# INDICES

## Exam Questions

**Question 1 (**)**

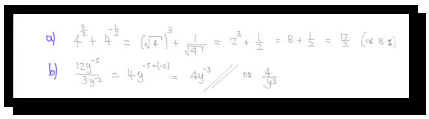
**a)** Evaluate the following indicial expression, giving the final answer as an exact simplified fraction.

$$4^{\frac{3}{2}}+4^{-\frac{1}{2}}.$$

**b)** Simplify fully the following expression

$$\frac{12y^{-5}}{3y^{-2}}.$$

$\square$, $\boxed{\frac{17}{2}}$, $\boxed{4y^{-3}=\frac{4}{y^3}}$



**Question 2 (**)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

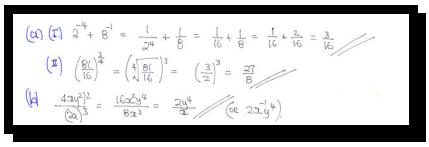
**i.** $2^{-4}+8^{-1}$.

**ii.** $\left(\frac{81}{16}\right)^{\frac{3}{4}}$.

**b)** Simplify fully the following expression

$$\frac{\left(4xy^2\right)^2}{\left(2x\right)^3}.$$

$\square$, $\boxed{\frac{3}{16}}$, $\boxed{\frac{27}{8}}$, $\boxed{2y^4x^{-1}=\frac{2y^4}{x}}$



**Question 3 (**+)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

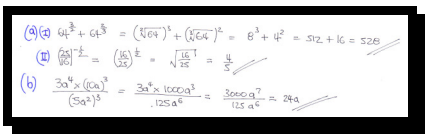
**i.** $64^{\frac{3}{2}}+64^{\frac{2}{3}}$.

**ii.** $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$.

**b)** Simplify fully the following expression

$$\frac{3a^4\times(10a)^3}{\left(5a^2\right)^3}.$$

$\boxed{528}$, $\boxed{\frac{4}{5}}$, $\boxed{24a}$



**Question 4 (**+)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

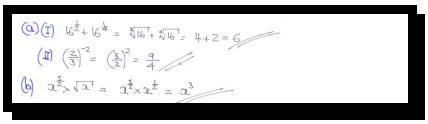
**i.** $16^{\frac{1}{2}}+16^{\frac{1}{4}}$.

**ii.** $\left(\frac{2}{3}\right)^{-2}$.

**b)** Simplify fully the following expression

$$x^{\frac{5}{2}}\times\sqrt{x}\,.$$

$\boxed{6}$, $\boxed{\frac{9}{4}}$, $\boxed{x^3}$



**Question 5 (**+)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

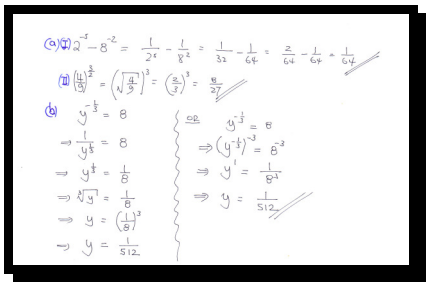
**i.** $2^{-5}-8^{-2}$.

**ii.** $\left(\frac{4}{9}\right)^{\frac{3}{2}}$.

**b)** Solve the equation

$$y^{-\frac{1}{3}}=8.$$

$\square$, $\boxed{\frac{1}{64}}$, $\boxed{\frac{8}{27}}$, $\boxed{\frac{1}{512}}$

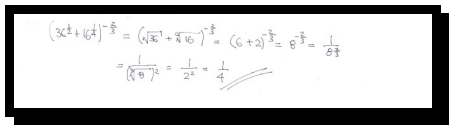


**Question 6 (**+)**

Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{-\frac{2}{3}}.$$

$\square$, $\boxed{\frac{1}{4}}$



**Question 7 (**+)**

**a)** Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

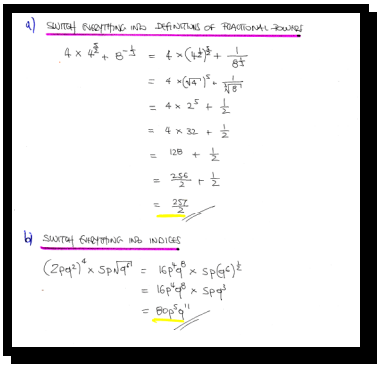
$$4\times 4^{\frac{5}{2}}+8^{-\frac{1}{3}}.$$

***The answer to this part of the question must be fully supported by a detailed method, justifying each step in the evaluation.***

**b)** Simplify fully the following expression

$$\left(2pq^2\right)^4\times 5p\sqrt{q^6}\ .$$

$\boxed{\phantom{00}}$, $\boxed{\frac{257}{2}}$, $\boxed{80p^5q^{11}}$



**Question 8 (***)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

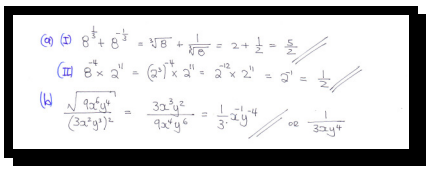
**i.** $8^{\frac{1}{3}}+8^{-\frac{1}{3}}$.

**ii.** $8^{-4}\times 2^{11}$.

**b)** Simplify fully

$$\frac{\sqrt{9x^6y^4}}{\left(3x^2y^3\right)^2}.$$

$\square$, $\boxed{\frac{5}{2}}$, $\boxed{\frac{1}{2}}$, $\boxed{\frac{1}{3xy^4}}$



**Question 9 (***)**

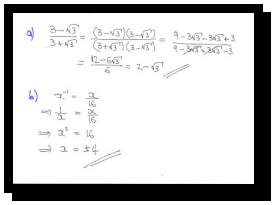
**a)** Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where $a$ and $b$ are integers

$$\frac{3-\sqrt{3}}{3+\sqrt{3}}.$$

**b)** Solve the equation

$$x^{-1}=\frac{x}{16}, \quad x\neq 0.$$

$\boxed{\phantom{00}}$, $\boxed{2-\sqrt{3}}$, $\boxed{x=\pm 4}$



**Question 10 (***)**

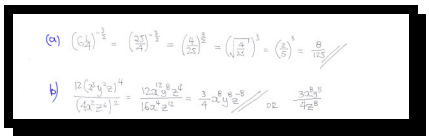
**a)** Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(6\frac{1}{4}\right)^{-\frac{3}{2}}.$$

**b)** Simplify fully the following expression

$$\frac{12\left(x^3y^2z\right)^4}{\left(4x^2z^6\right)^2}.$$

$\boxed{\frac{8}{125}}$, $\boxed{\frac{3}{4}x^8y^8z^{-8}=\frac{3x^8y^8}{4z^8}}$



**Question 11 (***)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions, where appropriate.

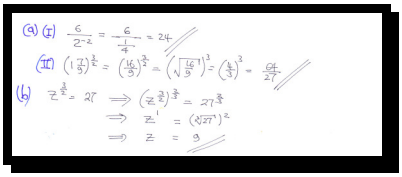
**i.** $\dfrac{6}{2^{-2}}$.

**ii.** $\left(1\dfrac{7}{9}\right)^{\frac{3}{2}}$.

**b)** Solve the equation

$$z^{\frac{3}{2}}=27\,.$$

$\square$, $\boxed{24}$, $\boxed{\dfrac{64}{27}}$, $\boxed{z=9}$



**Question 12 (***)**

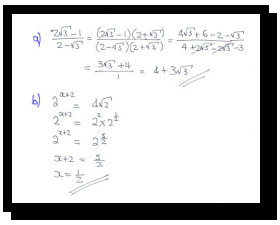
**a)** Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where $a$ and $b$ are integers

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}}.$$

**b)** Solve the equation

$$2^{x+2}=4\sqrt{2}.$$

$\square$, $\boxed{4+3\sqrt{3}}$, $\boxed{x=\frac{1}{2}}$



**Question 13 (***)**

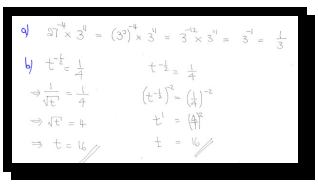
**a)** Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$27^{-4}\times 3^{11}.$$

**b)** Solve the equation

$$t^{-\frac{1}{2}}=\frac{1}{4}.$$

$\boxed{\frac{1}{3}}$, $\boxed{t=16}$



**Question 14 (***)**

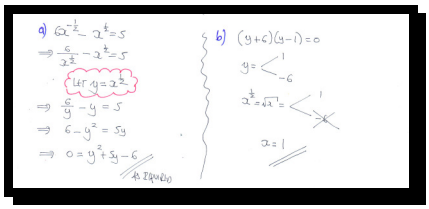
$$6x^{-\frac{1}{2}} - x^{\frac{1}{2}} = 5 .$$

**a)** Show that the substitution $y = x^{\frac{1}{2}}$ transforms the above indicial equation into the quadratic equation

$$y^2 + 5y - 6 = 0 .$$

**b)** Solve the quadratic equation and hence find the root of the **indicial** equation.

$$\boxed{x = 1}$$



**Question 15 (***)**

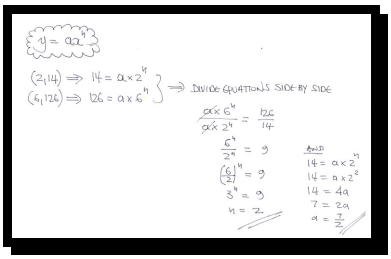
The points $(2,14)$ and $(6,126)$ lie on the curve with equation

$$y = ax^n , \ x \in \mathbb{R}$$

where $a$ and $n$ are non zero constants.

Find the value of $a$ and the value of $n$.

$$\boxed{a = \frac{7}{2}} , \ \boxed{n = 2}$$



**Question 16 (***)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

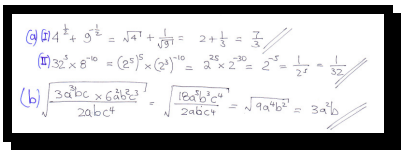
**i.** $4^{\frac{1}{2}}+9^{-\frac{1}{2}}$.

**ii.** $32^{5}\times 8^{-10}$.

**b)** Simplify fully the following expression

$$\sqrt{\frac{3a^3bc\times 6a^2b^2c^3}{2abc^4}}\,.$$

$\boxed{\frac{7}{3}}$, $\boxed{\frac{1}{32}}$, $\boxed{3a^2b}$



**Question 17 (***)**

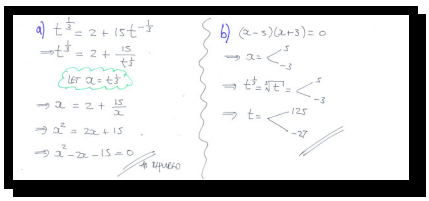
$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}.$$

**a)** Show that the substitution $x = t^{\frac{1}{3}}$ transforms the above indicial equation into the quadratic equation

$$x^2 - 2x - 15 = 0.$$

**b)** Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$$\boxed{t = -27, \ \ t = 125}$$



**Question 18 (***)**

**a)** Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

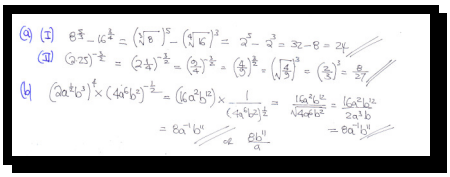
**i.** $8^{\frac{5}{3}}-16^{\frac{3}{4}}$.

**ii.** $(2.25)^{-\frac{3}{2}}$.

**b)** Simplify fully the following expression

$$\left(2a^{\frac{1}{2}}b^3\right)^4\times\left(4a^6b^2\right)^{-\frac{1}{2}}.$$

$\boxed{24}$, $\boxed{\frac{8}{27}}$, $\boxed{8a^{-1}b^{11}=\frac{8b^{11}}{a}}$



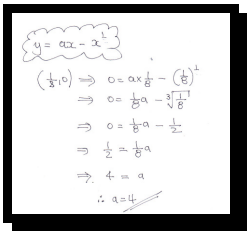
**Question 19 (***)**

Given that the curve with equation

$$y=ax-x^{\frac{1}{3}},\ x\geq 0,$$

passes though the point with coordinates $\left(\frac{1}{8},0\right)$, find the value of the constant $a$.

$\boxed{a=4}$



**Question 20 (***)**

**a)** Evaluate the following indicial expressions, giving the answers as integers.

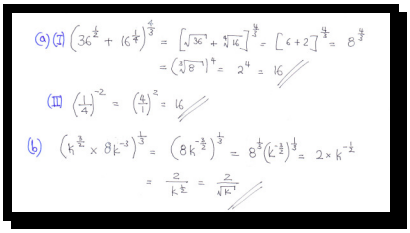
**i.** $\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{\frac{4}{3}}$.

**ii.** $\left(\frac{1}{4}\right)^{-2}$.

**b)** Simplify fully the following expression

$$\left(k^{\frac{3}{2}}\times 8k^{-3}\right)^{\frac{1}{3}}.$$

$\boxed{16}$, $\boxed{16}$, $\boxed{2k^{-\frac{1}{2}}=\frac{2}{\sqrt{k}}}$



**Question 21 (***+)**

**a)** Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{3}$.

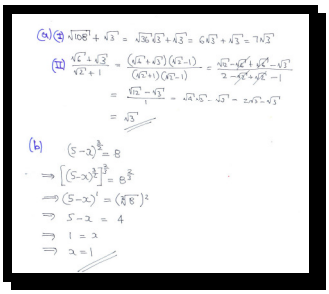
**i.** $\sqrt{108}+\sqrt{3}$.

**ii.** $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1}$.

**b)** Solve the equation

$$(5-x)^{\frac{3}{2}}=8.$$

$\boxed{7\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{x=1}$



**Question 22 (***+)**

**a)** Evaluate the following indicial expressions, giving the answers as integers.

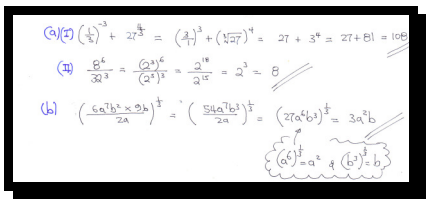
**i.** $\left(\frac{1}{3}\right)^{-3}$.

**ii.** $\frac{8^6}{32^3}$.

**b)** Simplify fully the following expression

$$\left(\frac{6a^7b^2\times 9b}{2a}\right)^{\frac{1}{3}}.$$

$\boxed{108}$, $\boxed{8}$, $\boxed{3a^2b}$



**Question 23 (***+)**

**a)** Simplify fully each of the following expressions, writing the final answer as a single simplified surd.

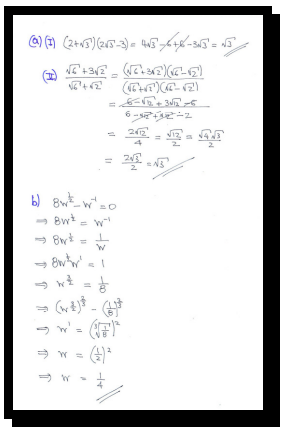
**i.** $\left(2+\sqrt{3}\right)\left(2\sqrt{3}-3\right)$.

**ii.** $\dfrac{\sqrt{6}+3\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.

**b)** Solve the equation

$$8w^{\frac{1}{2}}-w^{-1}=0, \quad w\neq 0.$$

$\square$, $\boxed{\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{w=\frac{1}{4}}$

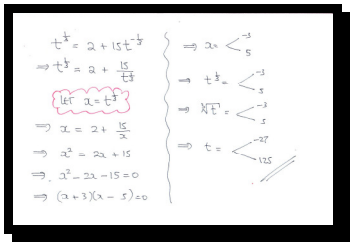


**Question 24 (***+)**

$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}, \quad t \neq 0.$$

Use the substitution $x = t^{\frac{1}{3}}$ to solve the above indicial equation.

$$\boxed{t = -27, \ t = 125}$$



**Question 25 (***+)**

An exponential curve has equation

$$y = ab^x, \ x \in \mathbb{R},$$

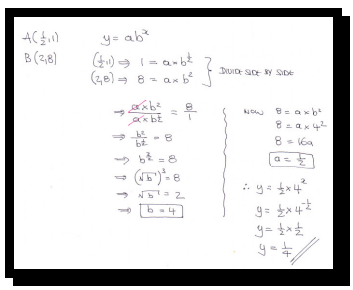
where $a$ and $b$ are non zero constants.

The points $A\left(\frac{1}{2},1\right)$, $B(2,8)$ and $C\left(-\frac{1}{2},k\right)$ lie on this curve.

**a)** Find the values of $a$ and $b$.

**b)** Find the value of $k$.

$$\boxed{a = \frac{1}{2}}, \ \boxed{b = 4}, \ \boxed{k = \frac{1}{4}}$$



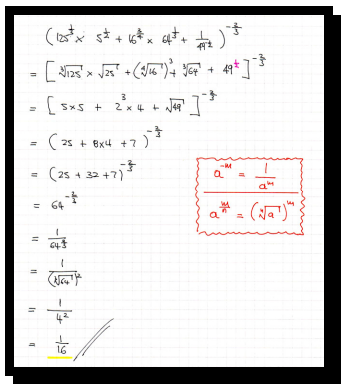
**Question 26 (***+)**

$$\left(125^{\frac{1}{3}}\times 25^{\frac{1}{2}}+16^{\frac{3}{4}}\times 64^{\frac{1}{3}}+\frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}.$$

Evaluate the above indicial expression, giving the final answer as a simplified fraction.

***You may not use any calculating aid in the above question, and detailed workings must support the answer.***

$\square$, $\boxed{\frac{1}{16}}$



**Question 27 (***+)**

**a)** Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{2}$.

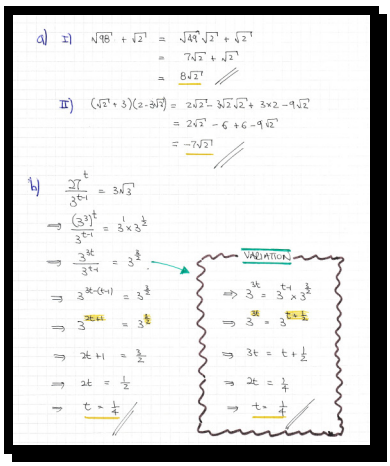
**i.** $\sqrt{98}+\sqrt{2}$.

**ii.** $\left(\sqrt{2}+3\right)\left(2-3\sqrt{2}\right)$.

**b)** Solve the equation

$$\frac{27^t}{3^{t-1}}=3\sqrt{3}.$$

$\boxed{\phantom{00}}$, $\boxed{8\sqrt{2}}$, $\boxed{-7\sqrt{2}}$, $\boxed{t=\frac{1}{4}}$



**Question 28** **(***+)**

**a)** Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$16^{\frac{1}{2}}+16^{-\frac{3}{4}}.$$

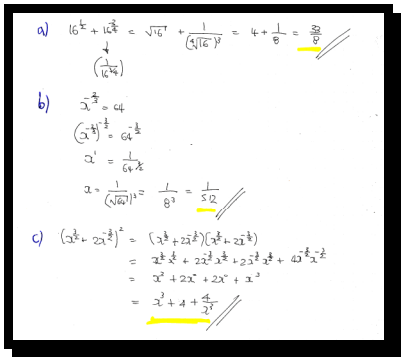
**b)** Solve the equation

$$x^{-\frac{2}{3}}=64.$$

**c)** Simplify fully

$$\left(x^{\frac{3}{2}}+2x^{-\frac{3}{2}}\right)^2.$$

$\boxed{\phantom{00}}$, $\boxed{\frac{33}{8}}$, $\boxed{x=\frac{1}{512}}$, $\boxed{x^3+4+\frac{4}{x^3}}$



**Question 29 (***+)**

An exponential curve has equation

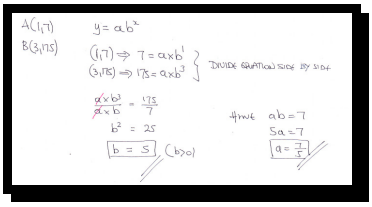
$$y = ab^x, \ x \in \mathbb{R}$$

where $a$ and $b$ are non zero constants.

The points $A(1,7)$ and $B(3,175)$ lie on this curve.

Given that $b > 0$, find the values of $a$ and $b$.

$\boxed{a = 1.4}$, $\boxed{b = 5}$

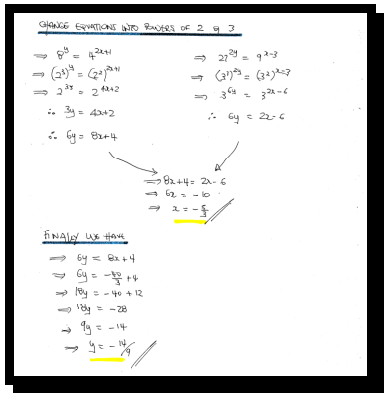



**Question 30 (***+)**

Solve the following simultaneous equations without using a calculator

$$8^y = 4^{2x+1}$$
$$27^{2y} = 9^{x-3}$$

$\boxed{\phantom{00}}$, $\boxed{\left(-\frac{5}{3}, -\frac{14}{9}\right)}$



**Question 31 (***+)**

**a)** Solve the equation

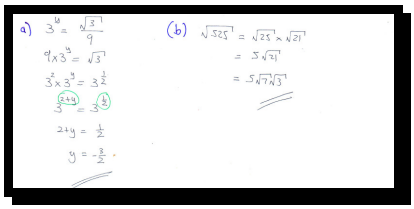
$$3^y = \frac{\sqrt{3}}{9}.$$

**b)** Express

$$\sqrt{525},$$

in the form $a\sqrt{b}\sqrt{c}$, where $a$, $b$ and $c$ are prime numbers.

$\square$, $\boxed{y = -\frac{3}{2}}$, $\boxed{5\sqrt{3}\sqrt{7}}$

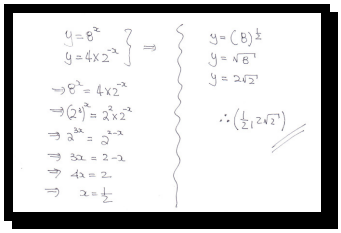


**Question 32 (***+)**

Find the exact coordinates of the point of intersection between the curves with equations

$$y = 8^x \quad \text{and} \quad y = 4 \times 2^{-x}.$$

$\boxed{\left(\frac{1}{2}, 2\sqrt{2}\right)}$



**Question 33 (***+)**

**a)** If $x$ is a real number solve the following indicial equation

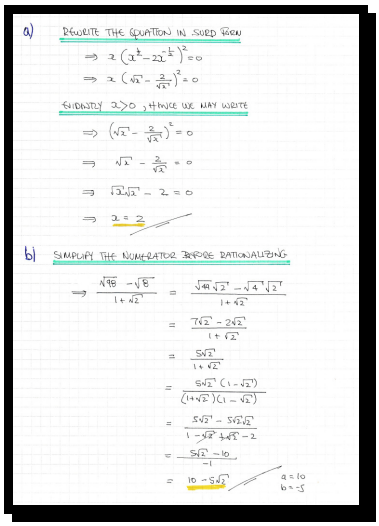
$$x\left(x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)^2=0\,.$$

**b)** Express

$$\frac{\sqrt{98}-\sqrt{8}}{1+\sqrt{2}}\,,$$

in the form $a+b\sqrt{2}$, where $a$ and $b$ are integers.

$\boxed{\phantom{x}}$, $\boxed{x=2}$, $\boxed{10-5\sqrt{2}}$

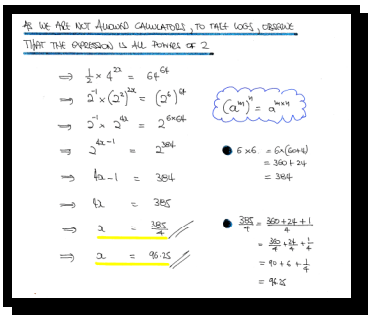


**Question 34 (***+)**

Find, **without** the use of any calculating aid, the solution of the equation

$$\frac{1}{2}\times 4^{2x}=64^{64}.$$

$\boxed{\phantom{x=96.25}}$, $\boxed{x=96.25}$




**Question 35 (***+)**

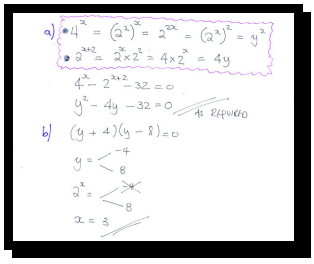
$$4^x-2^{x+2}=32.$$

**a)** Show that the substitution $y=2^x$ transforms the above indicial equation into the quadratic equation

$$y^2-4y-32=0.$$

**b)** Solve the quadratic equation and hence find the root of the **indicial** equation.

$\boxed{x=3}$

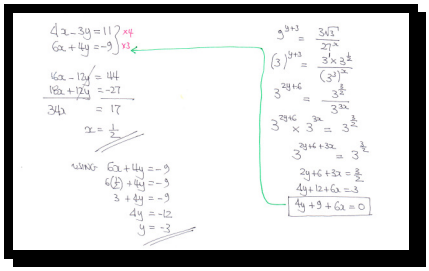


**Question 36 (****)**

Solve the following simultaneous equations without using a calculator.

$$4x-3y=11$$
$$9^{y+3}=\frac{3\sqrt{3}}{27^{x}}$$

$\left(\frac{1}{2},-3\right)$



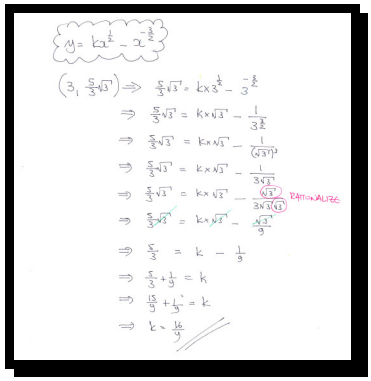
**Question 37 (****)**

Given that the curve with equation

$$y=kx^{\frac{1}{2}}-x^{-\frac{3}{2}},\ x\geq 0,$$

passes though the point with coordinates $\left(3,\frac{5}{3}\sqrt{3}\right)$, show clearly that $k=\frac{16}{9}$.

proof



**Question 38 (****)**

The indicial equation

$$2^{x+1}+2^{3-x}=17\,,\ x\in\mathbb{R}\,,$$

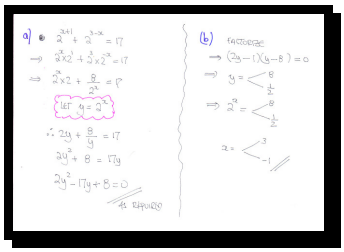
is to be solved by a suitable substitution.

**a)** Show clearly that the substitution $y=2^x$ transforms the above indicial equation into the quadratic equation

$$2y^2-17y+8=0\,.$$

**b)** Solve the quadratic equation by factorization and hence determine the two solutions of the indicial equation.

$\boxed{\phantom{x}}$, $\boxed{x=-1,\ 3}$

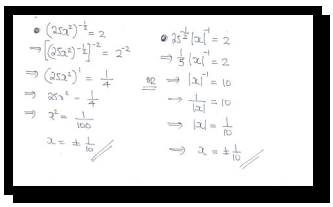


**Question 39 (****)**

Solve the equation

$$\left(25x^2\right)^{-\frac{1}{2}}=2\,,\quad x\neq 0\,.$$

$\boxed{x=\pm\frac{1}{10}}$



**Question 40** **(****)**

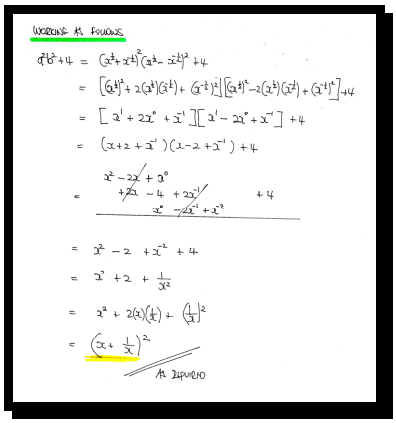
Given that

$$a = x^{\frac{1}{2}} + x^{-\frac{1}{2}} \quad \text{and} \quad b = x^{\frac{1}{2}} - x^{-\frac{1}{2}},$$

show clearly that

$$a^2b^2 + 4 \equiv \left( x + \frac{1}{x} \right)^2.$$

[ ], [proof]



**Question 41 (****)**

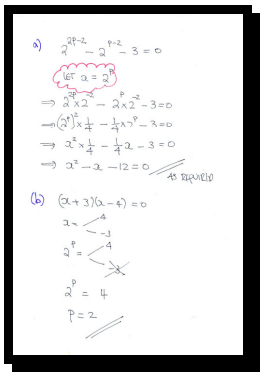
$$2^{2p-2}-2^{p-2}-3=0,\ p\in\mathbb{R},$$

**a)** Show clearly that the substitution $x=2^p$ transforms the above indicial equation into the quadratic equation

$$x^2-x-12=0.$$

**b)** Solve the quadratic equation and hence determine the value of $p$.

$$\boxed{p=2}$$



**Question 42** **(****)**

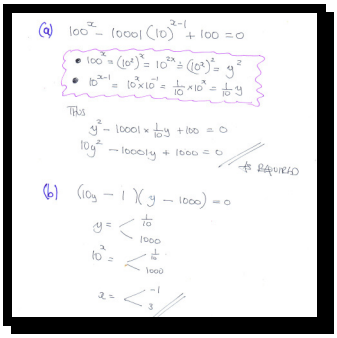
$$100^x - 10001\left(10^{x-1}\right) + 100 = 0 .$$

**a)** Show that the substitution $y = 10^x$ transforms the above indicial equation into the quadratic equation

$$10y^2 - 10001y + 1000 = 0 .$$

**b)** Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$\boxed{\phantom{x}}$, $\boxed{x = -1, \quad x = 3}$



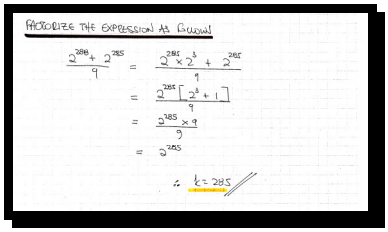
**Question 43** **(****)**

Determine the value of $k$.

$$\frac{2^{288} + 2^{285}}{9} = 2^k .$$

You must show full workings.

$\boxed{\phantom{x}}$, $\boxed{k = 285}$



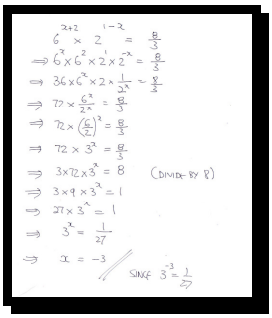
**Question 44 (****+)**

Solve the following indicial equation

$$6^{x+2}\times 2^{1-x}=\frac{8}{3}.$$

You must show full workings.

$\boxed{\phantom{x}}$, $\boxed{x=-3}$

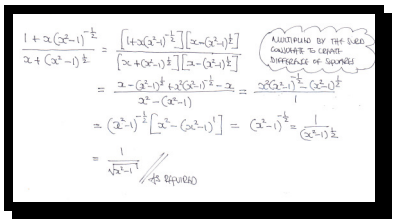


**Question 45 (****+)**

Show clearly that

$$\frac{1+x\left(x^2-1\right)^{-\frac{1}{2}}}{x+\left(x^2-1\right)^{\frac{1}{2}}} \equiv \frac{1}{\sqrt{x^2-1}}.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\text{proof}}$

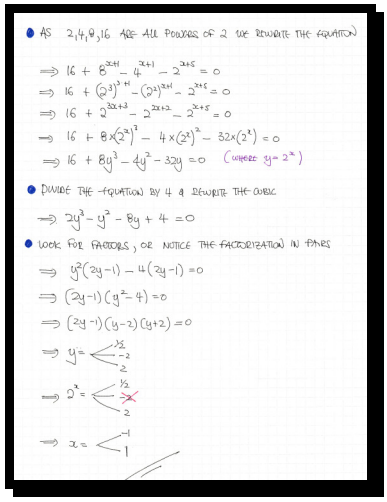


**Question 46 (****+)**

Solve the following exponential equation

$$16+8^{x+1}-4^{x+1}-2^{x+5}=0,\ x\in\mathbb{R}.$$

$\boxed{\ }$, $\boxed{x=\pm 1}$



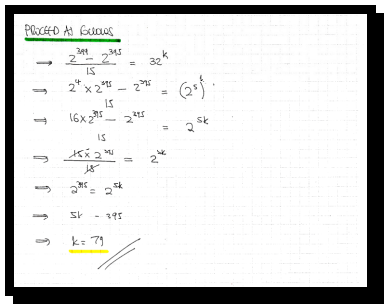

**Question 47 (****+)**

Determine the value of $k$.

$$\frac{2^{399}-2^{395}}{15}=32^{k}.$$

You must show full workings.

$\boxed{\ }$, $\boxed{k=79}$

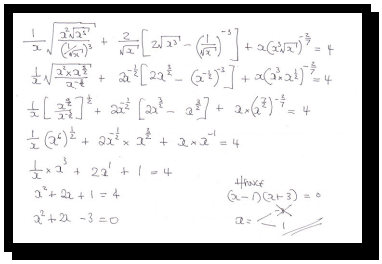


**Question 48 (****+)**

Solve the following equation

$$\frac{1}{x}\times\sqrt{\frac{x^2\sqrt{x^5}}{\left(\frac{1}{\sqrt{x}}\right)^3}}+\frac{2}{\sqrt{x}}\times\left[2\sqrt{x^3}-\left(\frac{1}{\sqrt{x}}\right)^{-3}\right]+x\left(x^3\sqrt{x}\right)^{-\frac{2}{7}}=4\,.$$

$\boxed{x=1}$



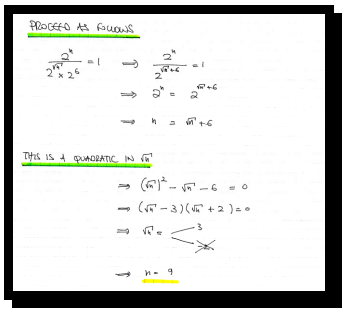
**Question 49 (****+)**

Solve the following indicial equation

$$\frac{2^n}{2^{\sqrt{n}}\times 2^6}=1\,.$$

You must show full workings.

$\boxed{\phantom{n=9}}$, $\boxed{n=9}$



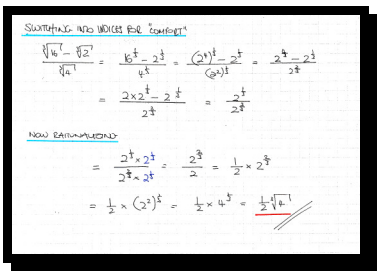
**Question 50** **(****+)**

Show with a detailed method that

$$\frac{\sqrt[3]{16}-\sqrt[3]{2}}{\sqrt[3]{4}} = k\sqrt[3]{4}$$

where $k$ is a constant to be found.

$\boxed{\phantom{k=\frac{1}{2}}}$, $\boxed{k=\frac{1}{2}}$



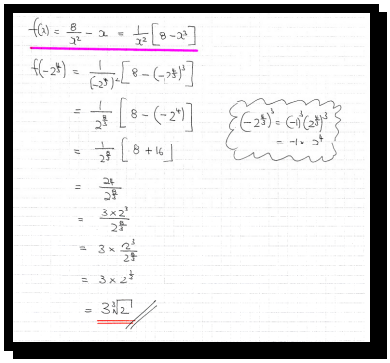
**Question 51** **(*****)**

$$f(x)\equiv\frac{8}{x^2}-x,\ x\neq 0.$$

Show that $f\left(-2^{\frac{4}{3}}\right)=3\sqrt[3]{2}$.

You must show detailed workings.

$\boxed{\phantom{\text{proof}}}$, $\boxed{\text{proof}}$

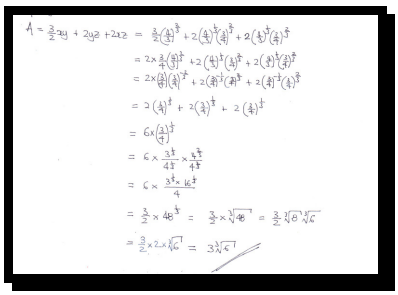


**Question 52 (*****)**

$$A=\frac{3}{2}xy+2yz+2xz\,.$$

Given that $x=\left(\frac{4}{3}\right)^{\frac{1}{3}}$, $y=\left(\frac{4}{3}\right)^{\frac{1}{3}}$ and $z=\left(\frac{3}{4}\right)^{\frac{2}{3}}$, show clearly that $A=3\sqrt[3]{6}$

$\boxed{\phantom{x}}$, $\boxed{\text{proof}}$



**Question 53 (*****)**

A curve has equation

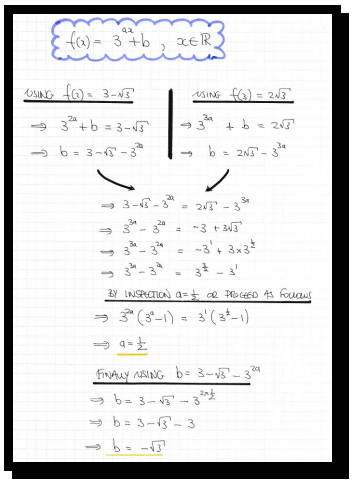
$$f(x) \equiv 3^{ax} + b, \ x \in \mathbb{R},$$

where $a$ and $b$ are non zero constants.

Find the value of $a$ and the value of $b$, given further that

$$f(2) = 3 - \sqrt{3} \quad \text{and} \quad f(3) = 2\sqrt{3}.$$

$\square$, $\boxed{a = \frac{1}{2}}$, $\boxed{b = -\sqrt{3}}$



**Question 54 (*****)**

A curve has equation

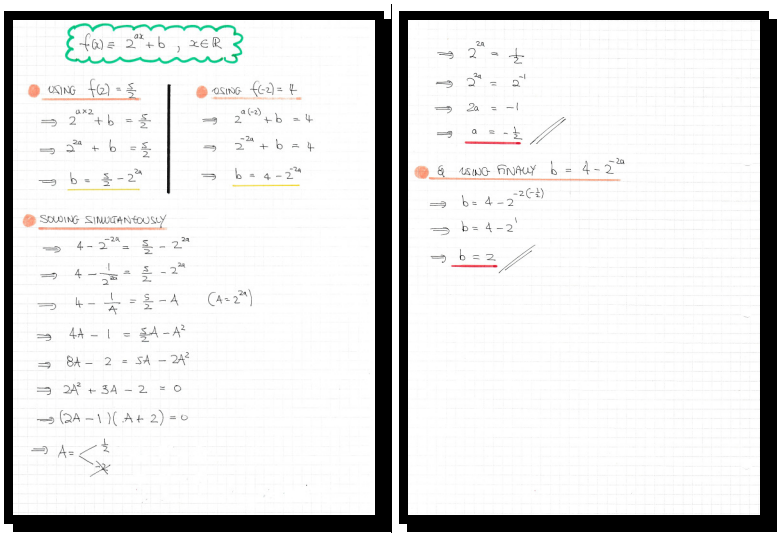
$$f(x) \equiv 2^{ax} + b, \ x \in \mathbb{R},$$

where $a$ and $b$ are non zero constants.

Find the value of $a$ and the value of $b$, given further that

$$f(2) = \frac{5}{2} \quad \text{and} \quad f(-2) = 4.$$

$\square$, $\boxed{a = -\frac{1}{2}}$, $\boxed{b = 2}$



**Question 55 (*****)**

A curve has equation

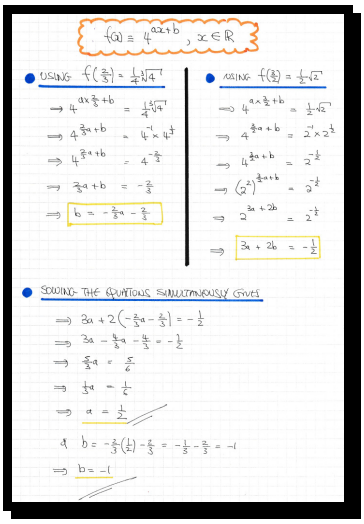
$$f(x) \equiv 4^{ax+b}, \ x \in \mathbb{R},$$

where $a$ and $b$ are non zero constants.

Find the value of $a$ and the value of $b$, given further that

$$f\left(\frac{2}{3}\right) = \frac{1}{4}\sqrt[3]{4} \quad \text{and} \quad f\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{2}.$$

$\square$, $\boxed{a = \frac{1}{2}}$, $\boxed{b = -1}$

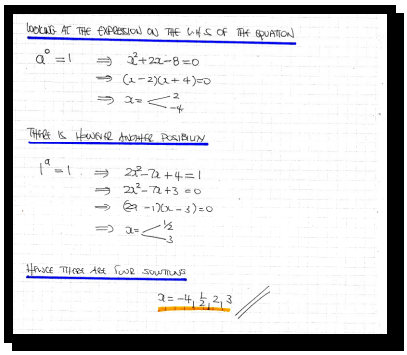


**Question 56 (*****)**

Find the solutions for the following equation.

$$\left(2x^2-7x+4\right)^{x^2+2x-8}=1.$$

V, [ ], $x=-4, \frac{1}{2}, 2, 3$

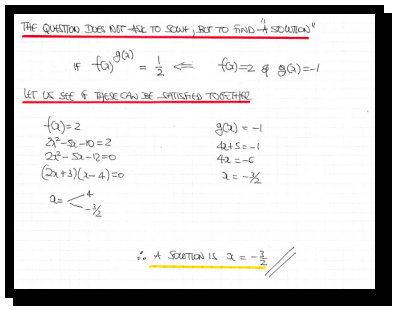


**Question 57 (*****)**

Find a solution for the following equation.

$$\left(2x^2-5x-10\right)^{4x+5}=\frac{1}{2}.$$

V, [ ], $\boxed{x=-\frac{3}{2}}$

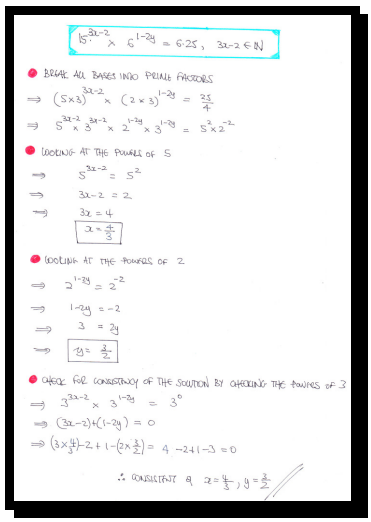


**Question 58 (*****)**

Determine the value of $x$ and the value of $y$ in the following equation

$$15^{3x-2}\times 6^{1-2y}=6.25\,,\quad (3x-2)\in\mathbb{N}\,.$$

$\square$, $\boxed{x=\frac{4}{3},\ y=\frac{3}{2}}$

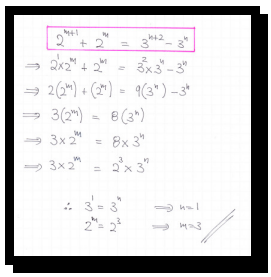


**Question 59 (*****)**

$$2^{m+1}+2^{m}=3^{n+2}-3^{n}\,.$$

Given that $m$ and $n$ are positive integers, find the value of $m$ and the value of $n$.

$\square$, $\boxed{m=3}$, $\boxed{n=1}$

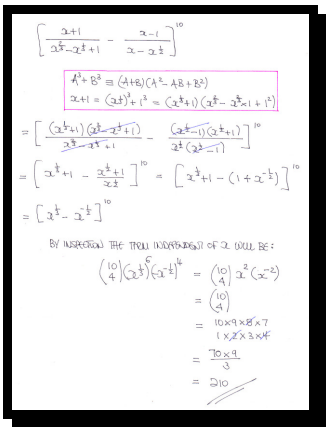


**Question 60 (*****)**

Find the term independent of $x$ in the expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}.$$

$\square$, $\boxed{210}$



$$\left[\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right]^{10}$$

$$A^3+B^3\equiv(A+B)(A^2-AB+B^2)$$

$$x+1=(x^{\frac{1}{3}})^3+1^3\equiv(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}}-x^{\frac{1}{3}}\times 1+1^2)$$

$$=\left[\frac{(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1)}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)}{x^{\frac{1}{2}}(x^{\frac{1}{2}}-1)}\right]^{10}$$

$$=\left[x^{\frac{1}{3}}+1-\frac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}}\right]^{10}=\left[x^{\frac{1}{3}}+1-(1+x^{-\frac{1}{2}})\right]^{10}$$

$$=\left[x^{\frac{1}{3}}-x^{-\frac{1}{2}}\right]^{10}$$

BY INSPECTION THE TERM INDEPENDENT OF $x$ WILL BE:

$$\binom{10}{4}(x^{\frac{1}{3}})^6(x^{-\frac{1}{2}})^4=\binom{10}{4}x^2(x^{-2})$$

$$=\binom{10}{4}$$

$$=\frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4}$$

$$=\frac{70\times 9}{3}$$

$$=210$$

**Question 61 (*****)**

Simplify the following expression.

$$\frac{x^6-yz^4}{x^4+xz^2\sqrt{y}}$$

Give the answer in the form $x^2-x^r y^s z^t$, where $r$, $s$ and $t$ are constants.

$\square$, $x^2-x^{-1}y^{\frac{1}{2}}z^2$

