INTEGRATION
INTRODUCTION

STANDARD INTEGRATION PRACTICE

Question 1

Integrate the following expressions with respect to x.

a) $\displaystyle\int 2\sqrt{x}-\frac{1}{x^2}\,dx$

$$\boxed{\frac{4}{3}x^{\frac{3}{2}}+\frac{1}{x}+C}$$

b) $\displaystyle\int 4\sqrt{x}-2\sqrt{x^5}\,dx$

$$\boxed{\frac{8}{3}x^{\frac{3}{2}}-\frac{4}{7}x^{\frac{7}{2}}+C}$$

c) $\displaystyle\int \frac{3}{4\sqrt{x}}+\frac{1}{x^3}\,dx$

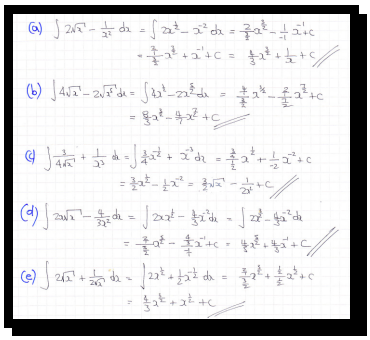
$$\boxed{\frac{3}{2}\sqrt{x}-\frac{1}{2x^2}+C}$$

d) $\displaystyle\int 2x\sqrt{x}-\frac{4}{3x^2}\,dx$

$$\boxed{\frac{4}{5}x^{\frac{5}{2}}+\frac{4}{3}x^{-1}+C}$$

e) $\displaystyle\int 2\sqrt{x}+\frac{1}{2\sqrt{x}}\,dx$

$$\boxed{\frac{4}{3}x^{\frac{3}{2}}+\sqrt{x}+C}$$



Question 2

Integrate the following expressions with respect to x.

a) $\displaystyle\int \frac{4}{x^3}+12x^{\frac{2}{3}}\,dx$

$$\boxed{-2x^{-2}+\frac{36}{5}x^{\frac{5}{3}}+C}$$

b) $\displaystyle\int 14x^{\frac{3}{4}}-\frac{3}{2x^4}\,dx$

$$\boxed{8x^{\frac{7}{4}}+\frac{1}{2}x^{-3}+C}$$

c) $\displaystyle\int 4x-\frac{6}{x^3}+4\sqrt{x}-1\,dx$

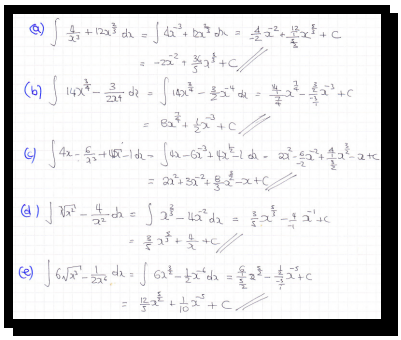
$$\boxed{2x^2-3x^{-2}+\frac{8}{3}x^{\frac{3}{2}}-x+C}$$

d) $\displaystyle\int \sqrt[3]{x^2}-\frac{4}{x^2}\,dx$

$$\boxed{\frac{3}{5}x^{\frac{5}{3}}+\frac{4}{x}+C}$$

e) $\displaystyle\int 6\sqrt{x^3}-\frac{1}{2x^6}\,dx$

$$\boxed{\frac{12}{5}x^{\frac{5}{2}}+\frac{1}{10}x^{-5}+C}$$



Question 3

Integrate the following expressions with respect to x.

a) $\displaystyle\int \frac{2+5x}{3x^3}\,dx$

$$\boxed{-\frac{1}{3x^2}-\frac{5}{3x}+C}$$

b) $\displaystyle\int \frac{2x^2+x}{2\sqrt{x}}\,dx$

$$\boxed{\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{3}x^{\frac{3}{2}}+C}$$

c) $\displaystyle\int \frac{2x+\sqrt{x}}{3x^3}\,dx$

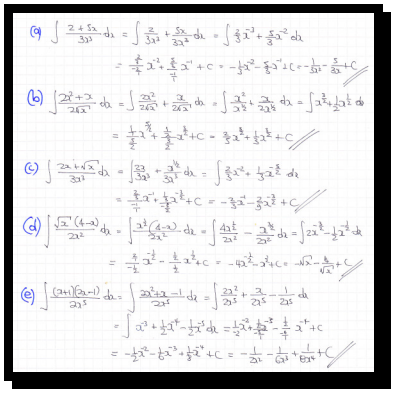
$$\boxed{-\frac{2}{3}x^{-1}-\frac{2}{9}x^{-\frac{3}{2}}+C}$$

d) $\displaystyle\int \frac{\sqrt{x}(4-x)}{2x^2}\,dx$

$$\boxed{-\sqrt{x}-\frac{4}{\sqrt{x}}+C}$$

e) $\displaystyle\int \frac{(x+1)(2x-1)}{2x^5}\,dx$

$$\boxed{-\frac{1}{2x^2}-\frac{1}{6x^3}+\frac{1}{8x^4}+C}$$



Question 4

Integrate the following expressions with respect to x.

a) $\displaystyle\int \frac{x+x^2}{\sqrt{x}}\,dx$

$\boxed{\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}+C}$

b) $\displaystyle\int \frac{4x^3+\sqrt{x}}{2x^2}\,dx$

$\boxed{x^2-x^{-\frac{1}{2}}+C}$

c) $\displaystyle\int \frac{x^2+2}{x^4}\,dx$

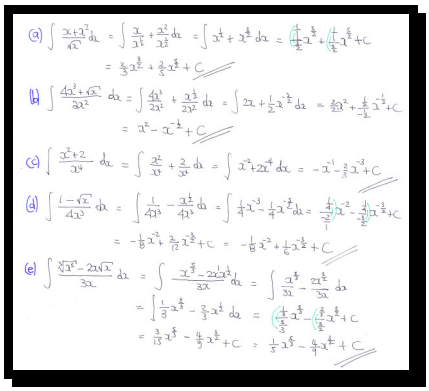
$\boxed{-x^{-1}-\frac{2}{3}x^{-3}+C}$

d) $\displaystyle\int \frac{1-\sqrt{x}}{4x^3}\,dx$

$\boxed{-\frac{1}{8}x^{-2}+\frac{1}{6}x^{-\frac{3}{2}}+C}$

e) $\displaystyle\int \frac{\sqrt[3]{x^5}-2x\sqrt{x}}{3x}\,dx$

$\boxed{\frac{1}{5}x^{\frac{5}{3}}-\frac{4}{9}x^{\frac{3}{2}}+C}$



Question 5

Integrate the following expressions with respect to x.

a) $\displaystyle\int x\left(\sqrt{x}+x^{-4}\right)dx$

$$\frac{2}{5}x^{\frac{5}{2}}-\frac{1}{2}x^{-2}+C$$

b) $\displaystyle\int \frac{1}{\sqrt{x}}\left(\frac{2}{x}-\frac{3}{4x^2}\right)dx$

$$-4x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}+C$$

c) $\displaystyle\int 4x^{\frac{7}{2}}\left(\frac{6}{x^2}-\frac{5}{\sqrt{x}}\right)dx$

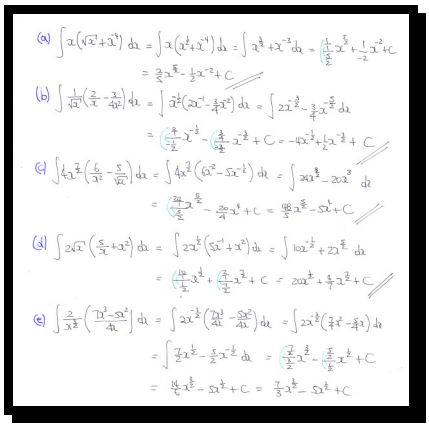
$$\frac{48}{5}x^{\frac{5}{2}}-5x^4+C$$

d) $\displaystyle\int 2\sqrt{x}\left(\frac{5}{x}+x^2\right)dx$

$$20x^{\frac{1}{2}}+\frac{4}{7}x^{\frac{7}{2}}+C$$

e) $\displaystyle\int \frac{2}{x^{\frac{3}{2}}}\left(\frac{7x^3-5x^2}{4x}\right)dx$

$$\frac{7}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C$$



Question 6

Integrate the following expressions with respect to x.

a) $\displaystyle\int \frac{(2x-1)(2x-3)}{2x^{\frac{3}{2}}}\,dx$

$$\boxed{\frac{4}{3}x^{\frac{3}{2}}-8x^{\frac{1}{2}}-3x^{-\frac{1}{2}}+C}$$

b) $\displaystyle\int \frac{\left(1+2\sqrt{x}\right)^2}{2x^3}\,dx$

$$\boxed{-\frac{1}{4}x^{-2}-\frac{4}{3}x^{-\frac{3}{2}}-2x^{-1}+C}$$

c) $\displaystyle\int \frac{2x^3+\sqrt{x^3}}{\sqrt{x}}\,dx$

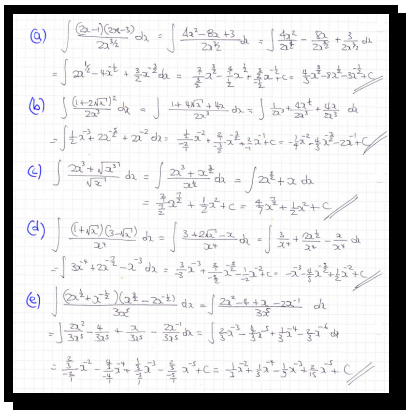
$$\boxed{\frac{4}{7}x^{\frac{7}{2}}+\frac{1}{2}x^2+C}$$

d) $\displaystyle\int \frac{\left(1+\sqrt{x}\right)\left(3-\sqrt{x}\right)}{x^4}\,dx$

$$\boxed{-x^{-3}-\frac{4}{5}x^{-\frac{5}{2}}+\frac{1}{2}x^{-2}+C}$$

e) $\displaystyle\int \frac{\left(2x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)\left(x^{\frac{3}{2}}-2x^{-\frac{1}{2}}\right)}{3x^5}\,dx$

$$\boxed{-\frac{1}{3}x^{-2}+\frac{1}{3}x^{-4}-\frac{1}{9}x^{-3}+\frac{2}{15}x^{-5}+C}$$



Question 7

Evaluate the following integrals.

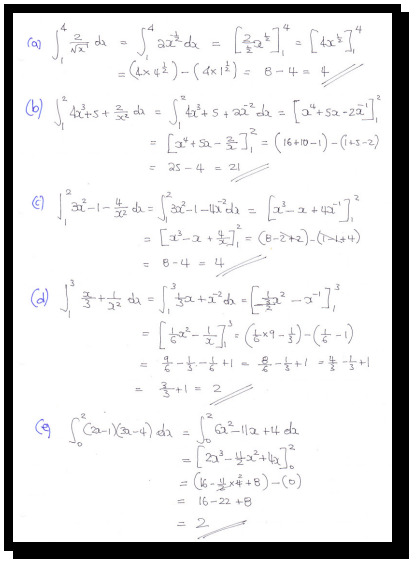
a) $\displaystyle\int_1^4 \frac{2}{\sqrt{x}}\ dx$ [4]

b) $\displaystyle\int_1^2 4x^3+5+\frac{2}{x^2}\ dx$ [21]

c) $\displaystyle\int_1^2 3x^2-1-\frac{4}{x^2}\ dx$ [4]

d) $\displaystyle\int_1^3 \frac{x}{3}+\frac{1}{x^2}\ dx$ [2]

e) $\displaystyle\int_0^2 (2x-1)(3x-4)\ dx$ [2]



(a) $\int_1^4 \frac{2}{\sqrt{x}}\,dx = \int_1^4 2x^{-\frac{1}{2}}\,dx = \left[\frac{2}{\frac{1}{2}}x^{\frac{1}{2}}\right]_1^4 = \left[4x^{\frac{1}{2}}\right]_1^4$
$= \left(4\times 4^{\frac{1}{2}}\right) - \left(4\times 1^{\frac{1}{2}}\right) = 8-4 = 4$

(b) $\int_1^2 4x^3+5+\frac{2}{x^2}\,dx = \int_1^2 4x^3+5+2x^{-2}\,dx = \left[x^4+5x-2x^{-1}\right]_1^2$
$= \left[x^4+5x-\frac{2}{x}\right]_1^2 = \left(16+10-1\right) - \left(1+5-2\right)$
$= 25-4 = 21$

(c) $\int_1^2 3x^2-1-\frac{4}{x^2}\,dx = \int_1^2 3x^2-1-4x^{-2}\,dx = \left[x^3-x+4x^{-1}\right]_1^2$
$= \left[x^3-x+\frac{4}{x}\right]_1^2 = (8-2+2)-(1-1+4)$
$= 8-4 = 4$

(d) $\int_1^3 \frac{x}{3}+\frac{1}{x^2}\,dx = \int_1^3 \frac{1}{3}x+x^{-2}\,dx = \left[\frac{1}{2}x^2-x^{-1}\right]_1^3$
$= \left[\frac{1}{6}x^2-\frac{1}{x}\right]_1^3 = \left(\frac{1}{6}\times 9-\frac{1}{3}\right)-\left(\frac{1}{6}-1\right)$
$= \frac{9}{6}-\frac{1}{3}-\frac{1}{6}+1 = \frac{8}{6}-\frac{1}{3}+1 = \frac{4}{3}-\frac{1}{3}+1$
$= \frac{3}{3}+1 = 2$

(e) $\int_0^2 (2x-1)(3x-4)\,dx = \int_0^2 6x^2-11x+4\,dx$
$= \left[2x^3-\frac{11}{2}x^2+4x\right]_0^2$
$= \left(16-\frac{11}{2}\times 4+8\right)-(0)$
$= 16-22+8$
$= 2$

Question 8

Evaluate the following integrals.

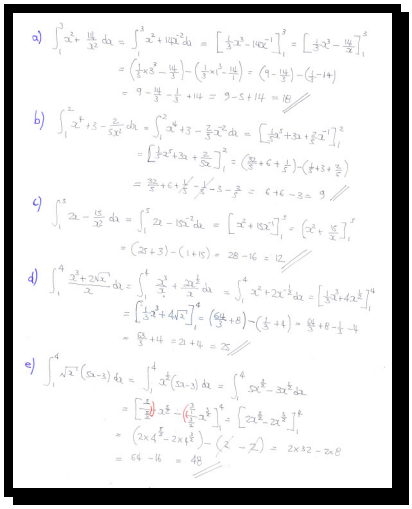
a) $\int_1^3 x^2+\frac{14}{x^2}\,dx$ 18

b) $\int_1^2 x^4+3-\frac{2}{5x^2}\,dx$ 9

c) $\int_1^5 2x-\frac{15}{x^2}\,dx$ 12

d) $\int_1^4 \frac{x^3+2\sqrt{x}}{x}\,dx$ 25

e) $\int_1^4 \sqrt{x}\,(5x-3)\,dx$ 48



Question 9

Evaluate the following integrals.

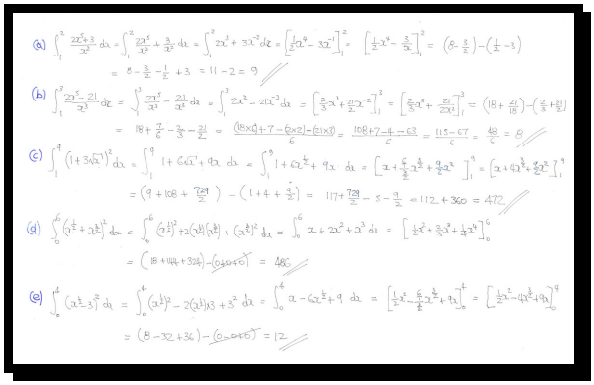
a) $\displaystyle\int_1^2 \frac{2x^5+3}{x^2}\,dx$ 9

b) $\displaystyle\int_1^3 \frac{2x^5-21}{x^3}\,dx$ 8

c) $\displaystyle\int_1^9 \left(1+3\sqrt{x}\right)^2\,dx$ 472

d) $\displaystyle\int_0^6 \left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)^2\,dx$ 486

e) $\displaystyle\int_0^4 \left(x^{\frac{1}{2}}-3\right)^2\,dx$ 12



Question 10

Evaluate the following integrals.

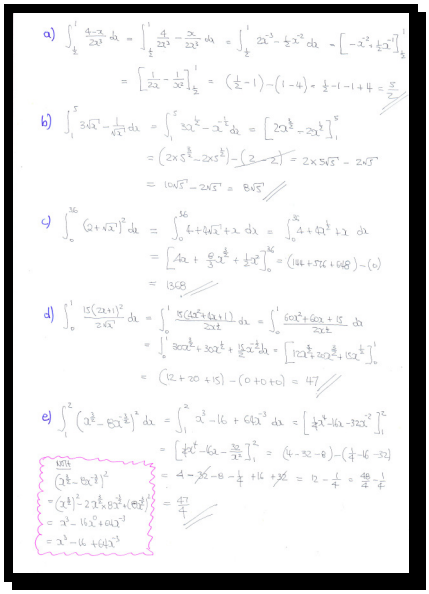
a) $\displaystyle\int_{\frac{1}{2}}^{1} \frac{4-x}{2x^3}\,dx$ $\boxed{\frac{5}{2}}$

b) $\displaystyle\int_{1}^{5} 3\sqrt{x}-\frac{1}{\sqrt{x}}\,dx$ $\boxed{8\sqrt{5}}$

c) $\displaystyle\int_{0}^{36} \left(2+\sqrt{x}\right)^2\,dx$ $\boxed{1368}$

d) $\displaystyle\int_{0}^{1} \frac{15(2x+1)^2}{2\sqrt{x}}\,dx$ $\boxed{47}$

e) $\displaystyle\int_{1}^{2} \left(x^{\frac{3}{2}}-8x^{-\frac{3}{2}}\right)^2\,dx$ $\boxed{\frac{47}{4}}$



VARIOUS INTEGRATION QUESTIONS

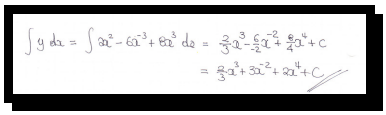
Question 1

$$y = 2x^2 - \frac{6}{x^3} + 8x^3, \ x \neq 0 .$$

Find an expression for

$$\int y \, dx .$$

$$\boxed{\frac{2}{3}x^3 + 3x^{-2} + 2x^4 + C}$$



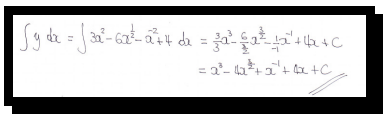
Question 2

$$y = 3x^2 - 6\sqrt{x} - \frac{1}{x^2} + 4, \ x > 0 .$$

Find an expression for

$$\int y \, dx .$$

$$\boxed{4x + x^3 - 4x^{\frac{3}{2}} + x^{-1} + C}$$



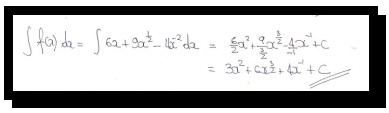
Question 3

$$f(x)=6x+9\sqrt{x}-\frac{4}{x^2},\ x>0.$$

Find an expression for

$$\int f(x)\,dx.$$

$$\boxed{3x^2+6x^{\frac{3}{2}}+4x^{-1}+C}$$



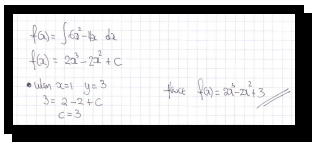
Question 4

The point $P(1,3)$ lies on the curve with equation $y=f(x)$, whose gradient function is given by

$$f'(x)=6x^2-4x,\ x\in\mathbb{R}.$$

Find an equation for $f(x)$.

$$\boxed{f(x)=2x^3-2x^2+3}$$



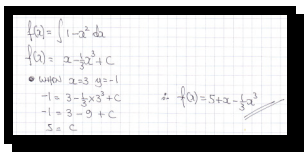
Question 5

The point $P(3,-1)$ lies on the curve with equation $y=f(x)$, whose gradient function is given by

$$f'(x)=1-x^2,\ x\in\mathbb{R}.$$

Find an equation for $f(x)$.

$$\boxed{f(x)=5+x-\frac{1}{3}x^3}$$

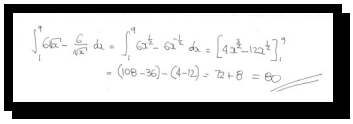


Question 6

By showing clear workings, find the value of

$$\int_1^9 6\sqrt{x}-\frac{6}{\sqrt{x}}\,dx.$$

$$\boxed{80}$$



Question 7

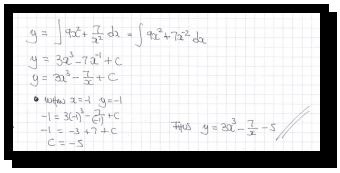
The curve C with equation $y = f(x)$ has gradient function

$$\frac{dy}{dx} = 9x^2 + \frac{7}{x^2}, \ x \neq 0.$$

The point $A(-1,-1)$ lies on C.

Find an equation for C.

$$\boxed{y = 3x^3 - \frac{7}{x} - 5}$$



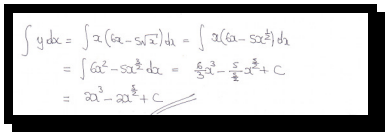
Question 8

$$y = x\left(6x - 5\sqrt{x}\right), \ x \geq 0.$$

By showing all steps in the workings, find an expression for

$$\int y \, dx.$$

$$\boxed{2x^3 - 2x^{\frac{5}{2}} + C}$$



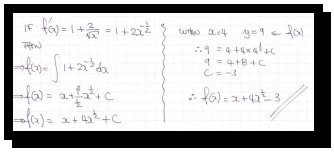
Question 9

The point $P(4,9)$ lies on the curve with equation $y = f(x)$, whose gradient function is given by

$$f'(x) = 1 + \frac{2}{\sqrt{x}}, \ x > 0.$$

Find an equation for $f(x)$.

$$\boxed{f(x) = x + 4\sqrt{x} - 3}$$

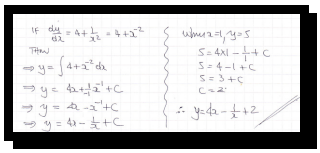


Question 10

$$\frac{dy}{dx} = 4 + \frac{1}{x^2}, \ x \neq 0.$$

Given that $y = 5$ when $x = 1$, express y in terms of x.

$$\boxed{y = 4x - \frac{1}{x} + 2}$$

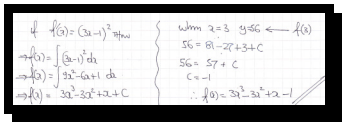


Question 11

$$f'(x)=(3x-1)^2.$$

Given that $f(3)=56$, find an expression for $f(x)$.

$$\boxed{f(x)=3x^3-3x^2+x-1}$$



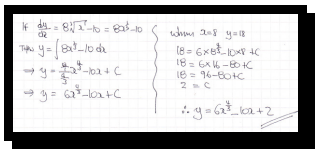
Question 12

The point $P(8,18)$ lies on the curve C, whose gradient function is given by

$$\frac{dy}{dx}=8\sqrt[3]{x}-10,\ x\geq 0.$$

Find an equation for C.

$$\boxed{y=6x^{\frac{4}{3}}-10x+2}$$



Question 13

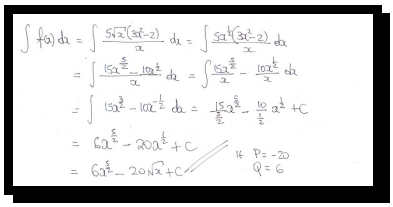
$$f(x)=\frac{5\sqrt{x}\left(3x^2-2\right)}{x},\ x>0.$$

Show clearly that

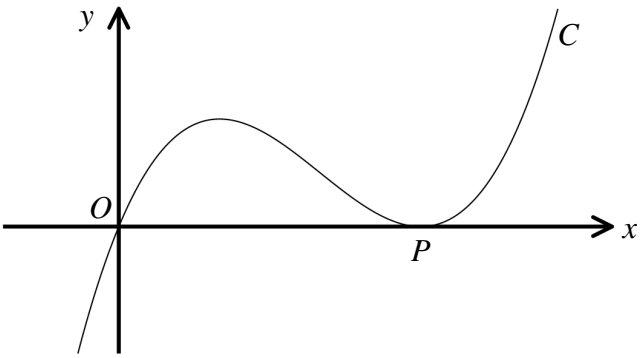
$$\int f(x)\,dx=P\sqrt{x}+Qx^{\frac{5}{2}}+C,$$

where P and Q are integers to be found, and C is an arbitrary constant.

$\boxed{P=-20}$, $\boxed{Q=6}$



Question 14



The figure above shows the cubic curve C which meets the coordinates axes at the origin O and at the point P.

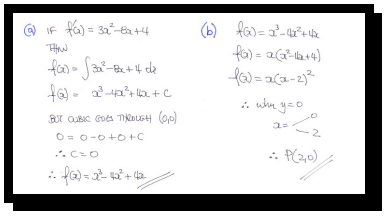
The gradient function of C is given by

$$f'(x)=3x^2-8x+4\,.$$

a) Find an equation for C.

b) Determine the coordinates of P.

$\boxed{f(x)=x^3-4x^2+4x}$, $\boxed{P(2,0)}$



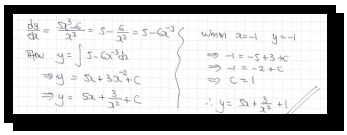
Question 15

The point $P(-1,-1)$ lies on the curve C, whose gradient function is given by

$$\frac{dy}{dx}=\frac{5x^3-6}{x^3},\ x\neq 0.$$

Find an equation for C.

$$\boxed{y=5x+\frac{3}{x^2}+1}$$



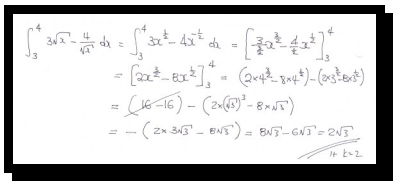
Question 16

Show clearly that

$$\int_3^4 3\sqrt{x}-\frac{4}{\sqrt{x}}\ dx=k\sqrt{3},$$

where k is an integer to be found.

$$\boxed{k=2}$$



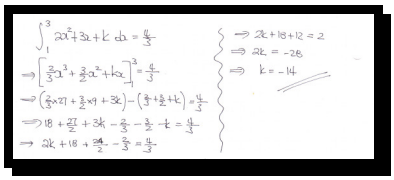
Question 17

$f(x) = 2x^2 + 3x + k$, where k is a constant.

Find the value of k, given that

$$\int_1^3 f(x)\ dx = \frac{4}{3}.$$

$\boxed{k = -14}$



$\int_1^3 2x^2+3x+k\ dx = \frac{4}{3}$

$\Rightarrow \left[\frac{2}{3}x^3 + \frac{3}{2}x^2 + kx\right]_1^3 = \frac{4}{3}$

$\Rightarrow \left(\frac{2}{3}\times 27 + \frac{3}{2}\times 9 + 3k\right) - \left(\frac{2}{3}+\frac{3}{2}+k\right) = \frac{4}{3}$

$\Rightarrow 18 + \frac{27}{2} + 3k - \frac{2}{3} - \frac{3}{2} - k = \frac{4}{3}$

$\Rightarrow 2k + 18 + \frac{24}{2} - \frac{2}{3} = \frac{4}{3}$

$\Rightarrow 2k + 18 + 12 = 2$

$\Rightarrow 2k = -28$

$\Rightarrow k = -14$

Question 18

The cubic equation C passes through the origin O and its gradient function is

$$\frac{dy}{dx} = 6x^2 - 6x - 20 .$$

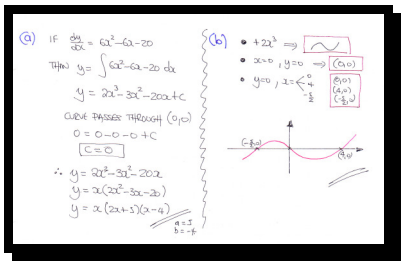
a) Show clearly that the equation of C can be written as

$$y = x(2x + a)(x + b),$$

where a and b are constants.

b) Sketch the graph of C, indicating clearly the coordinates of the points where the graph meets the coordinate axes.

$\boxed{a = 5}$, $\boxed{b = -4}$



Question 19

The gradient of every point on the curve C, with equation $y = f(x)$, satisfies

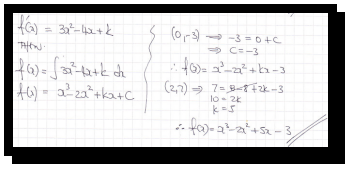
$$f'(x) = 3x^2 - 4x + k,$$

where k is a constant.

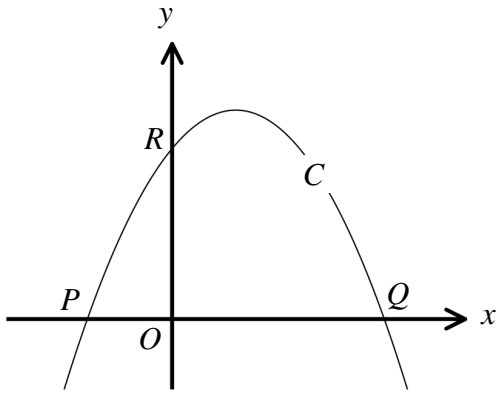
The points $P(0,-3)$ and $Q(2,7)$ both lie on C.

Find an equation for C.

$$\boxed{y = x^3 - 2x^2 + 5x - 3}$$



Question 20



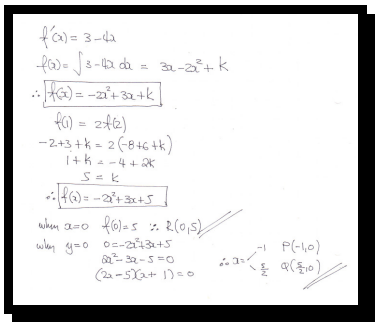
The figure above shows the curve C which meets the coordinates axes at the points P, Q and R.

Given the gradient function of C is given by

$$f'(x)=3-4x,$$

and that $f(1)=2f(2)$, determine the coordinates of P, Q and R.

$\boxed{P(-1,0)}$, $\boxed{P\left(\frac{5}{2},0\right)}$, $\boxed{R(0,5)}$



Question 21

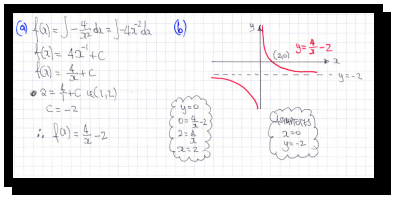
The curve C with equation $y = f(x)$ satisfies

$$f'(x) = -\frac{4}{x^2}, \ x \neq 0.$$

a) Given that $f(1) = 2$, find an expression for $f(x)$.

b) Sketch the graph of $f(x)$, indicating clearly the asymptotes of the curve and the coordinates of any points where the curve crosses the coordinate axes.

$$\boxed{f(x) = \frac{4}{x} - 2}, \ \boxed{(2,0)}$$



Question 22

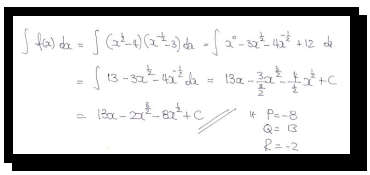
$$f(x) = \left(x^{\frac{1}{2}} - 4\right)\left(x^{-\frac{1}{2}} - 3\right), \ x > 0.$$

Show clearly that

$$\int f(x)\,dx = P\sqrt{x} + Qx + Rx^{\frac{3}{2}} + C,$$

where P, Q and R are integers to be found, and C is an arbitrary constant.

$$\boxed{P = -8}, \ \boxed{Q = 13}, \ \boxed{R = -2}$$

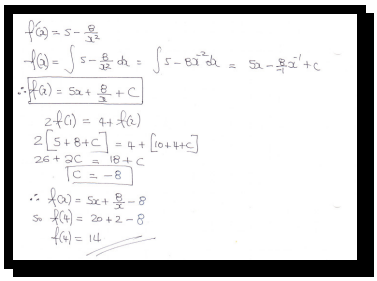


Question 23

$$f'(x)=5-\frac{8}{x^2},\ x\neq 0.$$

Find the value of $f(4)$, given that $2f(1)=4+f(2)$.

$\boxed{f(4)=14}$



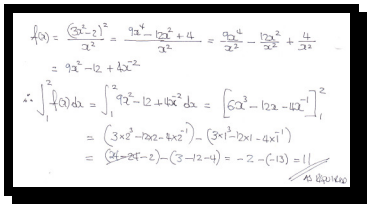
Question 24

$$f(x)=\frac{\left(3x^2-2\right)^2}{x^2},\ x\neq 0.$$

Show clearly that

$$\int_1^2 f(x)\ dx=11.$$

proof



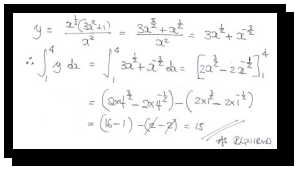
Question 25

$$y=\frac{x^{\frac{1}{2}}(3x^2+1)}{x^2},\ x>0.$$

Show clearly that

$$\int_1^4 y\ dx=15.$$

proof



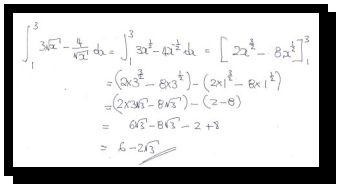
Question 26

Find the exact value of

$$\int_1^3 3\sqrt{x}-\frac{4}{\sqrt{x}}\,dx,$$

giving the answer in the form $p+q\sqrt{3}$, where p and q are integers.

$6-2\sqrt{3}$



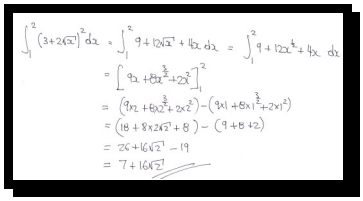
Question 27

Find the exact value of

$$\int_1^2 \left(3+2\sqrt{x}\right)^2 \, dx,$$

giving the answer in the form $a+b\sqrt{2}$, where a and b are integers.

$$\boxed{7+16\sqrt{2}}$$



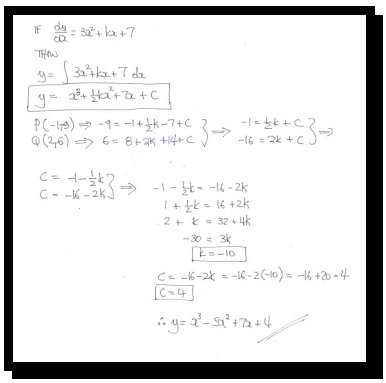
Question 28

A cubic curve passes through the points $P(-1,-9)$ and $Q(2,6)$ and its gradient function is given by

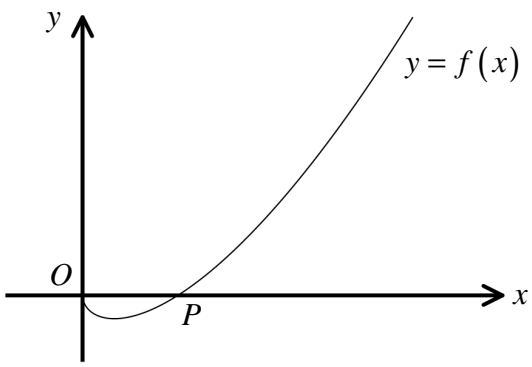
$$\frac{dy}{dx}=3x^2+kx+7, \text{ where } k \text{ is a constant.}$$

Find an equation for this cubic curve.

$$\boxed{y=x^3-5x^2+7x+4}$$



Question 29



The figure above shows a curve with equation $y=f\left(x\right)$ which meets the x axis at the origin O and at the point P.

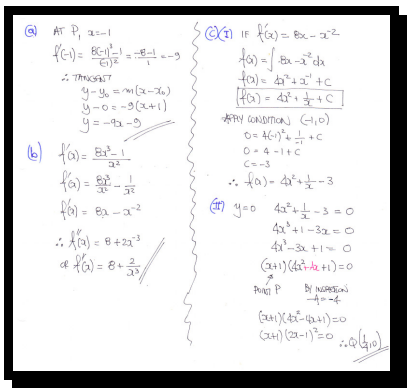
The gradient function of the curve is given by

$$f'\left(x\right)=\frac{12x-1}{\sqrt{x}},\ x>0.$$

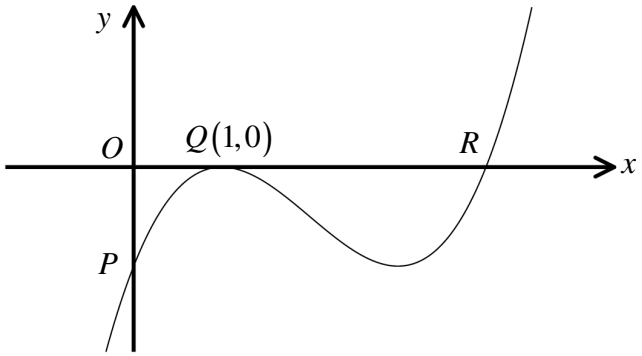
a) Find an equation of the curve.

b) Determine the coordinates of P.

$$\boxed{f\left(x\right)=8x^{\frac{3}{2}}-2\sqrt{x}},\ \boxed{P\left(\frac{1}{4},0\right)}$$



Question 30



The figure above shows the graph of a cubic curve, which touches the x axis at the point $Q(1,0)$.

a) Determine an equation for the cubic curve , given its gradient is given by

$$\frac{dy}{dx} = 3x^2 - 12x + 9 .$$

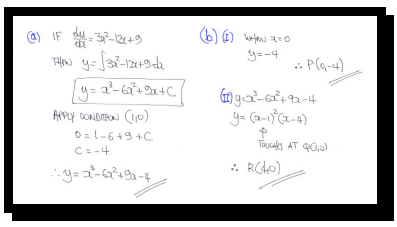
The cubic curve crosses the x axis and the y axis at the points R and P, respectively.

b) Determine the coordinates …

i. … of the point P.

ii. … of the point R.

$\boxed{y = x^3 - 6x^2 + 9x - 4}$, $\boxed{P(0,-4)}$, $\boxed{R(4,0)}$



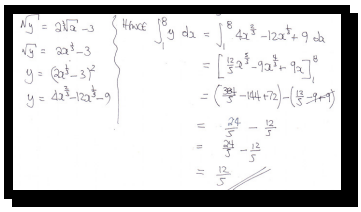
Question 31

$$\sqrt{y}=2\sqrt[3]{x}-3\,,\ x>0\,.$$

Show clearly that

$$\int_1^8 y\ dx=\frac{12}{5}\,.$$

proof



Question 32

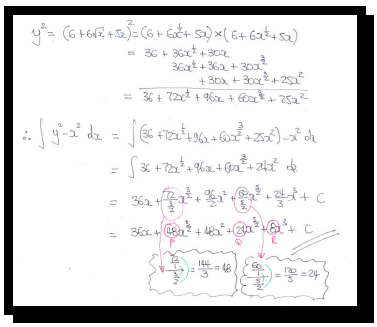
$$y=6+6\sqrt{x}+5x\,,\ x\geq 0\,.$$

Show clearly that

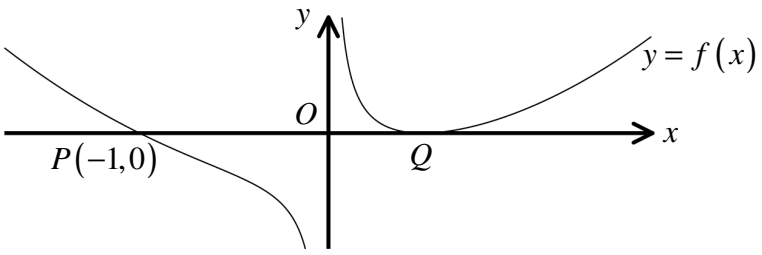
$$\int\left(y^2-x^2\right)dx=36x+Px^{\frac{3}{2}}+48x^2\sqrt{x}+Qx^{\frac{5}{2}}+Rx^3+C\,,$$

where P, Q and R are constants to be found, and C is an arbitrary constant.

$P=48$, $Q=24$, $R=8$



Question 33



The figure above shows a curve with equation $y = f(x)$.

The curve meets the x axis at the points $P(-1,0)$ and Q, and its gradient function is given by

$$f'(x) = \frac{8x^3 - 1}{x^2}, \ x \neq 0.$$

a) Find an equation of the tangent to the curve at P.

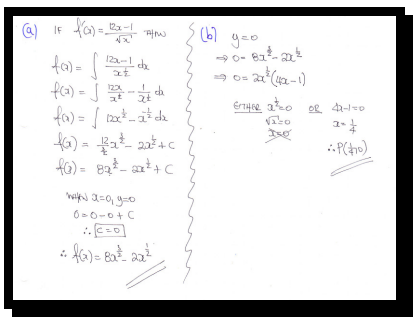
b) Find an expression for $f''(x)$.

c) Determine …

i. … an equation of the curve.

ii. … the coordinates of Q.

$$\boxed{y = -9x - 9}, \ \boxed{f''(x) = 8 + 2x^{-3}}, \ \boxed{y = 4x^2 + \frac{1}{x} - 3}, \ \boxed{Q\left(\frac{1}{2}, 0\right)}$$



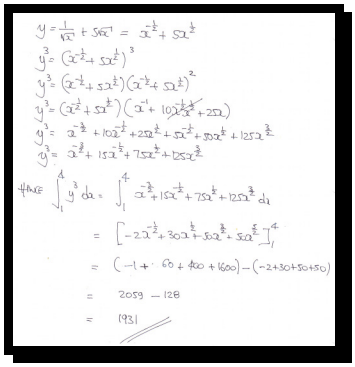
Question 34

$$y=\frac{1}{\sqrt{x}}+5\sqrt{x}\,,\ x>0\,.$$

Show clearly that

$$\int_1^4 y^3\ dx=1931\,.$$

proof



Question 35

A quadratic curve C passes through the points $P(a,b)$ and $Q(2a,2b)$, where a and b are constants.

The gradient at any given point on C is given by

$$\frac{dy}{dx}=2x-6.$$

Find an equation for C, in terms of a.

$$\boxed{y=x^2-6x+2a^2}$$

