55 DISCRIMINANT EXAM QUESTIONS

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BASIC QUESTIONS

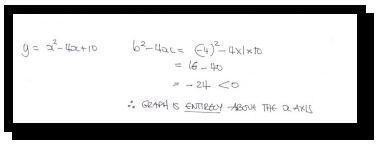
Question 1 (**)

Show by using the discriminant that the graph of the curve with equation

$$y = x^2 - 4x + 10,$$

does not cross the x axis.

proof



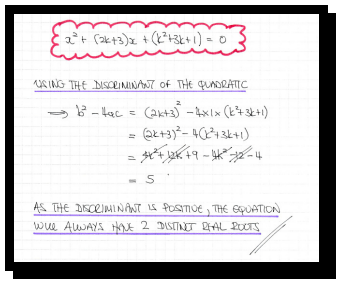
Question 2 (**)

Show that the quadratic equation

$$x^2 + (2k+3)x + k^2 + 3k + 1 = 0$$

has two distinct real roots in x, for all values of the constant k.

[ ], proof



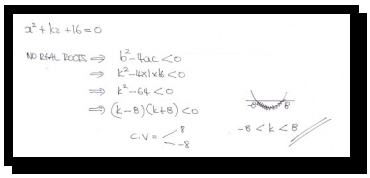
Question 3 (+)**

Find the range of values of the constant k so that the equation

$$x^2+kx+16=0,$$

has no real roots.

$$\boxed{-8<k<8}$$



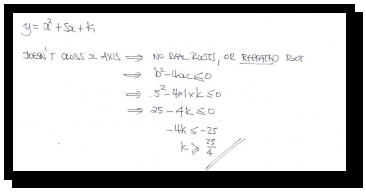
Question 4 (+)**

Find the range of values of the constant k so that the graph of the curve with equation

$$y=x^2+5x+k,$$

does not **cross** the x axis.

$$\boxed{x\geq\frac{25}{4}}$$



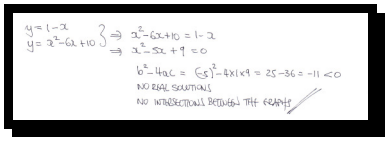
Question 5 (+)**

Use an algebraic method to show that the graphs

$$y = 1 - x \quad \text{and} \quad y = x^2 - 6x + 10,$$

do **not** intersect.

, proof



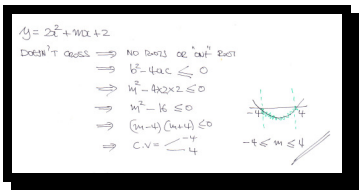
Question 6 (***)

Find the range of values of the constant m so that the graph of the curve with equation

$$y = 2x^2 + mx + 2,$$

does not **cross** the x axis.

$-4 \le m \le 4$



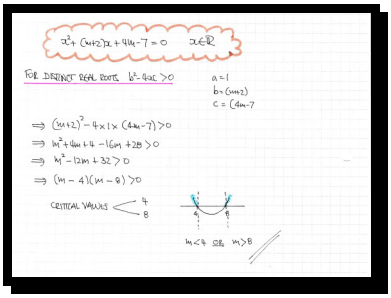
Question 7 (***)

The following quadratic equation, where m is a constant, has two distinct real roots.

$$x^2+(m+2)x+4m-7=0,\ x\in\mathbb{R}\ .$$

Determine the range of the possible values of m.

$\square$, $m<4 \cup m>8$



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STANDARD QUESTIONS

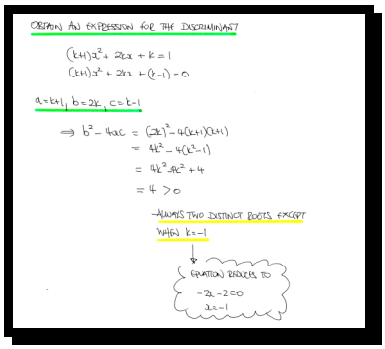
Question 1 **(***+)**

Show that the quadratic equation

$$(k+1)x^2+2kx+k=1$$

has two distinct real roots for all real values of the constant k, except for one value which must be stated.

$\boxed{}$, $\boxed{k\neq -1}$



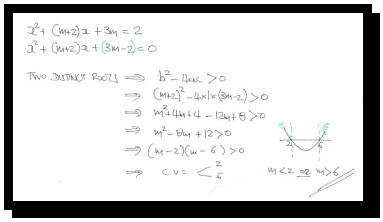
Question 2 **(***+)**

Find the range of the values of the constant m so that the equation

$$x^2+(m+2)x+3m=2\,,$$

has two distinct real roots.

$\boxed{\phantom{m<2 \text{ or } m>6}}$, $\boxed{m<2 \text{ or } m>6}$



Question 3 (*+)**

The straight line L and the curve C have respective equations

$$L:\ 2y=7x+10\,,$$

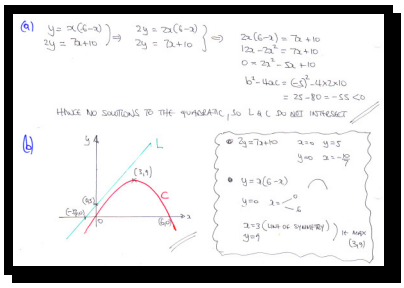
$$C:\ y=x(6-x)\,.$$

a) Show that L and C do not intersect.

b) Find the coordinates of the maximum point of C.

c) Sketch on the same diagram the graph of L and the graph of C, showing clearly the coordinates of any points where each of the graphs meet the coordinate axes.

$\square$, $\boxed{\max(3,9)}$



Question 4 (*+)**

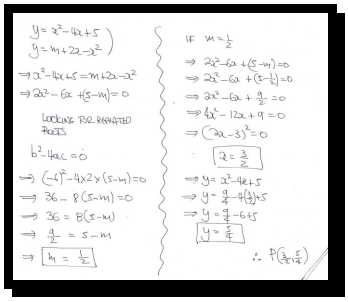
The quadratic curves with equations

$$y = x^2 - 4x + 5 \quad \text{and} \quad y = m + 2x - x^2,$$

where m is a constant, **touch** each other at the point P.

Determine the coordinates of P.

$\square$, $\boxed{P\left(\frac{3}{2}, \frac{5}{4}\right)}$



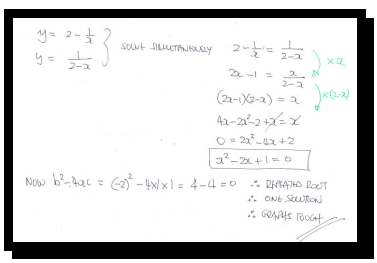
Question 5 (*+)**

Use the discriminant of a suitable quadratic equation to show that the graphs of the curves with equations

$$y = 2 - \frac{1}{x} \quad \text{and} \quad y = \frac{1}{2 - x},$$

touch each other.

$\square$, proof



Question 6 **(***+)**

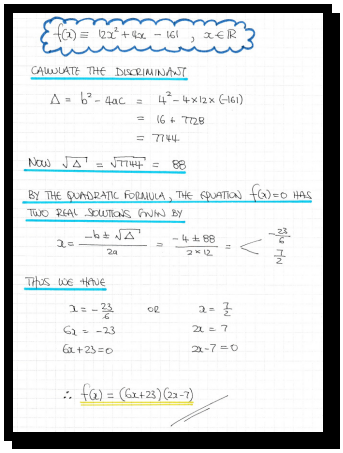
A quadratic curve has equation

$$f(x) \equiv 12x^2 + 4x - 161, \ x \in \mathbb{R}.$$

Express the above equation as the product of two linear factors.

A detailed method must be shown in this question.

$\boxed{}$, $\boxed{f(x) \equiv (6x+23)(2x-7)}$



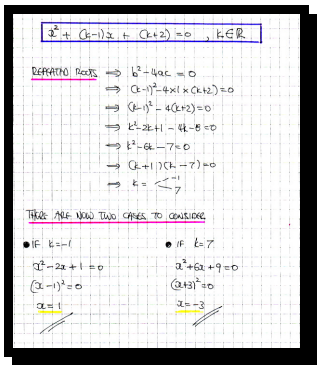
Question 7 **(***+)**

Find the possible solutions of the quadratic equation

$$x^2+(k-1)x+k+2=0,$$

where k is a constant, given that the equation has repeated roots.

$\boxed{}$, $\boxed{x=1 \cup x=-3}$



Question 8 **(****)**

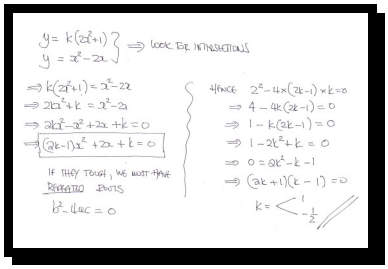
The quadratic curves with equations

$$y=k\left(2x^2+1\right) \quad \text{and} \quad y=x^2-2x,$$

where k is a constant, **touch** each other.

Determine the possible values of k.

$\boxed{}$, $\boxed{k=-\frac{1}{2},\ k=1}$



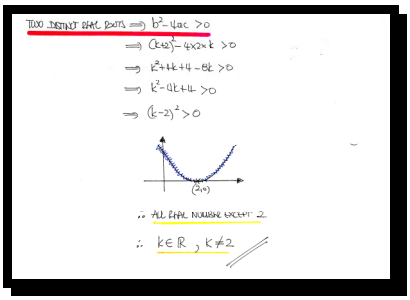
Question 9 **(****)**

Find the range of values that the constant k can take so that

$$2x^2+(k+2)x+k=0$$

has two distinct real roots.

$\boxed{}$, $\boxed{k\in\mathbb{R},\ k\neq 2}$



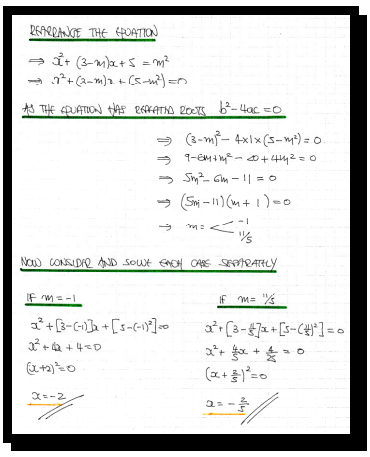
Question 10 **(****)**

Find the possible solutions of the quadratic equation

$$x^2+(3-m)x+5=m^2,$$

where m is a constant, given that the equation has repeated roots.

$\boxed{}$, $\boxed{x=-2\ \cup\ x=-\frac{2}{5}}$



Question 11 (****)

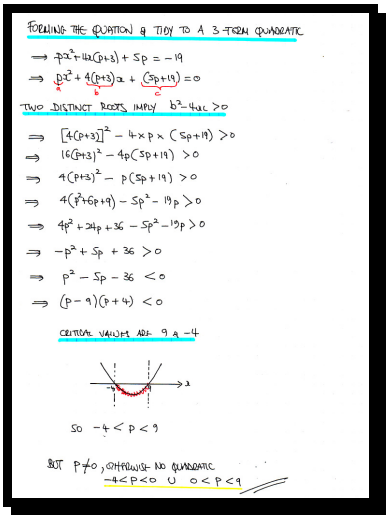
$$f(x)=px^2+4x(p+3)+5p,$$

where p is a non zero constant.

The equation $f(x)=-19$ has two distinct real roots.

Find the range of the possible values of p.

$\boxed{}$, $\boxed{-4<p<0 \quad \cup \quad 0<p<9}$



Question 12 (****)

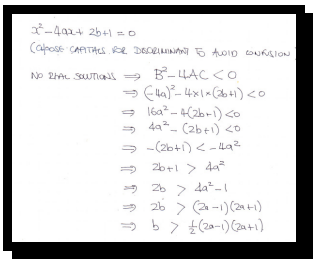
$$x^2-4ax+2b+1=0\,.$$

The above quadratic equation, where a and b are constants, has no real solutions.

Show clearly that

$$b>\frac{1}{2}(2a+1)(2a-1)\,.$$

[ ], [proof]



Question 13 (****)

The curve C has equation

$$y = 4x^2 - 7x + 11 .$$

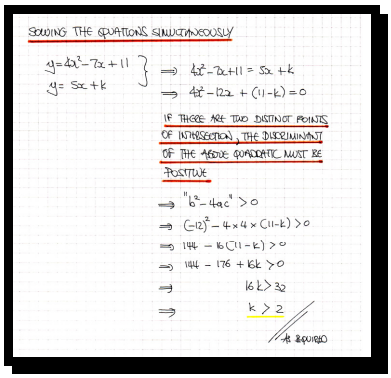
The straight line L has equation

$$y = 5x + k ,$$

where k is a constant.

Given that C and L intersect at two distinct points, show that $k > 2$.

[ ], [proof]



Question 14 (****)

The straight line L has equation

$$y = kx - 9 ,$$

where k is a constant.

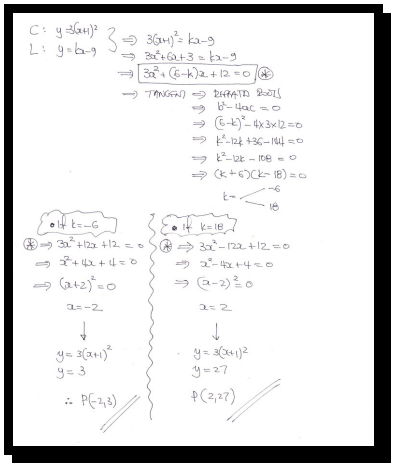
The curve C has equation

$$y = 3(x+1)^2 .$$

It is further given that L is a tangent to C at the point P.

Determine the possible coordinates of P.

$\square$, $\boxed{(-2,3) \text{ or } (2,27)}$



Question 15 (****)

The curve C has equation

$$y = 3x^2 - 4x + 7 .$$

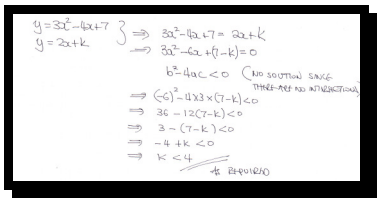
The straight line L has equation

$$y = 2x + k ,$$

where k is a constant.

Given that C and L do not intersect, show that $k < 4$

proof



Question 16 (****)

The straight line with equation

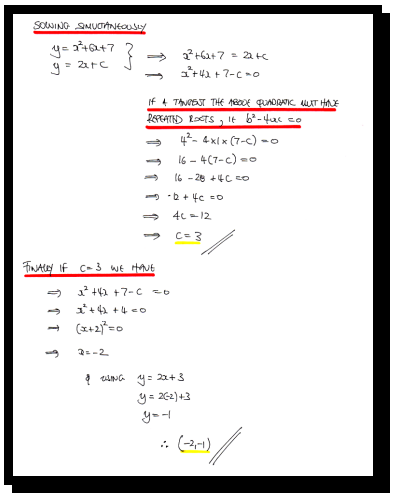
$$y = 2x + c$$

is a tangent to the curve with equation

$$y = x^2 + 6x + 7 .$$

By **using the discriminant** of a suitable quadratic, determine the value of the constant c and find the point of contact between the tangent and the curve.

$\boxed{}$, $\boxed{c = 3}$, $\boxed{(-2,-1)}$



Question 17 (****)

A circle has equation

$$x^2+y^2=8y\,.$$

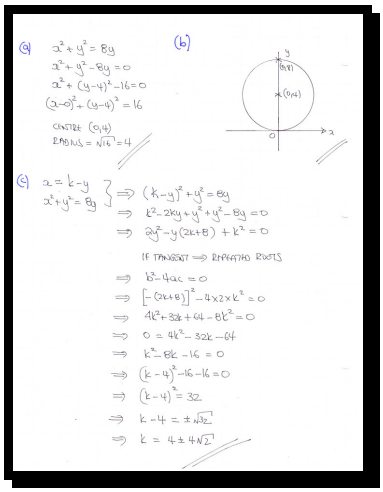
a) Find the coordinates of the centre of the circle and the size of its radius.

b) Sketch the circle.

The line with equation $x+y=k$, where k is a constant, is a tangent to this circle.

c) Determine, as exact surds, the possible values of k.

$\boxed{(0,4)}$, $\boxed{r=4}$, $\boxed{k=4\pm4\sqrt{2}}$



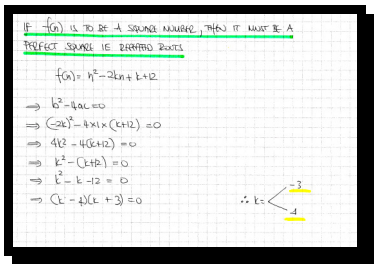
Question 18 (****)

$$f(n)=n^2-2kn+k+12,\ n\in\mathbb{N},$$

where k is a constant.

Given that $f(n)=n^2-2kn+k+12$ is a square number for all values of n, determine the possible values of the constant k.

$\boxed{}$, $\boxed{k=-3\ \cup\ k=4}$



Question 19 (****)

The straight line with equation

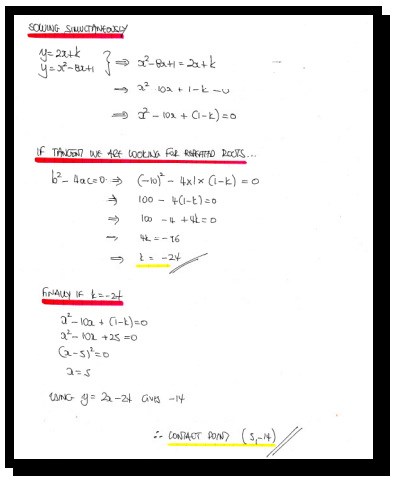
$$y = 2x + k\,,$$

where k is constant, is a tangent to the curve with equation

$$y = x^2 - 8x + 1\,.$$

By **using the discriminant** of a suitable quadratic, determine the value of the constant k and hence find the point of contact between the tangent and the curve.

$\boxed{}$, $\boxed{k=-24}$, $\boxed{(5,-14)}$



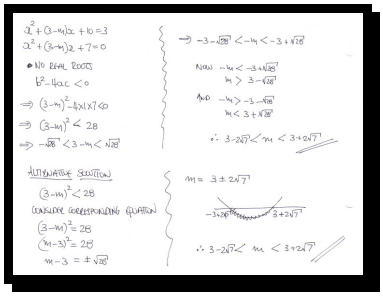
Question 20 (****)

Find, in surd form, the range of values of m for which the quadratic equation

$$x^2+(3-m)x+10=3$$

has no real roots.

$$\boxed{3-2\sqrt{7}<m<3+2\sqrt{7}}$$



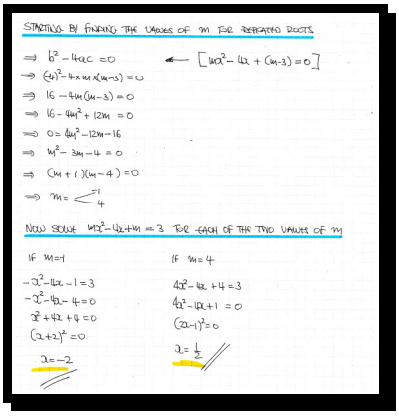
Question 21 (****)

Find the possible roots of the following quadratic equation

$$mx^2-4x+m=3,$$

where m is a non zero constant, given that it has repeated roots.

$$\boxed{}, \quad \boxed{x=-2,\frac{1}{2}}$$



Question 22 (****)

The cubic curve with equation

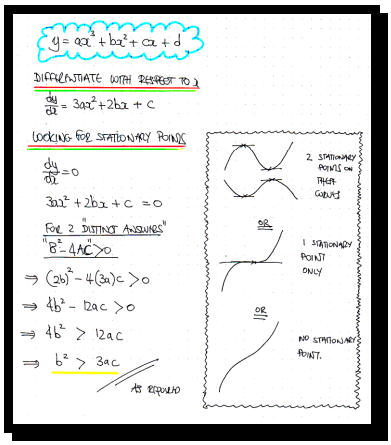
$$y = ax^3 + bx^2 + cx + d \ ,$$

where a, b, c are non zero constants and d is a constant, has one local maximum and one local minimum.

Show clearly that

$$b^2 > 3ac$$

, proof



Question 23 **(****)**

The straight line with equation

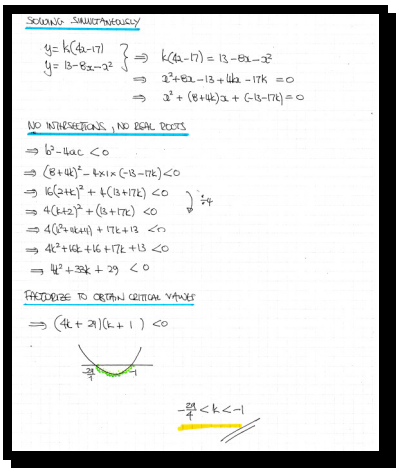
$$y=k\left(4x-17\right),$$

does **not** intersect with the quadratic with equation

$$y=13-8x-x^2.$$

Find the range of possible values of k.

$\boxed{}$, $\boxed{-\frac{29}{4}<k<-1}$

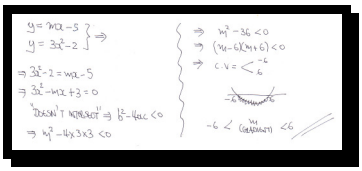


Question 24 **(****)**

A straight line crosses the y axis at $\left(0,-5\right)$ and **does not cross** the curve $y=3x^2-2$.

Find the range of the possible values of the **gradient** of the line.

$\boxed{-6<\text{gradient}<6}$



Question 25 (****)

The straight line with equation

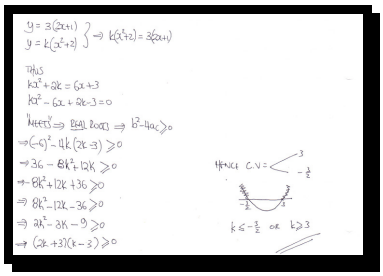
$$y = 3(2x+1)$$

meets the curve with equation

$$y = k\left(x^2+2\right).$$

By **using the discriminant** of a suitable quadratic, determine the range of the possible values of the constant k.

$$\boxed{k \le -\frac{3}{2} \text{ or } k \ge 3}$$



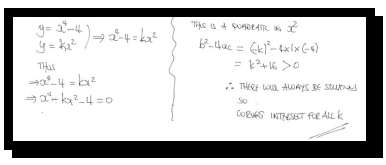
Question 26 (****)

Show that the curves with equations

$$y = x^4 - 4 \quad \text{and} \quad y = kx^2$$

intersect for all values of the constant k.

$$\boxed{\text{proof}}$$



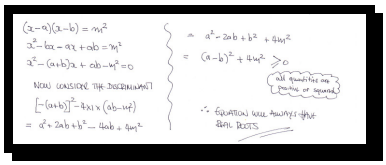
Question 27 (****)

$$(x-a)(x-b)=m^2,$$

where a, b and m are constants.

By using discriminant considerations, show that the above quadratic equation will always have real solutions.

$\boxed{}$, proof



Question 28 (****)

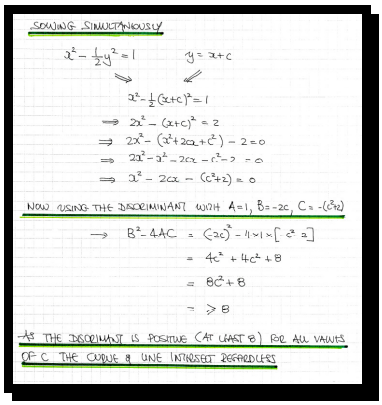
The curve C and the straight line L have respective equations

$$x^2-\frac{y^2}{2}=1 \qquad \text{and} \qquad y=x+c,$$

where c is a constant.

Show that C and L, intersect for all values of c.

$\boxed{}$, proof



Question 29 **(****)**

A curve C has equation

$$y=\frac{1}{x-1},\ x\neq 1.$$

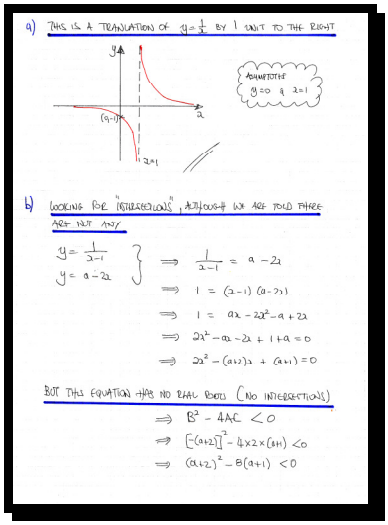
a) Sketch the graph of C, clearly labelling its asymptotes and the coordinates of any point where C meets the coordinate axes.

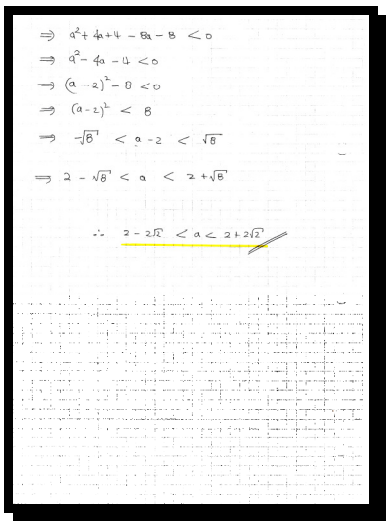
The line with equation $y=a-2x$, where a is a constant, does not meet C.

b) Show clearly that

$$2-2\sqrt{2}<a<2+2\sqrt{2}.$$

$\boxed{}$, $\boxed{\text{asymptotes } x=1,\ y=0}$, $\boxed{(0,-1)}$





Question 30 (****)

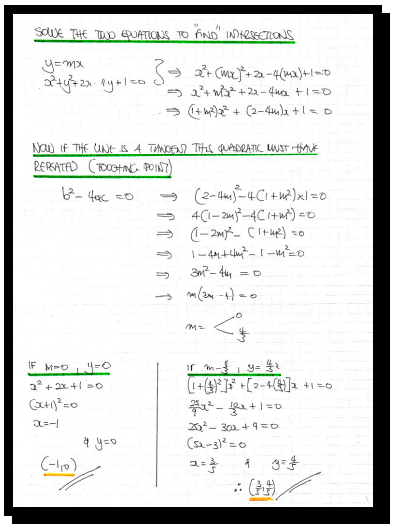
A circle C has equation

$$x^2+y^2+2x-4y+1=0$$

The straight line L with equation $y=mx$ is a tangent to C.

Find the possible values of m and hence determine the possible coordinates at which L meets C.

$\boxed{}$, $\boxed{m=0,\ \ m=\frac{4}{3}}$, $\boxed{(-1,0),\ \left(\frac{3}{5},\frac{4}{5}\right)}$



9 HARD QUESTIONS

Question 1 (**+)**

The straight line L crosses the y axis at $(0,-1)$.

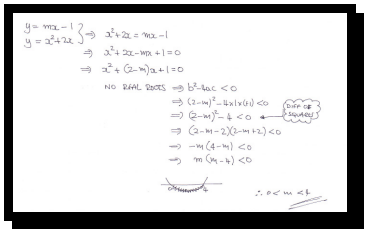
The curve with equation

$$y = x^2 + 2x$$

has **no intersections** with L.

Determine the range of the possible values of the gradient of L.

$\boxed{}$, $\boxed{0 < m < 4}$



Question 2 (**+)**

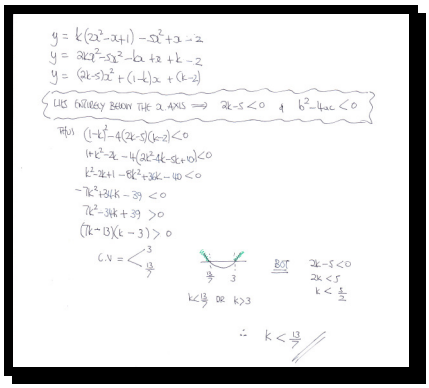
The equation of a quadratic curve C is

$$y = k\left(2x^2 - x + 1\right) - 5x^2 + x - 2,$$

where k is a constant.

Given that the graph of C lies below the x axis, determine the range of the possible values of k.

$\square$, $\boxed{k < \frac{13}{7}}$



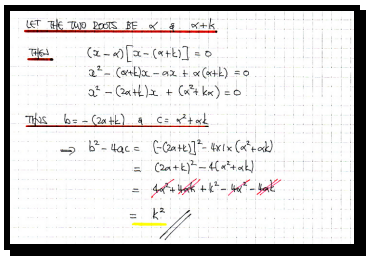
Question 3 (**+)**

A quadratic equation has two real roots differing by k, where k is a positive constant.

Determine, in terms of k, an exact simplified expression for the discriminant of this quadratic.

You may assume that the coefficient of the quadratic term of the equation is one.

$\boxed{}$, $\boxed{b^2-4ac \ = \ k^2}$



Question 4 (**+)**

$$f(x)=k+12x-4x^2,$$

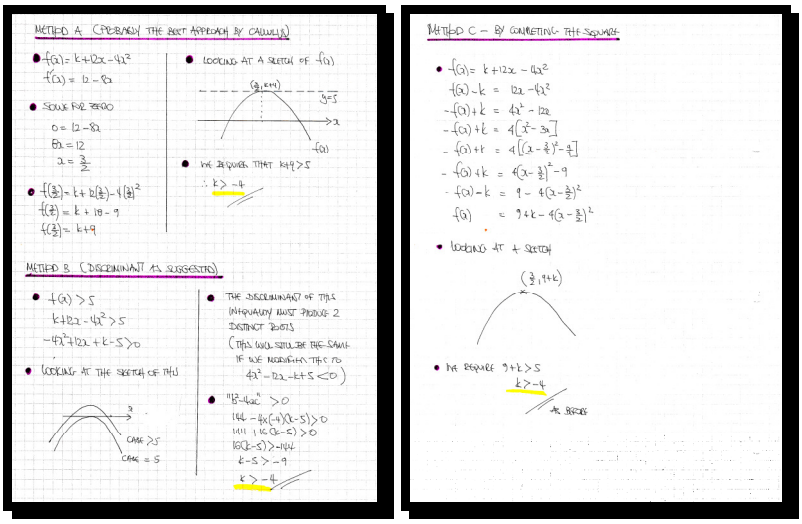
where k is a constant.

It is further given that $f(x)>5$ for **some** values of x.

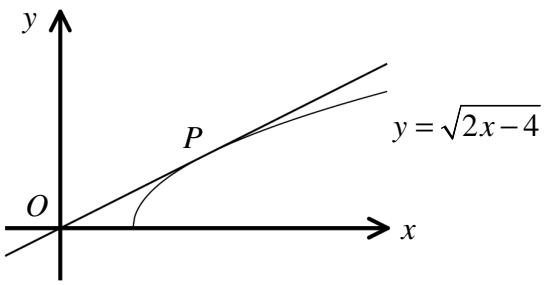
Show by suitable discriminant calculations, or otherwise, that

$$k>-4.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 5 (**+)**



The figure above shows the graph of the curve C with equation

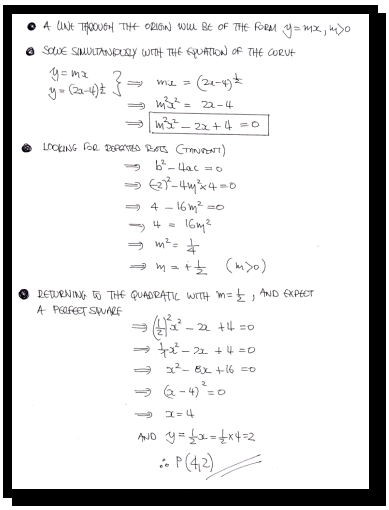
$$y=\sqrt{2x-4}\ ,\ x\geq 2\ .$$

The point P lies on C, so that the tangent to C at P passes through the origin O.

Determine the coordinates of P.

You may not use calculus in this question

$\square$, $\boxed{P(4,2)}$



Question 6 (**+)**

A curve C has equation

$$y = 2x^2 + 4(p+2)x + 8p + q + 8,$$

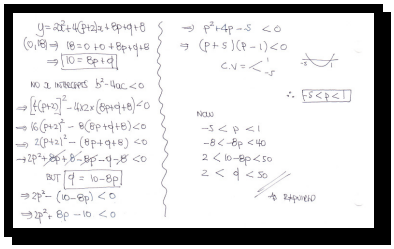
where p and q are constants.

The curve meets the y axis at $y = 18$.

Given further that C has no x intercepts, show that

$$2 < q < 50.$$

$\square$, proof



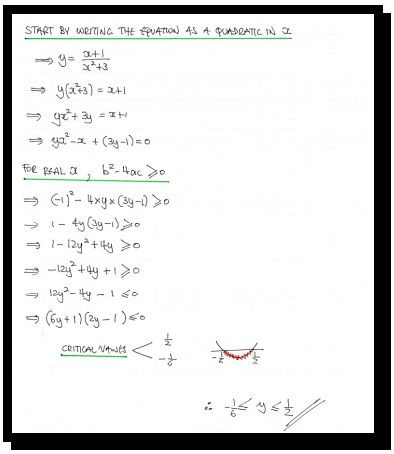
Question 7 (**+)**

The curve C has equation

$$y=\frac{x+1}{x^2+3},\ x\in\mathbb{R}.$$

By considering the discriminant of a suitable quadratic equation, determine the range of the possible values of y.

$\square$, $\boxed{-\frac{1}{6}<y<\frac{1}{2}}$



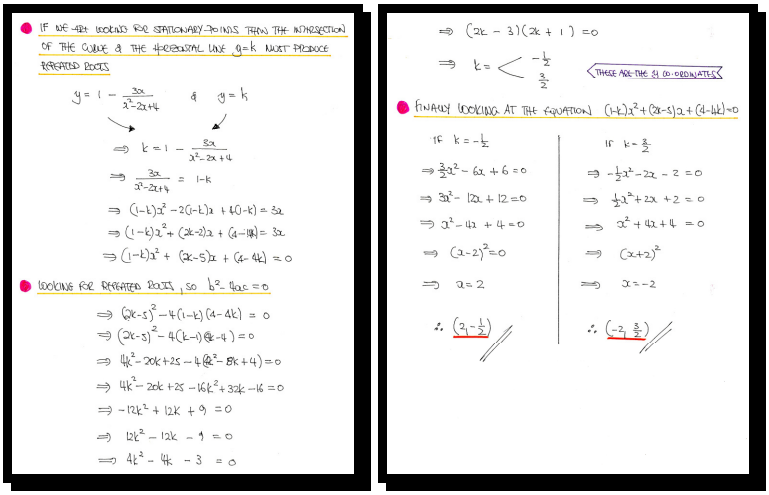
Question 8 **(****+)**

The curve C has equation

$$y=1-\frac{3x}{x^2-2x+4},\ x\in\mathbb{R}.$$

Use a non differentiation method to find the coordinates of the stationary points of C.

$\square$, $\boxed{\left(-2,\frac{3}{2}\right),\left(2,-\frac{1}{2}\right)}$



Question 9 **(****+)**

A quadratic curve has equation

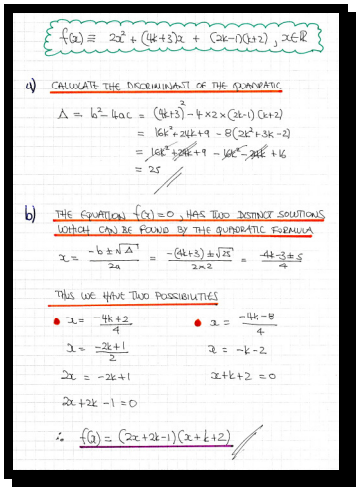
$$f(x)\equiv 2x^2+(4k+3)x+(2k-1)(k+2),\ x\in\mathbb{R},$$

where k is a constant.

a) Evaluate the discriminant of $f(x)$.

b) Express $f(x)$ as the product of two linear factors.

$\boxed{}$, $\boxed{b^2-4ac=25}$, $\boxed{f(x)\equiv(2x+2k-1)(x+k+2)}$



9

ENRICHMENT QUESTIONS

Question 1 (*****)

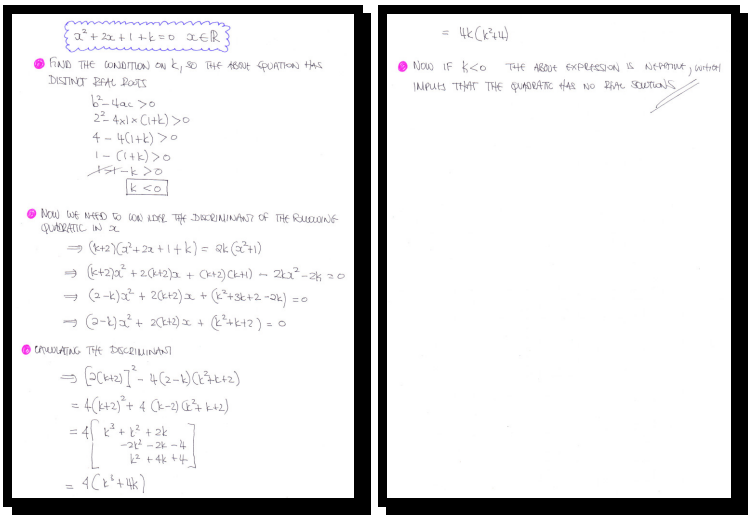
$$x^2+2x+1+k=0,\ x\in\mathbb{R},$$

where k is a real constant.

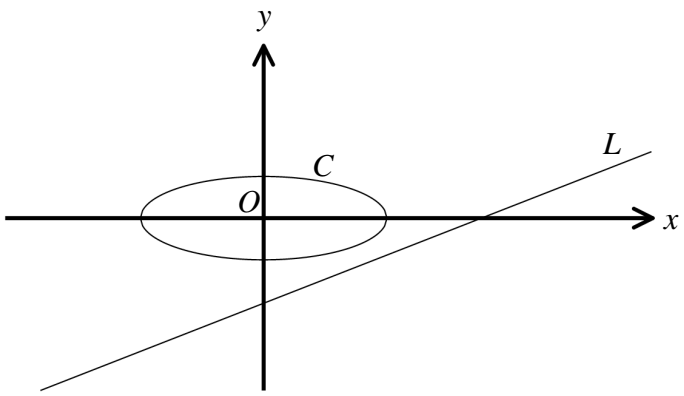
Given that the above equation has distinct real roots, determine the nature of the roots of the following equation

$$(k+2)\left(x^2+2x+1+k\right)=2k\left(x^2+1\right).$$

$\square$, no real solutions



Question 2 (*****)



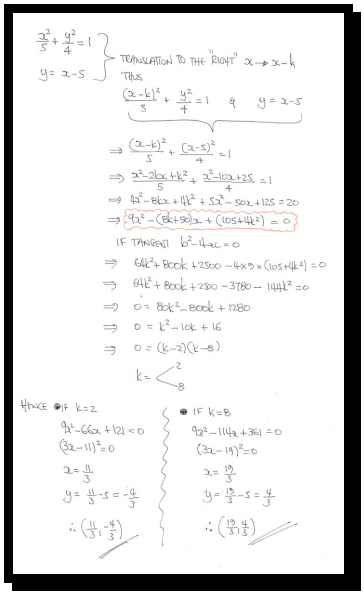
The figure above shows the graph of the curve C and the straight line L with respective equations

$$\frac{x^2}{5}+\frac{y^2}{4}=1 \quad \text{and} \quad y=x-5\,.$$

When C is translated in the positive x direction, L becomes a tangent to C, at some point P.

Determine the exact coordinates of P.

$\square$, $\boxed{P\left(\frac{11}{3},-\frac{4}{3}\right) \quad \text{or} \quad P\left(\frac{19}{3},\frac{4}{3}\right)}$



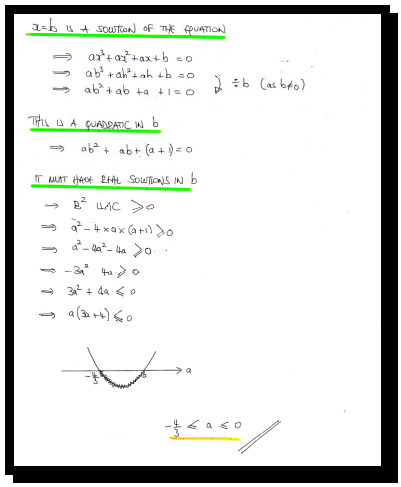
Question 3 (*****)

$$ax^3+ax^2+ax+b=0,$$

where a and b are non zero real constants.

Given that $x=b$ is a root of the above equation, determine the range of the possible values of a.

$\square$, $\boxed{-\frac{4}{3}\le a<0}$



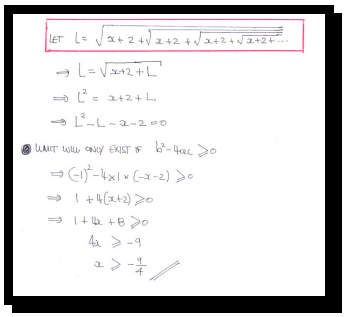
Question 4 (*****)

$$\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+\sqrt{x+2+...}}}}} \ ,$$

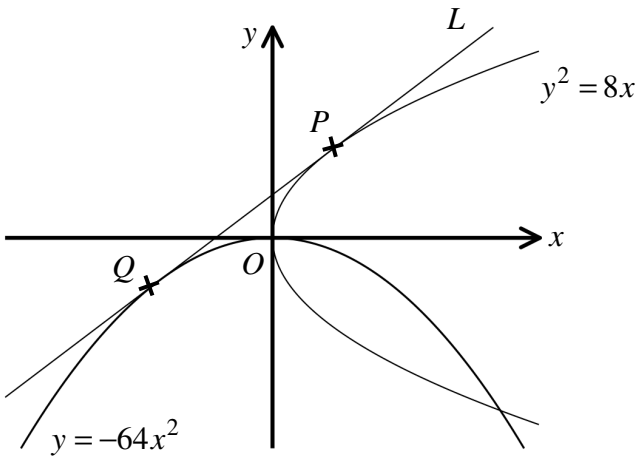
It is given that the above nested radical converges to a limit L, $L \in \mathbb{R}$.

Determine the range of possible values of x.

$\square$, $\boxed{x \geq -\frac{9}{4}}$



Question 5 **(*****)**



The straight line L is a tangent at the point P to the curve with equation

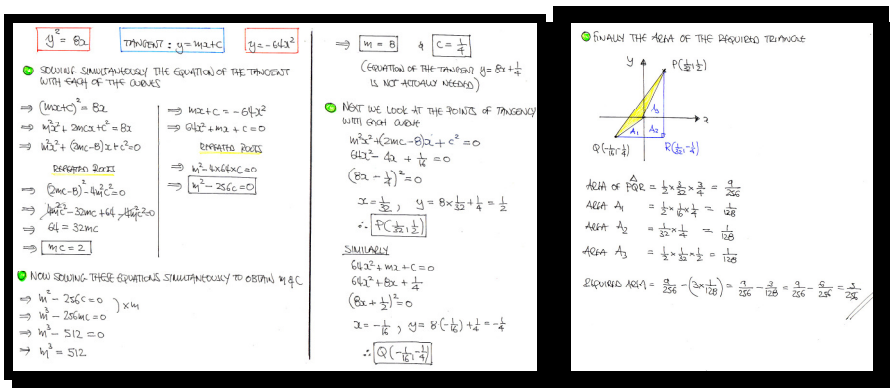
$$y^2 = 8x\,.$$

The straight line L is also a tangent at the point Q to the curve with equation

$$y = -64x^2\,.$$

Determine the exact area of the triangle POQ, where O is the origin.

$\square$, $\boxed{\text{area} = \dfrac{3}{256}}$

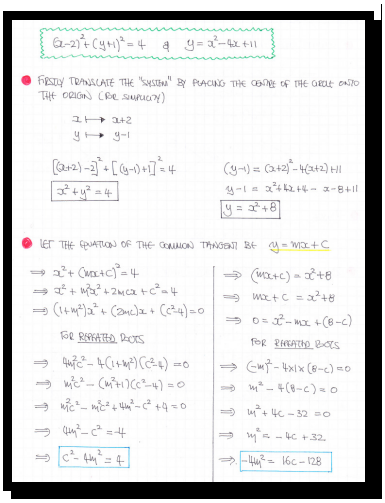


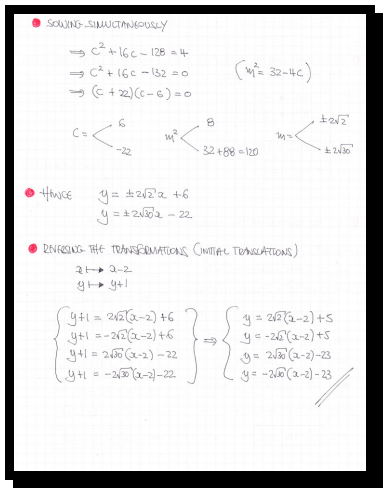
Question 6 (*****)

Find in exact form the equations of the common tangents to the curves with equations

$$(x-2)^2+(y+1)^2=4 \qquad \text{and} \qquad y=x^2-4x+11.$$

$\square$, $\boxed{y=2\sqrt{2}(x-2)+5}$, $\boxed{y=-2\sqrt{2}(x-2)+5}$, $\boxed{y=2\sqrt{30}(x-2)-23}$, $\boxed{y=-2\sqrt{30}(x-2)-23}$





Question 7 (*****)

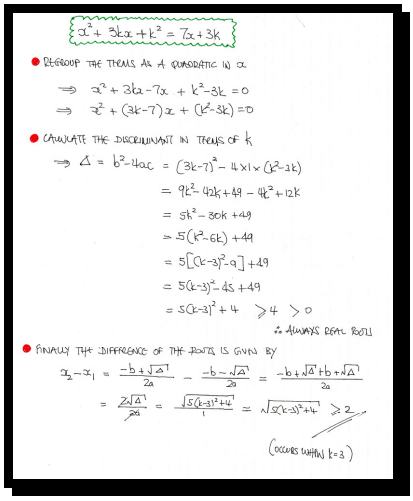
The following quadratic in x is given below

$$x^2+3kx+k^2=7x+3k\,,$$

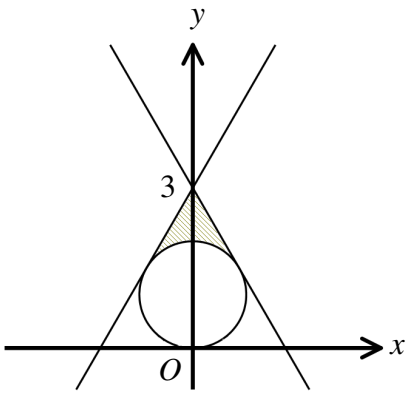
where k is a constant.

Show that the above quadratic has real solutions whose difference is at least 2.

$\boxed{}$, $\boxed{\text{proof}}$



Question 8 **(*****)**



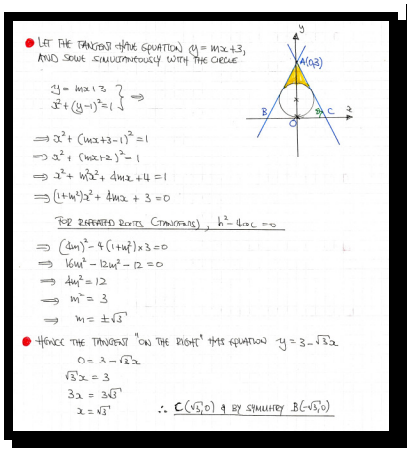
A circle with equation

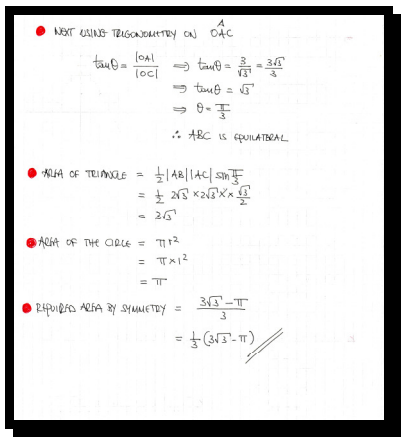
$$x^2+(y-1)^2=1.$$

Two tangents to the circle are drawn so both are passing though the point $(0,3)$.

Determine in exact simplified form the value of the finite region between the circle and the two tangents, shown shaded in the figure above.

$\square$, $\boxed{\text{area}=\frac{1}{3}\left(3\sqrt{3}-\pi\right)}$





Question 9 (*****)

The points P and Q are the points of tangency of the common tangent to each of the curves with equations

$$y^2 = 4ax \quad \text{and} \quad ay = 2x^2 ,$$

where a is a positive constant.

Show that $|PQ|$ is $7\frac{1}{2}$ times the distance of the common tangent from the origin O.

$\boxed{}$, $\boxed{\text{proof}}$

