QUADRATIC INEQUALITIES

EXAM QUESTIONS

Question 1 **(**)**

$$f(x)=2x^2-9x+4,\ x\in\mathbb{R}.$$

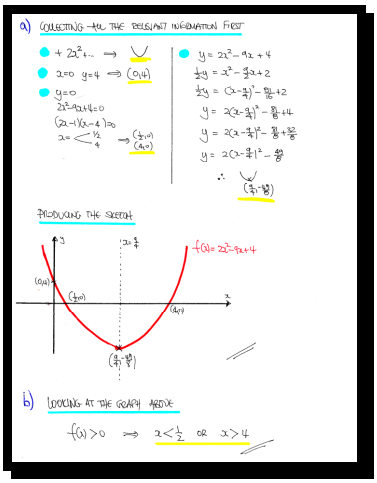
a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

b) Solve the inequality

$$f(x)>0.$$

$\square$, $\boxed{x<\frac{1}{2}\cup x>4}$



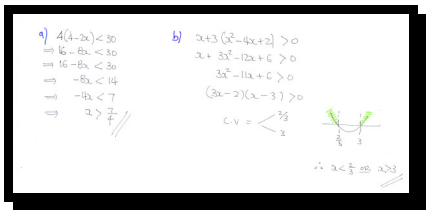
Question 2 **(**+)**

Solve each of the following inequalities.

a) $4(4-2x)<30$.

b) $x+3\left(x^2-4x+2\right)>0$.

$\square$, $\boxed{x>\frac{7}{4}}$, $\boxed{x<\frac{2}{3}\ \cup\ x>3}$



Question 3 **(**+)**

a) Solve the linear inequality

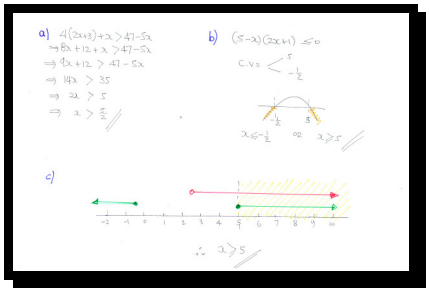
$$4(2x+3)+x>47-5x.$$

b) Solve the quadratic inequality

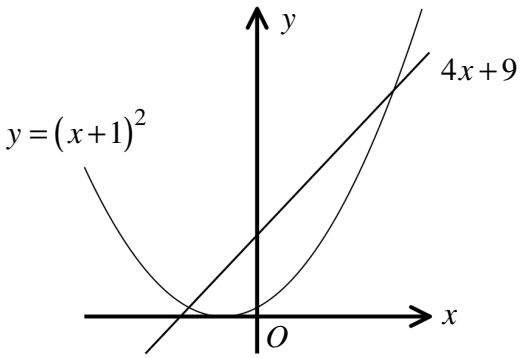
$$(5-x)(2x+1)\le 0.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part **(a)** and part **(b)**.

$\square$, $\boxed{x>\frac{5}{2}}$, $\boxed{x\le -\frac{1}{2}\ \cup\ x\ge 5}$, $\boxed{x\ge 5}$



Question 4 **(**+)**



The figure above shows the graphs of $y=(x+1)^2$ and $y=4x+9$.

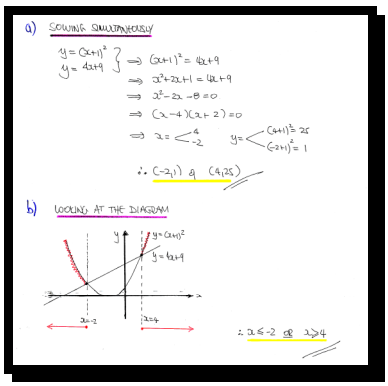
a) Find the coordinates of the points of intersection between the two graphs.

b) Hence solve the inequality

$$(x+1)^2 \geq 4x+9,$$

fully justifying the answer.

$\boxed{}$, $\boxed{(-2,1),(4,25)}$, $\boxed{x \leq -2 \ \cup \ x \geq 4}$



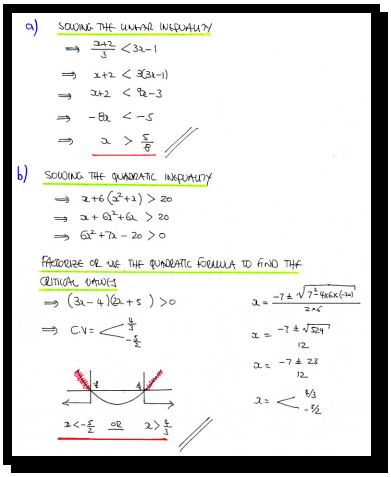
Question 5 (+)**

Solve each of the following inequalities.

a) $\dfrac{x+2}{3}<3x-1$.

b) $x+6\left(x^2+x\right)>20$.

$\boxed{}$, $\boxed{x<\frac{5}{8}}$, $\boxed{x<-\frac{5}{2}\ \cup\ x>\frac{4}{3}}$

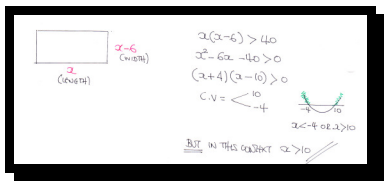


Question 6 (***)

A rectangle is such so that its length is 6 cm greater than its width.

Given the area of the rectangle is at least 40 cm^2, determine the range of the possible values of the **length** of the rectangle.

$\square$, $\boxed{\text{length} > 10}$



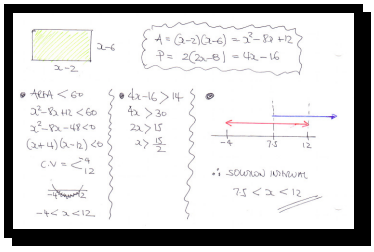
Question 7 (***)



The rectangle shown above measures $(x-2)$ cm by $(x-6)$ cm.

Given the area of the rectangle is at most 60 cm^2, and its perimeter at least 14 cm, determine the range of the possible values of x.

$\square$, $\boxed{7.5 < x < 12}$



Question 8 (***)

a) Solve the linear inequality

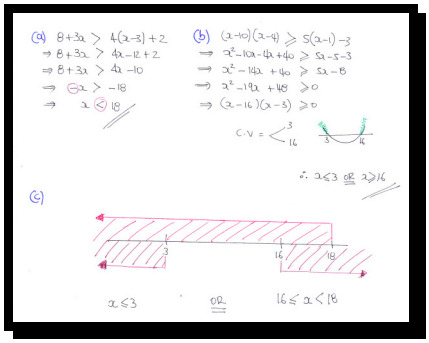
$$8+3x>4(x-3)+2.$$

b) Solve the quadratic inequality

$$(x-10)(x-4)\geq 5(x-1)-3.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part **(a)** and part **(b)**.

$\boxed{}$, $\boxed{x<18}$, $\boxed{x\leq 3\ \cup\ x\geq 16}$, $\boxed{x\leq 3\ \cup\ 16\leq x<18}$



Question 9 (***)

a) Solve the linear inequality

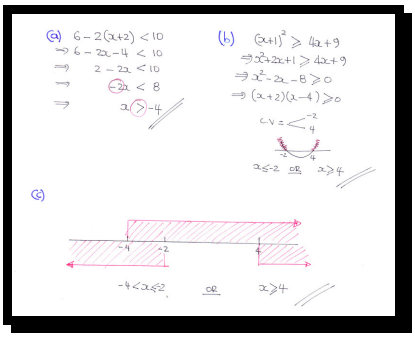
$$6-2(x+2)<10\,.$$

b) Solve the quadratic inequality

$$(x+1)^2\geq 4x+9\,.$$

c) Hence determine the range of values of x that satisfy both the inequalities of part (a) and part (b).

$\square$, $x>-4$, $x\leq -2\ \cup\ x\geq 4$, $-4<x\leq -2\ \cup\ x\geq 4$



Question 10 **(***+)**

The curve C has equation

$$y=(x-4)^2+2.$$

The line L has equation

$$y=13-2x$$

a) Sketch on the same diagram the graph of C and the graph of L.
The sketch must include the coordinates of any points where these graphs meet the coordinate axes.

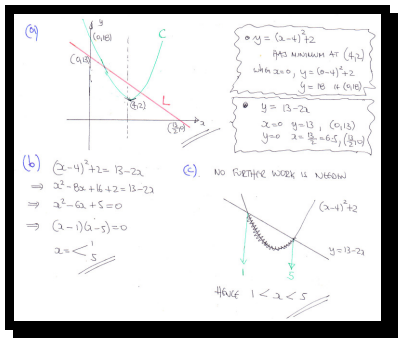
b) Solve the equation

$$(x-4)^2+2=13-2x.$$

c) Hence find the range of values of x for which

$$(x-4)^2+2<13-2x.$$

$\boxed{}$, $\boxed{x=1,5}$, $\boxed{1<x<5}$

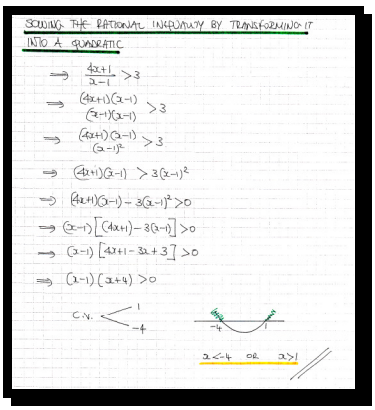


Question 11 (*+)**

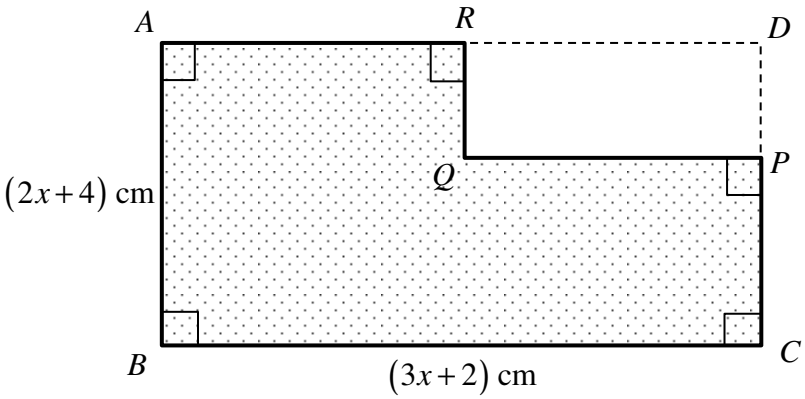
Find the set of values of x, that satisfy the following inequality.

$$\frac{4x+1}{x-1}>3.$$

$\square$, $\boxed{x<-4 \cup x>1}$



Question 12 **(***+)**



A rectangle $ABCD$ measures $(3x+2)$ cm by $(2x+4)$ cm.

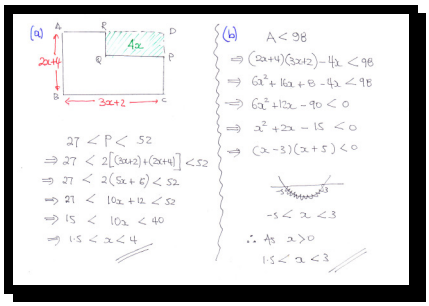
A second rectangle $PQRD$ is removed from the rectangle $ABCD$, as shown in the figure above. The perimeter of the composite shape $ABCPRQ$ is greater than 27 cm but less than 52 cm.

a) Find the range of the possible values of x.

The area of the rectangle $PQRD$ is $4x$ cm^2.

b) Given further that the area of the composite shape $ABCPRQ$ is less than 98 cm^2, determine an amended range of the possible values of x.

$\boxed{}$, $\boxed{1.5 < x < 4}$, $\boxed{1.5 < x < 3}$



Question 13 (*+)**

$$f(x)=x^2-2x-4,\ x\in\mathbb{R}.$$

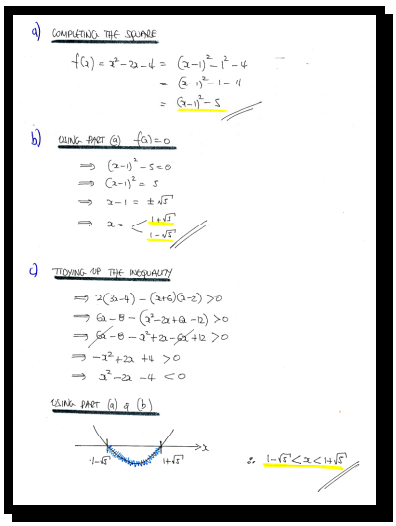
a) Express $f(x)$ in the form $(x+a)^2+b$, where a and b are constants.

b) Find in exact form the solutions of the equation $f(x)=0$.

c) Hence solve the inequality

$$2(3x-4)-(x+6)(x-2)>0.$$

$\square$, $\boxed{(x-1)^2-5}$, $\boxed{x=1\pm\sqrt{5}}$, $\boxed{1-\sqrt{5}<x<1+\sqrt{5}}$



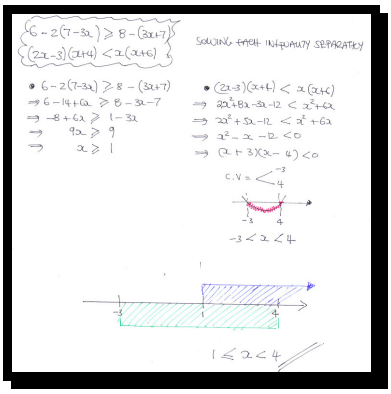
Question 14 **(***+)**

Determine the range of values of x that satisfy **both** the inequalities given below.

$$6-2(7-3x)\geq 8-(3x+7)$$

$$(2x-3)(x+4)<x(x+6).$$

$\square$, $\boxed{1\leq x<4}$

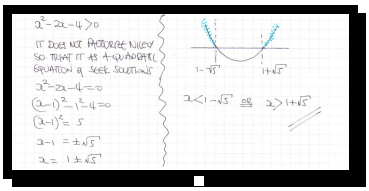


Question 15 **(***+)**

Solve the following quadratic inequality.

$$x^2-2x-4>0.$$

$\square$, $\boxed{x<1-\sqrt{5}\ \cup\ x>1+\sqrt{5}}$



Question 16 (*+)**

$$f(x)=x^2-12x+30,\ x\in\mathbb{R}.$$

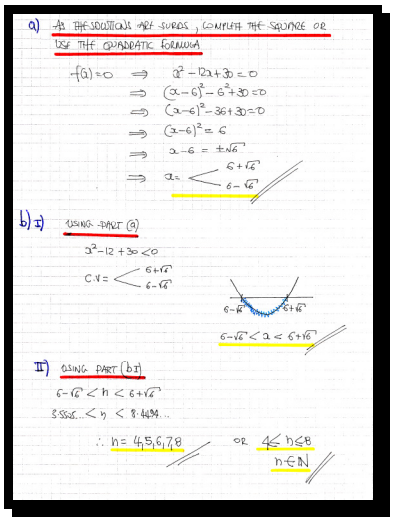
a) Find in exact surd form the solutions of the equation $f(x)=0$.

b) Hence solve the inequality …

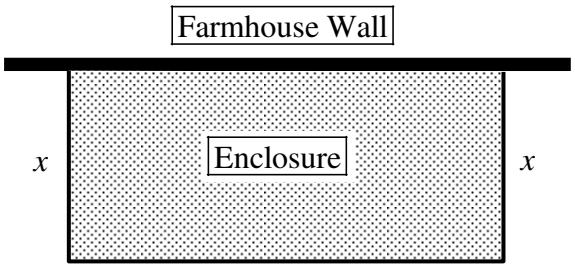
i. … $f(x)<0$.

ii. … $n^2-12n+30<0$, where n is an **integer**.

$\boxed{}$, $\boxed{x=6\pm\sqrt{6}}$, $\boxed{6-\sqrt{6}<x<6+\sqrt{6}}$, $\boxed{n=4,5,6,7,8}$



Question 17 (*+)**



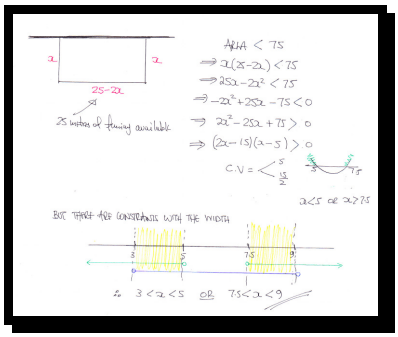
The figure above shows the plan of a rectangular enclosure to be built next to a farmhouse. One of the farmhouse's walls will form one of the sides of the enclosure and 25 metres of fencing will form the other three sides.

The width of the enclosure is x metres, as shown in the figure.

The area of the enclosure must be at most 75 m^2.

Given further that the width of the enclosure must be at least 3 metres but no more than 9 metres, determine the range of the possible values of x.

$\square$, $\boxed{3<x<5\ \cup\ 7.5<x<9}$



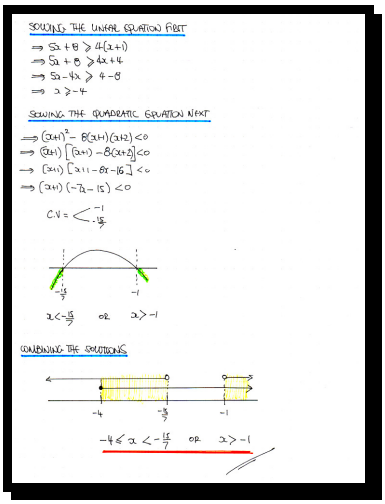
Question 18 (*+)**

Determine the range of values of x that satisfy **both** the inequalities given below.

$$5x+8\geq 4(x+1)$$

$$(x+1)^2-8(x+1)(x+2)<0\,.$$

$\square$, $-4\leq x<-\frac{15}{7}\ \cup\ x>-1$



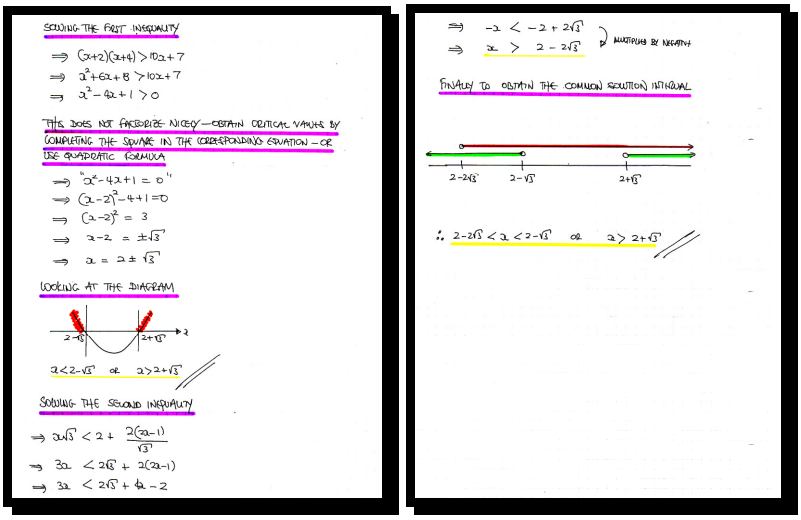
Question 19 (****)

Find, as exact simplified surds, the solution interval that satisfies both the following inequalities.

$$(x+2)(x+4)>10x+7$$

$$x\sqrt{3}<2+\frac{2(2x-1)}{\sqrt{3}}.$$

$\square$, $2-2\sqrt{3}<x<2-\sqrt{3}\ \cup\ x>2+\sqrt{3}$



Question 20 (****)

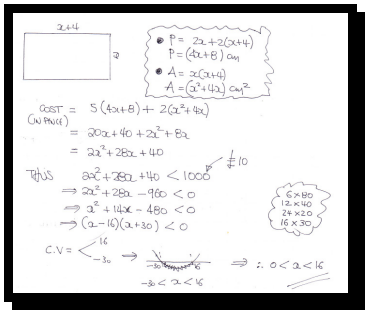
The cost for framing a picture is

- 2 pence per cm^2 of glass.
- 5 pence per cm of wooden frame.

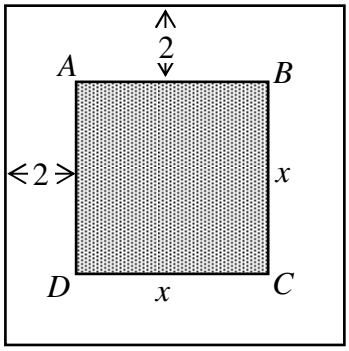
A rectangular picture is such so that its length is 4 cm greater than its width, x cm.

If a **maximum** of £10 is available for framing, determine the range of the possible values of x.

$\boxed{}$, $\boxed{0 < x < 16}$



Question 21 **(****)**



The figure above shows a square piece of lawn $ABCD$ of side length x metres.

The lawn is surrounded by a path which is 2 metres wide, as shown in the figure.

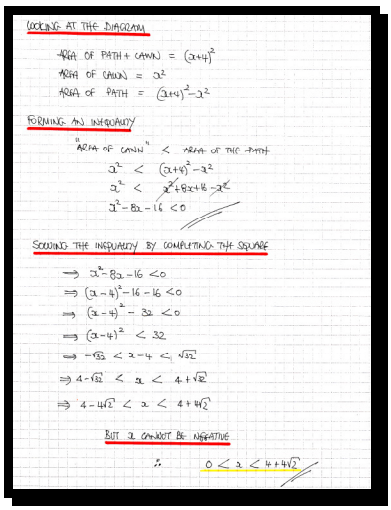
The area of the lawn must be less than the area of the path.

a) Show clearly that

$$x^2-8x-16<0\,.$$

b) Hence determine the range of the possible values of x, in terms of surds where appropriate.

$\boxed{}$, $\boxed{0<x<4+4\sqrt{2}}$


LOOKING AT THE DIAGRAM

AREA OF PATH + CAWN = $(x+4)^2$

AREA OF CAWN = x^2

AREA OF PATH = $(x+4)^2-x^2$

FORMING AN INEQUALITY

"AREA OF CAWN" < AREA OF THE PATH

$x^2 < (x+4)^2-x^2$

$x^2 < x^2+8x+16-x^2$

$x^2-8x-16<0$

SOLVING THE INEQUALITY BY COMPLETING THE SQUARE

$\Rightarrow x^2-8x-16<0$

$\Rightarrow (x-4)^2-16-16<0$

$\Rightarrow (x-4)^2-32<0$

$\Rightarrow (x-4)^2<32$

$\Rightarrow -\sqrt{32}<x-4<\sqrt{32}$

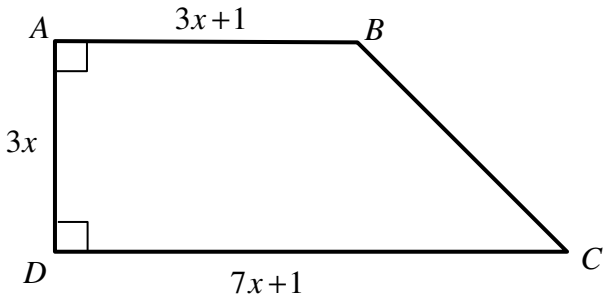
$\Rightarrow 4-\sqrt{32}<x<4+\sqrt{32}$

$\Rightarrow 4-4\sqrt{2}<x<4+4\sqrt{2}$

BUT x CANNOT BE NEGATIVE

$\therefore 0<x<4+4\sqrt{2}$

Question 22 **(****)**



The figure above shows a right angled trapezium $ABCD$ where $|AB|=3x+1$, $|AD|=3x$, $|DC|=7x+1$ and $\measuredangle DAB=\measuredangle CDA=90°$.

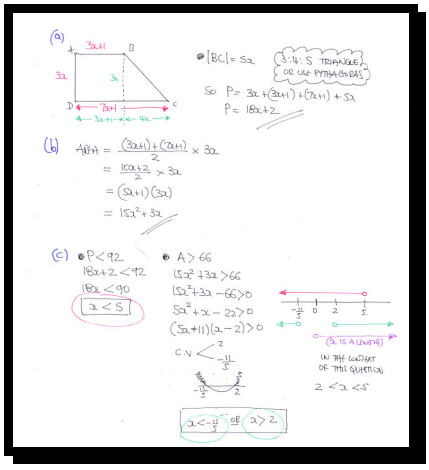
a) Express the perimeter of the trapezium in terms of x.

b) Show that the area of the trapezium is $15x^2+3x$.

The perimeter of the trapezium has to be less than 92 and its area greater than 66.

c) Determine the range of the possible values of x.

$\boxed{}$, $\boxed{P=18x+2}$, $\boxed{2<x<5}$



Question 23 (****)

A rectangular piece of card has length x cm and an area of 1200 cm^2.

A square of side length 5 cm is removed from each corner and the sides of the remaining card are folded upwards to form an **open** box of height 5 cm.

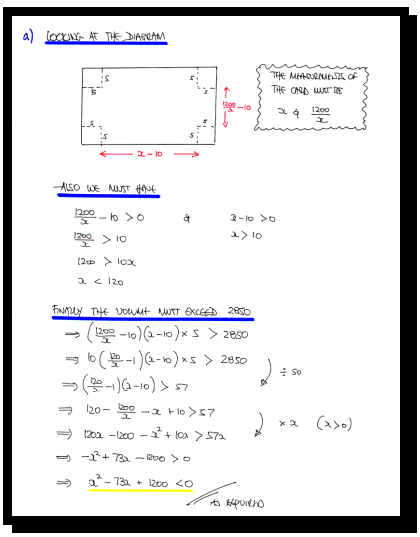
The resulting box must have a volume greater than 2850 cm^3.

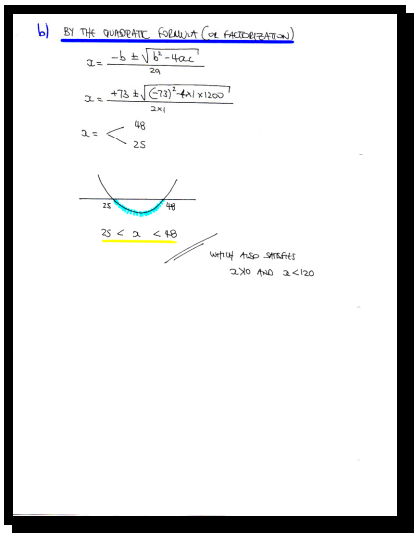
a) Show clearly that

$$x^2-73x+1200<0.$$

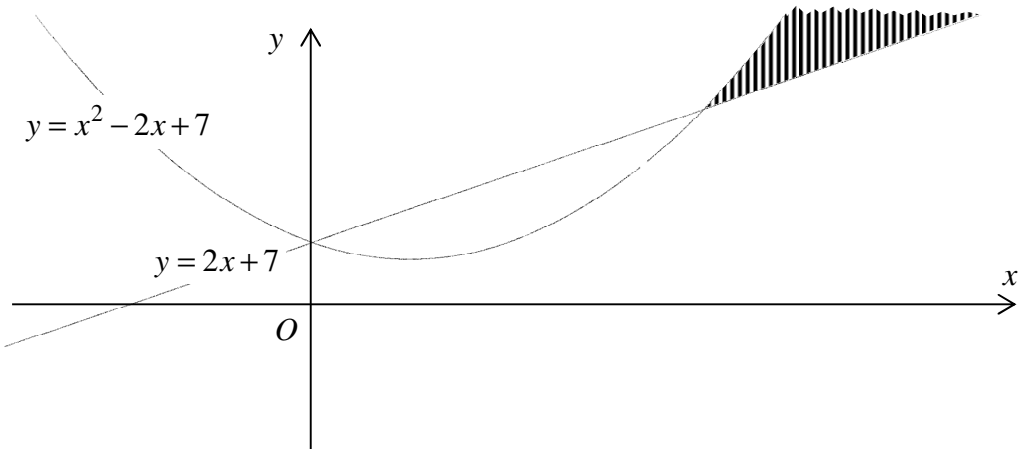
b) Hence determine the range of the possible values of x.

$\boxed{}$, $\boxed{25<x<48}$





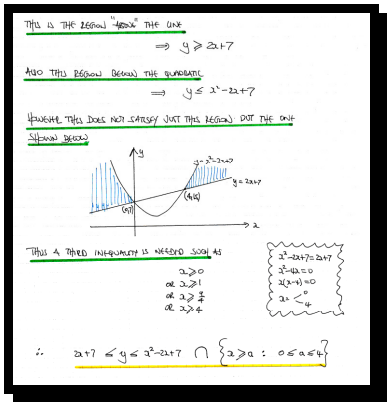
Question 24 (****)



The figure above shows the graphs of $y = x^2 - 2x + 7$ and $y = 2x + 7$.

Express the shaded region in the figure as a set of inequalities.

$\square$, $\boxed{2x + 7 \le y \le x^2 - 2x + 7 \cap \{x \ge a : 0 \le a \le 4\}}$



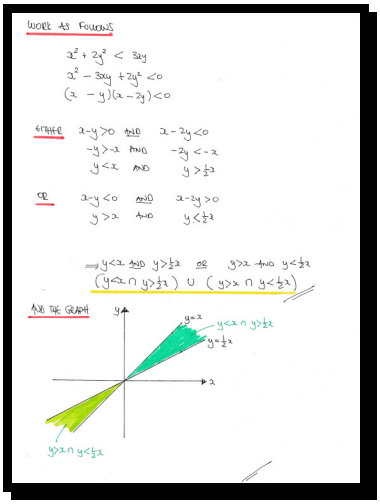
Question 25 (****)

Solve the inequality

$$x^2+2y^2<3xy\,,$$

and hence indicate the solution in a suitable sketch.

$\square$, $\boxed{\left(y<x\cap y>\frac{1}{2}x\right)\cup\left(y>x\cap y<\frac{1}{2}x\right)}$



Question 26 (**+)**

Consider the following inequalities

$$5x+13>4(x+2)$$

$$(x-2)^2-k(x-2)(x+3)<0,$$

where k is a non zero constant.

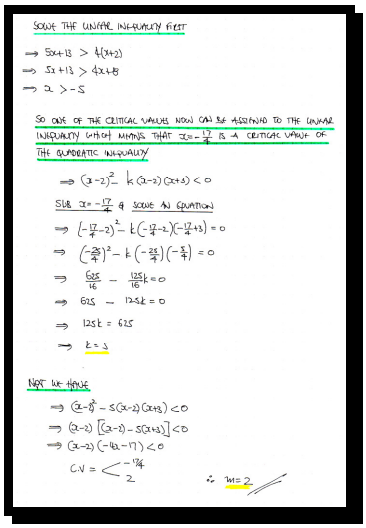
The common solution interval of both these inequalities is

$$-5<x<-\frac{17}{4} \quad \cup \quad x>m,$$

where m is a non zero constant.

Determine, in any order, the value of k and the value of m.

$\boxed{}$, $\boxed{m=2}$, $\boxed{k=5}$



Question 27 **(*****)**

Consider the following inequality

$$kx^2+2x+1 \le (x+1)(x-3),$$

where k is a real constant.

Find, in terms of k where appropriate, the solution intervals of the above inequality for all possible values of k.

$$\square,\begin{cases} k<1 & k<\dfrac{-2-\sqrt{2-k}}{k-1} \cup k>\dfrac{-2-\sqrt{2-k}}{k-1} \\ k=1 & x\le -1 \\ 1<k<2 & k<\dfrac{-2-\sqrt{2-k}}{k-1} \cup k>\dfrac{-2-\sqrt{2-k}}{k-1} \\ k=2 & x=-2 \\ k>2 & \text{no solutions} \end{cases}$$

