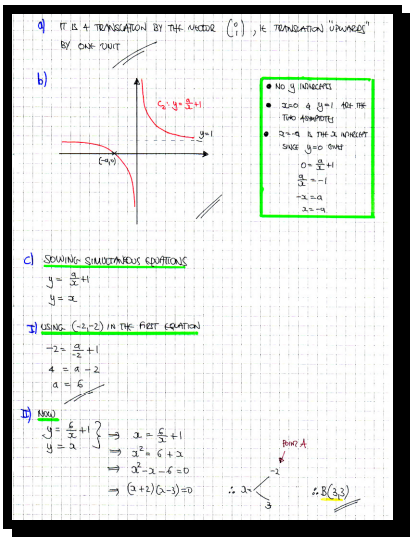
The line with equation $y=x$ intersects C_2 at the point $A(-2,-2)$ and B.

c) Determine …

i. … the value of a.

ii. … the coordinates of B.

$\boxed{}$, $\boxed{\text{translation "upwards" by 1 unit}}$, $\boxed{a=6}$, $\boxed{B(3,3)}$



Question 19 **(****)**

The curve C has equation

$$y=\frac{2x+3}{x-2},\ x\in\mathbb{R},\ x\neq 2.$$

a) Show clearly that

$$\frac{2x+3}{x-2}\equiv 2+\frac{7}{x-2}.$$

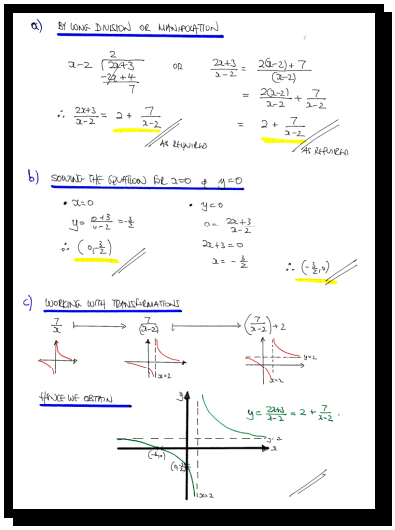
b) Find the coordinates of the points where C meets the coordinate axes.

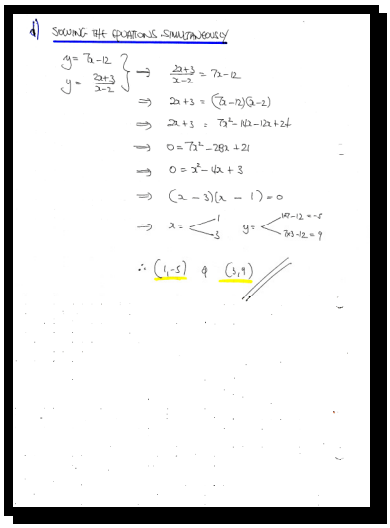
c) Sketch the graph of C showing clearly the equations of any asymptotes.

d) Determine the coordinates of the points of intersection of C and the straight line with equation

$$y=7x-12.$$

$\square$, $\left(0,-\frac{3}{2}\right),\left(-\frac{3}{2},0\right)$, $(1,-5),(3,9)$





Question 20 (****)

$$f(x)=\frac{1}{x-2},\ x\in\mathbb{R},\ x\neq 2.$$

$$g(x)=1+\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Describe mathematically the transformation that maps the graph of $y=\dfrac{1}{x}$ onto the graph of …

i. … $f(x)$.

ii. … $g(x)$.

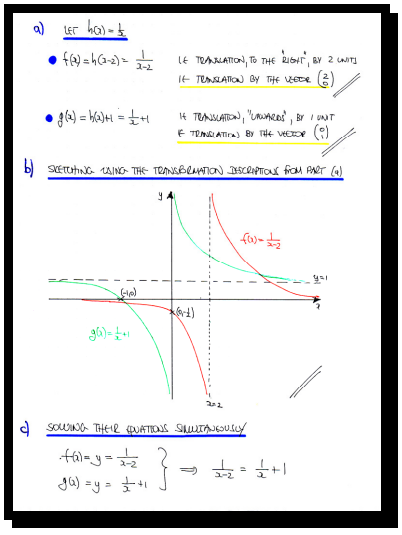
b) Sketch in the same diagram the graphs of $f(x)$ and $g(x)$.

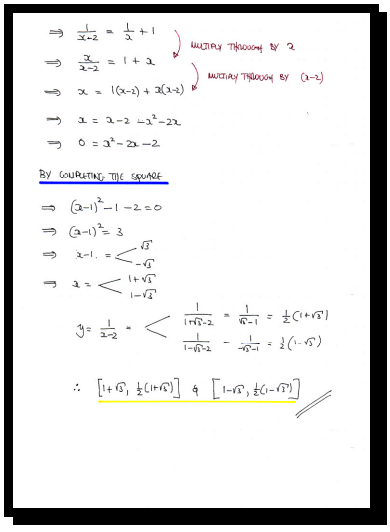
The sketch must include …

- … the coordinates of any the points where the curves meet the coordinate axes.
- … the equations of any asymptotes of the curves.

c) Find as exact surds the coordinates of the points of intersection of the graphs of $f(x)$ and $g(x)$.

[ ], [translation "right" by 2 units], [translation "upwards" by 1 unit], $\left[\left(1\pm\sqrt{3},\dfrac{1\pm\sqrt{3}}{2}\right)\right]$





Question 21 (****)

$$f(x)=\frac{x-2}{x-3},\ x\in\mathbb{R},\ x\neq 3.$$

a) Express $f(x)$ in the form

$$f(x)=a+\frac{1}{x+b},$$

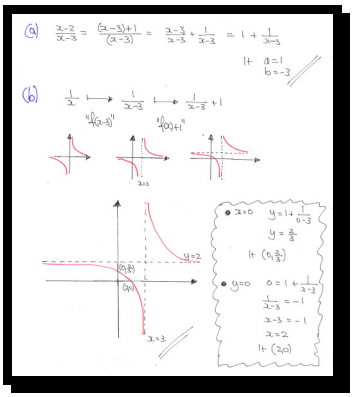
where a and b are integers.

b) By considering a series of transformations which map the graph of $\frac{1}{x}$ onto the graph of $f(x)$, sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

$\boxed{a=1}$, $\boxed{b=-3}$



Question 22 (****)

The curves C_1 and C_2 have respective equations

$$C_1: \ y=\frac{1}{x-1}, \ x\neq 1$$

$$C_2: \ y=1-\frac{3}{x+3}, \ x\neq -3$$

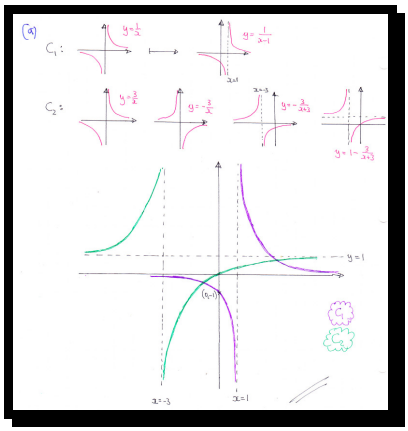
a) Sketch on the same diagram the graphs of C_1 and C_2.

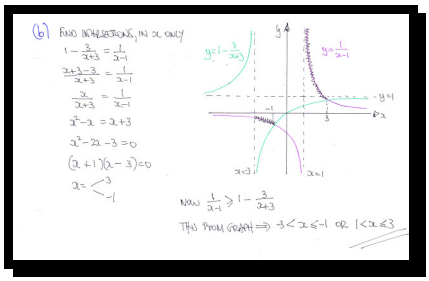
Indicate clearly any asymptotes and coordinates of any intersections with the coordinate axes.

b) By finding the intersections between C_1 and C_2, and considering the graphs sketched in part **(a)**, solve the inequality

$$\frac{1}{x-1}+\frac{3}{x+3}\geq 1.$$

$$\boxed{-3<x\leq -1 \ \cup \ 1<x\leq 3}$$





Question 23 **(****)**

$$f(x)=\frac{3x+3}{x-2},\ x\in\mathbb{R},\ x\neq 2.$$

a) Sketch the graph of $f(x)$.

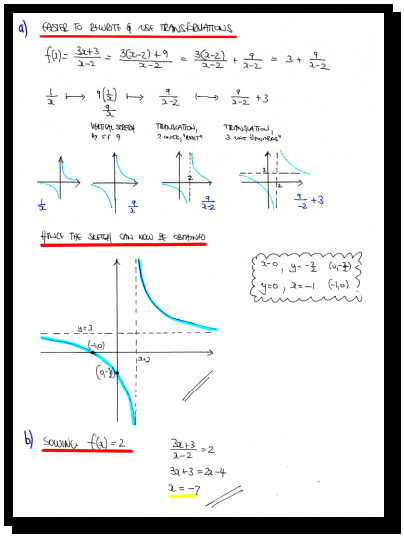
The sketch must include the coordinates of …

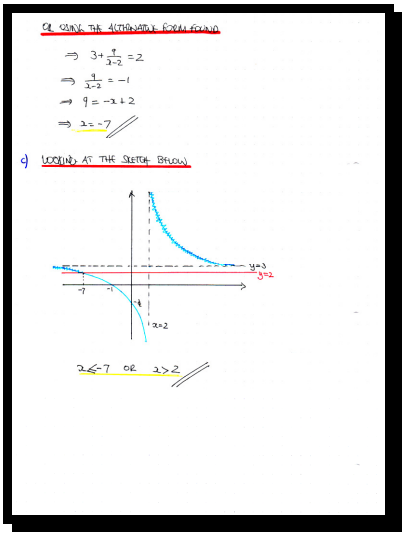
- … all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

b) Solve the equation $f(x)=2$.

c) Hence solve the inequality $f(x)\geq 2$.

$\boxed{}$, $\boxed{x=-7}$, $\boxed{x\leq -7\cup x>2}$





Question 24 **(****+)**

A curve has equation $y=f(x)$ given by

$$f(x)=\frac{3x-1}{x+2},\ x\in\mathbb{R},\ x\neq 2.$$

a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of …

- … all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

A different curve has equation $y=g(x)$ given by

$$g(x)=\frac{1}{x}+k,\ x\in\mathbb{R},\ x\neq 0,\ \text{where } k \text{ is a constant.}$$

The graph of $f(x)$ meets the graph of $g(x)$ at the points A and B.

b) Given that A lies on the x axis determine …

i. … the value of k.

ii. … the coordinates of B.

$\boxed{}$, $\boxed{k=-3}$, $\boxed{B(-1,-4)}$

