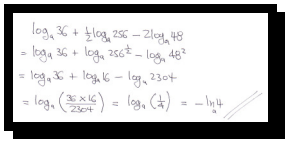
LOGARITHMS EXAM QUESTIONS

Question 1 (**)

Show clearly that

$$\log_a 36+\frac{1}{2}\log_a 256-2\log_a 48=-\log_a 4\,.$$

proof



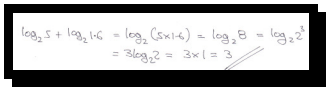
Question 2 (**)

Simplify

$$\log_2 5+\log_2 1.6\,,$$

giving the final answer as an integer.

3

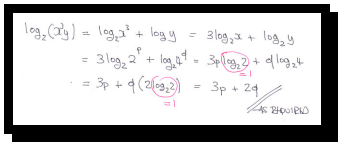


Question 3 (+)**

Given that $x=2^p$ and $y=4^q$, show clearly that

$$\log_2(x^3y)=3p+2q\,.$$

proof



Question 4 **(**+)**

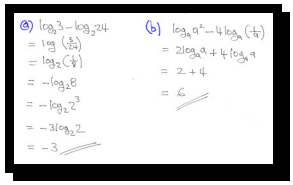
Simplify each of the following expressions, giving the final answer as an integer.

a) $\log_2 3-\log_2 24$.

b) $\log_a a^2-4\log_a\left(\frac{1}{a}\right)$, $a>0$, $a\neq 1$.

Full workings, justifying every step, must support each answer.

$\boxed{-3}$, $\boxed{6}$



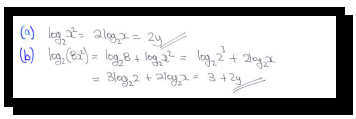
Question 5 **(**+)**

Given that $y=\log_2 x$, write each of the following expressions in terms of y.

a) $\log_2 x^2$

b) $\log_2\left(8x^2\right)$

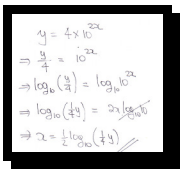
$\boxed{2y}$, $\boxed{3+2y}$



Question 6 (+)**

Given that $y = 4\times 10^{2x}$ express x in terms of y, giving an exact simplified answer in terms of logarithms base 10.

$$\boxed{x = \frac{1}{2}\log_{10}\left(\frac{1}{4}y\right)}$$



Question 7 (+)**

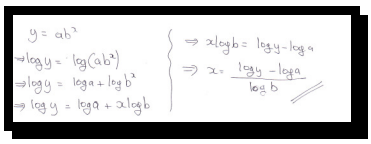
An exponential curve has equation

$$y = ab^x,\ x \in \mathbb{R},$$

where a and b are non zero constants.

Make x the subject of the above equation, giving the final answer in terms of logarithms base 10.

$$\boxed{x = \frac{\log y - \log a}{\log b}}$$

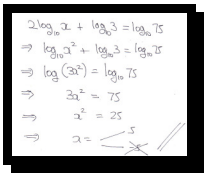


Question 8 (+)**

Solve the following logarithmic equation

$$2\log_{10} x+\log_{10} 3=\log_{10} 75 .$$

$\boxed{x=5,\ x\neq -5}$

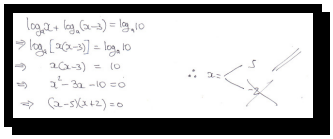


Question 9 (+)**

Solve the following logarithmic equation

$$\log_a x+\log_a (x-3)=\log_a 10 .$$

$\boxed{\ }$, $\boxed{x=5,\ x\neq -2}$



Question 10 (***)

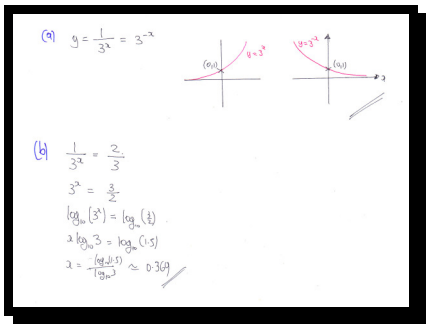
An exponential curve C has equation

$$y=\frac{1}{3^x},\ x\in\mathbb{R}.$$

a) Sketch the graph of C.

b) Solve the equation $y=\frac{2}{3}$, giving the answer correct to 3 significant figures.

$\square$, 0.369



Question 11 (***)

Given that

$$p=\log_a 4 \quad \text{and} \quad q=\log_a 5,$$

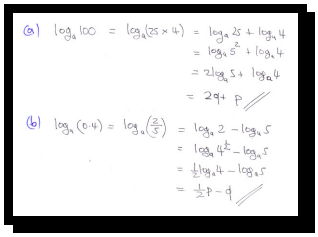
express each of the following logarithms in terms of p and q.

a) $\log_a 100$

b) $\log_a 0.4$

The final answers may not contain any logarithms.

$\square$, $\boxed{p+2q}$, $\boxed{\frac{1}{2}p-q}$

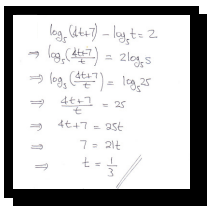


Question 12 (***)

Solve the following logarithmic equation

$$\log_5(4t+7)-\log_5 t=2.$$

$\square$, $\boxed{t=\frac{1}{3}}$



Question 13 (***)

Given that

$$p = \log_2 3 \quad \text{and} \quad q = \log_2 5,$$

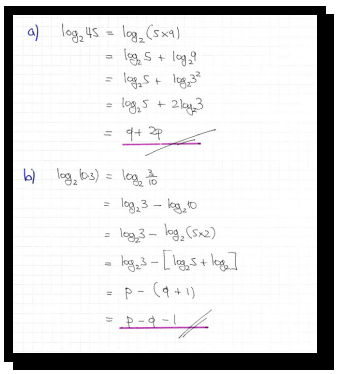
express each of the following logarithms in terms of p and q.

a) $\log_2 45$

b) $\log_2 0.3$

The final answers may not contain any logarithms.

$\boxed{}$, $\boxed{2p+q}$, $\boxed{p-q-1}$



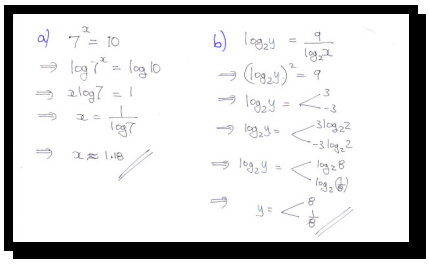
Question 14 (***)

Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $7^x = 10$.

b) $\log_2 y = \dfrac{9}{\log_2 y}$.

$\boxed{}$, $\boxed{x \approx 1.18}$, $\boxed{y = \frac{1}{8}, 8}$

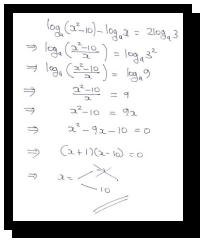


Question 15 (***)

Solve the following logarithmic equation for x.

$$\log_a (x^2 - 10) - \log_a x = 2\log_a 3 .$$

$\boxed{x = 10,\ x \neq -1}$

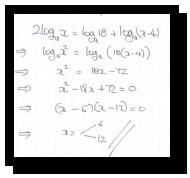


Question 16 (***)

Solve the following logarithmic equation for x.

$$2\log_a x = \log_a 18 + \log_a (x-4) .$$

$\square$, $\boxed{x = 6,\ 12}$

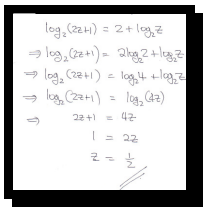


Question 17 (***)

Solve the following logarithmic equation

$$\log_2 (2z+1) = 2 + \log_2 z .$$

$\square$, $\boxed{z = \frac{1}{2}}$

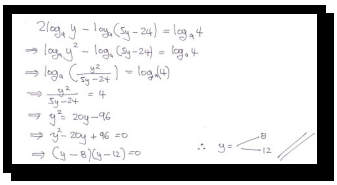


Question 18 (***)

Solve the following logarithmic equation for y.

$$2\log_a y - \log_a(5y-24) = \log_a 4 .$$

$$\boxed{x = 8,\ 12}$$



Question 19 (***)

It is given that x satisfies the logarithmic equation

$$\log_a x = 2(\log_a k - \log_a 2),$$

where $k > 0$, $a > 0$, $a \neq 1$.

a) Find x in terms of k, giving the answer in a form not involving logarithms.

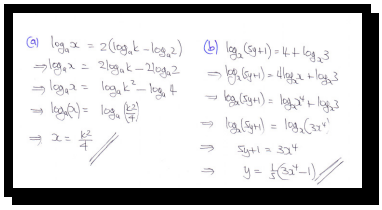
Suppose instead that x satisfies

$$\log_x(5y+1) = 4 + \log_x 3$$

where $x > 0$, $x \neq 1$ and $y > 0$, $y \neq 1$.

b) Solve the above equation expressing y in terms of x, giving the answer in a form not involving logarithms.

$$\boxed{x = \frac{k^2}{4}}, \quad \boxed{y = \frac{3x^4 - 1}{5}}$$

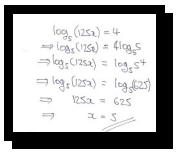


Question 20 (***)

Solve the following logarithmic equation

$$\log_5(125x)=4\,.$$

$$\boxed{x=5}$$

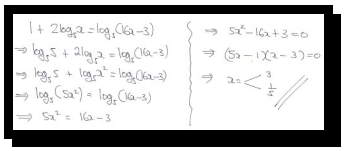


Question 21 (***)

Solve the following logarithmic equation

$$1+2\log_5 x=\log_5(16x-3)\,.$$

$$\boxed{x=3,\ x=\frac{1}{5}}$$



Question 22 (***)

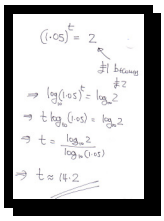
Every £1 invested in a saving scheme gains interest at the rate of 5% per annum so that the total value of this £1 investment after t years is £ y.

This is modelled by the equation

$$y = 1.05^t, \ t \geq 0.$$

Find after how many years the investment will double.

$$\boxed{t \approx 14.2}$$



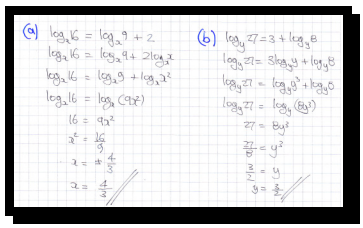
Question 23 (***)

Solve each of the following logarithmic equations.

a) $\log_x 16 = \log_x 9 + 2$.

b) $\log_y 27 = 3 + \log_y 8$.

$$\boxed{x = \frac{4}{3}, \ x \neq -\frac{4}{3}}, \quad \boxed{y = \frac{3}{2}}$$



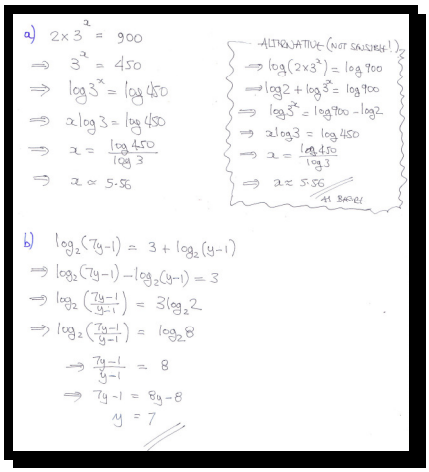
Question 24 (***)

Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $2\times 3^{x}=900$.

b) $\log_2(7y-1)=3+\log_2(y-1)$.

$\boxed{}$, $\boxed{x\approx 5.56}$, $\boxed{y=7}$



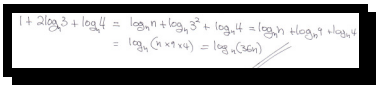
Question 25 (*+)**

Simplify fully

$$1+2\log_n 3+\log_n 4,$$

giving the final answer as a single logarithm.

$\boxed{\log_n(36n)}$



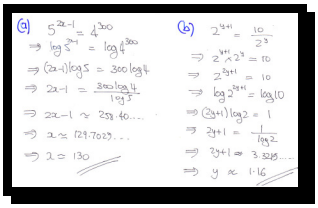
Question 26 (*+)**

Solve each of the following exponential equations, giving the final answers correct to 3 significant figures.

a) $5^{2x-1}=4^{300}$.

b) $2^{y+1}=\dfrac{10}{2^y}$.

$\boxed{}$, $\boxed{x\approx 130}$, $\boxed{y\approx 1.16}$

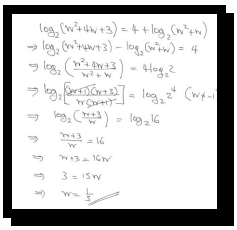


Question 27 (*+)**

Solve the following logarithmic equation

$$\log_2(w^2+4w+3)=4+\log_2(w^2+w)\,,\quad w\neq -1\,.$$

$\boxed{w=\dfrac{1}{5}}$



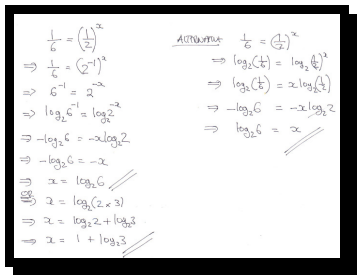
Question 28 (*+)**

Solve the following exponential equation

$$\frac{1}{6}=\left(\frac{1}{2}\right)^{x},$$

giving the answer as single logarithm of base 2.

$$\boxed{x=\log_2 6=1+\log_2 3}$$



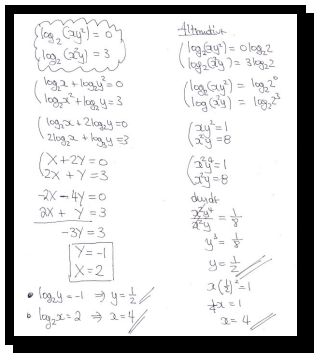
Question 29 (*+)**

Solve the following simultaneous logarithmic equations

$$\log_2\left(xy^2\right)=0$$

$$\log_2\left(x^2y\right)=3\,.$$

$$\square\,,\quad \boxed{x=4,\ \ y=\frac{1}{2}}$$

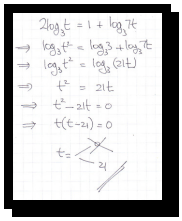


Question 30 (*+)**

Solve the following logarithmic equation

$$2\log_3 t = 1 + \log_3 7t .$$

$\boxed{t = 21,\ t \neq 0}$

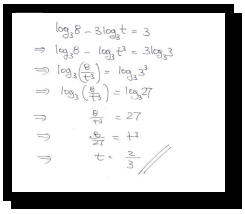


Question 31 (*+)**

Solve the following logarithmic equation

$$\log_3 8 - 3\log_3 t = 3 .$$

$\square$, $\boxed{t = \frac{2}{3}}$

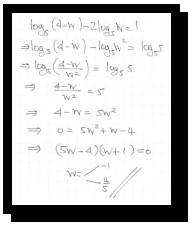


Question 32 (*+)**

Solve the following logarithmic equation

$$\log_5(4-w)-2\log_5 w=1.$$

$\square$, $\boxed{w=\frac{4}{5},\ w\neq -1}$

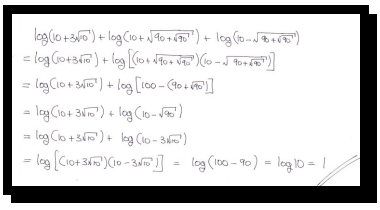


Question 33 (*+)**

Simplify fully the following logarithmic expression, showing clearly all the workings.

$$\log\left(10+3\sqrt{10}\right)+\log\left(10+\sqrt{90+\sqrt{90}}\right)+\log\left(10-\sqrt{90+\sqrt{90}}\right).$$

$\boxed{1}$

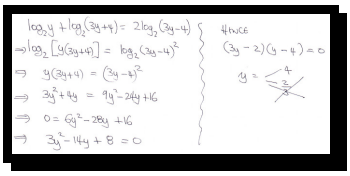


Question 34 (*+)**

Solve the following logarithmic equation

$$\log_2 y+\log_2(3y+4)=2\log_2(3y-4).$$

$$\boxed{y=4,\ y\neq\frac{2}{3}}$$

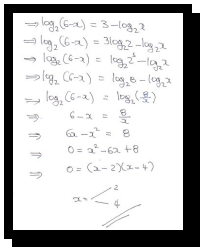


Question 35 (*+)**

Solve the following logarithmic equation

$$\log_2(6-x)=3-\log_2 x.$$

$$\boxed{x=2,\ 4}$$

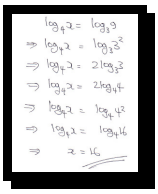


Question 36 (*+)**

Solve the following logarithmic equation

$$\log_4 x = \log_3 9 .$$

$\boxed{x = 16}$



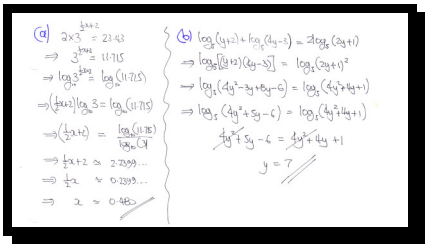
Question 37 (*+)**

Solve each of the following equations.

a) $2 \times 3^{\frac{1}{2}x+2} = 23.43$.

b) $\log_5 (y+2) + \log_5 (4y-3) = 2\log_5 (2y+1)$.

$\boxed{x \approx 0.480}$, $\boxed{y = 7}$



Question 38 (*+)**

The population P of a certain town in time t years is modelled by the equation

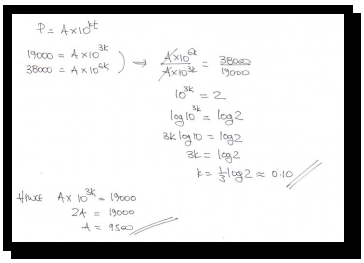
$$P = A\times 10^{kt},\ t \geq 0,$$

where A and k are non zero constants.

When $t = 3$, $P = 19000$ and when $t = 6$, $P = 38000$.

Find the value of A and the value of k, correct to 2 significant figures.

$\boxed{}$, $\boxed{A = 9500,\ k = 0.10}$

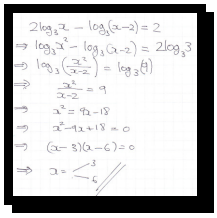


Question 39 (*+)**

Solve the following logarithmic equation

$$2\log_3 x - \log_3(x-2) = 2.$$

$\boxed{}$, $\boxed{x = 3,\ 6}$

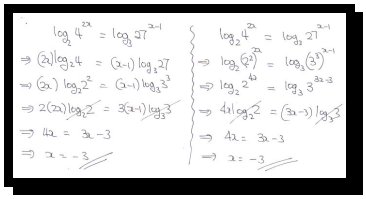


Question 40 (*+)**

Solve the following logarithmic equation

$$\log_2 4^{2x} = \log_3 27^{x-1}.$$

$\boxed{x=-3}$



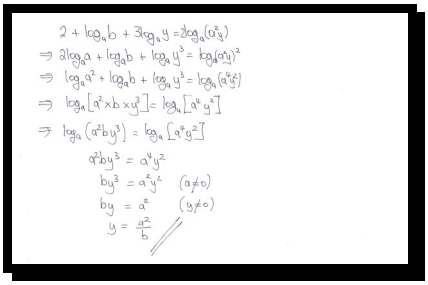
Question 41 (*+)**

Given that $a \neq 0$, $b \neq 0$, $y \neq 0$ and

$$2+\log_a b+3\log_a y = 2\log_a\left(a^2 y\right),$$

express y in terms of a and b, in a form **not** involving logarithms.

$\boxed{y=\dfrac{a^2}{b}}$

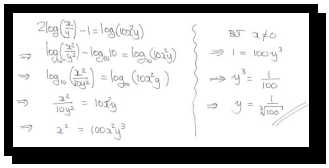


Question 42 (*+)**

$$2\log\left(\frac{x}{y}\right)-1=\log(10x^2y)\,,\ x\neq 0\,,\ y\neq 0\,.$$

Find the exact value of y.

$$\boxed{y=\frac{1}{\sqrt[3]{100}}}$$



Question 43 (*+) non calculator**

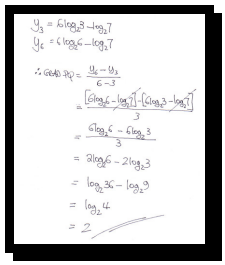
The points P and Q lie on the curve with equation

$$y=6\log_2 x-\log_2 7\,,\ x>0\,.$$

The x coordinates of P and Q are 3 and 6, respectively.

Find the gradient of the straight line segment PQ.

$$\boxed{2}$$



Question 44 **(***+)**

$$y = 3 \times 2^x .$$

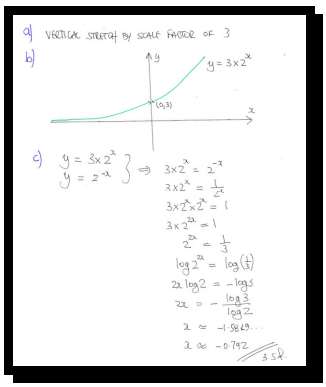
a) Describe the geometric transformation which maps the graph of the curve with equation $y = 2^x$, onto the graph of the curve with equation $y = 3 \times 2^x$.

b) Sketch the graph of $y = 3 \times 2^x$.

The curve with equation $y = 2^{-x}$ intersects the curve with equation $y = 3 \times 2^x$ at the point P.

c) Determine, correct to 3 decimal places, the x coordinate of P.

$\boxed{}$, $\boxed{\text{vertical stretch by scale factor } 3}$, $\boxed{x \approx -0.792}$



Question 45 (*+)**

It is given that $p=\log_6 25$ and $q=\log_6 2$.

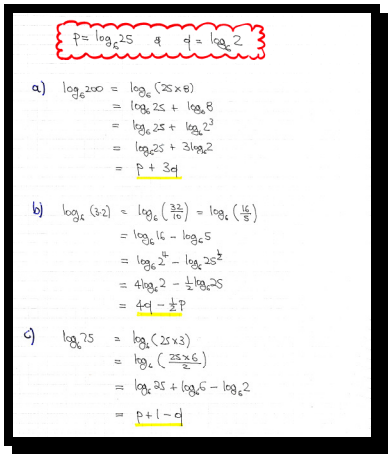
Express in terms of p and q each of the following expressions

a) $\log_6 200$

b) $\log_6 3.2$

c) $\log_6 75$

$\boxed{}$, $\boxed{\log_6 200=p+3q}$, $\boxed{\log_6 3.2=-\frac{1}{2}p+4q}$, $\boxed{\log_6 75=p-q+1}$



Question 46 (****)

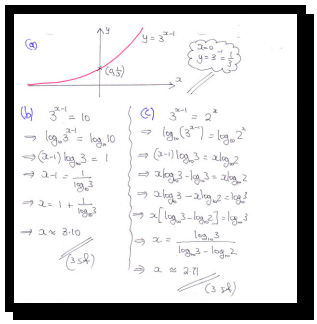
$$y = 3^{x-1}, \ x \in \mathbb{R}.$$

a) Sketch the graph of $y = 3^{x-1}$ showing the coordinates of all intercepts with the coordinate axes.

b) Find to 3 significant figures the x coordinate of the point where the curve $y = 3^{x-1}$ intersects with the straight line with equation $y = 10$.

c) Determine to 3 significant figures the x coordinate of the point where the curve $y = 3^{x-1}$ intersects with the curve $y = 2^x$.

$\boxed{3.10}$, $\boxed{2.71}$

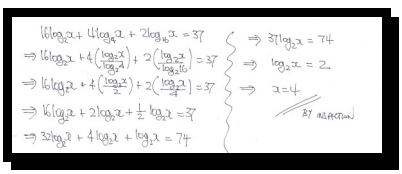


Question 47 (****)

Solve the following logarithmic equation

$$16\log_2 x + 4\log_4 x + 2\log_{16} x = 37, \quad x > 0.$$

$\boxed{x = 4}$



Question 48 (****)

In 1970 the average weekly pay of footballers in a certain club was £100.

The average weekly pay, £P, is modelled by the equation

$$P = A \times b^t ,$$

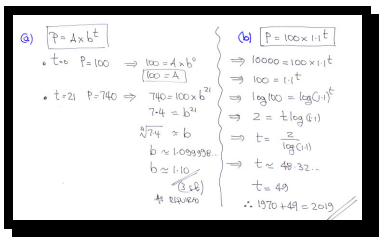
where t is the number of years since 1970, and A and b are positive constants.

In 1991 the average weekly pay of footballers in the same club had risen to £740.

a) Find the value of A and show that $b = 1.10$, correct to three significant figures.

b) Determine the year when the average weekly pay of footballers in this club will first exceed £10000.

$\boxed{}$, $\boxed{A = 100}$, $\boxed{2019}$



Question 49 (****)

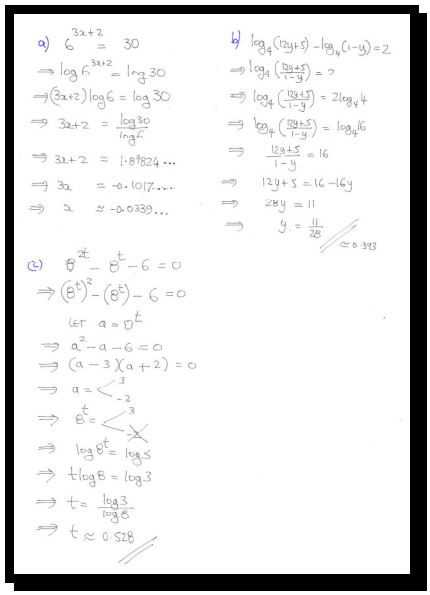
Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $6^{3x+2}=30$.

b) $\log_4(12y+5)-\log_4(1-y)=2$.

c) $8^{2t}-8^t-6=0$.

$$\boxed{x\approx -0.0339}\,,\quad \boxed{y=\frac{11}{28}\approx 0.393}\,,\quad \boxed{t\approx 0.528}$$



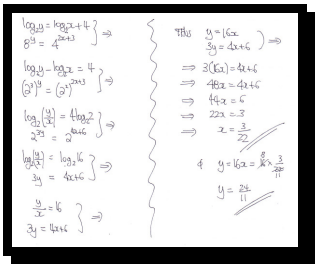
Question 50 (****)

Solve the following simultaneous equations, giving your answers as exact fractions

$$\log_2 y = \log_2 x + 4$$

$$8^y = 4^{2x+3}.$$

$$\boxed{x = \frac{3}{22},\ y = \frac{24}{11}}$$

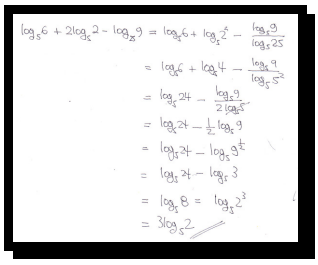


Question 51 (****)

Show clearly that

$$\log_5 6 + 2\log_5 2 - \log_{25} 9 = 3\log_5 2.$$

$$\boxed{\text{proof}}$$

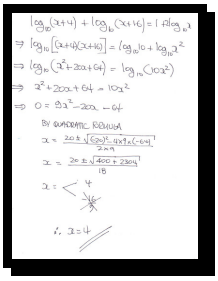


Question 52 (****)

Solve the following logarithmic equation

$$\log_{10}(x+4)+\log_{10}(x+16)=1+2\log_{10}x\,.$$

$$\boxed{x=4,\ x\neq-\frac{16}{9}}$$



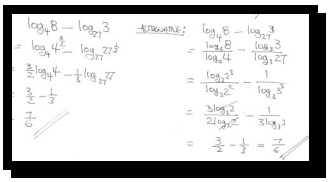
Question 53 (****)

Simplify

$$\log_4 8-\log_{27}3,$$

giving the final answer as a simplified fraction.

$$\boxed{\frac{7}{6}}$$



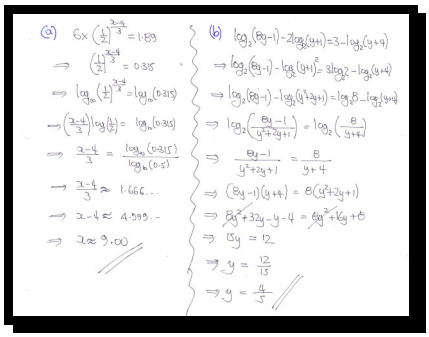
Question 54 **(****)**

Solve each of the following equations.

a) $6\times\left(\frac{1}{2}\right)^{\frac{x-4}{3}}=1.89$.

b) $\log_2(8y-1)-2\log_2(y+1)=3-\log_2(y+4)$.

$\boxed{}$, $\boxed{x\approx 9.00}$, $\boxed{y=\frac{4}{5}}$



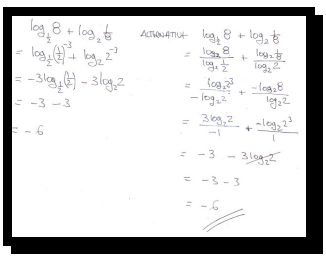
Question 55 **(****)**

Simplify

$$\log_{\frac{1}{2}}8+\log_2\frac{1}{8},$$

giving the final answer as an integer.

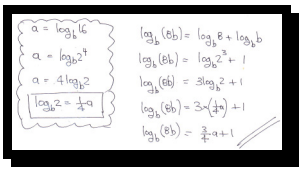
$\boxed{-6}$



Question 56 (****)

Given that $a = \log_b 16$, express $\log_b(8b)$ in terms of a.

$$1+\frac{3}{4}a$$

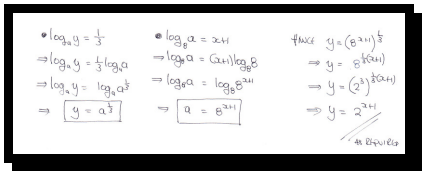


Question 57 (****)

$$\log_a y = \frac{1}{3} \quad \text{and} \quad \log_8 a = x+1 .$$

Show clearly that $y = 2^{x+1}$

proof



Question 58 **(****)**

It is given that

$$p=\log_6 25 \qquad \text{and} \qquad q=\log_6 2 .$$

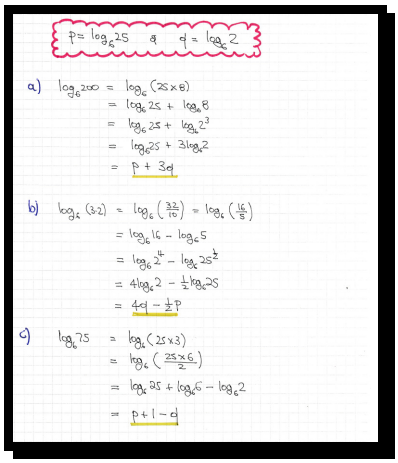
Simplify each of the following logarithms, giving the final answers in terms of p, q and positive integers, where appropriate.

i. $\log_6(200)$.

ii. $\log_6(3.2)$.

iii. $\log_6(75)$.

$\boxed{}$, $\boxed{p+3q}$, $\boxed{4q-\frac{1}{2}p}$, $\boxed{p+1-q}$

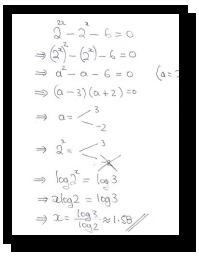


Question 59 (****)

Solve the following exponential equation, giving the answer correct to 3 s.f.

$$2^{2x}-2^{x}-6=0\,.$$

$\boxed{x\approx 1.58}$



Question 60 (****)

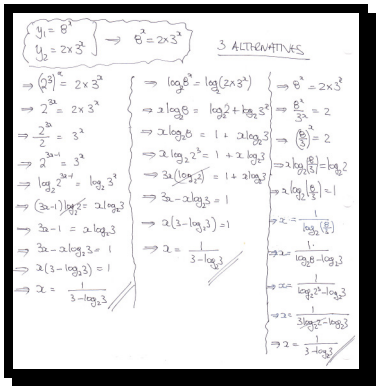
Two curves C_1 and C_2 are defined for all values of x and have respective equations

$$y_1=8^x \qquad \text{and} \qquad y_2=2\times 3^x\,.$$

Show that the x coordinate of the point of intersection of the two curves is given by

$$\frac{1}{3-\log_2 3}\,.$$

$\boxed{\text{proof}}$

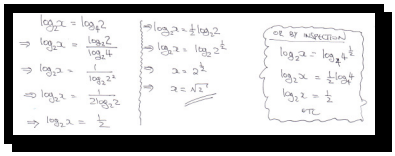


Question 61 (****)

Solve the following logarithmic equation

$$\log_2 x = \log_4 2 .$$

$$\boxed{x = \sqrt{2}}$$



Question 62 **(****)**

The functions f and g are defined as

$$f(x)=3\left(2^{-x}\right)-1,\quad x\in\mathbb{R},\quad x\geq 0$$

$$g(x)=\log_2 x,\quad x\in\mathbb{R},\quad x\geq 1.$$

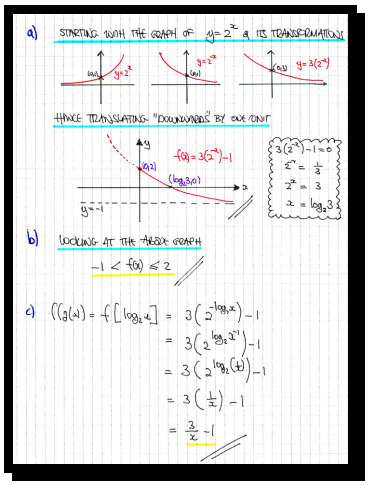
a) Sketch the graph of f.

- Mark clearly the exact coordinates of any points where the curve meets the coordinate axes. Give the answers, where appropriate, in exact form in terms of logarithms base 2.
- Mark and label the equation of the asymptote to the curve.

b) State the range of f.

c) Find $f\left(g(x)\right)$ in its simplest form.

$\boxed{}$, $\boxed{(0,2)}$, $\boxed{(\log_2 3,0)}$, $\boxed{y=-1}$, $\boxed{-1<f(x)\leq 2}$, $\boxed{f\left(g(x)\right)=\frac{6}{x}-1}$

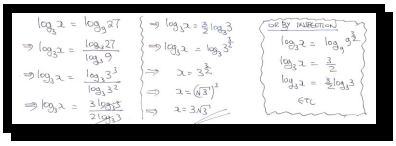


Question 63 (****)

Solve the following logarithmic equation

$$\log_3 x = \log_9 27 .$$

$$\boxed{x = 3\sqrt{3}}$$



Question 64 (****)

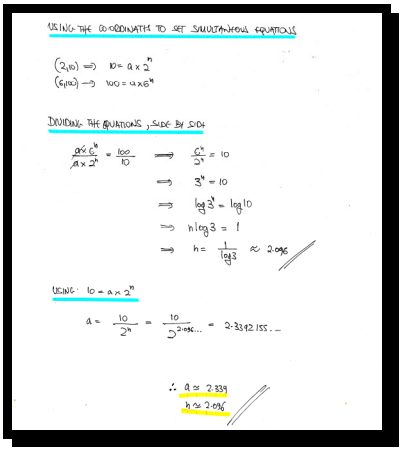
The points $(2,10)$ and $(6,100)$ lie on the curve with equation

$$y = ax^n ,$$

where a and n are non zero constants.

Find, to three decimal places, the value of a and the value of n.

$$\boxed{}, \ \boxed{a = 2.339}, \ \boxed{n \approx 2.096}$$

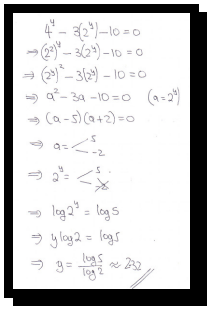


Question 65 (****)

Solve the following exponential equation, giving the answer correct to 3 s.f.

$$4^y - 3\left(2^y\right) - 10 = 0 .$$

$\boxed{}$, $\boxed{y \approx 2.32}$

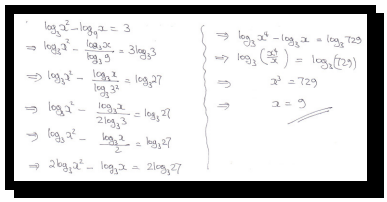


Question 66 (****)

Solve the following logarithmic equation

$$\log_3 x^2 - \log_9 x = 3 .$$

$\boxed{x = 9, x \neq 0}$



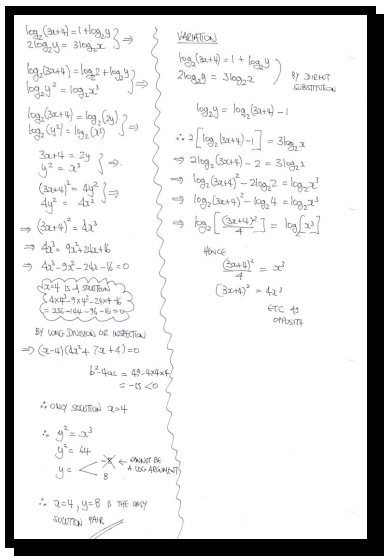
Question 67 (****)

Show that $x=4$ and $y=8$ is the only solution pair of the following logarithmic simultaneous equations

$$\log_2(3x+4)=1+\log_2 y\,.$$

$$2\log_2 y=3\log_2 x\,.$$

proof



Question 68 (****)

A population P of an endangered species of animals was introduced to a park.

The population obeys the equation

$$P=\frac{125ka^t}{k+2a^t},\ t\geq 0,$$

where k and a are positive constants, and t is the time in years since the species was introduced to the park .

Initially 100 individual animals were introduced to the park, and this population doubled after 5 years.

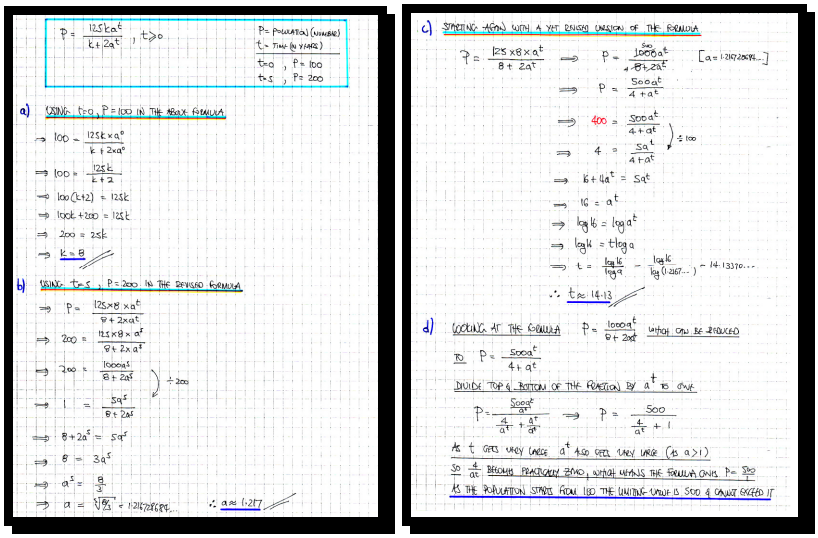
a) Show that $k=8$.

b) Find the value of a, correct to 4 significant figures.

c) Determine the value of t when the $P=400$.

d) Explain why this population cannot exceed 500.

$\boxed{}$, $\boxed{a\approx 1.217}$, $\boxed{t\approx 14.13}$

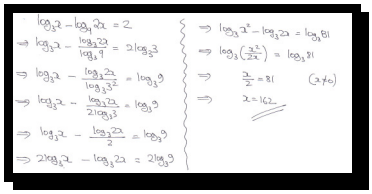


Question 69 (****)

Solve the following logarithmic equation

$$\log_3 x - \log_9 2x = 2 .$$

$$\boxed{x = 162,\ x \neq 0}$$

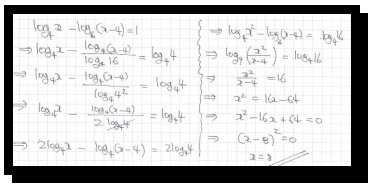


Question 70 (****)

Solve the following logarithmic equation

$$\log_4 x - \log_{16}(x-4) = 1 .$$

$$\boxed{\ \ \ \ }, \ \boxed{x = 8}$$



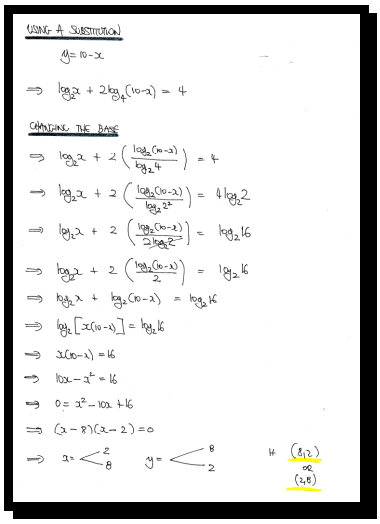
Question 71 (****)

Solve the following simultaneous equations

$$\log_2 x+2\log_4 y=4$$

$$x+y=10$$

$\boxed{}$, $\boxed{x=2,\ y=8 \quad \text{or} \quad x=8,\ y=2}$

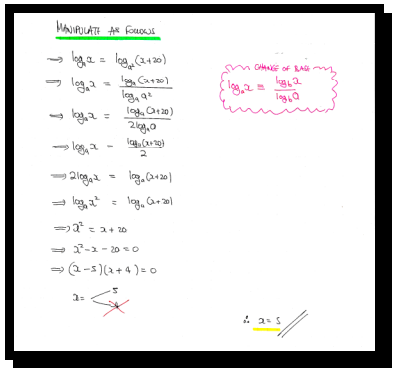


Question 72 (****)

Given that a is positive constant greater than 1, solve the following logarithmic equation

$$\log_a x=\log_{a^2}(x+20).$$

$\boxed{}$, $\boxed{x=5}$



Question 73 **(****)**

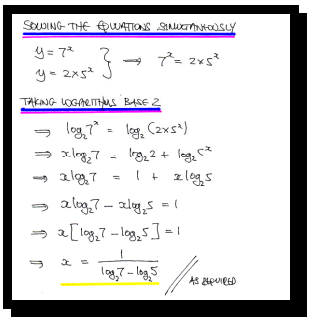
Two curves C_1 and C_2 are defined for all values of x and have respective equations

$$y_1 = 7^x \quad \text{and} \quad y_2 = 2\times 5^x .$$

Show that the x coordinate of the point of intersection of the two curves is given by

$$\frac{1}{\log_2 7 - \log_2 5} .$$

$\boxed{}$, $\boxed{\text{proof}}$

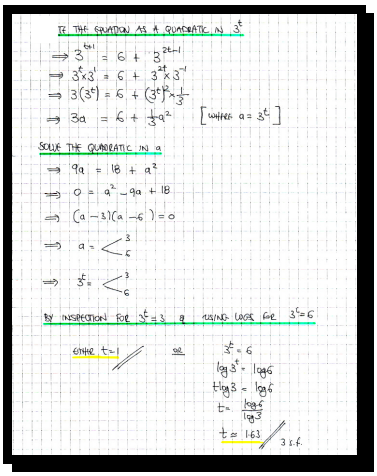


Question 74 **(****)**

Solve the following exponential equation, giving the answer correct to 3 s.f.

$$3^{t+1} = 6 + 3^{2t-1}$$

$\boxed{}$, $\boxed{t = 1 \text{ or } t \approx 1.63}$



Question 75 (****)

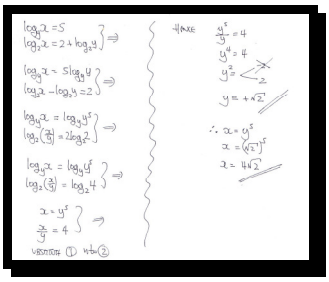
Solve the following simultaneous logarithmic equations

$$\log_y x = 5$$

$$\log_2 x = 2 + \log_2 y \, .$$

Give the answer as exact simplified surds.

$\boxed{}$, $\boxed{x = 4\sqrt{2}, \quad y = \sqrt{2}}$

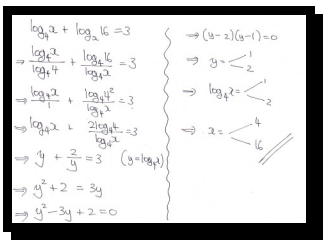


Question 76 (****)

Solve the following logarithmic equation

$$\log_4 x + \log_x 16 = 3 \, , \ x > 0 \, , \ x \neq 1 \, .$$

$\boxed{x = 4, \ 16}$

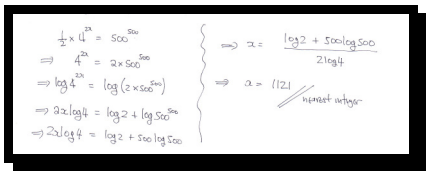


Question 77 (****)

Find, to the nearest integer, the solution of the following exponential equation

$$\frac{1}{2}\times 4^{2x}=500^{500}.$$

$$\boxed{x\approx 1121}$$



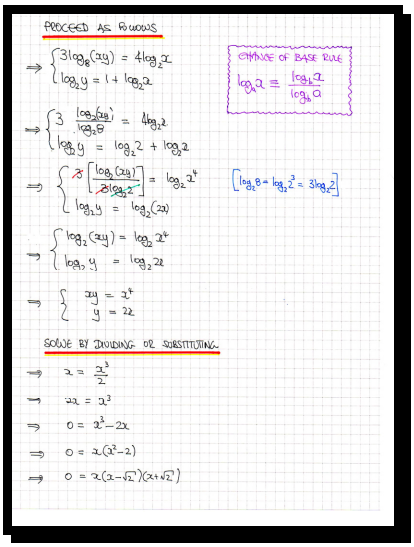
Question 78 (****)

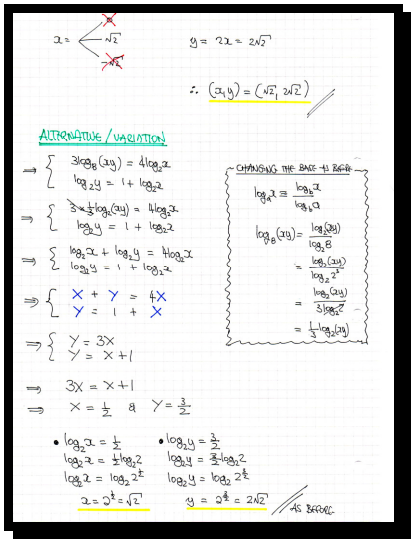
Solve the following simultaneous logarithmic equations.

$$3\log_8(xy)=4\log_2 x$$

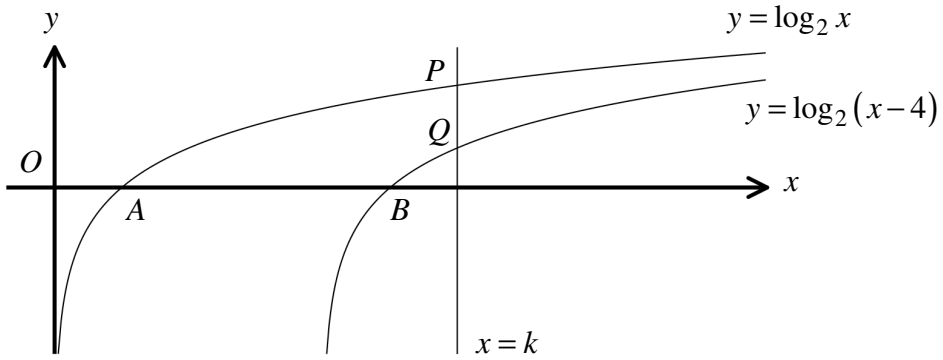
$$\log_2 y=1+\log_2 x$$

$$\boxed{\quad},\ \boxed{x=\sqrt{2},\quad y=\sqrt{8}}$$





Question 79 **(****)**



The figure above shows the graphs of the curves with equations

$$y = \log_2 x, \quad \text{and} \quad y = \log_2 (x-4).$$

The points A and B are the respective x intercepts of the two graphs.

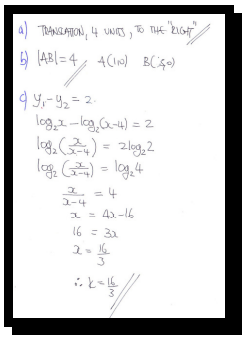
a) Describe the geometric transformation which maps the graph of $y = \log_2 x$ onto the graph of $y = \log_2 (x-4)$.

b) State the distance AB.

The straight line with equation $x = k$, where k is a positive constant, meets the graph of $y = \log_2 x$ at the point P and the graph of $y = \log_2 (x-4)$ at the point Q.

c) Given that the distance PQ is 2 units determine the value of k.

$\boxed{}$, $\boxed{\text{translation, 4 units to the "right"}}$, $\boxed{|AB| = 4}$, $\boxed{k = \frac{16}{3}}$



Question 80 (****)

The radioactive decay of a phosphorus isotope is modelled by the equation

$$m = m_0 \times 2^{-0.2t}, \ t \geq 0$$

where m is the mass of phosphorus left, in grams, and t is the time in days since the decay started. The initial mass of phosphorus is m_0.

a) Find the mass of the phosphorus left, when an initial mass of 20 grams is left to decay for 10 days, according to this model.

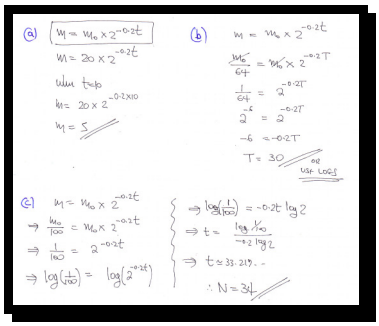
An initial mass, m_0 grams, of this type of phosphorus decays to $\dfrac{m_0}{64}$ grams in T days.

b) Find the value of T.

After N days have elapsed, less than 1% of this type of phosphorus remains from its initial mass m_0.

c) Find the smallest integer value of N.

$\boxed{}$, $\boxed{m = 5}$, $\boxed{T = 30}$, $\boxed{N = 34}$

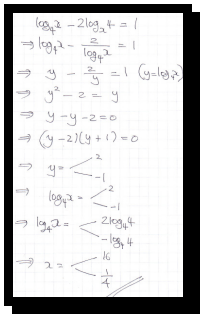


Question 81 (****)

Solve the following logarithmic equation

$$\log_4 x - 2\log_x 4 = 1 .$$

$\square$, $\boxed{x = \frac{1}{4}, 16}$



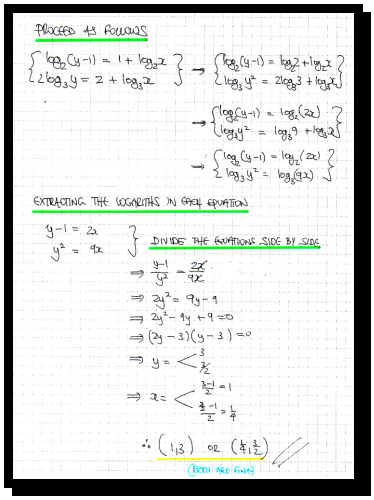
Question 82 (****)

Solve the following simultaneous logarithmic equations.

$$\log_2 (y-1) = 1 + \log_2 x$$

$$2\log_3 y = 2 + \log_3 x .$$

$\square$, $\boxed{x = 1,\ y = 3 \text{ or } x = \frac{1}{4}, y = \frac{3}{2}}$

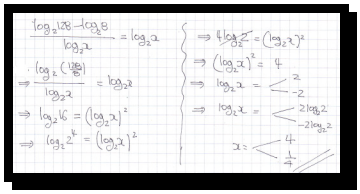


Question 83 (****)

Solve the following logarithmic equation

$$\frac{\log_2 128-\log_2 8}{\log_2 x}=\log_2 x\,.$$

$\square$, $\boxed{x=4,\ \frac{1}{4}}$



Question 84 (****)

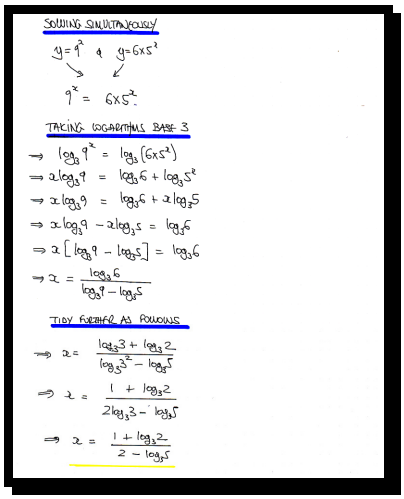
Two curves C_1 and C_2 are defined for all values of x and have respective equations

$$y_1=9^x \quad \text{and} \quad y_2=6\times 5^x\,.$$

Show that the x coordinate of the point of intersection of the two curves is given by :

$$\frac{1+\log_3 2}{2-\log_3 5}$$

$\square$, $\boxed{\text{proof}}$



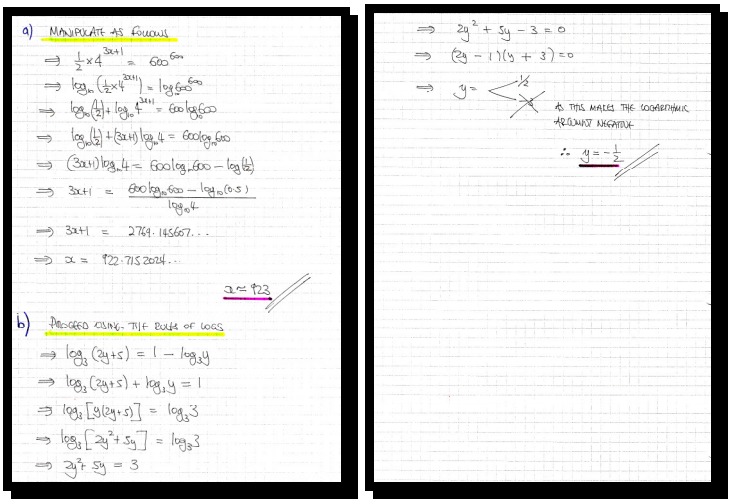
Question 85 (**+)**

Solve each of the following equations.

a) $\frac{1}{2}\times 4^{3x+1}=600^{600}$.

b) $\log_3(2y+5)=1-\log_3 y$.

$\boxed{}$, $\boxed{x=922.7152024...}$, $\boxed{y=\frac{1}{2}}$



Question 86 (**+)**

$$y=\left(\frac{1}{2}\right)^x.$$

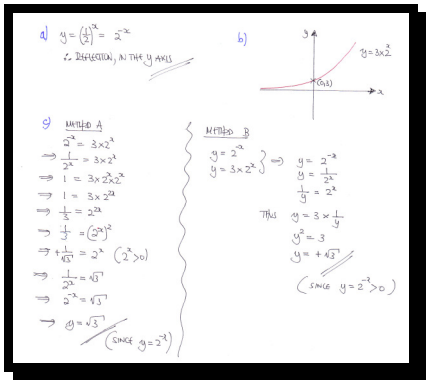
a) Describe the geometric transformation which maps the graph of the curve with equation $y=2^x$, onto the graph of the curve with equation $y=\left(\frac{1}{2}\right)^x$.

b) Sketch the graph of $y=3\times 2^x$.

The curve with equation $y=\left(\frac{1}{2}\right)^x$ intersects the curve with equation $y=3\times 2^x$ at the point P.

c) Determine, as an exact simplified surd, the y coordinate of P.

$\boxed{}$, $\boxed{\text{reflection in the } y \text{ axis}}$, $\boxed{y=\sqrt{3}}$



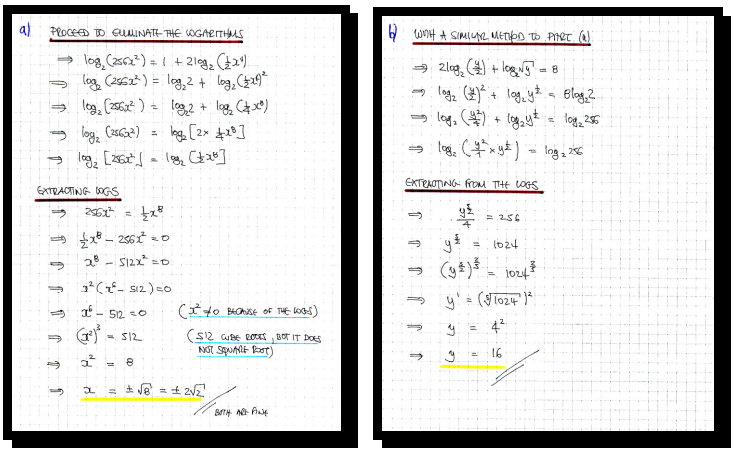
Question 87 (**+)**

Solve each of the following logarithmic equations, giving the answers in exact simplified form where appropriate.

a) $\log_2(256x^2)=1+2\log_2\left(\frac{1}{2}x^4\right)$.

b) $2\log_2\left(\frac{y}{2}\right)+\log_2\sqrt{y}=8$.

$\boxed{}$, $\boxed{x=\pm\sqrt{8}}$, $\boxed{y=16}$



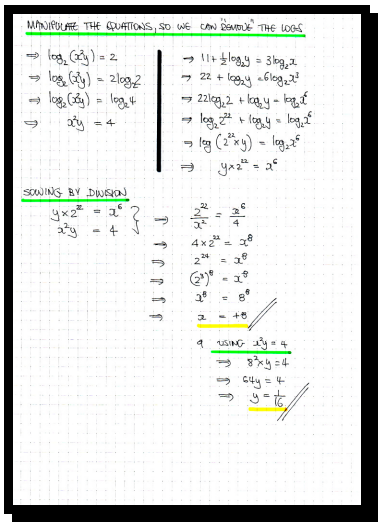
Question 88 (**+)**

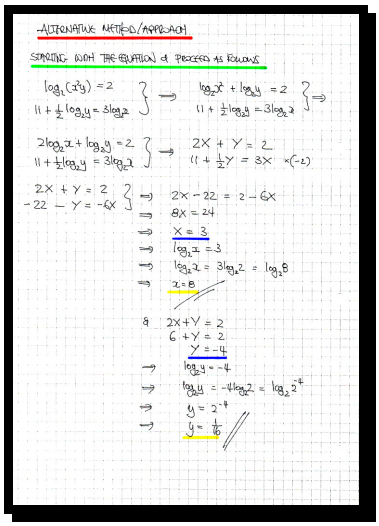
Solve the following simultaneous logarithmic equations.

$$\log_2\left(x^2y\right)=2$$

$$11+\frac{1}{2}\log_2 y=3\log_2 x\,.$$

$\square$, $\boxed{x=8,\quad y=\frac{1}{16}}$





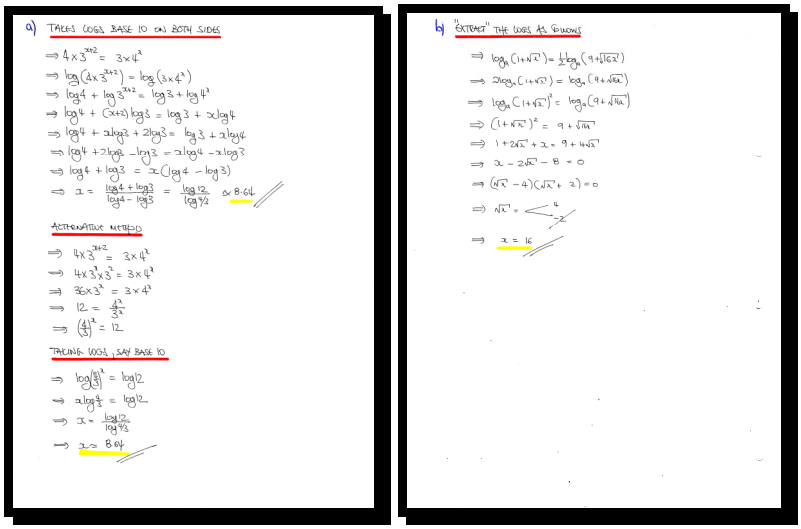
Question 89 (**+)**

Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $4\times 3^{x+2}=3\times 4^{x}$.

b) $\log_a\left(1+\sqrt{x}\right)=\frac{1}{2}\log_a\left(9+\sqrt{16x}\right)$.

$\boxed{}$, $\boxed{x\approx 8.64}$, $\boxed{y=16}$



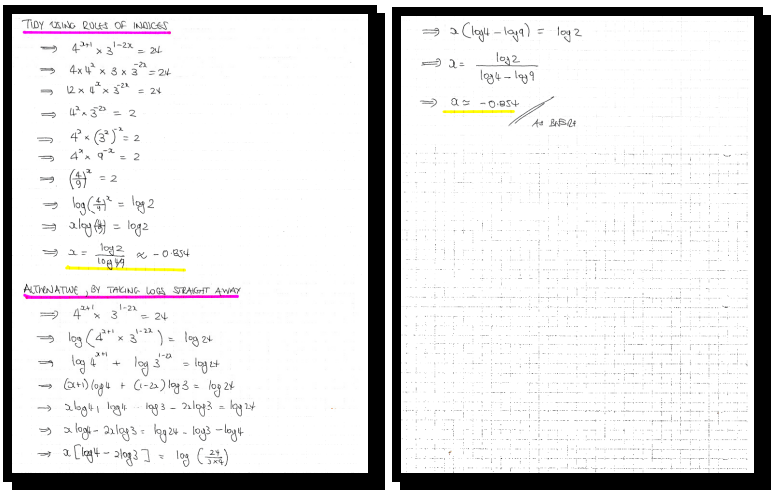
Question 90 (**+)**

Solve the following exponential equation.

$$4^{x+1}\times 3^{1-2x}=24\,.$$

Give the answer correct to 3 decimal places.

$\boxed{}$, $\boxed{x\approx -0.855}$

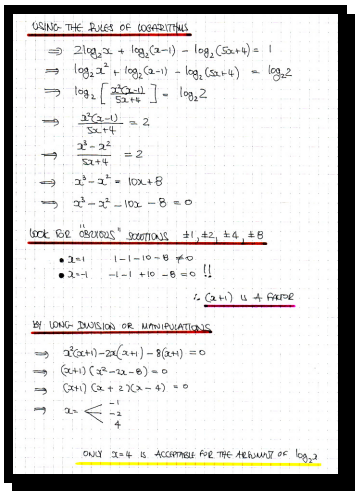


Question 91 (**+)**

Solve the following logarithmic equation.

$$2\log_2 x+\log_2(x-1)-\log_2(5x+4)=1.$$

$\boxed{}$, $\boxed{x=4}$



Question 92 (**+)**

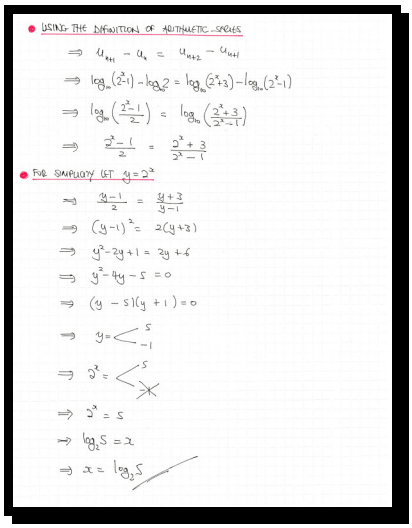
The following three numbers

$$\log_{10} 2\,, \quad \log_{10}\left(2^x-1\right), \quad \log_{10}\left(2^x+3\right),$$

are consecutive terms in an arithmetic progression.

Determine the value of x as an exact logarithm, of base 2.

$\boxed{}$, $\boxed{x=\log_2 5}$



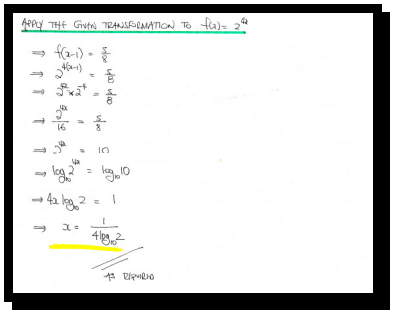
Question 93 (**+)**

$$f(x)=2^{4x}$$

Show that the solution of the equation $f(x-1)=\frac{5}{8}$ is given by

$$x=\frac{1}{4\log_{10}2}.$$

[ ] , [proof]



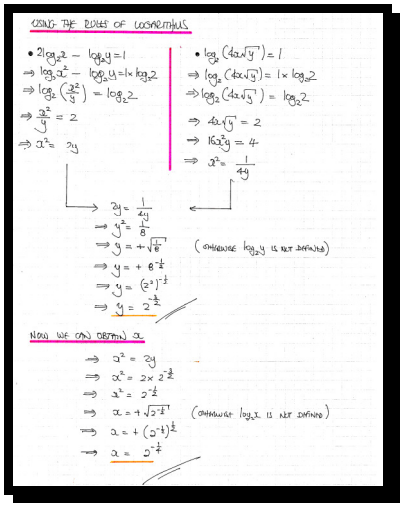
Question 94 **(****+)**

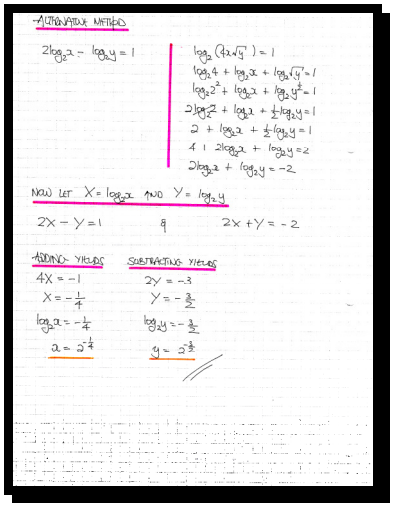
$$2\log_2 x-\log_2 y=1$$

$$\log_2\left(4x\sqrt{y}\right)=1\,.$$

Solve the above simultaneous logarithmic equations, giving the final answers as exact powers of 2.

$\square$, $\boxed{x=2^{-\frac{1}{4}},\ y=2^{-\frac{3}{2}}}$



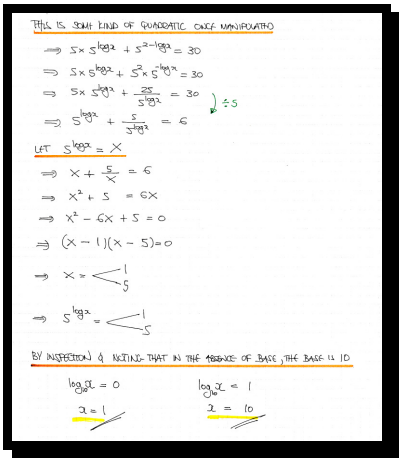


Question 95 (**+)**

Solve the following logarithmic equation

$$5\times 5^{\log x}+5^{2-\log x}=30, \quad x>0.$$

$\boxed{}$, $\boxed{x=1,\ x=10}$



Question 96 (**+)**

It is given that

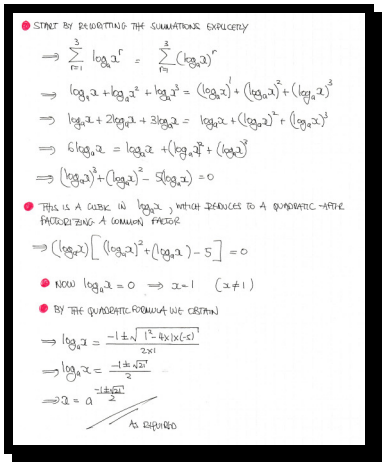
$$\sum_{r=1}^{3} \log_a x^r = \sum_{r=1}^{3} (\log_a x)^r ,$$

where a and x are positive numbers such that $x \neq a$, $x \neq 1$ and $a > 1$.

Show clearly that

$$x = a^{\frac{-1\pm\sqrt{21}}{2}} .$$

, proof

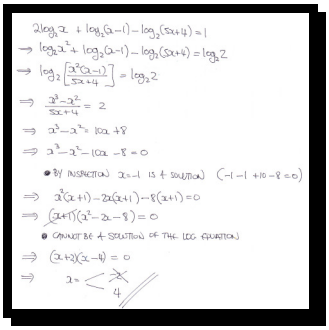


Question 97 (**+)**

$$2\log_2 x+\log_2(x-1)-\log_2(5x+4)=1.$$

Find the only real root of the above logarithmic equation.

$\boxed{}$, $\boxed{x=4}$



Question 98 (**+)**

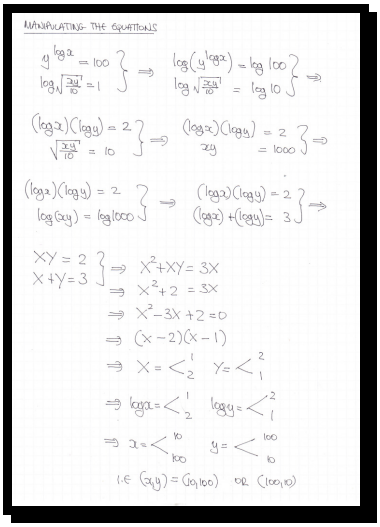
Solve the following simultaneous logarithmic equations

$$y^{\log x}=100$$

$$\log\sqrt{\frac{xy}{10}}=1,$$

given further that x, $y\in\mathbb{R}$, with $x>0$, $y>0$.

$\boxed{\quad}$, $\boxed{x=10,\ y=100,\ \text{or the other way round}}$

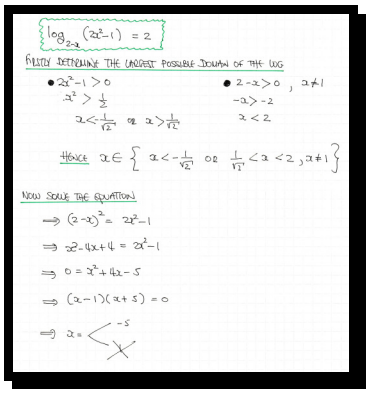


Question 99 (**+)**

Solve the following logarithmic equation, over the largest real domain

$$\log_{2-x}\left[2x^2-1\right]=2\,,\ x\in\mathbb{R}\,.$$

$\boxed{\ }$, $\boxed{x=-5}$



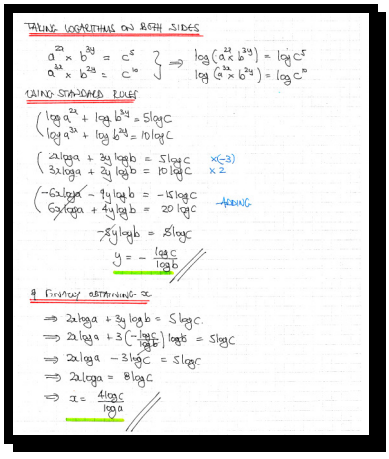
Question 100 (**+)**

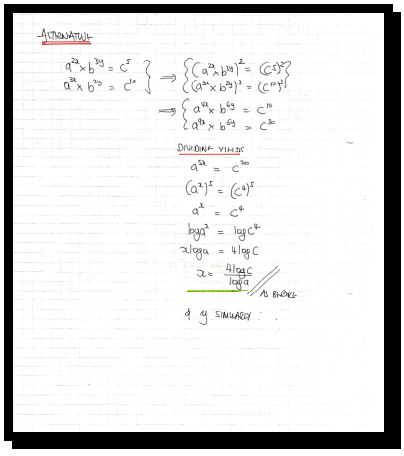
Solve the following simultaneous equations

$$a^{2x}\times b^{3y}=c^5$$
$$a^{3x}\times b^{2y}=c^{10}$$

Give the answers in exact form in terms of $\log a$, $\log b$ and $\log c$.

$\boxed{\ }$, $\boxed{x=\dfrac{4\log c}{\log a}}$, $\boxed{y=-\dfrac{\log c}{\log b}}$



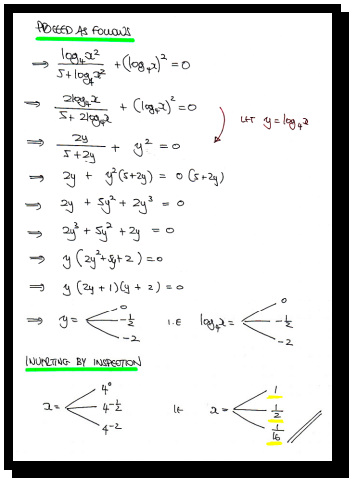


Question 101 (**+)**

Solve the following logarithmic equation

$$\frac{\log_4 x^2}{5+\log_4 x^2}+\left(\log_4 x\right)^2=0.$$

$\square$, $\boxed{x=1,\frac{1}{2},\frac{1}{16}}$



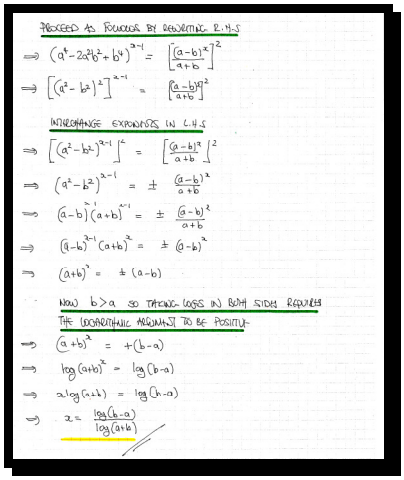
Question 102 **(****+)**

Solve the following equation

$$\left(a^4-2a^2b^2+b^4\right)^{x-1}=(a-b)^{2x}(a+b)^{-2}$$

Give the answers in exact form in terms of a and b.

$\square$, $\boxed{x=\dfrac{\log(b-a)}{\log(b+a)}}$

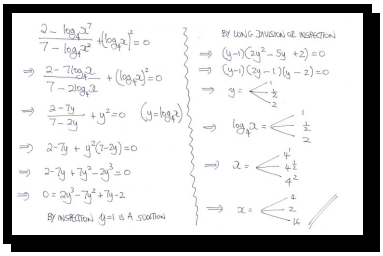


Question 103 **(*****)**

Solve the following logarithmic equation

$$\frac{2-\log_4 x^7}{7-\log_4 x^2}+\left(\log_4 x\right)^2=0$$

$\square$, $\boxed{x=2,\ x=4,\ x=16}$

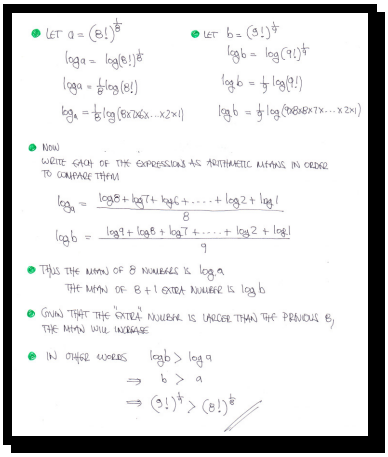


Question 104 (*****)

Show by valid mathematical arguments that

$$\sqrt[8]{8!} < \sqrt[9]{9!}$$

[ ], [proof]



Question 105 **(*****)**

i. Simplify the following expression.

$$9\log_{24} 2+\log_{24} 27 .$$

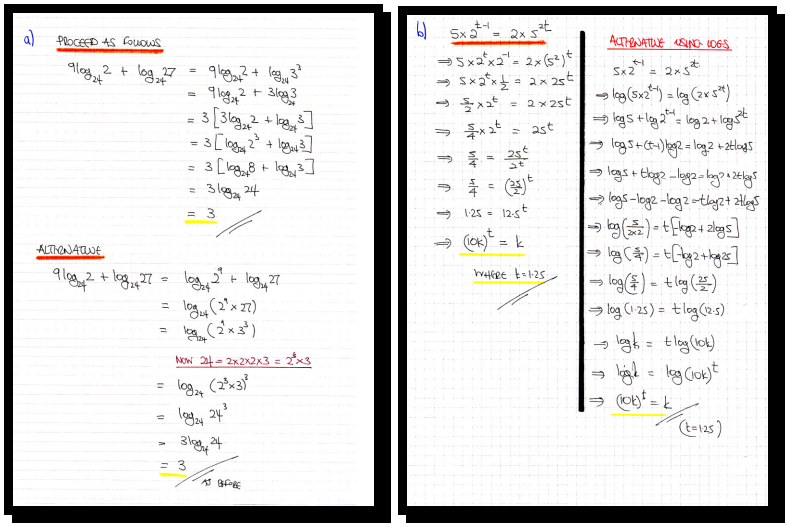
Show detailed workings in this simplification.

ii. It is given that

$$5\times 2^{t-1} = 2\times t^{2t} \quad \Rightarrow \quad (10k)^t = k$$

Determine the value of k.

$\boxed{}$, $\boxed{3}$, $\boxed{k=1.25}$



Question 106 (*****)

On the 1st January 2000 a rare stamp was purchased at an auction for £16384 and by the 1st January 2010 its value was four times as large as its purchase price.

The future value of this stamp, £V, t years after the 1st January 2000 is modelled by the equation

$$V = Ap^t , \ t \geq 0 ,$$

where A and p are positive constants.

On the 1st January 1990 a different stamp was purchased for £2.

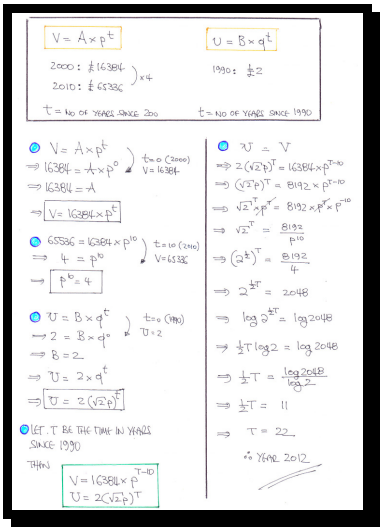
The future value of this stamp, £U, t years after the 1st January 1990 is modelled by the equation

$$U = Bq^t , \ t \geq 0 ,$$

where B and q are positive constants.

Given further that $q = p\sqrt{2}$, determine the year when the two stamps will achieve the same value according to their modelling equations.

$\boxed{}$, $\boxed{2012}$



Question 107 (*****)

It is given that

- a and x are positive real numbers such that $x \neq a$, $x \neq 1$ and $a > 1$.
- n is a positive integer such that $n > 1$.

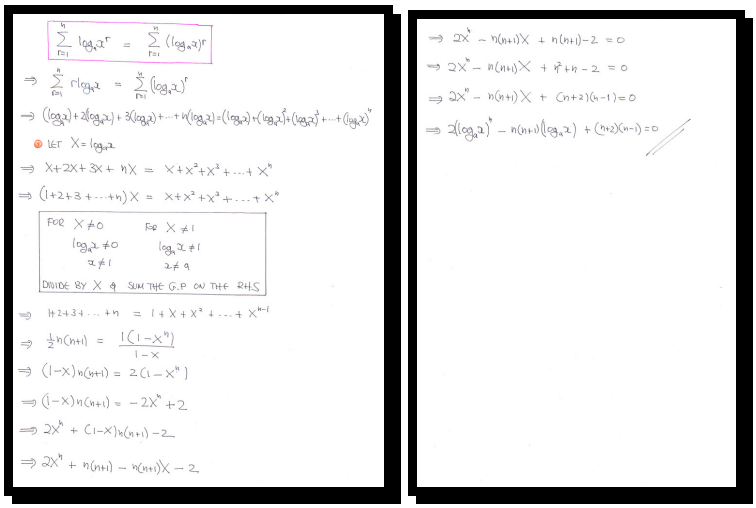
Show that the equation

$$\sum_{r=1}^{n} \log_a x^r = \sum_{r=1}^{n} (\log_a x)^r ,$$

can be written as

$$2(\log_a x)^n - n(n+1)\log_a x + (n-1)(n+2) = 0 .$$

, proof



Question 108 **(*****)**

It is given that

$$a^{\log b} \equiv b^{\log a}\,,\ a>0\,,\ b>0\,.$$

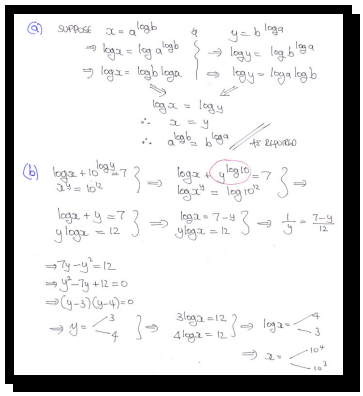
i. Prove the validity of the above result.

ii. Hence, or otherwise, solve the following simultaneous equations

$$\log x + 10^{\log y} = 7\,,$$

$$x^{y} = 10^{12}\,.$$

$\boxed{}$, $\boxed{(10000,3),(1000,4)}$



Question 109 (*****)

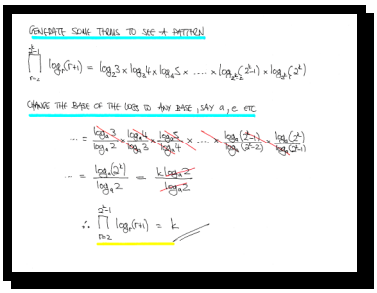
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

Given that $k \in \mathbb{N}$, use a detailed method to find the value of

$$\prod_{r=2}^{2^k-1}\left[\log_r(r+1)\right].$$

$\square$, $\boxed{k}$

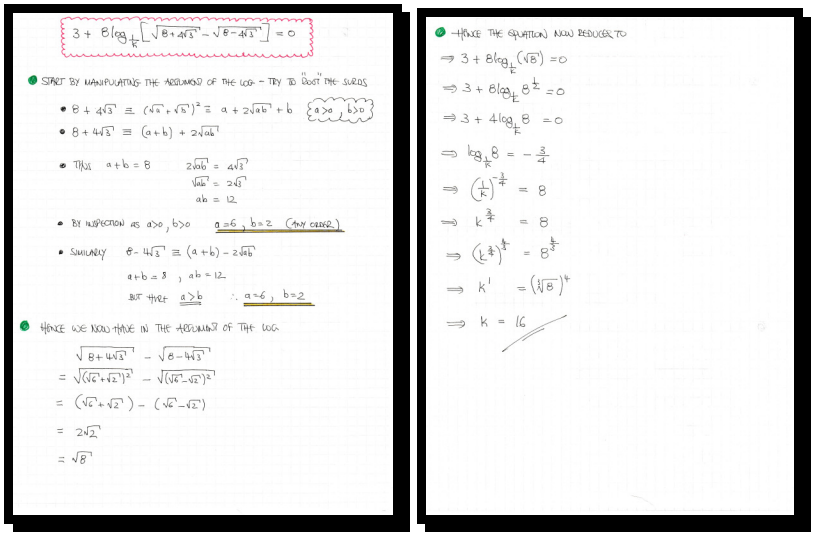


Question 110 (*****)

Solve the following logarithmic equation.

$$3+8\log_{\frac{1}{k}}\left[\sqrt{8+4\sqrt{3}}-\sqrt{8-4\sqrt{3}}\right]=0, \qquad k>0, \ k\neq 1.$$

$\square$, $\boxed{k=16}$

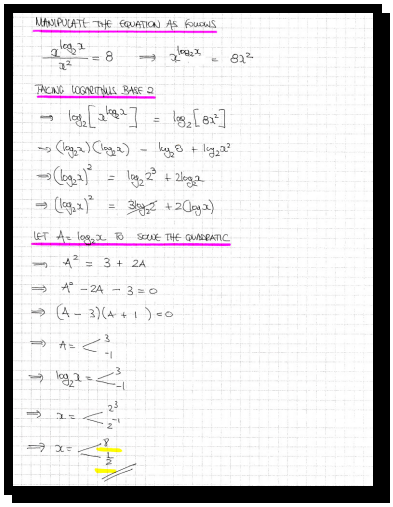


Question 111 (*****)

Solve the following logarithmic equation.

$$\frac{x^{\log_2 x}}{x^2}=8\,,\ x\in(0,\infty)$$

$\square$, $\boxed{x=\frac{1}{2}\ \cup\ x=8}$



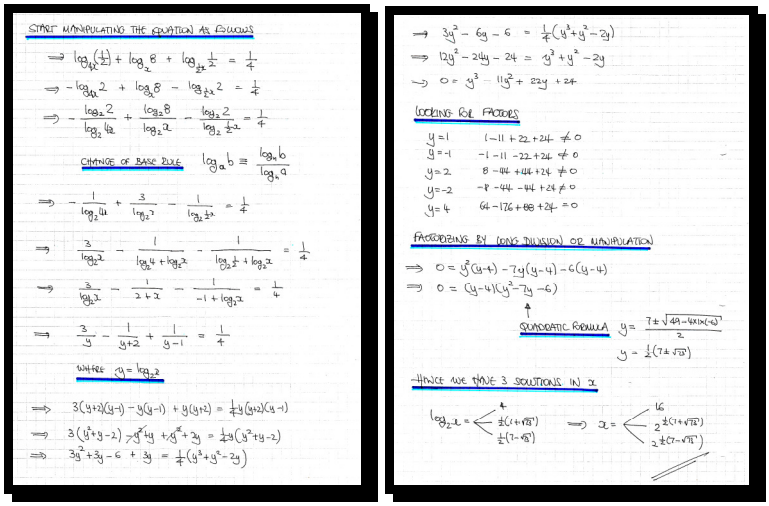
Question 112 (*****)

Solve the following logarithmic equation.

$$\log_{4x}\left(\frac{1}{2}\right)+\log_x 8+\log_{\frac{1}{2}x}\left(\frac{1}{2}\right)=\frac{1}{4}\,,\ x>0\,,\ x\neq 1\,.$$

Give the answers in exact simplified form where appropriate.

$\square$, $\boxed{x=16,\quad x=2^{\frac{1}{2}\left(7\pm\sqrt{73}\right)}}$



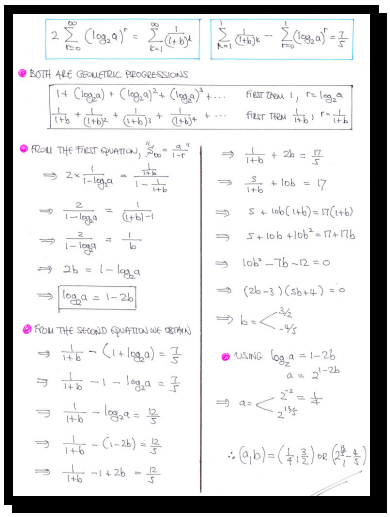
Question 113 (*****)

Solve the following simultaneous equations.

$$2\sum_{r=0}^{\infty}\left[\log_2 a\right]^r = \sum_{k=1}^{\infty}(1+b)^{-k} \qquad \text{and} \qquad \sum_{k=1}^{1}(1+b)^{-k} - \sum_{r=0}^{1}\left[\log_2 a\right]^r = \frac{7}{5}.$$

You may leave the answers as indices in their simplest form, where appropriate.

$\square$, $\boxed{[a,b]=\left[\frac{3}{2},\frac{1}{4}\right]=\left[-\frac{4}{5},2^{\frac{13}{5}}\right]}$



Question 114 **(*****)**

It is given that

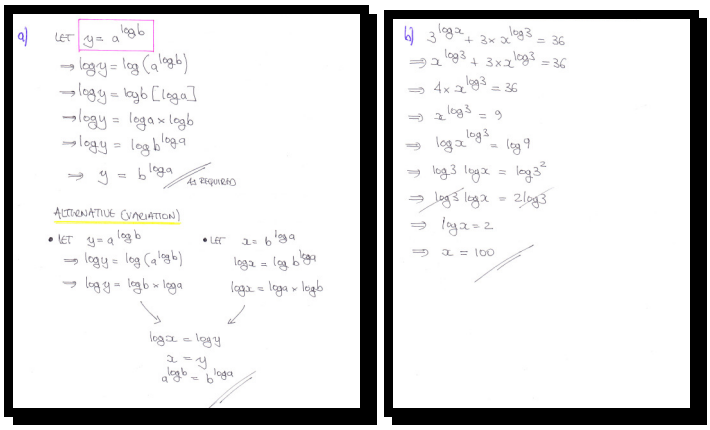
$$a^{\log b} \equiv b^{\log a}\,,\ a>0\,,\ b>0\,.$$

a) Prove the validity of the above result.

b) Hence, or otherwise, solve the equation

$$3^{\log x}+3\times x^{\log 3}=36\,.$$

$\boxed{}$, $\boxed{x=100}$



Question 115 (*****)

It is given that

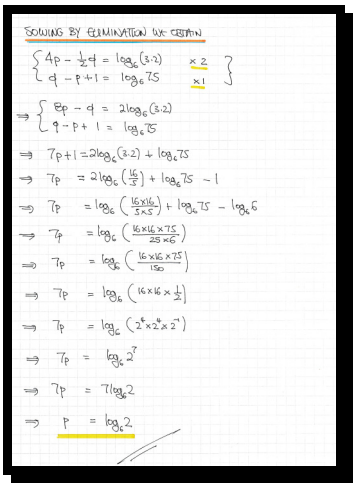
$$4p-\frac{1}{2}q=\log_6(3.6) \qquad \text{and} \qquad q-p+1=\log_6(75).$$

Solve these simultaneous equations, to show that

$$p=\log_6 k\,,$$

where k is a positive integer to be found.

$\boxed{}$, $\boxed{k=2}$

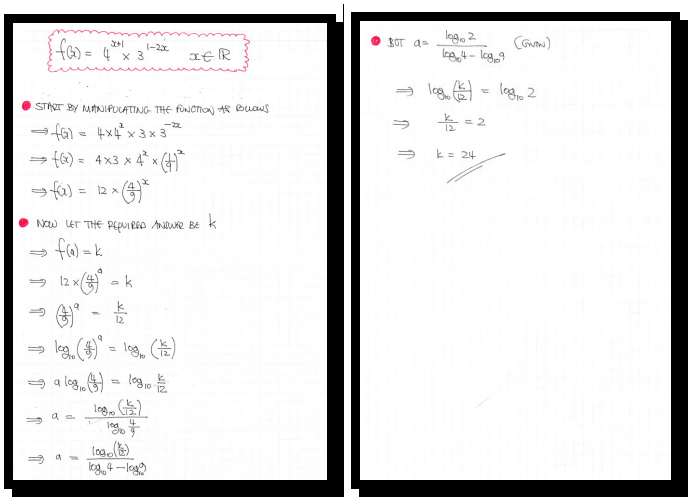


Question 116 (*****)

$$f(x) \equiv 4^{x+1} \times 3^{1-2x}, \ x \in \mathbb{R}.$$

Determine the value of $f(a)$, where $a = \dfrac{\log_{10} 2}{\log_{10} 4 - \log_{10} 9}$.

$\square$, $\boxed{f(a) = 24}$



Question 117 (*****)

The positive solution of the quadratic equation $x^2 - x - 1 = 0$ is denoted by ϕ, and is commonly known as the golden section or golden number.

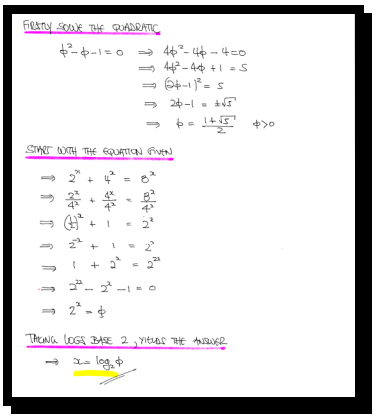
Show, with a detailed method, that the real solution of the following exponential equation

$$2^x + 4^x = 8^x,$$

can be written in exact form as

$$\log_2 \phi .$$

$\square$, proof

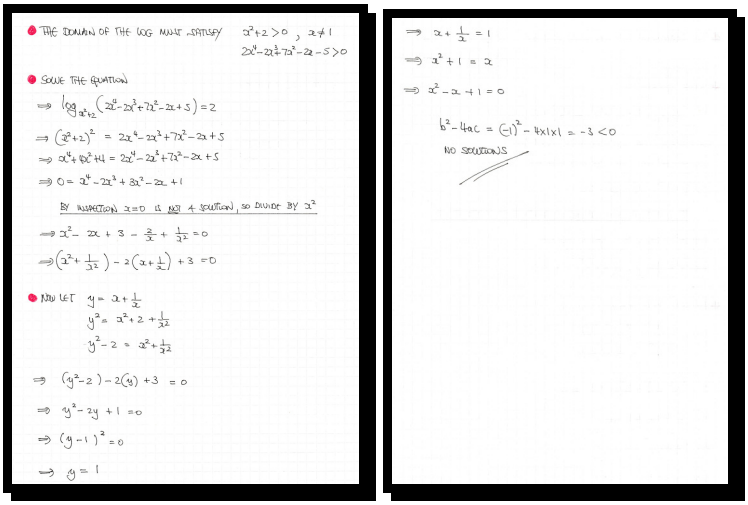


Question 118 (*****)

Show that the following logarithmic equation has no real solutions.

$$\log_{x^2+2}\left[2x^4-2x^3+7x^2-2x+5\right]=2\,,\ x\in\mathbb{R}\,.$$

[ ], [proof]



Question 119 (*****)

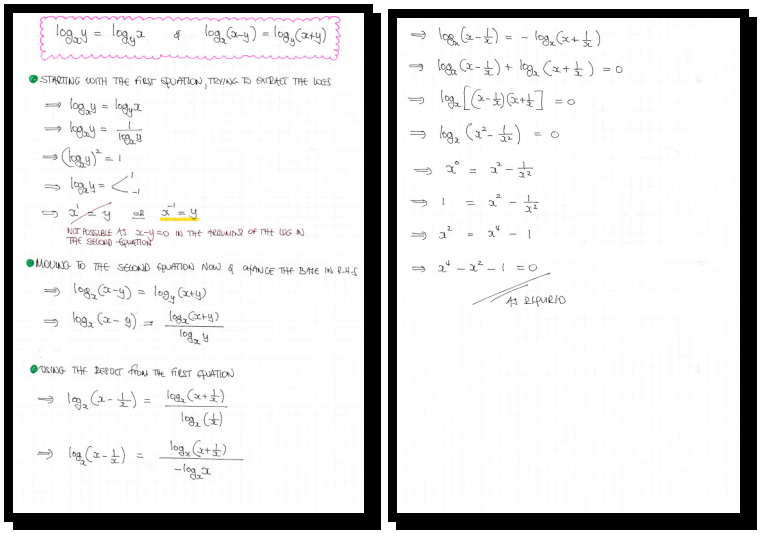
It is given that for $x>0$, $x\neq 1$ and $y>0$, $y\neq 1$

$$\log_x y=\log_y x \qquad \text{and} \qquad \log_x\left(x-y\right)=\log_y\left(x+y\right).$$

Show that

$$x^4-x^2-1=0.$$

[ ], [proof]



Question 120 (*****)

$$\log_{\sin x\cos x}(\sin x)\times\log_{\sin x\cos x}(\cos x)=\frac{1}{4}.$$

Show that the solution of the above equation is given by

$$x=\frac{1}{4}\pi(8n-7),\ n\in\mathbb{N}.$$

, proof

