INDICES

NUMBER INDICES

(Non Calculator)

Question 1

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $4^{-1}+2^{-3}$

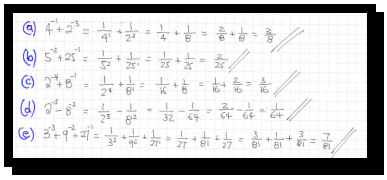
b) $5^{-2}+25^{-1}$

c) $2^{-4}+8^{-1}$

d) $2^{-5}-8^{-2}$

e) $3^{-3}+9^{-2}+27^{-1}$

$$\boxed{\frac{3}{8}},\boxed{\frac{2}{25}},\boxed{\frac{3}{16}},\boxed{\frac{1}{64}},\boxed{\frac{7}{81}}$$



Question 2

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $4^{\frac{1}{2}}+9^{\frac{1}{2}}$

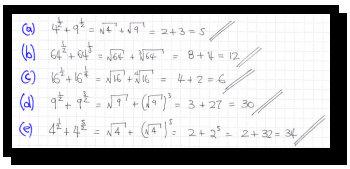
b) $64^{\frac{1}{2}}+64^{\frac{1}{3}}$

c) $16^{\frac{1}{2}}+16^{\frac{1}{4}}$

d) $9^{\frac{1}{2}}+9^{\frac{3}{2}}$

e) $4^{\frac{1}{2}}+4^{\frac{5}{2}}$

$\boxed{5}, \boxed{12}, \boxed{6}, \boxed{30}, \boxed{34}$



Question 3

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $9^{\frac{1}{2}}+9^{-\frac{1}{2}}$

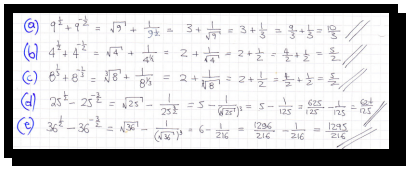
b) $4^{\frac{1}{2}}+4^{-\frac{1}{2}}$

c) $8^{\frac{1}{3}}+8^{-\frac{1}{3}}$

d) $25^{\frac{1}{2}}-25^{-\frac{3}{2}}$

e) $36^{\frac{1}{2}}-36^{-\frac{3}{2}}$

$\boxed{\frac{10}{3}}, \boxed{\frac{5}{2}}, \boxed{\frac{5}{2}}, \boxed{\frac{624}{125}}, \boxed{\frac{1295}{216}}$



Question 4

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $16^{\frac{3}{2}}+8^{\frac{2}{3}}$

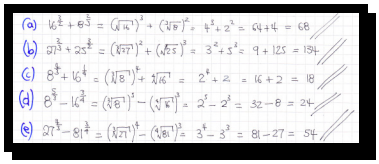
b) $27^{\frac{2}{3}}+25^{\frac{3}{2}}$

c) $8^{\frac{4}{3}}+16^{\frac{1}{4}}$

d) $8^{\frac{5}{3}}-16^{\frac{3}{4}}$

e) $27^{\frac{4}{3}}-81^{\frac{3}{4}}$

$\boxed{68}$, $\boxed{134}$, $\boxed{18}$, $\boxed{24}$, $\boxed{54}$



Question 5

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $9^{\frac{3}{2}}$

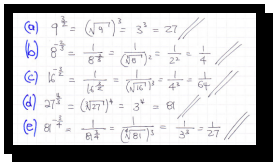
b) $8^{-\frac{2}{3}}$

c) $16^{-\frac{3}{2}}$

d) $27^{\frac{4}{3}}$

e) $81^{-\frac{3}{4}}$

$\boxed{27}, \boxed{\frac{1}{4}}, \boxed{\frac{1}{64}}, \boxed{81}, \boxed{\frac{1}{27}}$



Question 6

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\left(\frac{2}{3}\right)^{-2}$

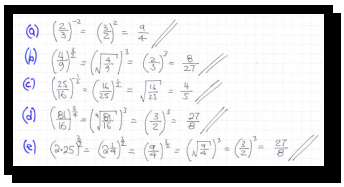
b) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

c) $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$

d) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

e) $(2.25)^{\frac{3}{2}}$

$\boxed{\frac{9}{4}}$, $\boxed{\frac{8}{27}}$, $\boxed{\frac{4}{5}}$, $\boxed{\frac{27}{8}}$, $\boxed{\frac{27}{8}}$



Question 7

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\left(1\frac{7}{9}\right)^{\frac{3}{2}}$

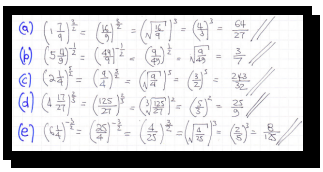
b) $\left(5\frac{4}{9}\right)^{-\frac{1}{2}}$

c) $\left(2\frac{1}{4}\right)^{\frac{5}{2}}$

d) $\left(4\frac{17}{27}\right)^{\frac{2}{3}}$

e) $\left(6\frac{1}{4}\right)^{-\frac{3}{2}}$

$\boxed{\frac{64}{27}}$, $\boxed{\frac{3}{7}}$, $\boxed{\frac{243}{32}}$, $\boxed{\frac{25}{9}}$, $\boxed{\frac{8}{125}}$



Question 8

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $32^5 \times 8^{-9} \times 2^8$

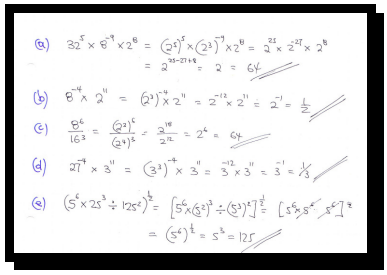
b) $8^{-4} \times 2^{11}$

c) $\dfrac{8^6}{16^3}$

d) $27^{-4} \times 3^{11}$

e) $\left(5^6 \times 25^3 \div 125^2\right)^{\frac{1}{2}}$

$\boxed{64}$, $\boxed{\frac{1}{2}}$, $\boxed{64}$, $\boxed{\frac{1}{3}}$, $\boxed{125}$



Question 9

Simplify the following without the use of a calculator, showing clearly all the steps in your calculations.

a) $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

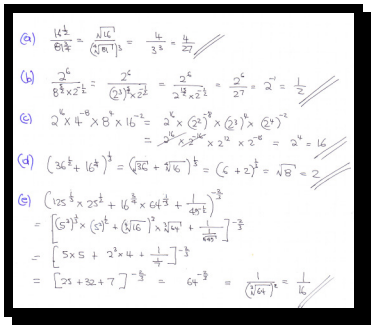
b) $\dfrac{2^6}{8^{\frac{5}{2}}\times 2^{-\frac{1}{2}}}$

c) $2^{16}\times 4^{-8}\times 8^4\times 16^{-2}$

d) $\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{\frac{1}{3}}$

e) $\left(125^{\frac{1}{3}}\times 25^{\frac{1}{2}}+16^{\frac{3}{4}}\times 64^{\frac{1}{3}}+\dfrac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$

$\boxed{\frac{4}{27}}$, $\boxed{\frac{1}{2}}$, $\boxed{16}$, $\boxed{2}$, $\boxed{\frac{1}{16}}$



ALGEBRAIC INDICES

(Non Calculator)

Question 1

Simplify **fully** each of the following expressions.

a) $4a^2b^3 \times 3ab^4$

b) $\left(2a^3b^2\right)^4$

c) $\dfrac{3a^3b^2c \times 6ab^2c^3}{2a^2bc^3}$

d) $\dfrac{\left(4xy^2\right)^2}{\left(2x\right)^3}$

e) $\dfrac{\sqrt{9x^6y^4}}{\left(3x^2y^3\right)^2}$

$\boxed{12a^3b^7}$, $\boxed{16a^{12}b^8}$, $\boxed{9a^2b^3c}$, $\boxed{\dfrac{2y^4}{x}}$, $\boxed{\dfrac{1}{3xy^4}}$

Question 2

Simplify **fully** each of the following expressions.

a) $\dfrac{x^6}{x^{-2}}$

b) $\dfrac{12y^{-5}}{3y^{-2}}$

c) $\left(3t^3q^4\right)^3$

d) $\dfrac{3z^4\times(10z)^3}{125z^5}$

$\boxed{x^8}$, $\boxed{4y^{-3} \text{ or } \dfrac{4}{y^3}}$, $\boxed{27t^9q^{12}}$, $\boxed{24z^2}$

Question 3

Simplify **fully** each of the following expressions.

a) $x^{\frac{5}{2}} \times \sqrt{x}$

b) $2y^3 \times 2y^{-1}$

c) $2w^{\frac{1}{2}} \times 3w^2$

d) $2t^{\frac{4}{3}} \times 4\sqrt[3]{t^2}$

e) $k^{\frac{3}{2}} \times 4k^{-3}$

$\boxed{x^3}$, $\boxed{4y^2}$, $\boxed{6w^{\frac{5}{2}}}$, $\boxed{8t^2}$, $\boxed{4k^{-\frac{3}{2}}}$

Question 4

Simplify **fully** each of the following expressions.

a) $\left(2k^{\frac{1}{2}}h^3\right)^4$

b) $\left(9a^6b^2\right)^{-\frac{1}{2}}$

c) $\left(2pq^2\right)^4 \times 5p\sqrt{q^6}$

d) $\dfrac{12\left(x^3y^2z\right)^4}{\left(4x^2z^6\right)^2}$

$\boxed{16k^2h^{12}}$, $\boxed{\frac{1}{3}a^{-3}b^{-1} \text{ or } \frac{1}{3a^3b}}$, $\boxed{80p^5q^{11}}$, $\boxed{\frac{3}{4}x^8y^8z^{-8} \text{ or } \frac{3x^8y^8}{4z^8}}$

Question 5

Simplify **fully** each of the following expressions.

a) $\left(2ab^2c^3\right)^3$

b) $\left(\frac{1}{2}x^3y^2\right)^3$

c) $\left(9a^6b^4\right)^{\frac{1}{2}}$

d) $\left(16p^8q^{-2}\right)^{\frac{1}{2}}$

$\boxed{8a^3b^6c^9}$, $\boxed{\frac{1}{8}x^9y^6}$, $\boxed{3a^3b^2}$, $\boxed{4p^4q^{-1} \text{ or } \frac{4p^4}{q}}$

Question 6

Simplify **fully** each of the following expressions.

a) $2a^3\left(2a^{-1}+a^{\frac{1}{2}}\right)$

b) $4b^{\frac{1}{2}}\left(2b+b^{\frac{1}{2}}\right)$

c) $c^{\frac{3}{2}}\left(3c^{-1}+c\right)$

d) $3d^{\frac{3}{2}}\left(4d^{-2}-2d^{-\frac{1}{2}}\right)$

$\boxed{4a^2+2a^{\frac{7}{2}}}$, $\boxed{8b^{\frac{3}{2}}+4b}$, $\boxed{3c^{\frac{1}{2}}+c^{\frac{5}{2}}}$, $\boxed{12d^{-\frac{1}{2}}-6d}$

Question 7

Simplify **fully** each of the following expressions.

a) $a\left(2a^{-1}-3a^{-\frac{1}{2}}\right)$

b) $3b^{2}\left(b^{-2}+2b^{-\frac{1}{2}}\right)$

c) $3c^{\frac{7}{2}}\left(2c^{-\frac{1}{2}}-c\right)$

d) $2d^{\frac{7}{2}}\left(2d^{-1}+d^{\frac{1}{2}}\right)$

$\boxed{2-3a^{\frac{1}{2}}}$, $\boxed{3+6b^{\frac{3}{2}}}$, $\boxed{6c^{3}-3c^{\frac{9}{2}}}$, $\boxed{4d^{\frac{5}{2}}+2d^{4}}$

Question 8

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\dfrac{1}{2\sqrt{x}}+\dfrac{4}{x^2}$

b) $x\sqrt{x}-\dfrac{1}{x^2}$

c) $\sqrt{x^3}-\dfrac{1}{2x^2}$

d) $\sqrt[3]{x^2}-\dfrac{3}{2x^3}$

e) $4\sqrt{x}+\dfrac{1}{4\sqrt{x}}$

$\boxed{\frac{1}{2}x^{-\frac{1}{2}}+4x^{-2}}$, $\boxed{x^{\frac{3}{2}}-x^{-2}}$, $\boxed{x^{\frac{3}{2}}-\frac{1}{2}x^{-2}}$, $\boxed{x^{\frac{2}{3}}-\frac{3}{2}x^{-3}}$, $\boxed{4x^{\frac{1}{2}}+\frac{1}{4}x^{-\frac{1}{2}}}$

Question 9

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\left(5-x^{-2}\right)\left(2x^3-x\right)$

b) $\left(1-x^{\frac{1}{2}}\right)\left(2-x^{\frac{1}{2}}\right)$

c) $\left(1+x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}+2\right)$

$\boxed{10x^3-7x+x^{-1}}$, $\boxed{x-3x^{\frac{1}{2}}+2}$, $\boxed{x^2+x^{\frac{3}{2}}+2x^{\frac{1}{2}}+2}$

Question 10

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\left(x^{\frac{3}{2}}+2x^{-\frac{3}{2}}\right)^2$

b) $\left(x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)^2$

c) $\left(3x^{-\frac{3}{2}}+2x^{\frac{1}{2}}\right)^2$

d) $\left(x^{\frac{5}{2}}+x^{\frac{1}{2}}\right)^2$

e) $\left(3\sqrt{x}-2\right)^2$

$\boxed{x^3+4+4x^{-3}}$, $\boxed{x-4+4x^{-1}}$, $\boxed{4x+12x^{-1}+9x^{-3}}$, $\boxed{x^5+2x^3+x}$, $\boxed{9x-12x^{\frac{1}{2}}+4}$

Question 11

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\left(2x^{\frac{1}{2}}+3\right)^2$

b) $\left(2x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)^2$

c) $\left(2x^{\frac{3}{2}}-3x^{-\frac{3}{2}}\right)^2$

d) $\left(x^{\frac{1}{2}}-2x^{-\frac{3}{2}}\right)^2$

e) $\left(x^{\frac{1}{2}}-4\right)\left(x^{-\frac{1}{2}}-1\right)$

$\boxed{4x+12x^{\frac{1}{2}}+9}$, $\boxed{4x-4+x^{-1}}$, $\boxed{4x^3-12+9x^{-3}}$, $\boxed{x-4x^{-1}+4x^{-3}}$, $\boxed{-x^{\frac{1}{2}}+5-4x^{-\frac{1}{2}}}$

Question 12

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\dfrac{4+x}{2x^3}$

b) $\dfrac{9\sqrt{x}+6x}{3x^3}$

c) $\dfrac{(x+2)(2x-3)}{4x^5}$

d) $\dfrac{x^2+3x}{2\sqrt{x}}$

e) $\dfrac{\sqrt{x}(2x-4)}{3x^2}$

$\boxed{2x^{-3}+\frac{1}{2}x^{-2}}$, $\boxed{3x^{-\frac{5}{2}}+2x^{-2}}$, $\boxed{\frac{1}{2}x^{-3}+\frac{1}{4}x^{-4}-\frac{3}{2}x^{-5}}$, $\boxed{\frac{1}{2}x^{\frac{3}{2}}+\frac{3}{2}x^{\frac{1}{2}}}$, $\boxed{\frac{2}{3}x^{-\frac{1}{2}}-\frac{4}{3}x^{-\frac{3}{2}}}$

Question 13

Write each of the following expressions as the sum of terms of the form kx^n, where k is a constant.

a) $\dfrac{(3x-2)(2x-1)}{2x^{\frac{3}{2}}}$

b) $\dfrac{(2\sqrt{x}+3)^2}{4x}$

c) $\dfrac{x^2(\sqrt{x}+4x)}{4\sqrt{x}}$

d) $\dfrac{\sqrt{x}\left(5x^2-8\right)}{4x}$

e) $\dfrac{(x^2-3)(\sqrt{x}+4x)}{3\sqrt{x}}$

$\boxed{3x^{\frac{1}{2}}-\frac{7}{2}x^{-\frac{1}{2}}+x^{-\frac{3}{2}}}$, $\boxed{1+3x^{-\frac{1}{2}}+\frac{9}{4}x^{-1}}$, $\boxed{x^{\frac{5}{2}}+\frac{1}{4}x^2}$, $\boxed{\frac{5}{4}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}}$, $\boxed{\frac{4}{3}x^{\frac{5}{2}}+x^2-4x^{\frac{1}{2}}-3}$

INDICIAL EQUATIONS

(Non Calculator)

Question 1

Solve each of the following equations without using a calculator.

a) $x^{\frac{1}{3}} = 2$

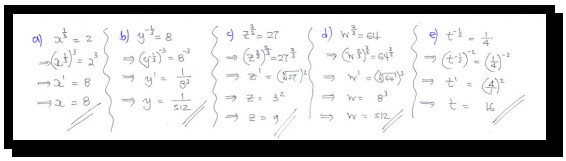
b) $y^{-\frac{1}{3}} = 8$

c) $z^{\frac{3}{2}} = 27$

d) $w^{\frac{2}{3}} = 64$

e) $t^{-\frac{1}{2}} = \frac{1}{4}$

$\boxed{x = 8}$, $\boxed{y = \frac{1}{512}}$, $\boxed{z = 9}$, $\boxed{w = 512}$, $\boxed{t = 16}$



Question 2

Solve each of the following equations without using a calculator.

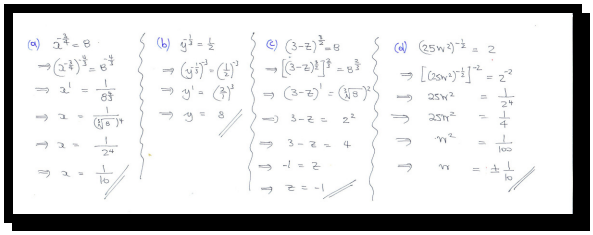
a) $x^{-\frac{3}{4}} = 8$

b) $y^{-\frac{1}{3}} = \frac{1}{2}$

c) $(3-z)^{\frac{3}{2}} = 8$

d) $\left(25w^2\right)^{-\frac{1}{2}} = 2$

$\boxed{x = \frac{1}{16}}$, $\boxed{y = 8}$, $\boxed{z = -1}$, $\boxed{w = (\pm)\frac{1}{10}}$



Question 3

Solve each of the following equations without using a calculator.

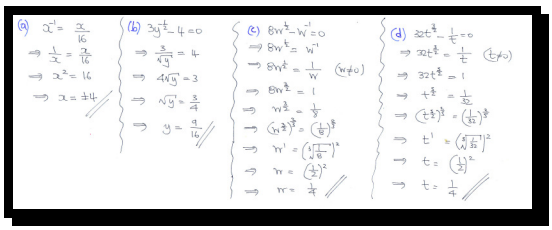
a) $x^{-1} = \dfrac{x}{16}$

b) $3y^{-\frac{1}{2}} - 4 = 0$

c) $8w^{\frac{1}{2}} - w^{-1} = 0$

d) $32t^{\frac{3}{2}} - \dfrac{1}{t} = 0$

$\boxed{x = \pm 4}$, $\boxed{y = \dfrac{9}{16}}$, $\boxed{w = \dfrac{1}{4}}$, $\boxed{t = \dfrac{1}{4}}$



Question 4

Solve each of the following equations without using a calculator.

a) $2^{3-x} = 4^{x}$

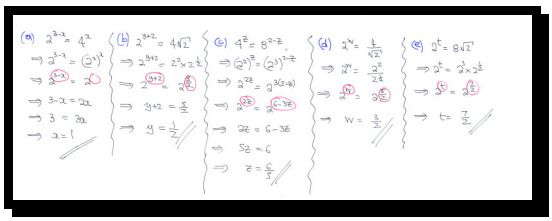
b) $2^{y+2} = 4\sqrt{2}$

c) $4^{z} = 8^{2-z}$

d) $2^{w} = \dfrac{4}{\sqrt{2}}$

e) $2^{t} = 8\sqrt{2}$

$\boxed{x=1}$, $\boxed{y=\frac{1}{2}}$, $\boxed{z=\frac{6}{5}}$, $\boxed{w=\frac{3}{2}}$, $\boxed{t=\frac{7}{2}}$



Question 5

Solve each of the following equations without using a calculator.

a) $3^{x+2} = 9^{x}$

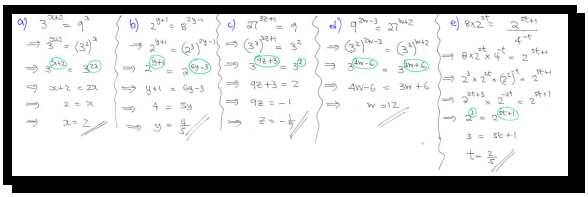
b) $2^{y+1} = 8^{2y-1}$

c) $27^{3z+1} = 9$

d) $9^{2w-3} = 27^{w+2}$

e) $8 \times 2^{2t} = \dfrac{2^{5t+1}}{4^{-t}}$

$\boxed{x = 2}$, $\boxed{y = \frac{4}{5}}$, $\boxed{z = -\frac{1}{9}}$, $\boxed{w = 12}$, $\boxed{t = \frac{2}{5}}$



Question 6

Solve each of the following equations without using a calculator.

a) $2^{x+2} = 4^x$

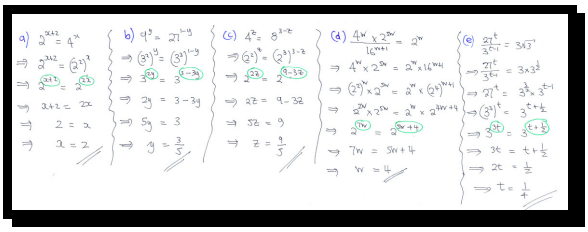
b) $9^y = 27^{1-y}$

c) $4^z = 8^{3-z}$

d) $\dfrac{4^w \times 2^{5w}}{16^w} = 2^w$

e) $\dfrac{27^t}{3^{t-1}} = 3\sqrt{3}$

$\boxed{x=2}$, $\boxed{y=\frac{3}{5}}$, $\boxed{z=\frac{9}{5}}$, $\boxed{w=4}$, $\boxed{t=\frac{1}{4}}$



Question 7

Solve each of the following equations without using a calculator.

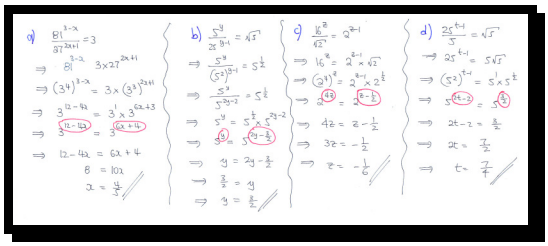
a) $\dfrac{81^{3-x}}{27^{2x+1}}=3$

b) $\dfrac{5^{y}}{25^{y-1}}=\sqrt{5}$

c) $\dfrac{16^{z}}{\sqrt{2}}=2^{z-1}$

d) $\dfrac{25^{t-1}}{5}=\sqrt{5}$

$\boxed{x=\frac{4}{5}}$, $\boxed{y=\frac{3}{2}}$, $\boxed{z=-\frac{1}{6}}$, $\boxed{t=\frac{7}{4}}$



Question 8

Solve each of the following equations without using a calculator.

a) $2^{3x+4} = 4\sqrt{2}$

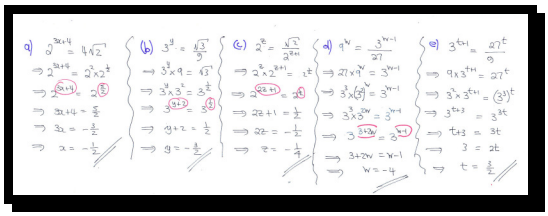
b) $3^{x} = \dfrac{\sqrt{3}}{9}$

c) $2^{z} = \dfrac{\sqrt{2}}{2^{z+1}}$

d) $9^{w} = \dfrac{3^{w-1}}{27}$

e) $3^{t+1} = \dfrac{27^{t}}{9}$

$\boxed{x = -\frac{1}{2}}$, $\boxed{y = -\frac{3}{2}}$, $\boxed{z = -\frac{1}{4}}$, $\boxed{w = -4}$, $\boxed{t = \frac{3}{2}}$



Question 9

Solve each of the following equations without using a calculator.

a) $3x^{\frac{1}{3}} = x^{-\frac{2}{3}}$

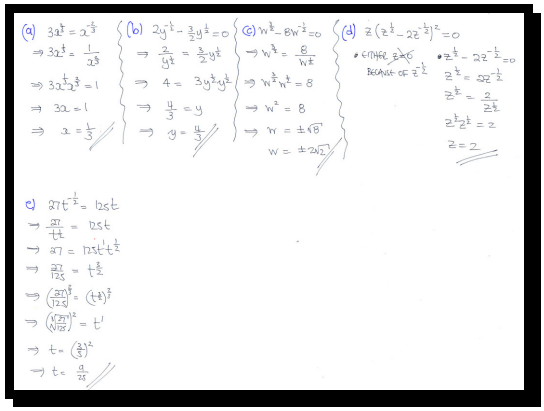
b) $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

c) $w^{\frac{3}{2}} - 8x^{-\frac{1}{2}} = 0$

d) $z\left(z^{\frac{1}{2}} - 2z^{-\frac{1}{2}}\right)^2 = 0$

e) $27t^{-\frac{1}{2}} = 125t$

$\boxed{x = \frac{1}{3}}$, $\boxed{y = \frac{4}{3}}$, $\boxed{w = \pm 2\sqrt{2}}$, $\boxed{z = 2}$, $\boxed{t = \frac{9}{25}}$



Question 10

Solve each of the following equations without using a calculator.

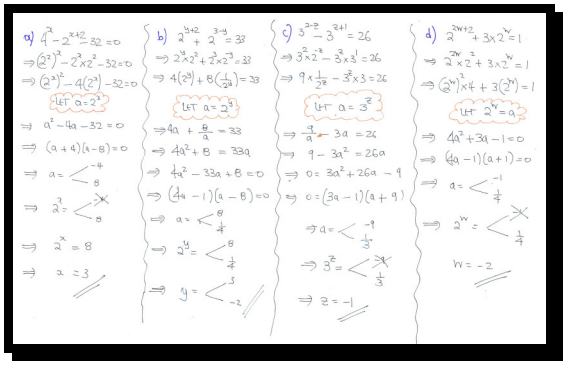
a) $4^x - 2^{x+2} - 32 = 0$

b) $2^{y+2} + 2^{3-y} = 33$

c) $3^{2-z} - 3^{z+1} = 26$

d) $2^{2w+2} + 3 \times 2^w - 1 = 0$

$\boxed{x = 3}$, $\boxed{y = -2, 3}$, $\boxed{z = -1}$, $\boxed{w = -2}$



Question 11

Solve each of the following equations without using a calculator.

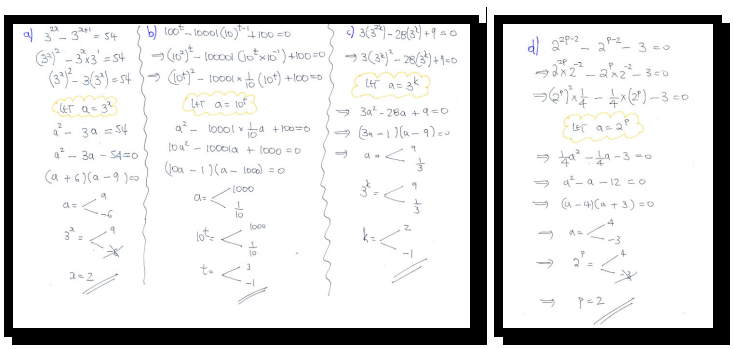
a) $3^{2x}-3^{x+1}=54$

b) $100^{t}-10001(10)^{t-1}+100=0$

c) $3\left(3^{2k}\right)-28\left(3^{k}\right)+9=0$

d) $2^{2p-2}-2^{p-2}-3=0$

$\boxed{x=2}$, $\boxed{t=3,\ -1}$, $\boxed{k=-1,\ 2}$, $\boxed{p=2}$



Question 12

Solve each of the following equations without using a calculator.

a) $2x^{\frac{2}{3}}+5x^{\frac{1}{3}}-12=0$

b) $y^{\frac{1}{4}}-y^{-\frac{1}{4}}=2$

c) $6z^{-\frac{1}{3}}-z^{\frac{1}{3}}=5$

d) $3w+w^{\frac{1}{2}}-2=0$

e) $t^{\frac{1}{3}}=2+15t^{-\frac{1}{3}}$

$\boxed{x=-64,\ \frac{27}{8}}$, $\boxed{y=3}$, $\boxed{z=1,-216}$, $\boxed{w=\frac{4}{9}}$, $\boxed{t=-27,\ 125}$

