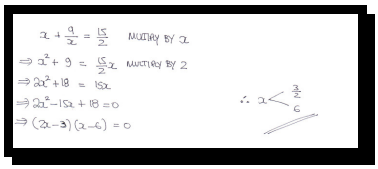
EQUATIONS EXAM QUESTIONS

Question 1 (**)

Solve the following equation

$$x+\frac{9}{x}=\frac{15}{2},\ x\neq 0.$$

$\square$, $\boxed{x=\frac{3}{2},6}$

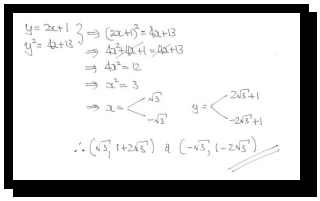


Question 2 (**)

Find as exact simplified surds the coordinates of the points of intersection between the graphs of

$$y=2x+1 \quad \text{and} \quad y^2=4x+13.$$

$\boxed{\left(\sqrt{3},1+2\sqrt{3}\right),\left(-\sqrt{3},1-2\sqrt{3}\right)}$

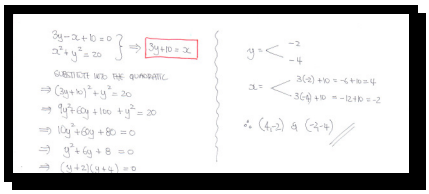


Question 3 (**)

Solve the simultaneous equations

$$3y - x + 10 = 0$$
$$x^2 + y^2 = 20$$

$\boxed{(4,-2) \ \& \ (-2,-4)}$

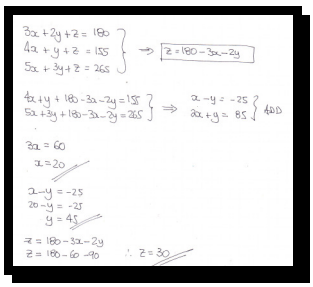


Question 4 (+)**

Solve the following system of simultaneous equations

$$\begin{aligned} 3x \quad +2y \quad +z \quad &= 180 \\ 4x \quad +y \quad +z \quad &= 155 \\ 5x \quad +3y \quad +z \quad &= 265 \end{aligned}$$

$\boxed{x = 20}$, $\boxed{y = 45}$, $\boxed{z = 30}$

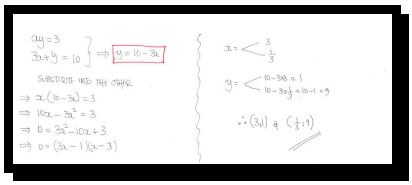


Question 5 (+)**

Solve the following simultaneous equations

$$xy = 3$$
$$3x + y = 10$$

$\square$, $\boxed{(3,1) \ \& \ \left(\frac{1}{3},9\right)}$

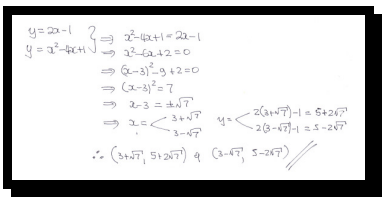


Question 6 (+)**

Find as exact simplified surds the coordinates of the points of intersection between the graphs of

$$y = 2x - 1 \quad \text{and} \quad y = x^2 - 4x + 1 .$$

$\boxed{\left(3+\sqrt{7},5+2\sqrt{7}\right),\left(3-\sqrt{7},5-2\sqrt{7}\right)}$

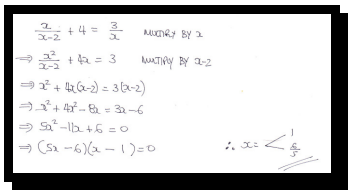


Question 7 (+)**

Solve the following equation

$$\frac{x}{x-2}+4=\frac{3}{x},\ x\neq 0.$$

$\boxed{x=1,\frac{6}{5}}$



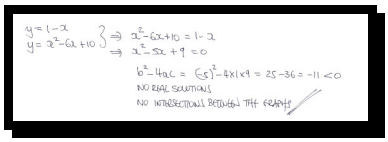
Question 8 (+)**

Use an algebraic method to show that the graphs

$$y=1-x \quad \text{and} \quad y=x^2-6x+10,$$

do **not** intersect.

proof

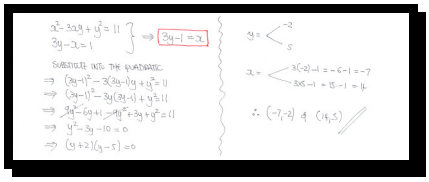


Question 9 (+)**

Solve the following simultaneous equations

$$x^2-3xy+y^2=11$$
$$3y-x=1$$

$\square$, $\boxed{(14,5)\ \&\ (-7,-2)}$

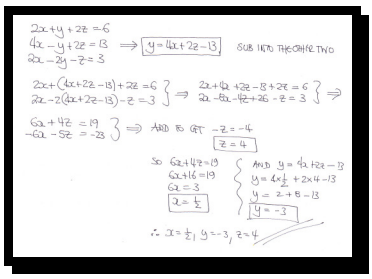


Question 10 (***)

Solve the following simultaneous equations

$$2x+y+2z=6$$
$$4x-y+2z=13$$
$$2x-2y-z=3$$

$\boxed{x=\frac{1}{2},\ y=-3,\ z=4}$

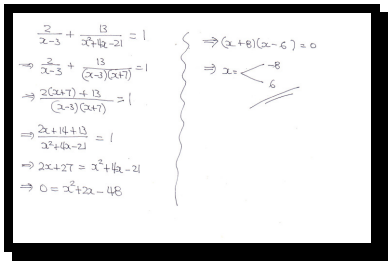


Question 11 (***)

Solve the following equation

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}=1,\ x\neq 3,\ x\neq 7.$$

$$\boxed{x=-8,6}$$

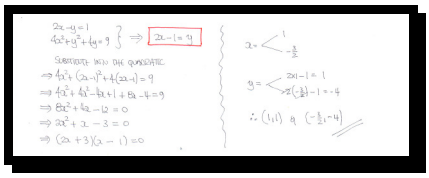


Question 12 (***)

Solve the following simultaneous equations

$$2x-y=1$$
$$4x^2+y^2+4y=9$$

$$\boxed{(1,1)\ \&\ \left(-\frac{3}{2},-4\right)}$$

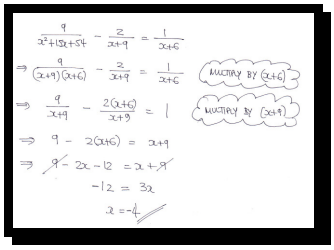


Question 13 (***)

Solve the following equation

$$\frac{9}{x^2+15x+54}-\frac{2}{x+9}=\frac{1}{x+6},\ x\neq -6,\ x\neq -9.$$

$$\boxed{x=-4}$$

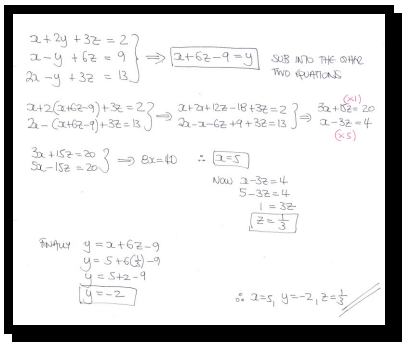


Question 14 (***)

Solve the following simultaneous equations

$$\begin{aligned} x+2y+3z&=2\\ x-y+6z&=9\\ 2x-y+3z&=13 \end{aligned}$$

$$\boxed{x=5,\ y=-2,\ z=\frac{1}{3}}$$

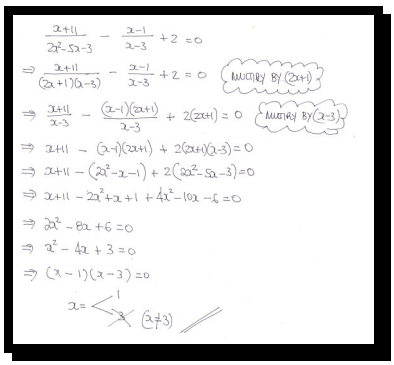


Question 15 (***)

Solve the following equation

$$\frac{x+11}{2x^2-5x-3}-\frac{x-1}{x-3}+2=0\,,\ x\neq -\frac{1}{2}\,,\ x\neq 3\,.$$

$$\boxed{x=1}$$

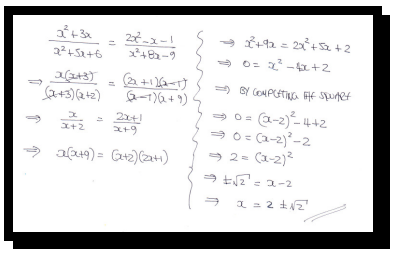


Question 16 (***)

Find, in exact surd form, the roots of the equation

$$\frac{x^2+3x}{x^2+5x+6}=\frac{2x^2-x-1}{x^2+8x-9}\,,\ x\neq -3\,,\ x\neq 1\,.$$

$$\boxed{x=2\pm\sqrt{2}}$$

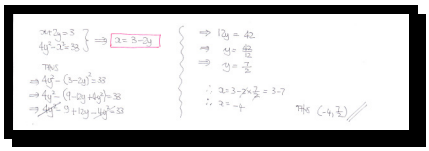


Question 17 (***)

Solve the following simultaneous equations

$$x+2y=3$$
$$4y^2-x^2=33$$

$\square$, $\boxed{\left(-4,\frac{7}{2}\right)}$

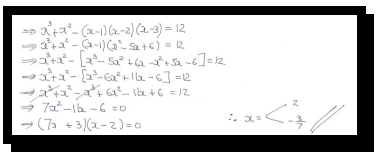


Question 18 (***)

Solve the following equation

$$x^3+x^2-(x-1)(x-2)(x-3)=12\,.$$

$\boxed{x=-\frac{3}{7},2}$

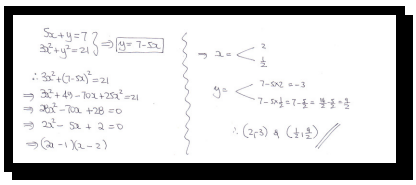


Question 19 (***)

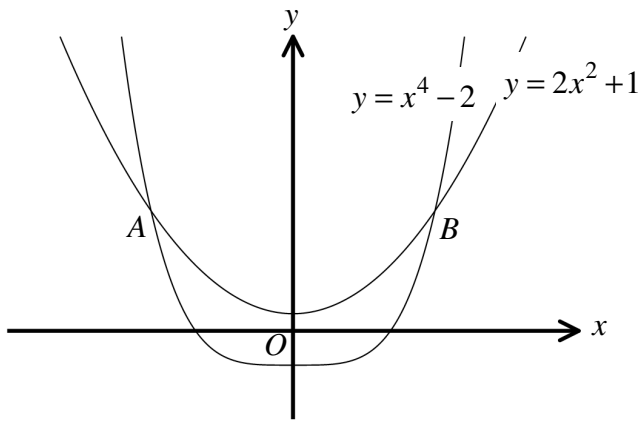
Solve the following simultaneous equations

$$5x + y = 7$$
$$3x^2 + y^2 = 21$$

$\square$, $\boxed{(2,-3) \ \& \ \left(\frac{1}{2},\frac{9}{2}\right)}$



Question 20 (*+)**



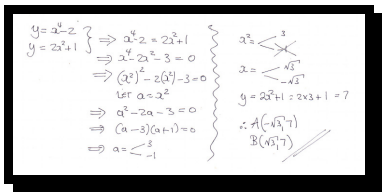
The figure above shows the graphs of the curves with equations

$$y = x^4 - 2 \quad \text{and} \quad y = 2x^2 + 1 .$$

The two curves intersect at the points A and B.

Find the exact coordinates of A and B.

$$\boxed{A\left(-\sqrt{3},7\right), B\left(\sqrt{3},7\right)}$$



Question 21 (*+)**

Three students are on the same tariff from a certain mobile company.

All calls cost X pence per minute, every text message costs Y pence each and every picture message costs Z pence each.

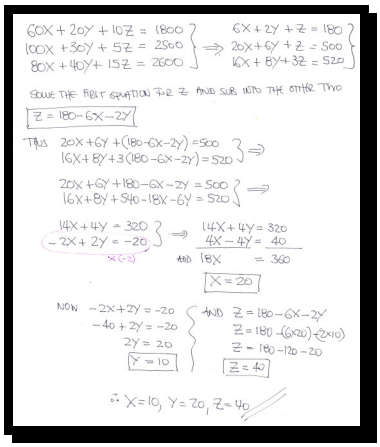
Abbie made 60 minutes of calls, sent 20 text messages and sent 10 picture messages. Her monthly bill came to £18.00 .

Beth made 100 minutes of calls, sent 30 text messages and sent 5 picture messages. Her monthly bill came to £25.00 .

Chiara made 80 minutes of calls, sent 40 text messages and sent 15 picture messages. Her monthly bill came to £26.00 .

Find the values of X , Y and Z .

$$\boxed{X=20,\ Y=10,\ Z=40}$$

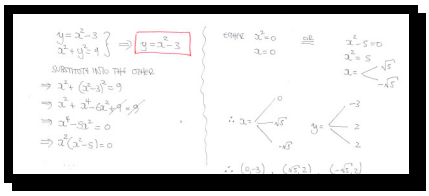


Question 22 (*+)**

Solve the following simultaneous equations

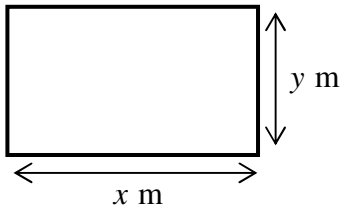
$$y = x^2 - 3$$
$$x^2 + y^2 = 9$$

$$\boxed{(0,-3)\ \&\ \left(\sqrt{5},2\right)\ \&\ \left(-\sqrt{5},2\right)}$$



Question 23 (*+)**

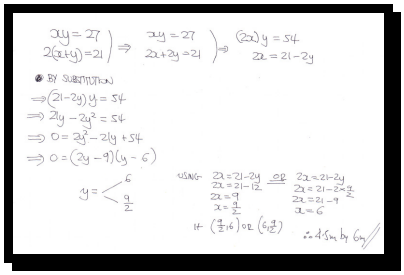
The figure below shows the plan of the floor of a room with a length of x m and a width of y m.



The floor has an area of $27\,\text{m}^2$ and a perimeter of 21 m.

Determine the measurements of the room.

$$\boxed{\text{6 by 4.5}}$$

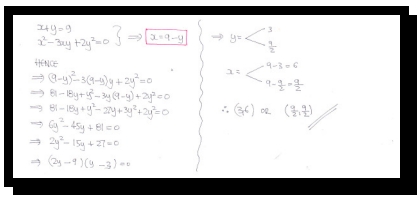


Question 24 (*+)**

Solve the following simultaneous equations

$$x+y=9$$
$$x^2-3xy+2y^2=0$$

$\square$, $\boxed{(6,3),\left(\frac{9}{2},\frac{9}{2}\right)}$

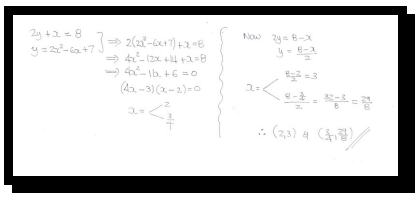


Question 25 (*+)**

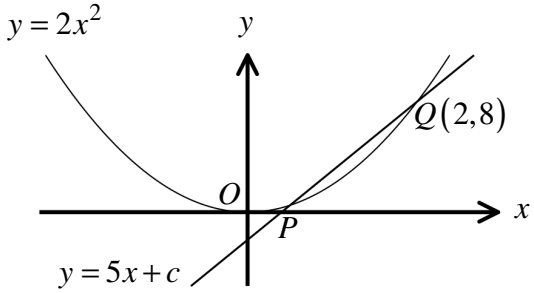
Solve the following simultaneous equations

$$2y+x=8$$
$$y=2x^2-6x+7$$

$\square$, $\boxed{(2,3)\ \&\ \left(\frac{3}{4},\frac{29}{8}\right)}$



Question 26 (*+)**

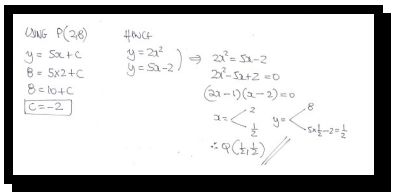


The figure above shows the graph of the curve with equation $y = 2x^2$ and the line with equation $y = 5x + c$, where c is a constant.

The line meets the curve at the point P and at the point $Q(2,8)$.

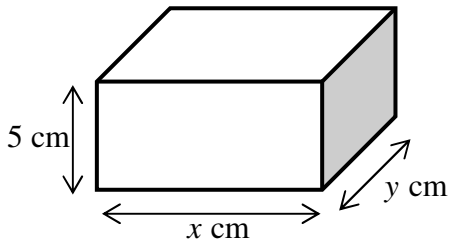
Determine the coordinates of P.

$$\boxed{P\left(\frac{1}{2},\frac{1}{2}\right)}$$



Question 27 (*+)**

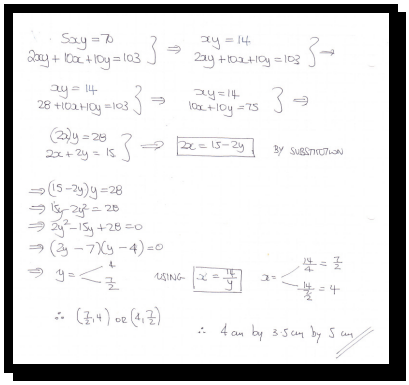
The figure below shows a cuboid of length x cm, width y cm and height 5 cm.



The cuboid has a volume of 70 cm^3 and a surface area of 103 cm^2.

Determine the measurements of the cuboid.

4 by 3.5 by 5

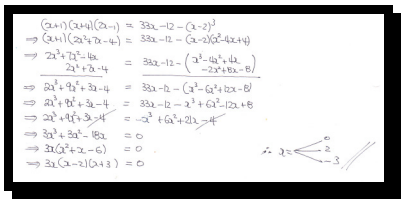


Question 28 (*+)**

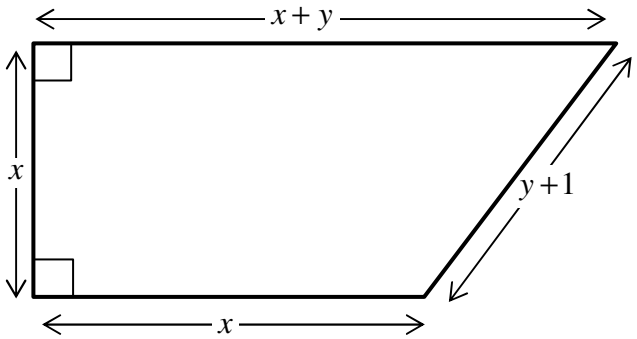
Solve the following equation

$$(x+1)(x+4)(2x-1)=33x-12-(x-2)^3.$$

$x=-3,0,2$



Question **(***+)**

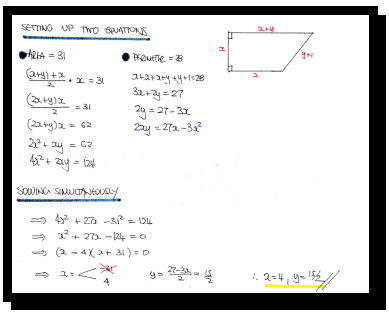


The figure above shows a right angled trapezium whose measurements are given in terms of x and y.

The trapezium has and a perimeter of 28 and an area of 31.

Determine the value x and the value of y.

$$\boxed{x = 4,\ \ y = 7.5}$$

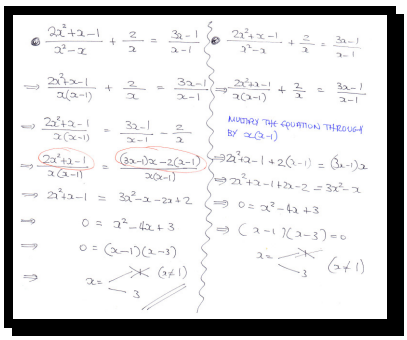


Question 29 (*+)**

Find the solution of the following equation

$$\frac{2x^2+x-1}{x^2-x}+\frac{2}{x}=\frac{3x-1}{x-1}.$$

$\boxed{x=3, x\neq 1}$



Question 30 (*+)**

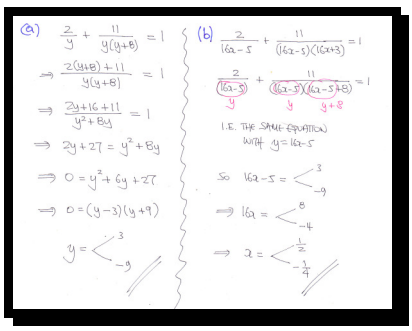
a) Find the solutions of the following equation

$$\frac{2}{y}+\frac{11}{y(y+8)}=1,\ y\neq -8,\ y\neq 0.$$

b) Hence, or otherwise, solve the equation

$$\frac{2}{16x-5}+\frac{11}{(16x-5)(16x+3)}=1,\ x\neq -\frac{3}{16},\ x\neq \frac{5}{16}.$$

$\boxed{x=-9,3}$, $\boxed{x=-\frac{1}{4},\frac{1}{2}}$



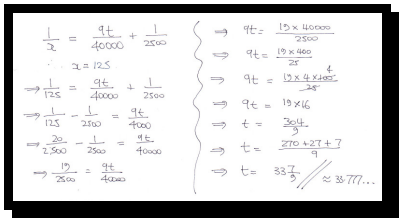
Question 31 (*+)**

A relationship between two variables is given below

$$\frac{1}{x}=\frac{9t}{40000}+\frac{1}{2500}.$$

Find the value of t when $x=125$.

$$\boxed{t=33\frac{7}{9}\approx 33.8}$$



Question 32 (*+)**

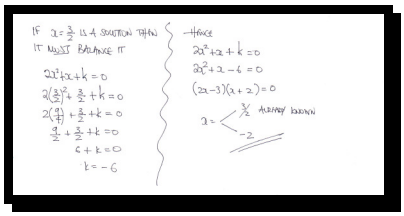
The quadratic equation given below

$$2x^2+x+k=0,$$

where k is a constant, has solutions $x=\frac{3}{2}$ and $x=x_0$.

Find the value of x_0.

$$\boxed{}, \quad \boxed{x_0=-2}$$



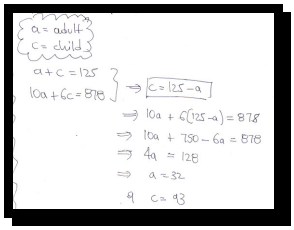
Question 33 (*+)**

In a cinema adult tickets cost £10 each while child tickets cost £6.

For a certain film there were 125 people in the cinema, having paid in total £878.

Find how many adults and how many children were watching this film?

93 children and 32 adults

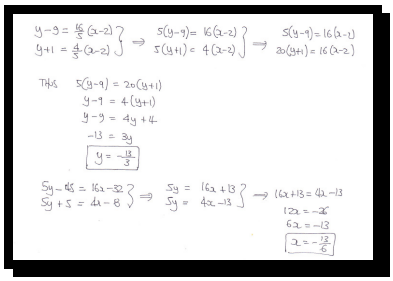


Question 34 (*+) non calculator**

Find the exact solution of the following simultaneous equations

$$y-9=\frac{16}{5}(x-2)$$
$$y+1=\frac{4}{5}(x-2)$$

$$\left(-\frac{13}{6},-\frac{13}{3}\right)$$

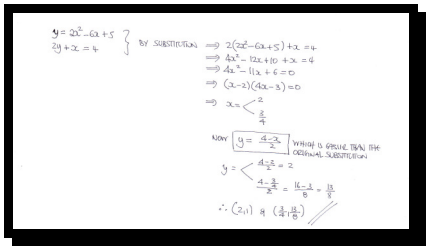


Question 35 (*+) non calculator**

Find the coordinates of the points of intersection between the graphs of

$$y = 2x^2 - 6x + 5 \quad \text{and} \quad 2y + x = 4 .$$

$$\boxed{(2,1),\ \left(\frac{3}{4},\frac{13}{8}\right)}$$

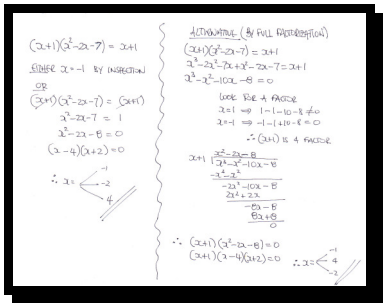


Question 36 (*+)**

Solve the following equation

$$(x+1)\left(x^2 - 2x - 7\right) = x + 1$$

$$\boxed{x = -2, -1, 4}$$

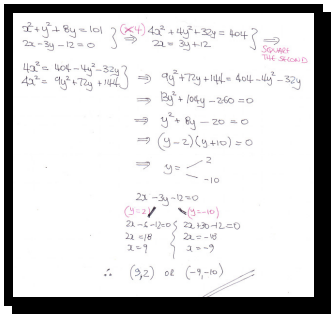


Question 37 (*+)**

Find the coordinates of the points of intersection between

$$x^2+y^2+8y=101 \quad \text{and} \quad 2x-3y-12=0\,.$$

$\square$, $\boxed{(9,2),(-9,10)}$

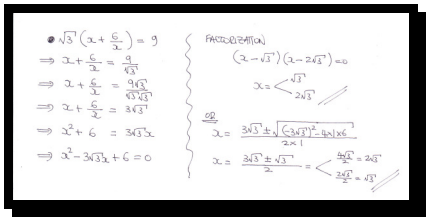


Question 38 (*+)**

Find in exact surd form the roots of the following equation

$$\sqrt{3}\left(x+\frac{6}{x}\right)=9\,,\ x\neq 0\,.$$

$\boxed{x=\sqrt{3},\ x=2\sqrt{3}}$



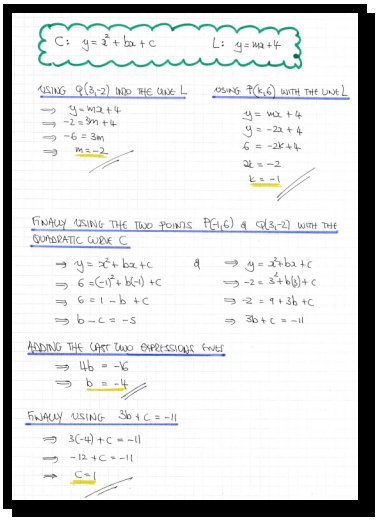
Question 39 (****)

$$C: \quad y = x^2 + bx + c$$
$$L: \quad y = mx + 4$$

The quadratic curve C intersects the straight line L at the points with coordinates $(k,6)$ and $(3,-2)$, where k, m, b and c are constants.

Find the value of k, m, b and c.

$\boxed{}$, $\boxed{m=-2,\ \ k=-1,\ \ b=-4,\ \ c=1}$



Question 40 (****)

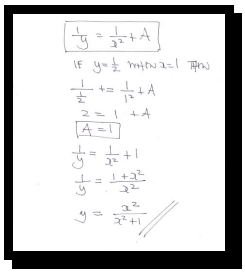
A relationship between three variables is given below

$$\frac{1}{y}=\frac{1}{x^2}+A\,.$$

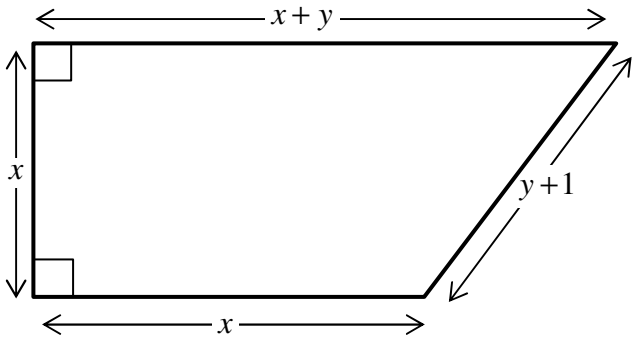
Given further that when $x=1$, $y=\frac{1}{2}$, show clearly that

$$y=\frac{x^2}{1+x^2}\,.$$

proof



Question 41 **(****)**

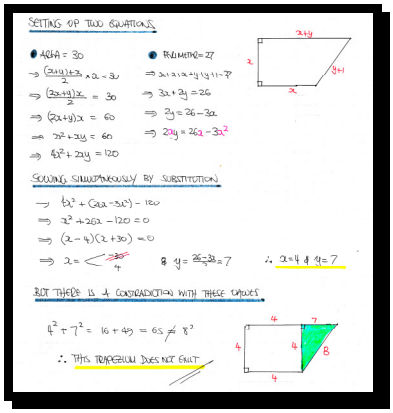


The figure above shows a right angled trapezium whose measurements are given in terms of x and y.

The trapezium has and a perimeter of 27 and an area of 30.

Determine the value x and the value of y, and hence show that the above trapezium does **not** exist.

$\boxed{}$, $\boxed{x = 4, \ y = 7}$



Question 42 **(****)**

$$f(x)=x^2(x-4),\ x\in\mathbb{R}.$$

$$g(x)=x(10-x),\ x\in\mathbb{R}.$$

a) Determine the coordinates of the points of intersection between the graphs of $f(x)$ and $g(x)$.

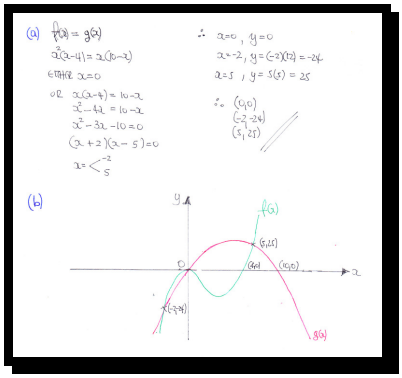
b) Sketch the graph of $f(x)$ and the graph of $g(x)$ in the same diagram.

The sketch must include …

… the coordinates of any points where the graph of $f(x)$ and the graph of $g(x)$ meet the coordinate axes.

… the coordinates of the points of intersection between the graph of $f(x)$ and the graph of $g(x)$.

$$\boxed{(0,0),(-2,-24),(5,25)}$$



Question 43 **(****)**

$$f(x)=x^3-9x^2+13x+2\,,\ x\in\mathbb{R}\,.$$

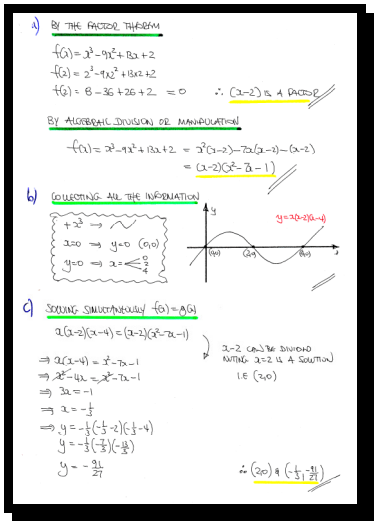
a) Show, by using the factor theorem, that $(x-2)$ is a factor of $f(x)$ and hence express $f(x)$ as a product of a linear and a quadratic factor.

$$g(x)=x(x-2)(x-4),\ x\in\mathbb{R}\,.$$

b) Sketch the graph of $g(x)$, indicating clearly the coordinates of any points where the graph of $g(x)$ meets the coordinate axes.

c) Determine the exact coordinates, where appropriate, of the points of intersection between the graph of $f(x)$ and the graph of $g(x)$.

$\square$, $\boxed{f(x)=(x-2)\left(x^2-7x-1\right)}$, $\boxed{(2,0),\left(-\frac{1}{3},-\frac{91}{27}\right)}$



Question 44 (****)

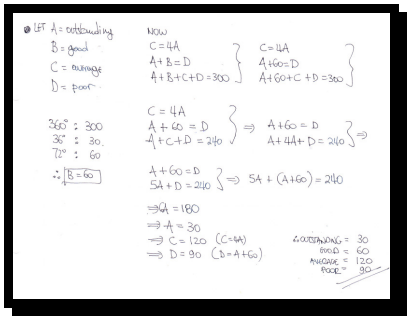
The 300 Year 11 pupils of a certain school are classed as "outstanding", "good", "average" or "poor".

- The following information is also available about these pupils.
- In a standard pie chart the sector that represents the "good" pupils is $72°$.
- The "poor" pupils are as many as the "good" and "outstanding" pupils added together.

There are four times as many "average" pupils as "outstanding" ones.

Determine the number of students in each class.

$$\boxed{O=30,\ G=60,\ A=120,\ P=90}$$



Question 45 (****)

Andrew and Bethany are preparing for a Mathematics exam by doing the same set of practice papers.

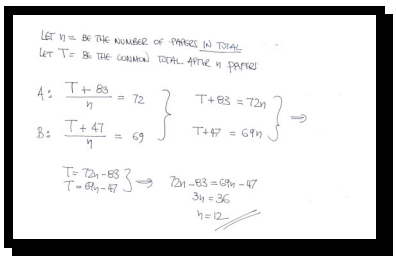
They both have one practice paper left to do and their mean scores are identical.

Andrew scores 83% on his last paper and his mean score is rises to 72%.

Bethany scores 47% on her last paper and her mean score is drops to 69%.

Determine the number of practice papers in the set.

$\boxed{n=12}$



Question 46 (****)

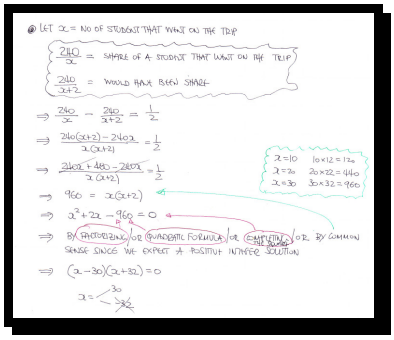
The students in a class hired a coach for a day trip, at a cost of £240 .

They agreed to share **equally** the cost of the coach hire among them.

On the day of the trip 2 students fell ill so the share of the remaining students increased by £0.50 .

How many students went on the school trip.

$$\boxed{30}$$

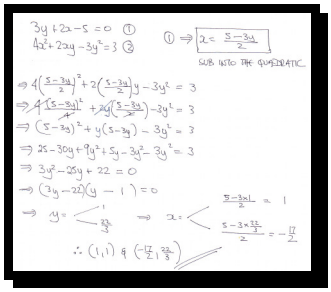


Question 47 (****)

Solve the following simultaneous equations

$$\begin{aligned} 3y+2x-5&=0 \\ 4x^2+2xy-3y^2&=3 \end{aligned}$$

$$\boxed{(1,1)\ \&\ \left(-\frac{17}{2},\frac{22}{3}\right)}$$



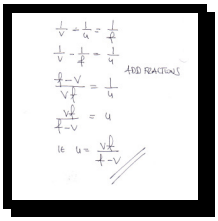
Question 48 (****)

Make u the subject of the equation

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\ .$$

Give the answer as a single simplified fraction.

$$\boxed{u=\frac{vf}{f-v}}$$



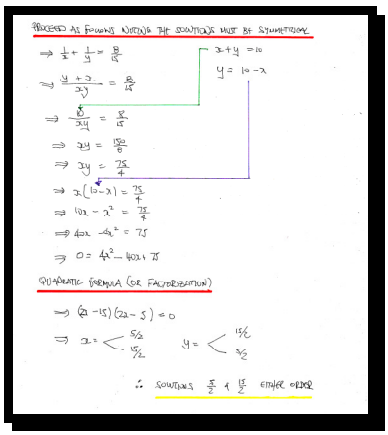
Question 49 (****)

Solve the following system of simultaneous equations

$$\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$$

$$x+y=10\ .$$

$$\boxed{\quad}\ ,\ \boxed{\left(\frac{5}{2},\frac{15}{2}\right),\left(\frac{15}{2},\frac{5}{2}\right)}$$



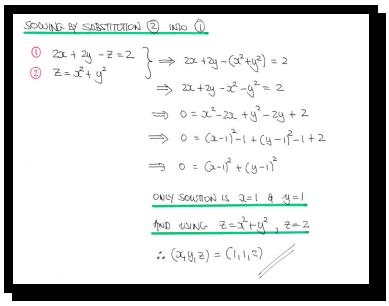
Question 50 (****)

Find the solution of the following simultaneous equations

$$2x+2y-z=2$$
$$z=x^2+y^2$$

assuming that x, y, z are all real numbers.

$\square$, $\boxed{(x,y,z)=(1,1,2)}$



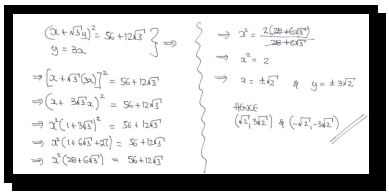
Question 51 (****)

Solve the following system of simultaneous equations

$$\left(x+y\sqrt{3}\right)^2=56+12\sqrt{3}$$

$$y=3x\,.$$

$\square$, $\boxed{\left(\sqrt{2},3\sqrt{2}\right),\ \left(-\sqrt{2},-3\sqrt{2}\right)}$

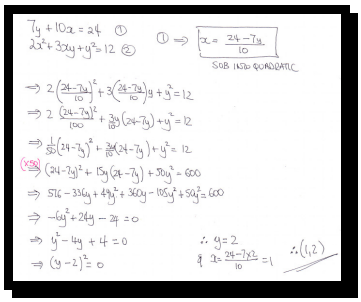


Question 52 (**+)**

Solve the following system of simultaneous equations

$$7y+10x=24$$
$$2x^2+3xy+y^2=12$$

$\boxed{(1,2)}$

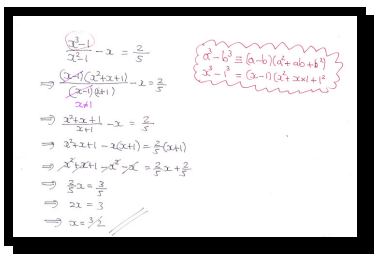


Question 53 (**+)**

Solve the following equation

$$\frac{x^3-1}{x^2-1}-x=\frac{2}{5},\ x\neq\pm1.$$

$\boxed{x=\frac{3}{2}}$



Question 54 (**+)**

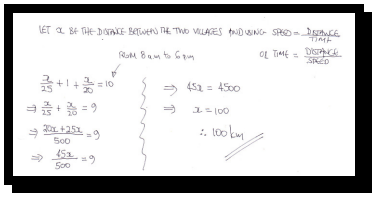
A cyclist leaves village A at 8 a.m. cycling towards village B at constant speed of $25\ \text{km}\,\text{h}^{-1}$.

After arriving at B the cyclist spends exactly 1 hour there before he cycles back to A, following exactly the same route he took on his outward journey.

On his return journey he cycles at a constant speed $20\ \text{km}\,\text{h}^{-1}$.

Given the cyclist returns back to village A at 6 p.m. determine the distance between the two villages.

$\boxed{100\text{ km}}$



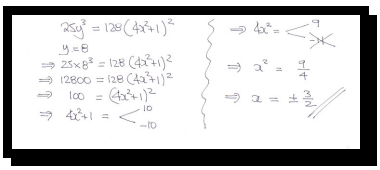
Question 55 (**+)**

A relationship between two variables is given below

$$25y^3=128\left(4x^2+1\right)^2.$$

Find the possible values of x when $y=8$.

$\boxed{\ }$, $\boxed{x=\pm\frac{3}{2}}$

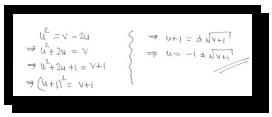


Question 56 (**+)**

Make u the subject of the equation

$$u^2 = v - 2u\,.$$

$$\boxed{u = -1 \pm \sqrt{v+1}}$$

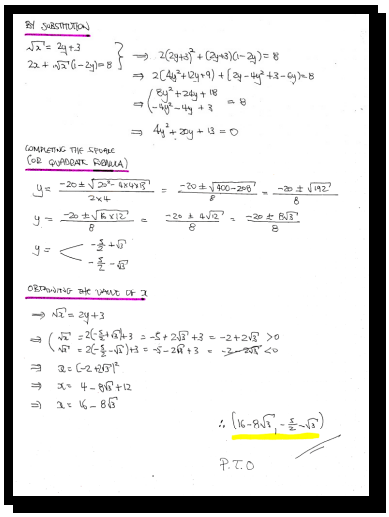


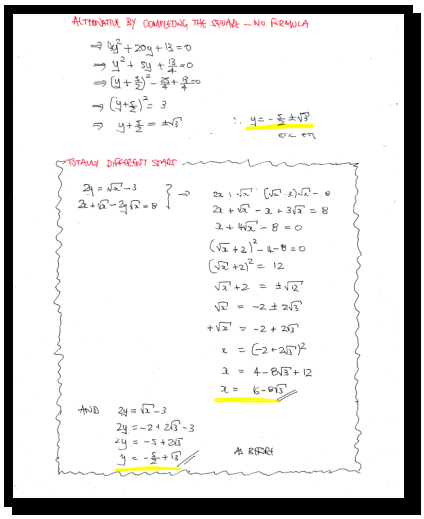
Question 57 (**+)**

Find as exact simplified surds the coordinates of the point of intersection between the graphs of

$$\sqrt{x} = 2y + 3 \quad \text{and} \quad 2x + \sqrt{x} - 2y\sqrt{x} = 8\,.$$

$$\boxed{}, \boxed{\left(16 - 8\sqrt{3}, -\frac{5}{2} + \sqrt{3}\right)}$$





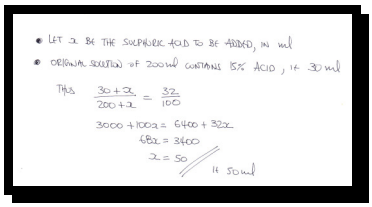
Question 58 (**+)**

Sulphuric acid is a colourless liquid which can be diluted with water.

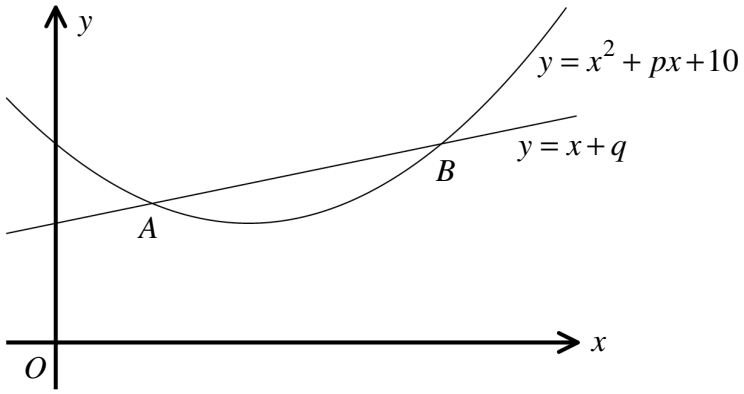
Pure sulphuric acid is to be added to a 200 ml water solution, which also contains sulphuric acid of concentration 15% by volume.

How many ml of pure sulphuric acid must be added so that the resulting solution contains sulphuric acid of concentration 32% by volume.

50 ml



Question 59 (**+)**



The figure above shows the graph of the curve with equation

$$y = x^2 + px + 10$$

and the straight line with equation

$$y = x + q\,,$$

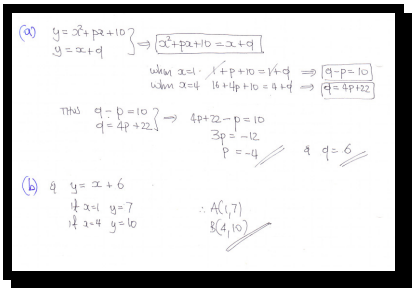
where p and q are constants.

The curve and the straight line intersect at the points A and B whose x coordinates are 1 and 4, respectively.

a) Determine the value of p and the value of q.

b) Find the coordinates of A and B.

$\boxed{}$, $\boxed{p=-4,\ q=6}$, $\boxed{A(1,7),\ B(4,10)}$



Question 60 (**+)**

A pupil is heard saying to another pupil …

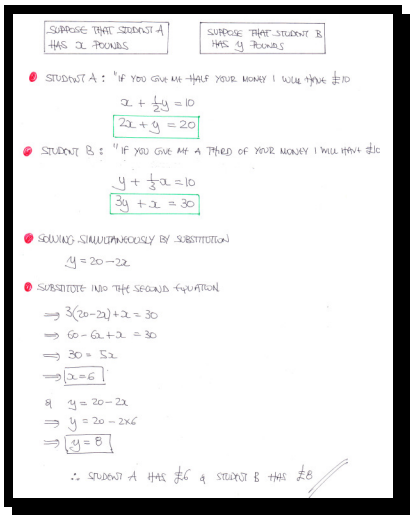
"… if you give me half your pocket money I will have £10."

The other pupil replied …

"…if you give me one third of your pocket money I will have £10."

Determine how much money each pupil has.

£6 and £8

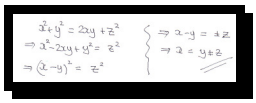


Question 61 (**+)**

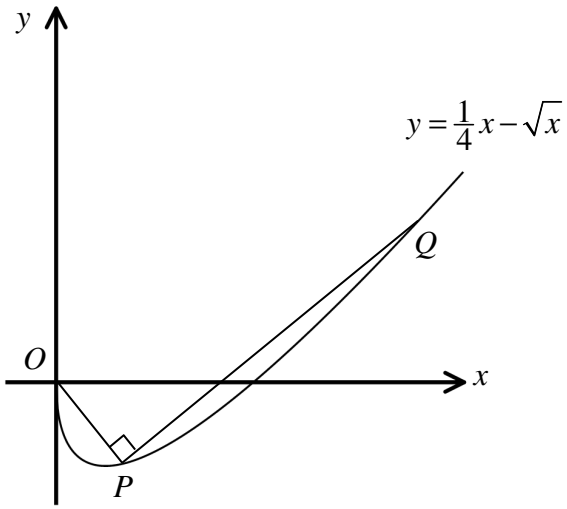
Make x the subject of the equation

$$x^2 + y^2 = 2xy + z^2 .$$

$\square$, $x = y \pm z$



Question 62 (**+)**



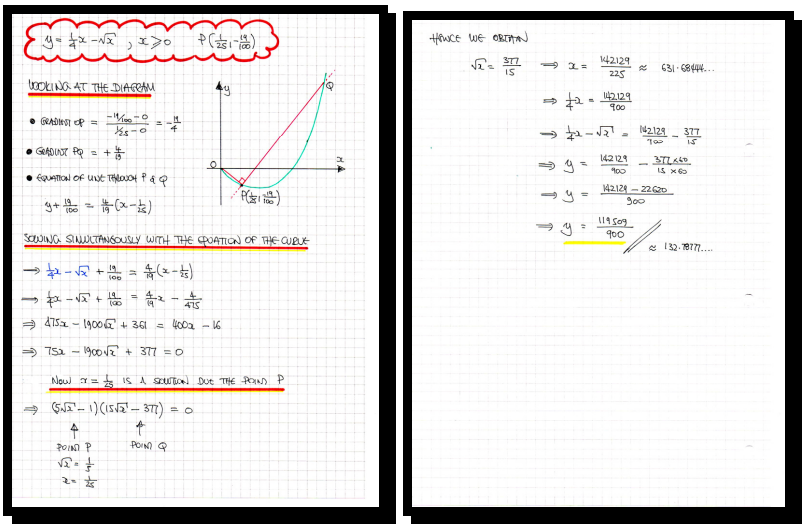
The figure above show the curve with equation

$$y=\frac{1}{4}x-\sqrt{x}\,,\quad x\in\mathbb{R}\,,\quad x\geq 0\,.$$

The points $P(0.04,-0.19)$ and Q lie on the curve, so that $\angle OPQ=90°$, where O is the origin.

Show that the y coordinate of Q is $\frac{k}{900}$, where k is a six digit integer.

$\boxed{}$, $\boxed{k=119509}$



Question 63 (*****)

Two joggers, A and B ran a standard route of 5 km, which consists of a downhill section to start with, a flat section in the middle of the run and an uphill section all the way to the finish line.

A ran the three sections with respective speeds $2.4\ \text{ms}^{-1}$, $3.2\ \text{ms}^{-1}$ and $2\ \text{ms}^{-1}$.

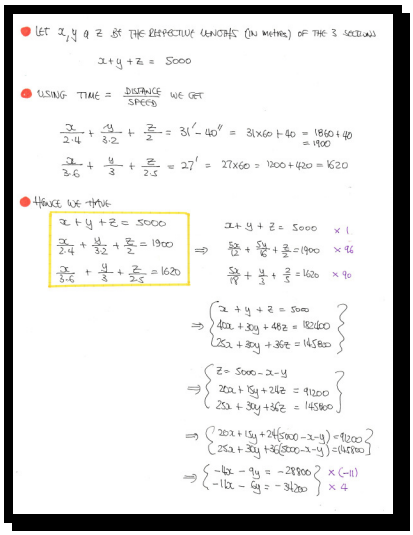
A took 31 minutes and 40 seconds to complete the run.

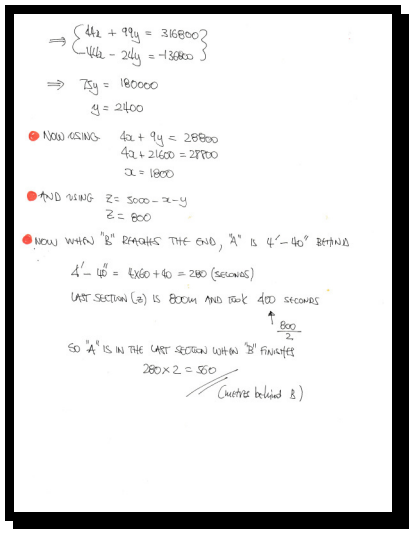
B ran the three sections with respective speeds $3.6\ \text{ms}^{-1}$, $3\ \text{ms}^{-1}$ and $2.5\ \text{ms}^{-1}$.

A took exactly 27 minutes to complete the run.

Assuming that both runners started at the same time, determine the distance between A and B, as B crosses the finish line.

$\square$, $\boxed{560\text{ m}}$





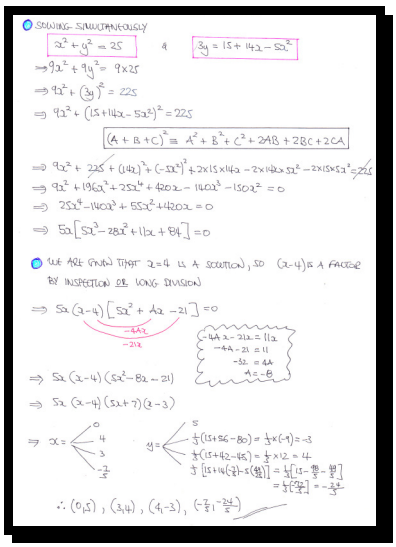
Question 64 (*****)

Find the coordinates of the points of intersections between

$$x^2+y^2=25 \qquad \text{and} \qquad 3y=15+14x-5x^2,$$

given further that the x coordinate of one of these points is 4.

$\square$, $\boxed{(0,5),(3,4),(4,-3),\left(-\frac{2}{5},-\frac{24}{5}\right)}$

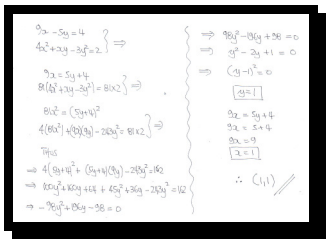


Question 65 (*****) non calculator

Solve the simultaneous equations

$$9x-5y=4$$
$$4x^2+xy-3y^2=2$$

$\boxed{(1,1)}$

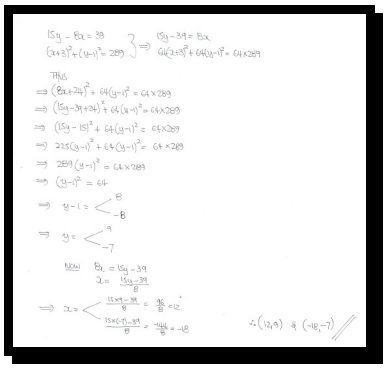


Question 66 (*****)

Solve the simultaneous equations

$$15y-8x=39$$
$$(x+3)^2+(y-1)^2=289$$

$\square$, $\boxed{(12,9)\ \ \&\ \ (-18,-7)}$

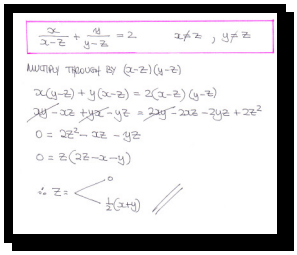


Question 67 (*****)

Solve the following equation for x.

$$\frac{x}{x-z}+\frac{y}{y-z}=2, \quad x\neq z,\ y\neq z.$$

$\square$, $\boxed{z=0,\quad z=\frac{1}{2}(x+y)}$

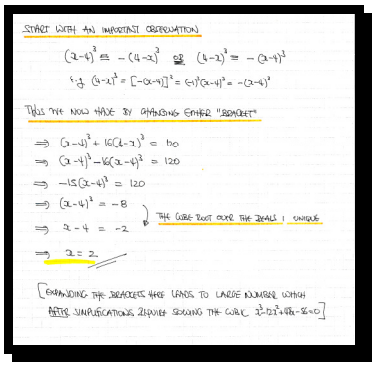


Question 68 (*****)

Use algebra to solve the equation

$$(x-4)^3+16(4-x)^3=120, \quad x\in\mathbb{R}.$$

V, [ ], $\boxed{x=2}$

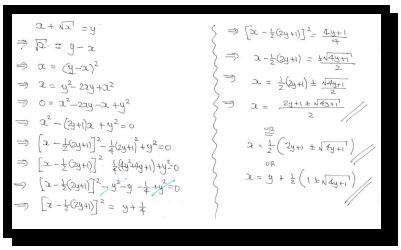


Question 69 (*****)

Make x the subject of the equation

$$x+\sqrt{x}=y \ .$$

$$\boxed{x=y+\frac{1}{2}\left[1\pm\sqrt{4y+1}\right]}$$

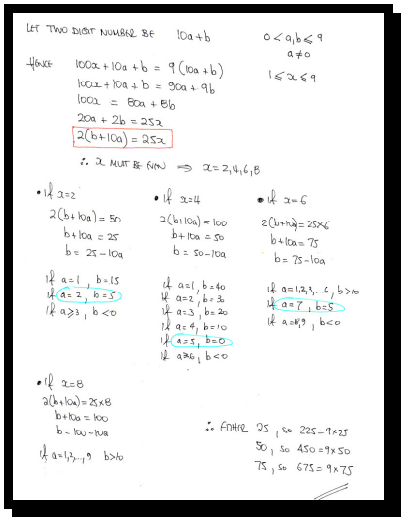


Question 70 (*****)

It is required to add a single digit at the front of a two digit number so that the resulting three digit number is nine times as large as the original two digit number.

Determine with full justification the three possible cases that satisfy this requirement.

$$25,\ 50,\ 75$$



Question 71 (*****)

When a man is asked how old he is, he replied.

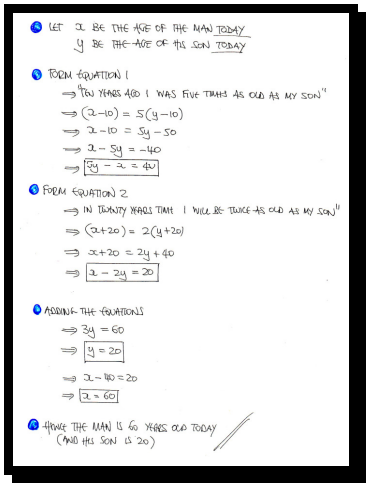
"Ten years ago I was five times as old as my son."

He continued …

"… in twenty years time I will be twice as old as my son."

Determine how old the man is.

[ ], [60 years old]



Question 72 (*****)

When a man is asked how old he is, he replied.

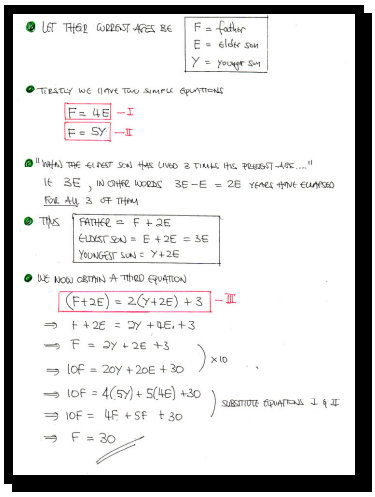
"I am four times as old as my eldest son and five times as old as my youngest son."

He continued …

"… when my eldest son is three times as old as he is now I will be exceeding twice my youngest son's age by three years."

Determine how old the man is.

[ ], [30 years old]



Question 73 (*****)

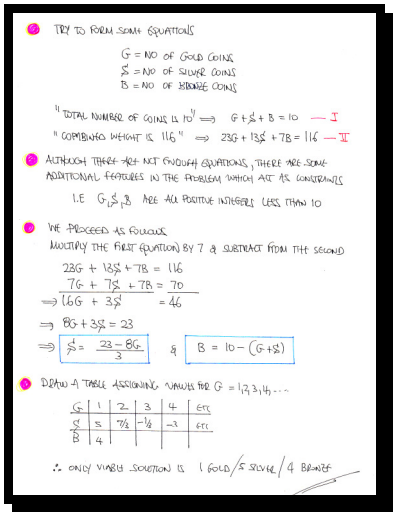
It is known that a box contains 10 coins of which some are gold, some are silver and some are bronze.

The combined weight of the 10 coins is 116 grams

Each gold coin weighs 23 grams, each silver coin weighs 13 grams and each bronze coin weighs 7 grams.

Determine the number of each type of coin.

$\boxed{}$, $\boxed{(G,S,B)=(1,5,4)}$



Question 74 (*****)

A water tank is full of water.

The tank has 3 outlet pipes, each having a constant drainage rate, when the water is allowed to flow out of the tank.

Let A, B and C be labels for each of the three outlet pipes.

If only A and B are turned on, it takes 12 hours to drain the tank.

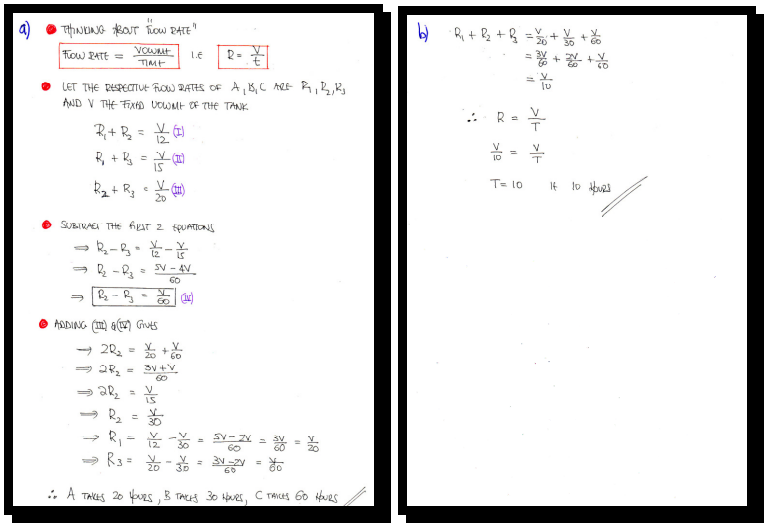
If only A and C are both turned on, it takes 15 hours to drain the tank.

If only B and C are both turned on, it takes 20 hours to drain the tank.

a) Find how long does each outlet pipe on its own take to drain a full tank.

b) Determine the time it takes to drain a full tank, if all three outlet pipes are turned on.

$\boxed{}$, $\boxed{(A,B,C)=(20,30,60)\ \text{ hours}}$, $\boxed{10\text{ hours}}$



Question 75 (*****)

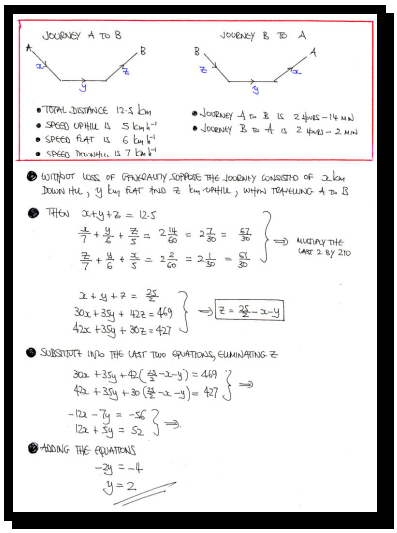
A man walked from his village to the nearby town in 2 hours and 14 minutes.

His return journey over the same route took him 2 hours and 2 minutes.

It is further known that the man always walks at 5 km h^{-1} uphill, at 6 km h^{-1} on flat ground and at 7 km h^{-1} downhill.

Given that the distance between the village and the town is 12.5 km, determine how long the flat distance between the village and the nearby town is.

$\square$, $\boxed{2\text{ km}}$



Question 76 (*****)

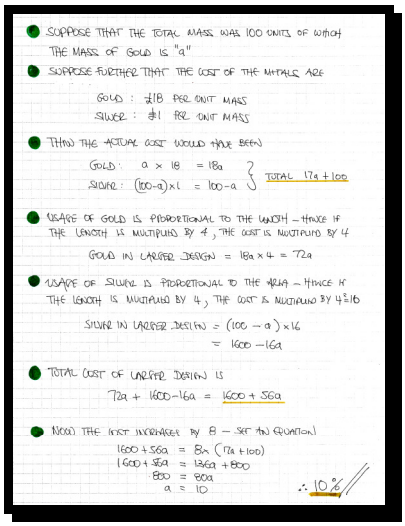
A square jewellery design is made of gold and silver.

The amount of gold used is proportional to the side of the square but the amount of silver used is proportional to the area of the square.

If the side of the square was to be enlarged by a factor of 8, the cost of the jewellery design would increase by a factor of 8.

Given that gold is 18 times more expensive than silver, determine the percentage of gold used in the standard design.

$\square$, 10%



Question 77 (*****)

Two walkers, A and B, start their walk at the point P, at the same time.

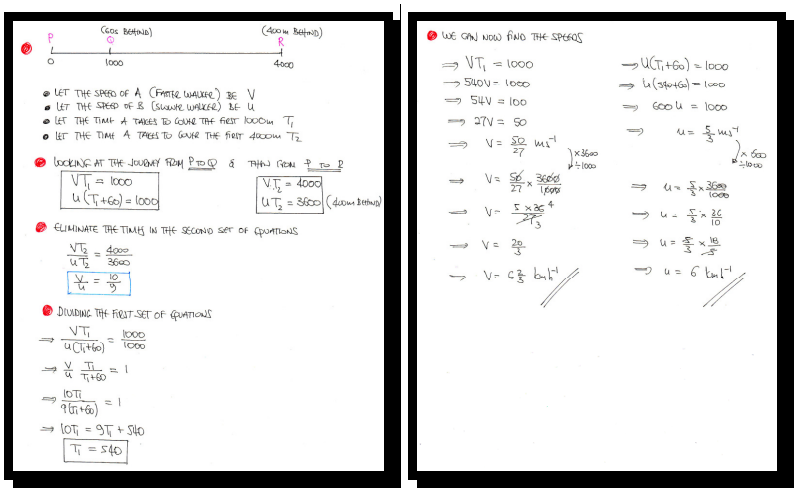
They both walk in the same direction along a straight road, each walker with different constant speed.

The points Q and R lies on that road so that $|PQ| = 1$ km and $|QR| = 3$ km.

- Walker B passes through Q 60 s after walker A passed through Q.
- When walker A passes through R, walker B is 400 m behind A.

Determine the speed of each of the two walkers, in km h^{-1}.

$\square$, $\boxed{V_A = 6\frac{2}{3} \text{ km h}^{-1}}$, $\boxed{V_B = 6 \text{ km h}^{-1}}$



Question 78 (*****)

Two thin rigid vertical poles AB and CD are standing on level horizontal ground.

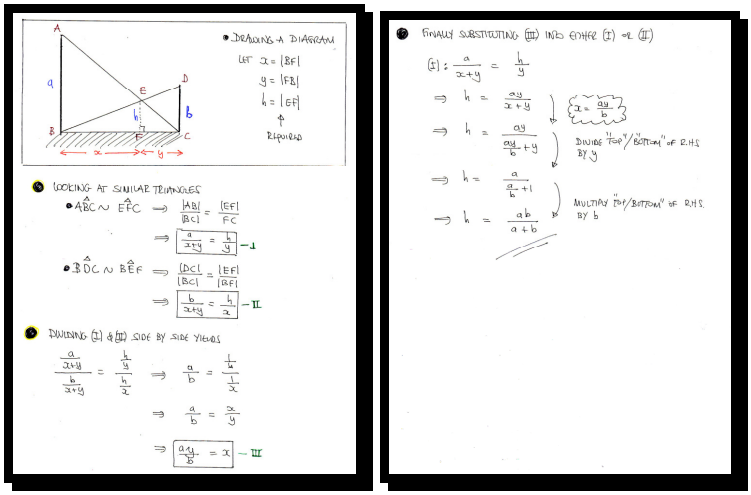
- AB has length a and the point B is level with the ground.
- CD has length b, $b < a$, and the point C is level with the ground.

A taut string is connecting A to C and another taut string is connecting B to D.

The two strings cross each other at the point E.

Find, in terms of a and b, the vertical height of E above the ground.

$\square$, $\boxed{h = \dfrac{ab}{a+b}}$



Question 79 (*****)

A water tank is fed by one inlet pipe which feeds into the tank at constant rate

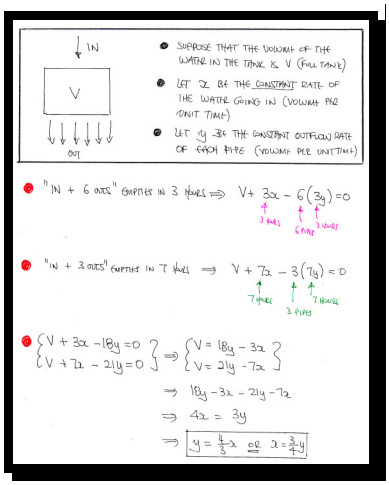
The tank has 6 outlet pipes, each having the same constant drainage rate. The drainage rate of one of the outlet pipes is greater than the inflow rate of the inlet pipe.

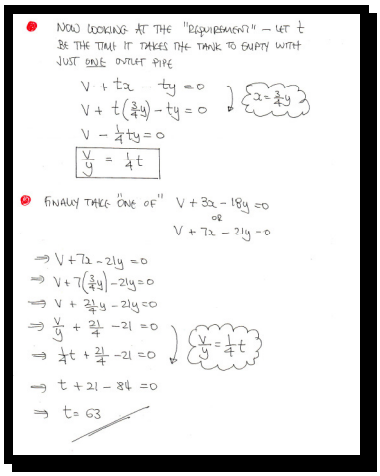
- When the inlet pipe and all 6 outlet pipes are turned on, it takes 3 hours to empty the full tank.

- When the inlet pipe and 3 outlet pipes are turned on, it takes 7 hours to empty the full tank.

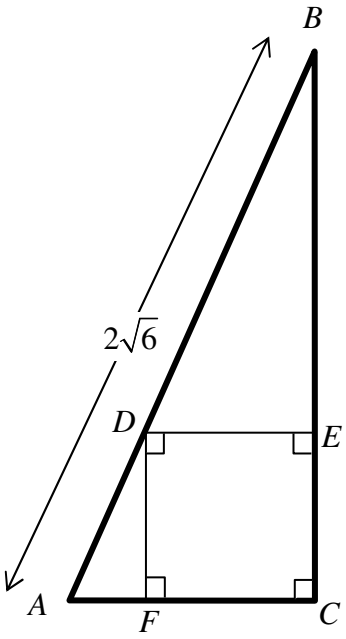
Determine the number of hours it takes to empty a full tank with the inlet pipe and just one of the outlet pipes turned on.

$\boxed{}$, $\boxed{63 \text{ hours}}$





Question 80 (*****)

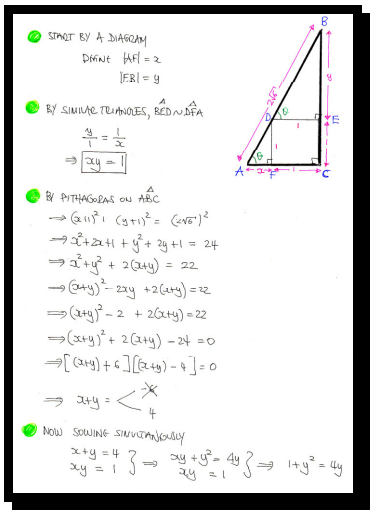


The figure above shows a right angled triangle ABC, where $|AB| = 2\sqrt{6}$.

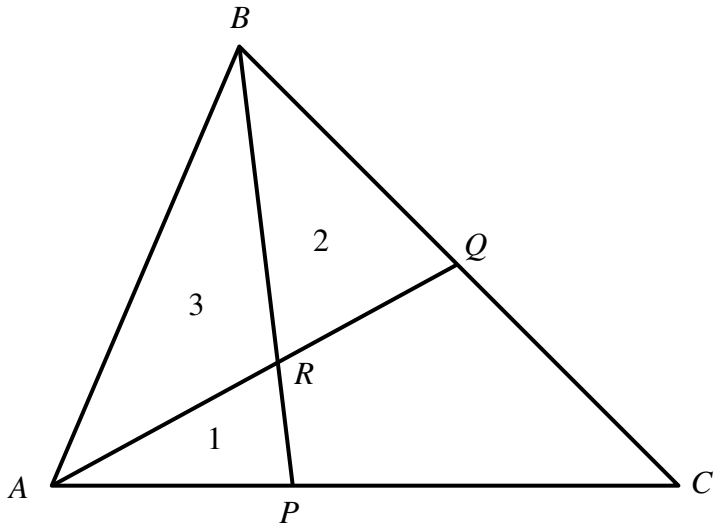
A square $DECF$, of side length 1, is drawn inside ABC, so that D lies on AB, E lies on BC and F lies on AC.

Determine, in exact simplified surd form, the possible values of the tangent of the angle BAC.

$\square$, $\boxed{2 \pm \sqrt{3}}$



Question 81 **(*****)**



The figure above shows a triangle ABC.

The point P lies on AC and the point Q lies on BC.

The point R is the intersection of BP and AQ.

Given that the respective areas of the triangles APR, BQR and ABR are 1, 2 and 3 square units, determine the exact area of the quadrilateral $CPRQ$.

$\square$, $\boxed{\frac{18}{7}}$

