DIFFERENTIATION PRACTICE

THE OPERATION OF DIFFERENTIATION

Question 1

Evaluate the following.

a) $\dfrac{d}{dx}\left(5x^6\right)$

$$\boxed{\frac{d}{dx}\left(5x^6\right)=30x^5}$$

b) $\dfrac{d}{dx}\left(2x^{\frac{3}{2}}\right)$

$$\boxed{\frac{d}{dx}\left(2x^{\frac{3}{2}}\right)=3x^{\frac{1}{2}}}$$

c) $\dfrac{d}{dx}\left(6x^4-x^3\right)$

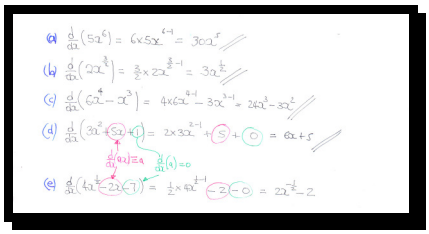
$$\boxed{\frac{d}{dx}\left(6x^4-x^3\right)=24x^3-3x^2}$$

d) $\dfrac{d}{dx}\left(3x^2+5x+1\right)$

$$\boxed{\frac{d}{dx}\left(3x^2+5x+1\right)=6x+5}$$

e) $\dfrac{d}{dx}\left(4x^{\frac{1}{2}}-2x-7\right)$

$$\boxed{\frac{d}{dx}\left(4x^{\frac{1}{2}}-2x-7\right)=2x^{-\frac{1}{2}}-2}$$



Question 2

Evaluate the following.

a) $\dfrac{d}{dx}\left(4x^3\right)$

$$\boxed{\dfrac{d}{dx}\left(4x^3\right)=12x^2}$$

b) $\dfrac{d}{dx}\left(7x^5\right)$

$$\boxed{\dfrac{d}{dx}\left(7x^5\right)=35x^4}$$

c) $\dfrac{d}{dx}\left(4x^2+3x^4\right)$

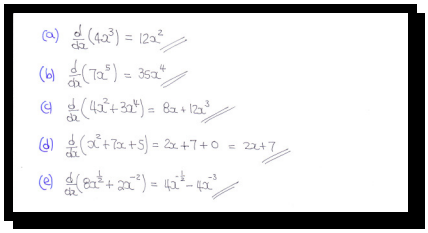
$$\boxed{\dfrac{d}{dx}\left(4x^2+3x^4\right)=8x+12x^3}$$

d) $\dfrac{d}{dx}\left(x^2+7x+5\right)$

$$\boxed{\dfrac{d}{dx}\left(x^2+7x+5\right)=2x+7}$$

e) $\dfrac{d}{dx}\left(8x^{\frac{1}{2}}+2x^{-2}\right)$

$$\boxed{\dfrac{d}{dx}\left(8x^{\frac{1}{2}}+2x^{-2}\right)=4x^{-\frac{1}{2}}-4x^{-3}}$$



Question 3

Differentiate the following expressions with respect to x

a) $y = x^2 - 4x^6$

$$\boxed{\frac{dy}{dx} = 2x - 24x^5}$$

b) $y = 5x^3 - 6x^{\frac{3}{2}}$

$$\boxed{\frac{dy}{dx} = 15x^2 - 9x^{\frac{1}{2}}}$$

c) $y = 9x^{-3} + 7x^{-2}$

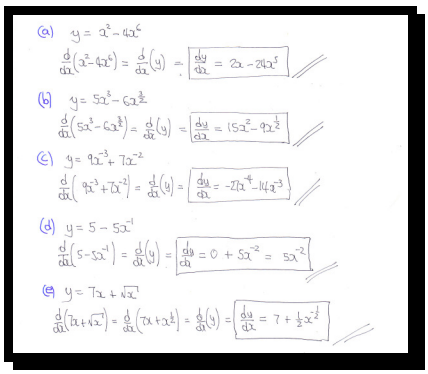
$$\boxed{\frac{dy}{dx} = -27x^{-4} - 14x^{-3}}$$

d) $y = 5 - 5x^{-1}$

$$\boxed{\frac{dy}{dx} = 5x^{-2}}$$

e) $y = 7x + \sqrt{x}$

$$\boxed{\frac{dy}{dx} = 7 + \frac{1}{2}x^{-\frac{1}{2}}}$$



Question 4

Differentiate the following expressions with respect to x

a) $y = x^6 - 7x^2$

$$\boxed{\frac{dy}{dx} = 6x^5 - 14x}$$

b) $y = 1 - 6x^{\frac{5}{2}}$

$$\boxed{\frac{dy}{dx} = 15x^{\frac{3}{2}}}$$

c) $y = 2x + 8x^{-2}$

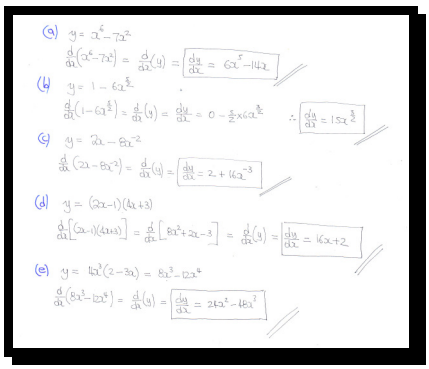
$$\boxed{\frac{dy}{dx} = 2 + 16x^{-3}}$$

d) $y = (2x-1)(4x+3)$

$$\boxed{\frac{dy}{dx} = 16x + 2}$$

e) $y = 4x^3(2-3x)$

$$\boxed{\frac{dy}{dx} = 24x^2 - 48x^3}$$



Question 5

Find $f'(x)$ for each of the following functions.

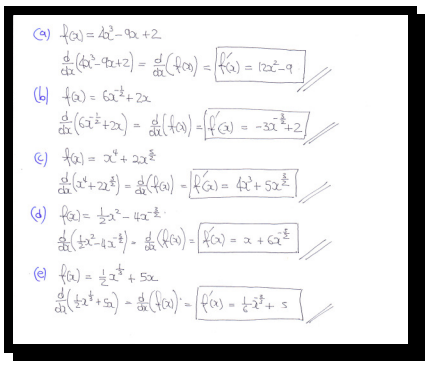
a) $f(x)=4x^3-9x+2$ $\quad f'(x)=12x^2-9$

b) $f(x)=6x^{-\frac{1}{2}}+2x$ $\quad f'(x)=-3x^{-\frac{3}{2}}+2$

c) $f(x)=x^4+2x^{\frac{5}{2}}$ $\quad f'(x)=4x^3+5x^{\frac{3}{2}}$

d) $f(x)=\frac{1}{2}x^2-4x^{-\frac{3}{2}}$ $\quad f'(x)=x+6x^{-\frac{5}{2}}$

e) $f(x)=\frac{1}{2}x^{\frac{1}{3}}+5x$ $\quad f'(x)=\frac{1}{6}x^{-\frac{2}{3}}+5$



Question 6

Differentiate each of the following functions with respect to x.

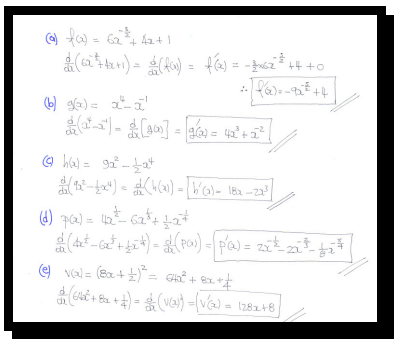
a) $f(x)=6x^{-\frac{3}{2}}+4x+1$ $\qquad$ $\boxed{f'(x)=-9x^{-\frac{5}{2}}+4}$

b) $g(x)=x^4-x^{-1}$ $\qquad$ $\boxed{g'(x)=4x^3+x^{-2}}$

c) $h(x)=9x^2-\frac{1}{2}x^4$ $\qquad$ $\boxed{h'(x)=18x-2x^3}$

d) $p(x)=4x^{\frac{1}{2}}-6x^{\frac{1}{3}}+\frac{1}{2}x^{-\frac{1}{4}}$ $\qquad$ $\boxed{p'(x)=2x^{-\frac{1}{2}}-2x^{-\frac{2}{3}}-\frac{1}{8}x^{-\frac{5}{4}}}$

e) $v(x)=\left(8x+\frac{1}{2}\right)^2$ $\qquad$ $\boxed{v'(x)=128x+8}$



Question 7

Carry out the following differentiations.

a) $\dfrac{d}{dt}\left(4t^2-7t+5\right)$

$$\boxed{\frac{d}{dt}\left(4t^2-7t+5\right)=8t-7}$$

b) $\dfrac{d}{dy}\left(y^{\frac{1}{2}}-\frac{2}{3}y^{-\frac{1}{2}}\right)$

$$\boxed{\frac{d}{dy}\left(y^{\frac{1}{2}}-\frac{2}{3}y^{-\frac{1}{2}}\right)=\frac{1}{2}y^{-\frac{1}{2}}+\frac{1}{3}y^{-\frac{3}{2}}}$$

c) $\dfrac{d}{dz}\left(2z^2-3z^{-1}+z\right)$

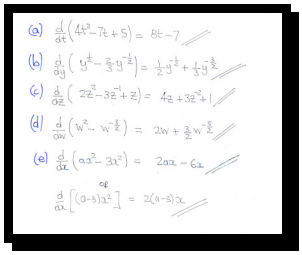
$$\boxed{\frac{d}{dz}\left(2z^2-3z^{-1}+z\right)=4z+3z^{-2}+1}$$

d) $\dfrac{d}{dw}\left(w^2-w^{-\frac{3}{2}}\right)$

$$\boxed{\frac{d}{dw}\left(w^2-w^{-\frac{3}{2}}\right)=2w+\frac{3}{2}w^{-\frac{5}{2}}}$$

e) $\dfrac{d}{dx}\left(ax^2-3x^2\right)$

$$\boxed{\frac{d}{dx}\left(ax^2-3x^2\right)=2ax-6x}$$



Question 8

Carry out the following differentiations.

a) $\dfrac{d}{dy}\left(4y^3+6y+2\right)$

$$\boxed{\frac{d}{dy}\left(4y^3+6y+2\right)=12y^2+6}$$

b) $\dfrac{d}{dt}\left(7t^2-4t^{\frac{1}{2}}\right)$

$$\boxed{\frac{d}{dt}\left(7t^2-4t^{\frac{1}{2}}\right)=14t-2t^{-\frac{1}{2}}}$$

c) $\dfrac{d}{dx}\left(ax^2+bx+c\right)$

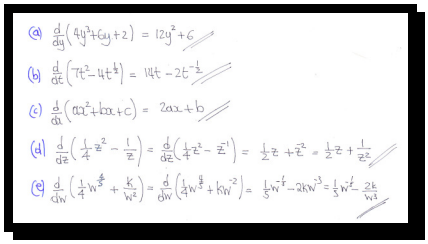
$$\boxed{\frac{d}{dx}\left(ax^2+bx+c\right)=2ax+b}$$

d) $\dfrac{d}{dz}\left(\dfrac{1}{4}z^2-\dfrac{1}{z}\right)$

$$\boxed{\frac{d}{dz}\left(\frac{1}{4}z^2-\frac{1}{z}\right)=\frac{1}{2}z+\frac{1}{z^2}}$$

e) $\dfrac{d}{dw}\left(\dfrac{1}{4}w^{\frac{4}{5}}+\dfrac{k}{w^2}\right)$

$$\boxed{\frac{d}{dw}\left(\frac{1}{4}w^{\frac{4}{5}}+\frac{k}{w^2}\right)=\frac{1}{5}w^{-\frac{1}{5}}-\frac{2k}{w^3}}$$



Question 9

a) If $A=\pi x^2-20x$, find the rate of change of A with respect to x.

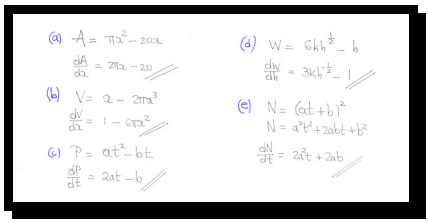
b) If $V=x-2\pi x^3$, find the rate of change of V with respect to x.

c) If $P=at^2-bt$, find the rate of change of P with respect to t.

d) If $W=6kh^{\frac{1}{2}}-h$, find the rate of change of W with respect to h.

e) If $N=(at+b)^2$, find the rate of change of N with respect to t.

$$\boxed{\frac{dA}{dx}=2\pi x-20},\ \boxed{\frac{dV}{dx}=1-6\pi x^2},\ \boxed{\frac{dP}{dt}=2at-b},\ \boxed{\frac{dW}{dh}=3kh^{-\frac{1}{2}}-1},\ \boxed{\frac{dN}{dt}=2a^2t+2ab}$$



DIFFERENTIATING INDICES

Question 1

Differentiate the following expressions with respect to x

a) $y = 4\sqrt{x} - \sqrt[3]{x}$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{3}x^{-\frac{2}{3}}$$

b) $y = 2\sqrt{x} - 4\sqrt{x^3}$

$$\frac{dy}{dx} = x^{-\frac{1}{2}} - 6x^{\frac{1}{2}}$$

c) $y = \dfrac{1}{2\sqrt{x}} + \dfrac{4}{x^2}$

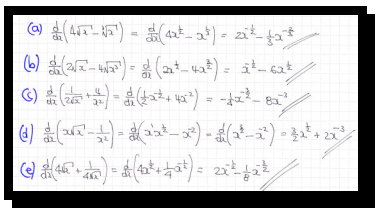
$$\frac{dy}{dx} = -\frac{1}{4}x^{-\frac{3}{2}} - 8x^{-3}$$

d) $y = x\sqrt{x} - \dfrac{1}{x^2}$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-3}$$

e) $y = 4\sqrt{x} + \dfrac{1}{4\sqrt{x}}$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{8}x^{-\frac{3}{2}}$$



Question 2

Find $f'(x)$ for each of the following functions.

a) $f(x)=\dfrac{2}{x^3}+5x^{\frac{2}{3}}$

$$\boxed{f'(x)=-6x^{-4}+\frac{10}{3}x^{-\frac{1}{3}}}$$

b) $f(x)=8x^{\frac{3}{4}}-\dfrac{2}{x^4}$

$$\boxed{f'(x)=6x^{-\frac{1}{4}}+8x^{-5}}$$

c) $f(x)=2x-\dfrac{3}{x^2}+4\sqrt{x}+2$

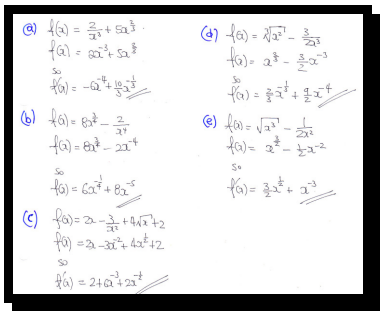
$$\boxed{f'(x)=2+6x^{-3}+2x^{-\frac{1}{2}}}$$

d) $f(x)=\sqrt[3]{x^2}-\dfrac{3}{2x^3}$

$$\boxed{f'(x)=\frac{2}{3}x^{-\frac{1}{3}}+\frac{9}{2}x^{-4}}$$

e) $f(x)=\sqrt{x^3}-\dfrac{1}{2x^2}$

$$\boxed{f'(x)=\frac{3}{2}x^{\frac{1}{2}}+x^{-3}}$$



Question 3

Differentiate the following expressions with respect to x

a) $y=\dfrac{4}{x^3}-\dfrac{4}{3x^2}$

$$\boxed{\frac{dy}{dx}=\frac{8}{3}x^{-3}-12x^{-4}}$$

b) $y=\dfrac{3}{4x^2}-\dfrac{12}{x^2\sqrt{x}}$

$$\boxed{\frac{dy}{dx}=30x^{-\frac{7}{2}}-\frac{3}{2}x^{-3}}$$

c) $y=\dfrac{1}{3x}+\dfrac{2x^3+1}{3\sqrt{x}}$

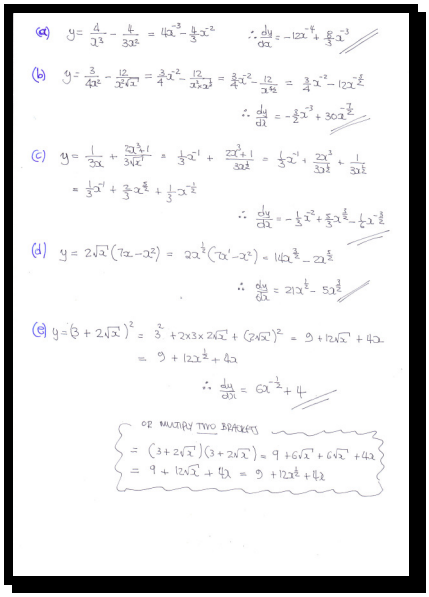
$$\boxed{\frac{dy}{dx}=-\frac{1}{3}x^{-2}+\frac{5}{3}x^{\frac{3}{2}}-\frac{1}{6}x^{-\frac{3}{2}}}$$

d) $y=2\sqrt{x}\left(7x-x^2\right)$

$$\boxed{\frac{dy}{dx}=21x^{\frac{1}{2}}-5x^{\frac{3}{2}}}$$

e) $y=\left(3+2\sqrt{x}\right)^2$

$$\boxed{\frac{dy}{dx}=6x^{-\frac{1}{2}}+4}$$



Question 4

Evaluate the following.

a) $\dfrac{d}{dx}\left(6x^{\frac{4}{3}}-2x^{\frac{5}{2}}\right)$

$$\boxed{8x^{\frac{1}{3}}-5x^{\frac{3}{2}}}$$

b) $\dfrac{d}{dx}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x}}\right)$

$$\boxed{-x^{-2}+\frac{1}{2}x^{-\frac{3}{2}}}$$

c) $\dfrac{d}{dx}\left(\sqrt[3]{x}-\dfrac{27}{x}\right)$

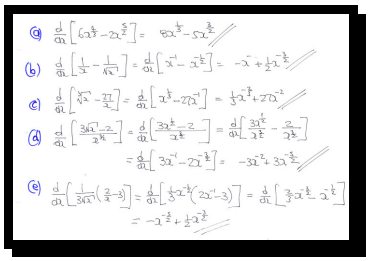
$$\boxed{\frac{1}{3}x^{-\frac{2}{3}}+27x^{-2}}$$

d) $\dfrac{d}{dx}\left(\dfrac{3\sqrt{x}-2}{x^{\frac{3}{2}}}\right)$

$$\boxed{-3x^{-2}+3x^{-\frac{5}{2}}}$$

e) $\dfrac{d}{dx}\left[\dfrac{1}{3\sqrt{x}}\left(\dfrac{2}{x}-3\right)\right]$

$$\boxed{-x^{-\frac{5}{2}}+\frac{1}{2}x^{-\frac{3}{2}}}$$



Question 5

Evaluate the following.

a) $\dfrac{d}{dx}\left(\dfrac{x+x^2}{\sqrt{x}}\right)$

$\boxed{\frac{1}{2}x^{-\frac{1}{2}}+\frac{3}{2}x^{\frac{1}{2}}}$

b) $\dfrac{d}{dx}\left(\dfrac{4x+\sqrt{x}}{2x^2}\right)$

$\boxed{-2x^{-2}-\frac{3}{4}x^{-\frac{5}{2}}}$

c) $\dfrac{d}{dx}\left(\dfrac{x^2+2}{x^3}\right)$

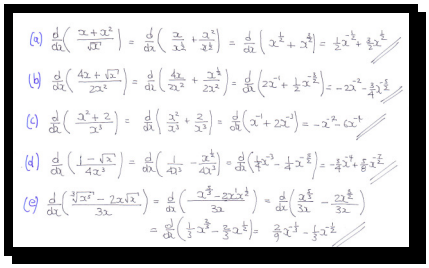
$\boxed{-x^{-2}-6x^{-4}}$

d) $\dfrac{d}{dx}\left(\dfrac{1-\sqrt{x}}{4x^3}\right)$

$\boxed{-\frac{3}{4}x^{-4}+\frac{5}{8}x^{-\frac{7}{2}}}$

e) $\dfrac{d}{dx}\left[\dfrac{\sqrt[3]{x^5}-2x\sqrt{x}}{3x}\right]$

$\boxed{\frac{2}{9}x^{-\frac{1}{3}}-\frac{1}{3}x^{-\frac{1}{2}}}$



Question 6

Differentiate the following expressions with respect to x

a) $y=\dfrac{4+x}{2x^3}$

$$\boxed{\frac{dy}{dx}=-6x^{-4}-x^{-3}}$$

b) $y=\dfrac{x^2+3x}{2\sqrt{x}}$

$$\boxed{\frac{dy}{dx}=\frac{3}{4}x^{-\frac{1}{2}}+\frac{3}{4}x^{\frac{1}{2}}}$$

c) $y=\dfrac{x+4\sqrt{x}}{2x^3}$

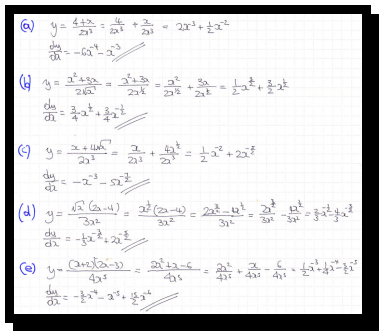
$$\boxed{\frac{dy}{dx}=-5x^{-\frac{7}{2}}-x^{-3}}$$

d) $y=\dfrac{\sqrt{x}\left(2x-4\right)}{3x^2}$

$$\boxed{\frac{dy}{dx}=-\frac{1}{3}x^{-\frac{3}{2}}+2x^{-\frac{5}{2}}}$$

e) $y=\dfrac{\left(x+2\right)\left(2x-3\right)}{4x^5}$

$$\boxed{\frac{dy}{dx}=-\frac{3}{2}x^{-4}-x^{-5}+\frac{15}{2}x^{-6}}$$



Question 7

Find $f'(x)$ for each of the following functions.

a) $f(x)=x\left(\sqrt{x}+x^{-4}\right)$

$$\boxed{f'(x)=\frac{3}{2}x^{\frac{1}{2}}-3x^{-4}}$$

b) $f(x)=\dfrac{1}{\sqrt{x}}\left(\dfrac{2}{x}-\dfrac{3}{4x^2}\right)$

$$\boxed{f'(x)=-3x^{-\frac{5}{2}}+\frac{15}{8}x^{-\frac{7}{2}}}$$

c) $f(x)=4x^{\frac{7}{2}}\left(\dfrac{6}{x^2}-\dfrac{5}{\sqrt{x}}\right)$

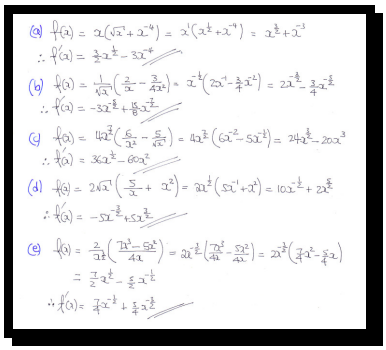
$$\boxed{f'(x)=36x^{\frac{1}{2}}-60x^2}$$

d) $f(x)=2\sqrt{x}\left(\dfrac{5}{x}+x^2\right)$

$$\boxed{f'(x)=-5x^{-\frac{3}{2}}+5x^{\frac{3}{2}}}$$

e) $f(x)=\dfrac{2}{x^{\frac{3}{2}}}\left(\dfrac{7x^3-5x^2}{4x}\right)$

$$\boxed{f'(x)=\frac{7}{4}x^{-\frac{1}{2}}+\frac{5}{4}x^{-\frac{3}{2}}}$$



Question 8

Differentiate the following expressions with respect to x

a) $y=\dfrac{(2x-1)(3x-2)}{2x^{\frac{3}{2}}}$

$$\frac{dy}{dx}=\frac{3}{2}x^{-\frac{1}{2}}+\frac{7}{4}x^{-\frac{3}{2}}-\frac{3}{2}x^{-\frac{5}{2}}$$

b) $y=\dfrac{\left(3+2\sqrt{x}\right)^2}{4x}$

$$\frac{dy}{dx}=-\frac{3}{2}x^{-\frac{3}{2}}-\frac{9}{4}x^{-2}$$

c) $y=\dfrac{4x^3+\sqrt{x^5}}{4\sqrt{x}}$

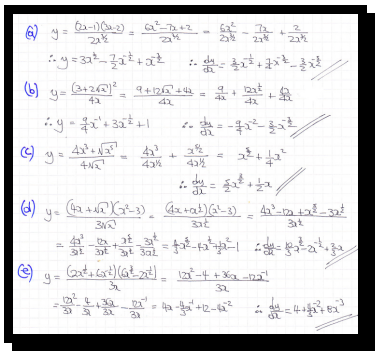
$$\frac{dy}{dx}=\frac{1}{2}x+\frac{5}{2}x^{\frac{3}{2}}$$

d) $y=\dfrac{\left(4x+\sqrt{x}\right)\left(x^2-3\right)}{3\sqrt{x}}$

$$\frac{dy}{dx}=\frac{2}{3}x+\frac{10}{3}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}$$

e) $y=\dfrac{\left(2x^{\frac{1}{2}}+6x^{-\frac{1}{2}}\right)\left(6x^{\frac{3}{2}}-2x^{-\frac{1}{2}}\right)}{3x}$

$$\frac{dy}{dx}=\frac{4}{3}x^{-2}+8x^{-3}+4$$



TANGENTS
&
NORMALS

Question 1 (non calculator)

For each of the following curves find an equation of the tangent to the curve at the point whose x coordinate is given.

a) $y = x^2 - 9x + 13$, where $x = 6$ $\boxed{y = 3x - 23}$

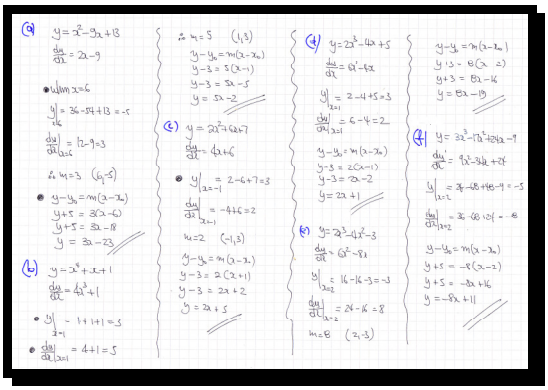
b) $y = x^4 + x + 1$, where $x = 1$ $\boxed{y = 5x - 2}$

c) $y = 2x^2 + 6x + 7$, where $x = -1$ $\boxed{y = 2x + 5}$

d) $y = 2x^3 - 4x + 5$, where $x = 1$ $\boxed{y = 2x + 1}$

e) $y = 2x^3 - 4x^2 - 3$, where $x = 2$ $\boxed{y = 8x - 19}$

f) $y = 3x^3 - 17x^2 + 24x - 9$, where $x = 2$ $\boxed{y = -8x + 11}$



Question 2 (non calculator)

For each of the following curves find an equation of the tangent to the curve at the point whose x coordinate is given.

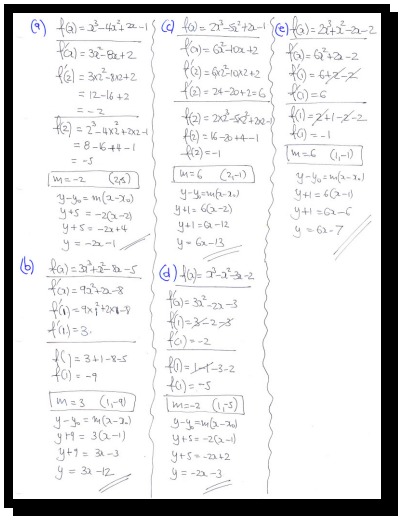
a) $f(x)=x^3-4x^2+2x-1$, where $x=2$ $\boxed{y=-2x-1}$

b) $f(x)=3x^3+x^2-8x-5$, where $x=1$ $\boxed{y=3x-12}$

c) $f(x)=2x^3-5x^2+2x-1$, where $x=2$ $\boxed{y=6x-13}$

d) $f(x)=x^3-x^2-3x-2$, where $x=1$ $\boxed{y=-2x-3}$

e) $f(x)=2x^3+x^2-2x-2$, where $x=1$ $\boxed{y=6x-7}$



Question 3 (non calculator)

For each of the following curves find an equation of the tangent to the curve at the point whose x coordinate is given.

a) $y = x^2 - \frac{3}{x} - \frac{1}{2}$, where $x = -2$ $\qquad \boxed{13x + 4y + 6 = 0}$

b) $y = x^3 - 6x + \frac{8}{x} + 1$, where $x = 2$ $\qquad \boxed{y = 4x - 7}$

c) $y = 4x^2 + \frac{5}{x} - 1$, where $x = 1$ $\qquad \boxed{y = 3x + 5}$

d) $y = 2\sqrt{x} - \frac{6}{\sqrt{x}}$, where $x = 4$ $\qquad \boxed{7x - 8y - 20 = 0}$

e) $y = 3x^{\frac{3}{2}} - \frac{32}{x}$, where $x = 4$ $\qquad \boxed{y = 11x - 28}$



Question 4 (non calculator)

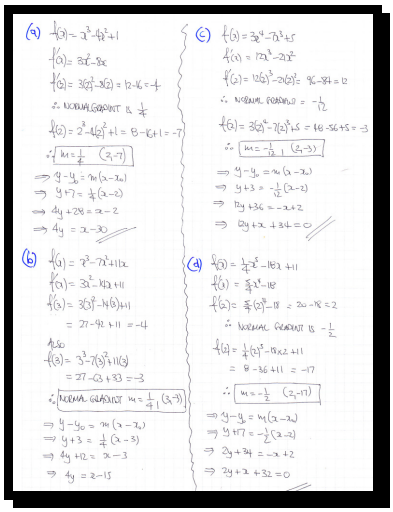
For each of the following curves find an equation of the normal to the curve at the point whose x coordinate is given.

a) $f(x)=x^3-4x^2+1$, where $x=2$ $\boxed{4y=x-30}$

b) $f(x)=x^3-7x^2+11x$, where $x=3$ $\boxed{4y=x-15}$

c) $f(x)=3x^4-7x^3+5$ where $x=2$ $\boxed{12y+x+34=0}$

d) $f(x)=\frac{1}{4}x^5-18x+11$ where $x=2$ $\boxed{2y+x+32=0}$



Question 5 (non calculator)

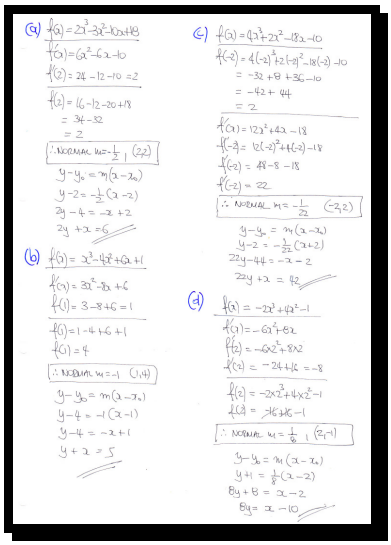
For each of the following curves find an equation of the normal to the curve at the point whose x coordinate is given.

a) $f(x)=2x^3-3x^2-10x+18$, where $x=2$ $\boxed{x+2y=6}$

b) $f(x)=x^3-4x^2+6x+1$, where $x=1$ $\boxed{x+y=5}$

c) $f(x)=4x^3+2x^2-18x-10$ where $x=-2$ $\boxed{22y+x=42}$

d) $f(x)=-2x^3+4x^2-1$, where $x=2$ $\boxed{8y=x-10}$



Question 6 (non calculator)

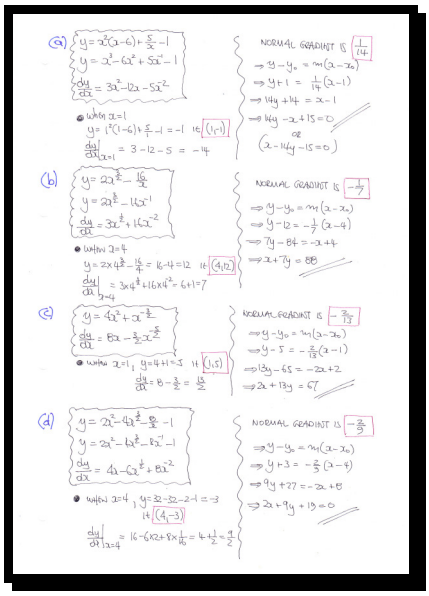
For each of the following curves find an equation of the normal to the curve at the point whose x coordinate is given.

a) $y = x^2(x-6) + \dfrac{5}{x} - 1$, where $x = 1$ $\qquad \boxed{x - 14y - 15 = 0}$

b) $y = 2x^{\frac{3}{2}} - \dfrac{16}{x}$, where $x = 4$ $\qquad \boxed{x + 7y = 88}$

c) $y = 4x^2 + x^{-\frac{3}{2}}$, where $x = 1$ $\qquad \boxed{2x + 13y = 67}$

d) $y = 2x^2 - 4x^{\frac{3}{2}} - \dfrac{8}{x} - 1$, where $x = 4$ $\qquad \boxed{2x + 9y + 19 = 0}$



STATIONARY POINTS

Question 1 (non calculator)

For each of the following cubic equations find the coordinates of their stationary points and determine their nature.

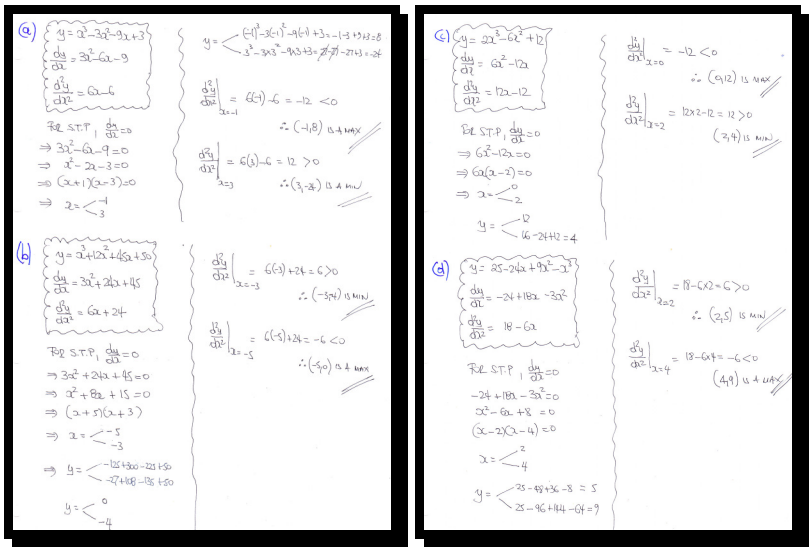
a) $y = x^3 - 3x^2 - 9x + 3$

b) $y = x^3 + 12x^2 + 45x + 50$

c) $y = 2x^3 - 6x^2 + 12$

d) $y = 25 - 24x + 9x^2 - x^3$

$\boxed{\min(3,-24), \max(-1,8)}$, $\boxed{\min(-3,-4), \max(-5,0)}$ $\boxed{\min(2,4), \max(0,12)}$, $\boxed{\min(2,5), \max(4,9)}$



Question 2

For each of the following equations find the coordinates of their stationary points and determine their nature.

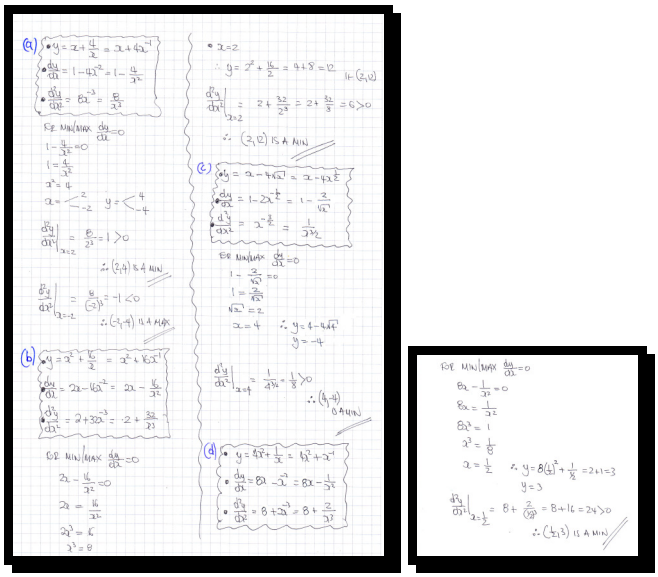
a) $y = x + \dfrac{4}{x}, \quad x \neq 0$

b) $y = x^2 + \dfrac{16}{x}, \quad x \neq 0$

c) $y = x - 4\sqrt{x}, \quad x > 0$

d) $y = 4x^2 + \dfrac{1}{x}, \quad x \neq 0$

$\boxed{\min(2,4), \max(-2,-4)}$, $\boxed{\min(2,12)}$, $\boxed{\min(4,-4)}$, $\boxed{\min\left(\frac{1}{2},3\right)}$



Question 3

For each of the following equations find the coordinates of their stationary points and determine their nature.

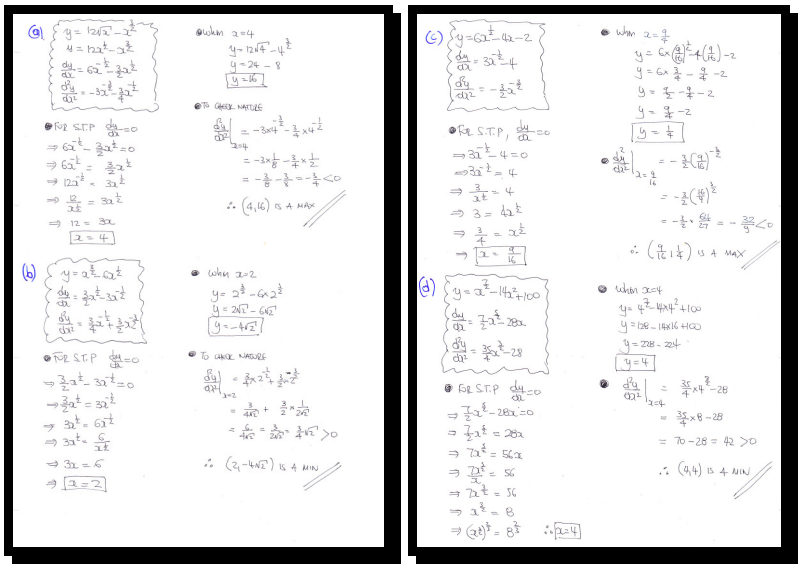
a) $y=12\sqrt{x}-x^{\frac{3}{2}},\ \ x>0$

b) $y=x^{\frac{3}{2}}-6x^{\frac{1}{2}},\ \ x>0$

c) $y=6x^{\frac{1}{2}}-4x-2,\ \ x>0$

d) $y=x^{\frac{7}{2}}-14x^{2}+100,\ \ x>0$

$\boxed{\max\left(4,16\right)}$, $\boxed{\min\left(2,-4\sqrt{2}\right)}$, $\boxed{\max\left(\frac{9}{16},\frac{1}{4}\right)}$, $\boxed{\min\left(4,4\right)}$



Question 4

For each of the following equations find the coordinates of their stationary points and determine their nature.

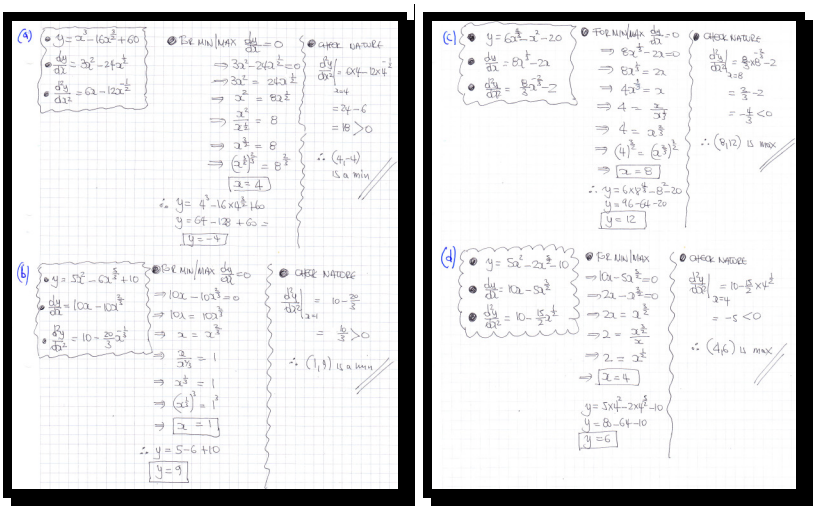
a) $y = x^3 - 16x^{\frac{3}{2}} + 60, \quad x > 0$

b) $y = 5x^2 - 6x^{\frac{5}{3}} + 10, \quad x > 0$

c) $y = 6x^{\frac{4}{3}} - x^2 - 20, \quad x > 0$

d) $y = 5x^2 - 2x^{\frac{5}{2}} - 10, \quad x > 0$

$\boxed{\min(4,-4)}$, $\boxed{\min(1,9)}$, $\boxed{\max(8,12)}$, $\boxed{\max(4,6)}$



Question 5

For each of the following equations find the coordinates of their stationary points and determine their nature.

a) $y=\dfrac{1}{x}-\dfrac{1}{\sqrt{x}},\quad x>0$

b) $y=\dfrac{3\sqrt{x}-2}{x^{\frac{3}{2}}},\quad x>0$

c) $y=\sqrt[3]{x}+\dfrac{27}{x},\quad x>0$

d) $y=\dfrac{1}{3\sqrt{x}}\left[\dfrac{2}{x}-3\right],\quad x>0$

$\boxed{\min\left(4,-\frac{1}{4}\right)}$, $\boxed{\max\left(1,1\right)}$, $\boxed{\min\left(27,4\right)}$, $\boxed{\min\left(2,-\frac{\sqrt{2}}{3}\right)}$



INCREASING and DECREASING FUNCTIONS

Question 1

For each of the following equations find the range of the values of x, for which y is increasing or decreasing.

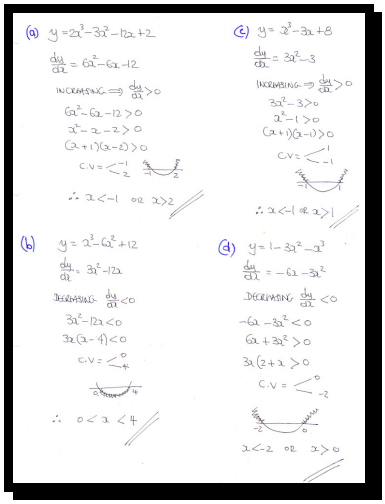
a) $y = 2x^3 - 3x^2 - 12x + 2$, increasing

b) $y = x^3 - 6x^2 + 12$, decreasing

c) $y = x^3 - 3x + 8$, increasing

d) $y = 1 - 3x^2 - x^3$, decreasing

$\boxed{x < -1 \text{ or } x > 2}$, $\boxed{0 < x < 4}$, $\boxed{x < -1 \text{ or } x > 1}$, $\boxed{x < -2 \text{ or } x > 0}$



Question 2

Find the range of the values of x, for which $f(x)$ is increasing or decreasing.

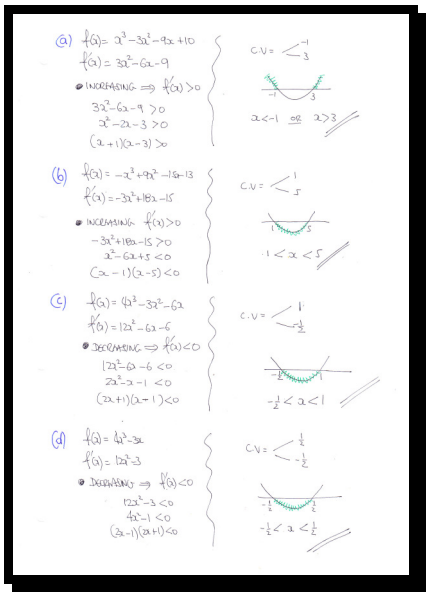
a) $f(x)=x^3-3x^2-9x+10$, increasing

b) $f(x)=-x^3+9x^2-15x-13$, increasing

c) $f(x)=4x^3-3x^2-6x$, decreasing

d) $f(x)=4x^3-3x$, decreasing

$\boxed{x<-1 \text{ or } x>3}$, $\boxed{1<x<5}$, $\boxed{-\frac{1}{2}<x<1}$, $\boxed{-\frac{1}{2}<x<\frac{1}{2}}$



DIFFERENTIATION PRACTICE IN CONTEXT

Question 1

The curve C has equation

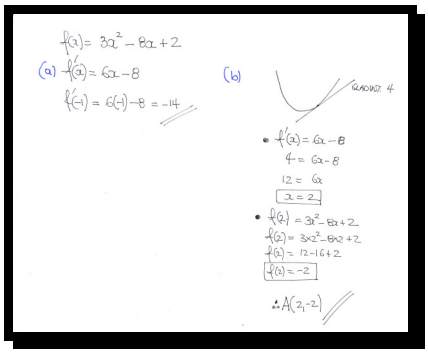
$$f(x) = 3x^2 - 8x + 2 .$$

a) Find the gradient at the point on C, where $x = -1$.

The point A lies on C and the gradient at that point is 4.

b) Find the coordinates of A.

$\boxed{-14}$, $\boxed{A(2,-2)}$



Question 2

The curve C has equation

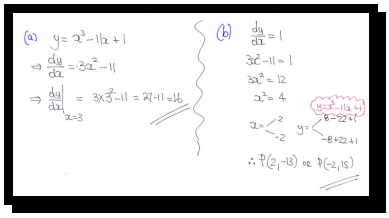
$$y = x^3 - 11x + 1.$$

a) Find the gradient at the point on C, where $x = 3$.

The point P lies on C and the gradient at that point is 1.

b) Find the possible coordinates of P.

$\boxed{16}$, $\boxed{P(2,-13) \text{ or } P(-2,15)}$



Question 3

The curve C has equation

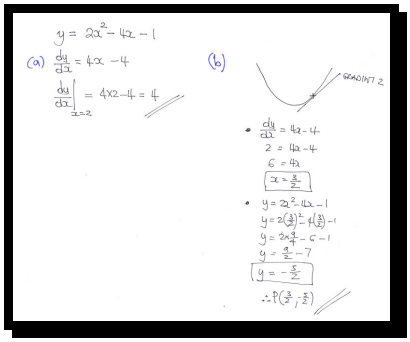
$$y = 2x^2 - 4x - 1 .$$

a) Find the gradient at the point on C, where $x = 2$.

The point P lies on C and the gradient at that point is 2.

b) Find the coordinates of P.

$\boxed{4}$, $\boxed{P\left(\frac{3}{2}, -\frac{5}{2}\right)}$



Question 4

The curve C has equation

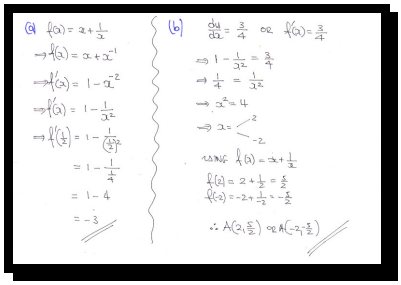
$$f(x)=x+\frac{1}{x},\ x\neq 0.$$

a) Find the gradient at the point on C, where $x=\frac{1}{2}$.

The point A lies on C and the gradient at that point is $\frac{3}{4}$.

b) Find the possible coordinates of A.

$\boxed{-3}$, $\boxed{A\left(2,\frac{5}{2}\right) \text{ or } A\left(-2,-\frac{5}{2}\right)}$



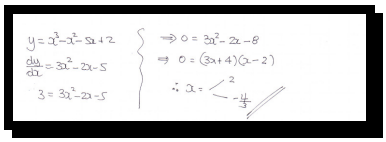
Question 5

The curve C has equation

$$y = x^3 - x^2 - 5x + 2 .$$

Find the x coordinates of the points on C with gradient 3.

$$\boxed{x = -\frac{4}{3}, 2}$$



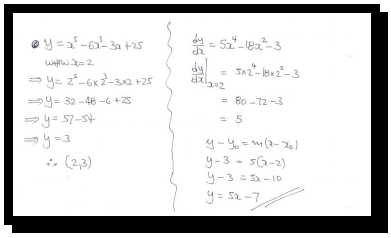
Question 6

The curve C has equation

$$y = x^5 - 6x^3 - 3x + 25 .$$

Find an equation of the tangent to C at the point where $x = 2$.

$$\boxed{y = 5x - 7}$$



Question 7

The curve C has equation

$$y=-x^2(x+1),\ x\in\mathbb{R}.$$

The curve meets the coordinate axes at the origin O and at the point A.

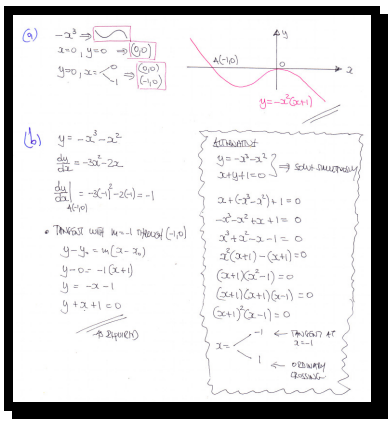
a) Sketch the graph of C, indicating clearly the coordinates of A.

b) Show that the straight line with equation

$$x+y+1=0,$$

is a tangent to C at A.

$$\boxed{A(-1,0)}$$



Question 8

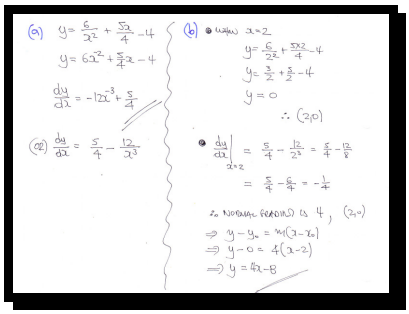
The curve C has equation

$$y=\frac{6}{x^2}+\frac{5x}{4}-4,\ x\neq 0.$$

a) Find an expression for $\dfrac{dy}{dx}$.

b) Determine an equation of the normal to the curve at the point where $x=2$.

$$\boxed{\frac{dy}{dx}=\frac{5}{4}-\frac{12}{x^3}},\ \boxed{y=4x-8}$$



Question 9

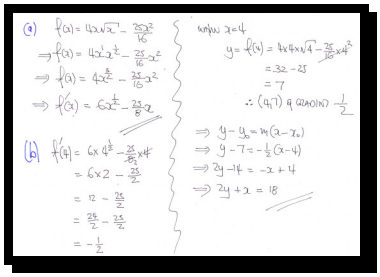
The curve C has equation

$$f(x)=4x\sqrt{x}-\frac{25x^2}{16},\ x\geq 0.$$

a) Find a simplified expression for $f'(x)$.

b) Determine an equation of the tangent to C at the point where $x=4$, giving the answer in the form $ax+by=c$, where a, b and c are integers.

$$\boxed{f'(x)=6x^{\frac{1}{2}}-\frac{25}{8}x},\ \boxed{x+2y=18}$$



Question 10

A curve has the following equation

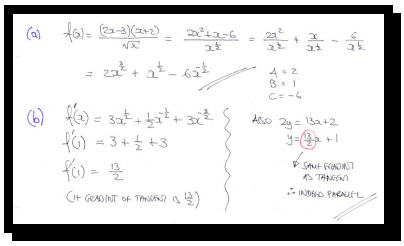
$$f(x)=\frac{(2x-3)(x+2)}{\sqrt{x}},\ x>0.$$

a) Express $f(x)$ in the form $Ax^{\frac{3}{2}}+Bx^{\frac{1}{2}}+Cx^{-\frac{1}{2}}$, where A, B and C are constants to be found.

b) Show that the tangent to the curve at the point where $x=1$ is parallel to the line with equation

$$2y=13x+2.$$

$$\boxed{A=2},\ \boxed{B=1},\ \boxed{C=-6}$$



Question 11

A cubic curve has equation

$$f(x)=2x^3-7x^2+6x+1.$$

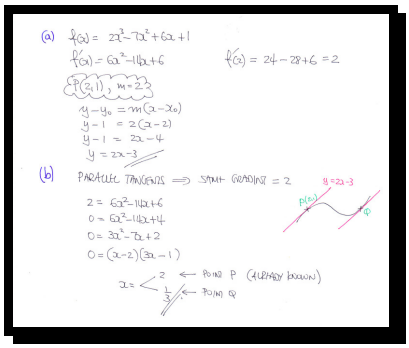
The point $P(2,1)$ lies on the curve.

a) Find an equation of the tangent to the curve at P.

The point Q lies on the curve so that the tangent to the curve at Q is parallel to the tangent to the curve at P.

b) Determine the x coordinate of Q.

$$\boxed{y=2x-3}\,,\quad \boxed{x_Q=\frac{1}{3}}$$



Question 12

The curve C has equation

$$y = 2x^3 - 9x^2 + 12x - 10 .$$

a) Find the coordinates of the two points on the curve where the gradient is zero.

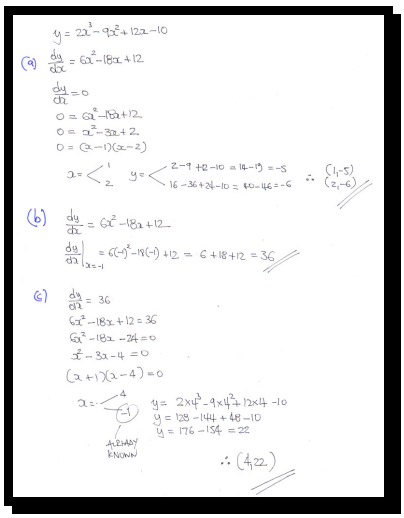
The point P lies on C and its x coordinate is -1.

b) Determine the gradient of C at the point P.

The point Q lies on C so that the gradient at Q is the same as the gradient at P.

c) Find the coordinates of Q.

$\boxed{(1,-5),(2,-6)}$, $\boxed{36}$, $\boxed{Q(4,22)}$



Question 13

The curve C has equation

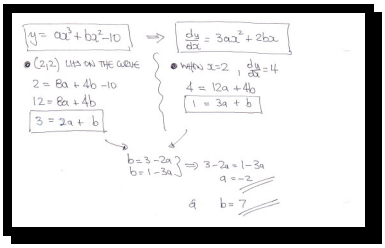
$$y = ax^3 + bx^2 - 10,$$

where a and b are constants.

The point $A(2,2)$ lies on C.

Given that the gradient at A is 4, determine the value of a and the value of b.

$\boxed{a = -2}$, $\boxed{b = 7}$



Question 14

The curve C has equation

$$y = x^3 - 4x^2 + 6x - 3 .$$

The point $P(2,1)$ lies on C and the straight line L_1 is the tangent to C at P.

a) Find an equation of L_1.

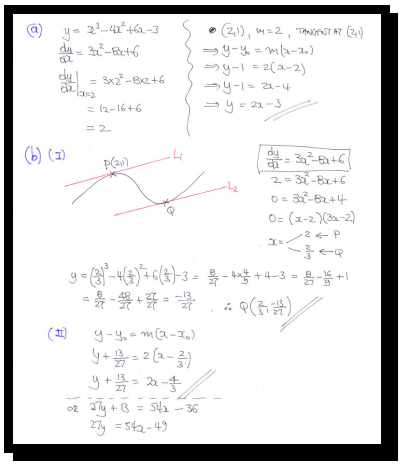
The straight line L_2 is a tangent to C at the point Q.

b) Given that L_2 is parallel to L_1, determine …

i. … the exact coordinates of Q.

ii. … an equation of L_2.

$$\boxed{y = 2x - 3} \ , \ \boxed{Q\left(\frac{2}{3}, -\frac{13}{27}\right)}, \ \boxed{27y = 54x - 49}$$



Question 15

A curve C and a straight line L have respective equations

$$y = 2x^2 - 6x + 5 \quad \text{and} \quad 2y + x = 4 .$$

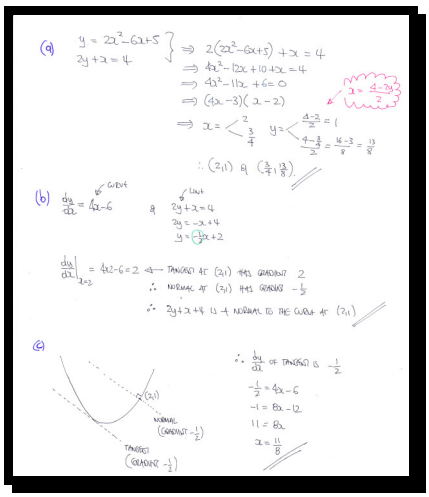
a) Find the coordinates of the points of intersection between C and L.

b) Show that L is a normal to C.

The tangent to C at the point P is parallel to L.

c) Determine the x coordinate of P.

$$\boxed{(2,1), \left(\frac{3}{4}, \frac{13}{8}\right)}, \quad \boxed{x_P = \frac{11}{8}}$$



Question 16

The curve C has equation

$$y = 2x^3 - 6x^2 + 3x + 5 .$$

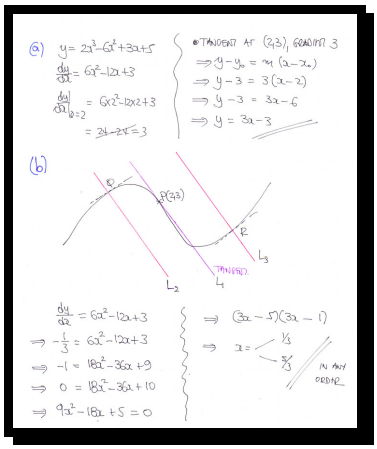
The point $P(2,3)$ lies on C and the straight line L_1 is the tangent to C at P.

a) Find an equation of L_1.

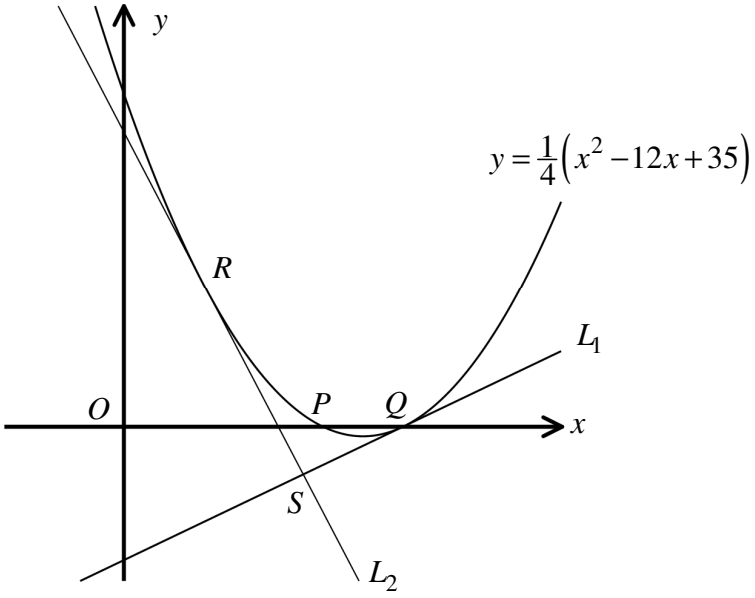
The straight lines L_2 and L_3 are parallel to L_1, and they are the respective normals to C at the points Q and R.

b) Determine the x coordinate of Q and the x coordinate of R.

$\boxed{y = 3x - 3}$, $\boxed{x = \frac{1}{3}, \frac{5}{3}}$



Question 17



The figure above shows the curve with equation

$$y=\frac{1}{4}\left(x^2-12x+35\right).$$

The curve crosses the x axis at the points $P\left(x_1,0\right)$ and $Q\left(x_2,0\right)$, where $x_2>x_1$.

The tangent to the curve at Q is the straight line L_1.

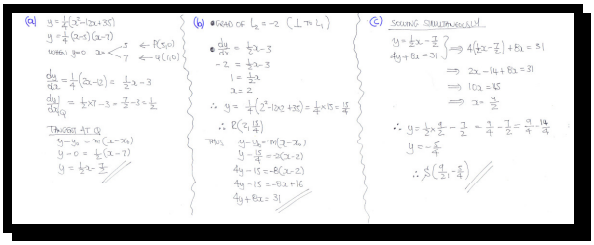
a) Find an equation of L_1.

The tangent to the curve at the point R is denoted by L_2. It is further given that L_2 meets L_1 at right angles, at the point S.

b) Find an equation of L_2.

c) Determine the exact coordinates of S.

$\square$, $\boxed{y=\frac{1}{2}x-\frac{7}{2}}$, $\boxed{4y+8x=31}$, $\boxed{S\left(\frac{9}{2},-\frac{5}{4}\right)}$



Question 18

The point $P(1,0)$ lies on the curve C with equation

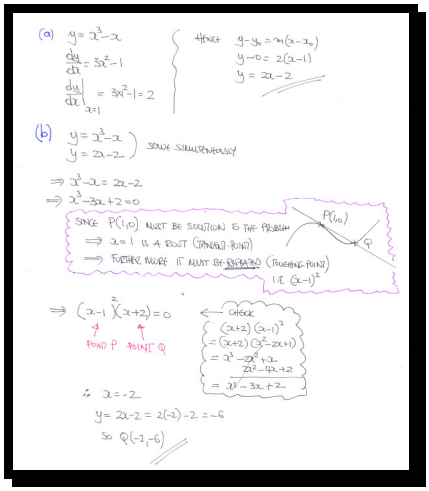
$$y = x^3 - x, \quad x \in \mathbb{R}.$$

a) Find an equation of the tangent to C at P, giving the answer in the form $y = mx + c$, where m and c are constants.

The tangent to C at P meets C again at the point Q.

b) Determine the coordinates of Q.

$\boxed{y = 2x - 2}$, $\boxed{Q(-2,-6)}$



Question 19

A curve C with equation

$$y = 4x^3 + 7x^2 + x + 11, \quad x \in \mathbb{R}.$$

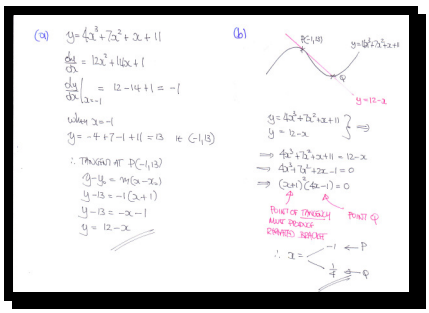
The point P lies on C, where $x = -1$.

a) Find an equation of the tangent to C at P.

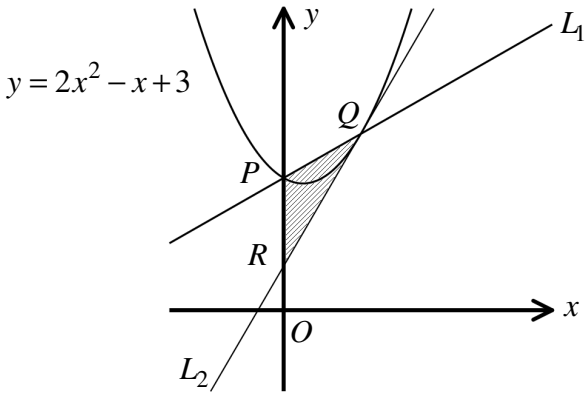
The tangent to C at P meets C again at the point Q.

b) Determine the x coordinate of Q.

$$\boxed{y = 12 - x}, \quad \boxed{x_Q = \frac{1}{4}}$$



Question 20



The figure above shows the curve C with equation

$$y = 2x^2 - x + 3.$$

C crosses the y axis at the point P. The normal to C at P is the straight line L_1.

a) Find an equation of L_1.

L_1 meets the curve again at the point Q.

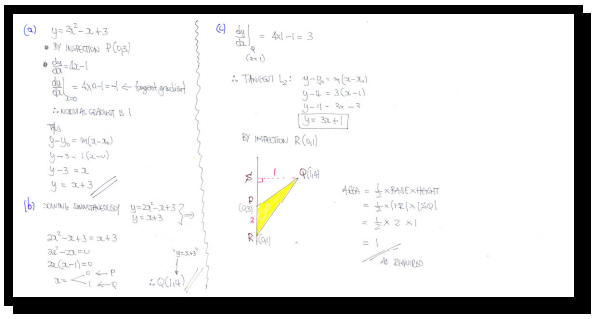
b) Determine the coordinates of Q.

The tangent to C at Q is the straight line L_2.

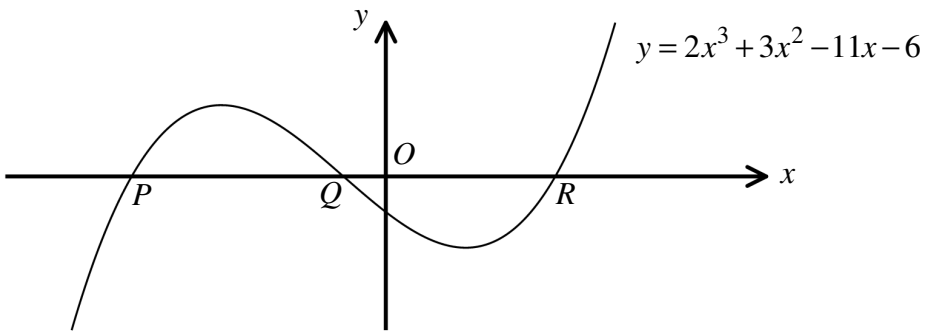
L_2 meets the y axis at the point R.

c) Show that the area of the triangle PQR is one square unit.

$\boxed{y = x + 3}$, $\boxed{Q(1,4)}$



Question 21



The figure above shows the curve C with equation

$$y = 2x^3 + 3x^2 - 11x - 6.$$

The curve crosses the x axis at the points P, Q and $R(2,0)$.

The tangent to C at R is the straight line L_1.

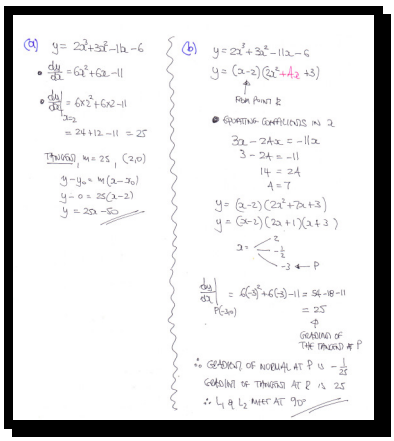
a) Find an equation of L_1.

The normal to C at P is the straight line L_2.

The straight lines L_1 and L_2 meet at the point S.

b) Show that $\angle PSR = 90°$.

$$\boxed{y = 25x - 50}$$



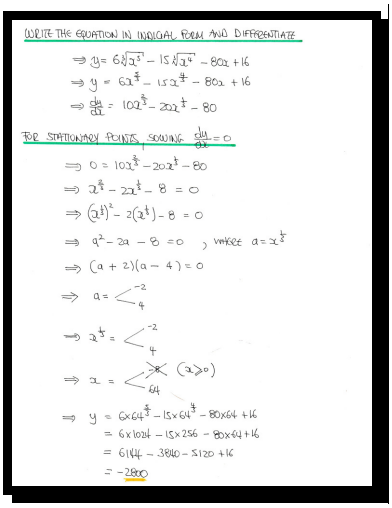
Question 22

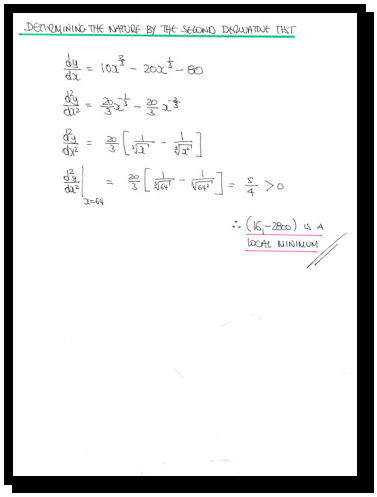
A curve has equation

$$y = 6\sqrt[3]{x^5} - 15\sqrt[3]{x^4} - 80x + 16, \qquad x \in \mathbb{R}, \qquad x \geq 0.$$

Find the coordinates of the stationary point of the curve and determine whether it is a local maximum, a local minimum or a point of inflexion.

$\boxed{\text{local minimum at } (16, -2800)}$





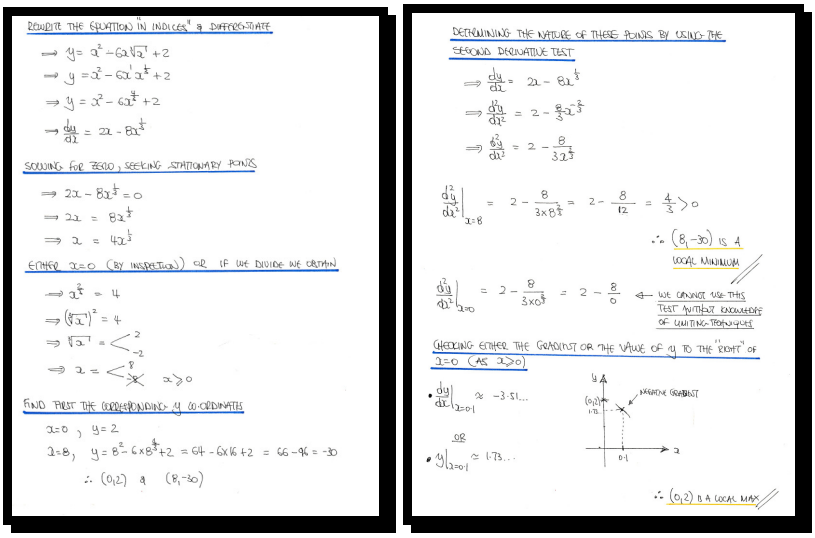
Question 23

A curve has equation

$$y = x^2 - 6x\sqrt[3]{x} + 2, \qquad x \in \mathbb{R}, \qquad x \geq 0.$$

Find the coordinates of the stationary points of the curve and classify them as local maxima, local minima or a points of inflexion.

$\boxed{\text{local minimum at } (8,-30)}$, $\boxed{\text{local maximum at } (0,2)}$



Question 24

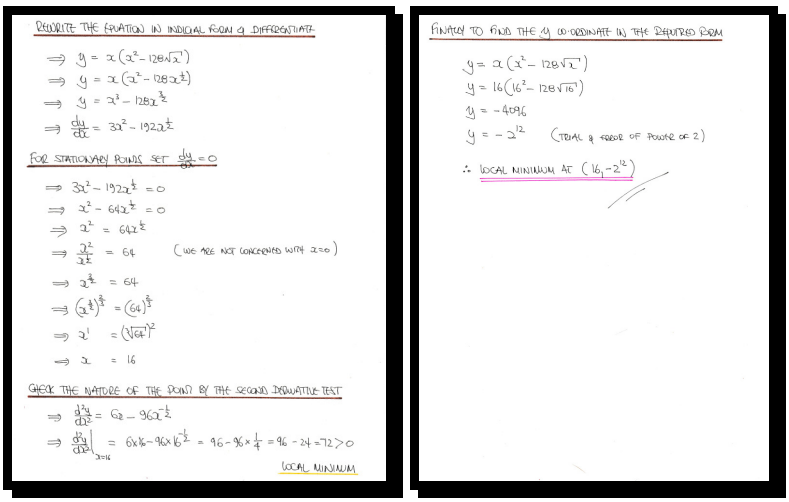
A curve has equation

$$y = x\left(x^2 - 128\sqrt{x}\right), \qquad x \in \mathbb{R}, \qquad x > 0 .$$

The curve has a single stationary point with coordinates $\left(2^{\alpha},\ -2^{\beta}\right)$, where α and β are positive integers.

Find the value of β and justify that the stationary point is a local minimum.

$\boxed{\beta = 12}$



Question 25

The point P, whose x coordinate is $\frac{1}{4}$, lies on the curve with equation

$$y=\frac{k+4x\sqrt{x}}{7x},\ x\in\mathbb{R},\ x>0,$$

where k is a non zero constant.

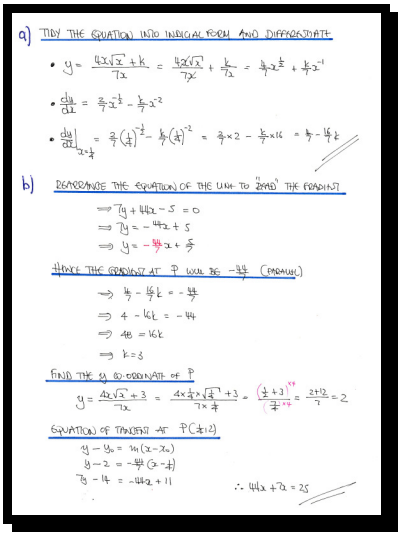
a) Determine, in terms of k, the gradient of the curve at P.

The tangent to the curve at P is parallel to the straight line with equation

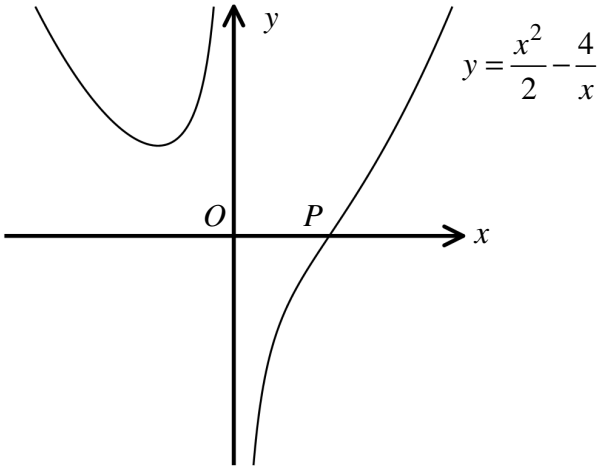
$$44x+7y-5=0.$$

b) Find an equation of the tangent to the curve at P.

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}}=\frac{4-16k}{7},\qquad \boxed{44x+7y=25}$$



Question 26



The figure above shows the curve C with equation

$$y=\frac{x^2}{2}-\frac{4}{x},\ x\neq 0.$$

The curve crosses the x axis at the point P.

The straight line L is the normal to C at P.

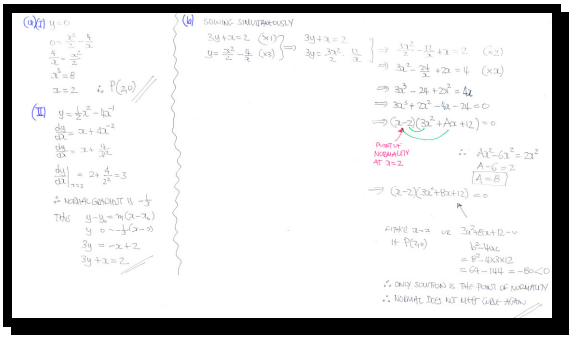
a) Find …

i. … the coordinates of P.

ii. … an equation of L.

b) Show that L does not meet C again.

$\boxed{P(2,0)}$, $\boxed{x+3y=2}$



Question 27

The curve C has equation

$$y=(x-1)\left(x^2+4x+5\right),\ x\in\mathbb{R}.$$

a) Show that C meets the x axis at only one point.

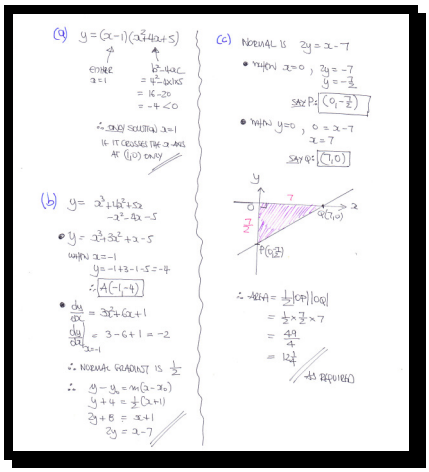
The point A, where $x=-1$, lies on C.

b) Find an equation of the normal to C at A.

The normal to C at A meets the coordinate axes at the points P and Q.

c) Show further that the area of the triangle OPQ, where O is the origin, is $12\frac{1}{4}$ square units.

$$\boxed{2y=x-7}$$



Question 28

A curve has equation

$$y = x - 8\sqrt{x}\,,\ x \in \mathbb{R}\,,\ x \geq 0\,.$$

The curve meets the coordinate axes at the origin and at the point P.

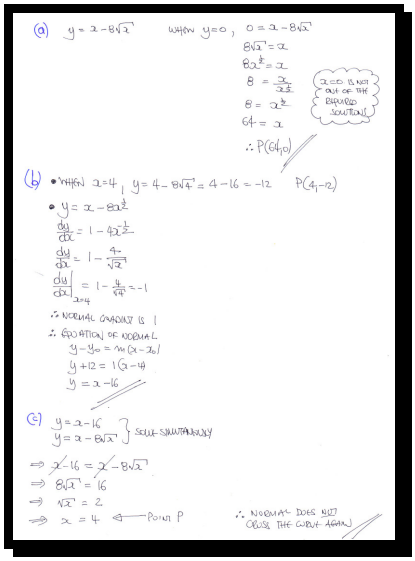
a) Determine the coordinates of P.

The point Q, where $x = 4$, lies on the curve.

b) Find an equation of the normal to curve at Q.

c) Show clearly that the normal to the curve at Q does not meet the curve again.

$$\boxed{P(64,0)}\,,\ \boxed{y = x - 16}$$



Question 29

The curve C has equation

$$y = x^3 - 9x^2 + 24x - 19\,,\ x \in \mathbb{R}\,.$$

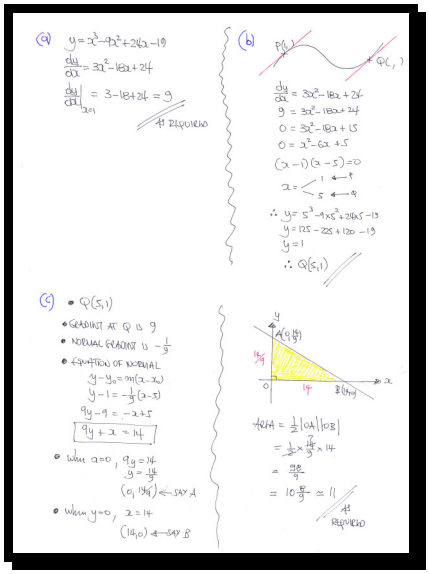
a) Show that the tangent to C at the point P, where $x = 1$, has gradient 9.

b) Find the coordinates of another point Q on C at which the tangent also has gradient 9.

The normal to C at Q meets the coordinate axes at the points A and B.

c) Show further that the **approximate** area of the triangle OAB, where O is the origin, is 11 square units.

$\boxed{Q(5,1)}$



Question 30

The point $A(2,1)$ lies on the curve with equation

$$y=\frac{(x-1)(x+2)}{2x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Find the gradient of the curve at A.

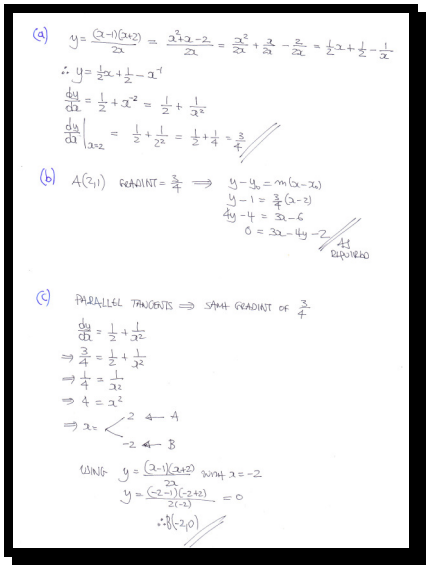
b) Show that the tangent to the curve at A has equation

$$3x-4y-2=0.$$

The tangent to the curve at the point B is parallel to the tangent to the curve at A.

c) Determine the coordinates of B.

$\boxed{\text{gradient at } A=\frac{3}{4}}$, $\boxed{B(-2,0)}$



Question 31

The curve C has equation $y = f(x)$ given by

$$f(x) = 2(x-2)^3, \quad x \in \mathbb{R}.$$

a) Sketch the graph of $f(x)$.

b) Find an expression for $f'(x)$.

The point $P(3,2)$ lies on C and the straight line l_1 is the tangent to C at P.

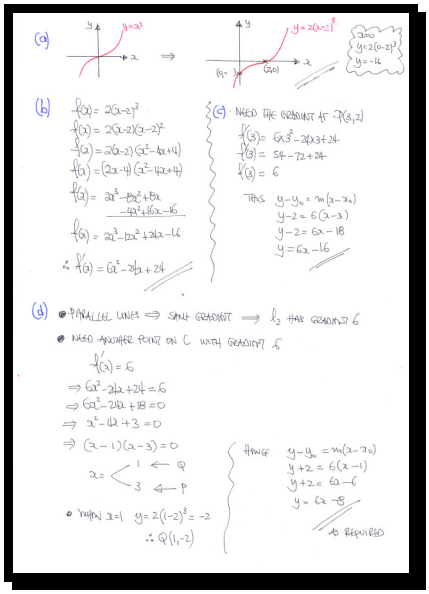
c) Find an equation of l_1.

The straight line l_2 is another tangent at a different point Q on C.

d) Given that l_1 is parallel to l_2 show that an equation of l_2 is

$$y = 6x - 8.$$

$\boxed{f'(x) = 6x^2 - 24x + 24}$, $\boxed{y = 6x - 16}$



Question 32

The point $P(2,9)$ lies on the curve C with equation

$$y = x^3 - 3x^2 + 2x + 9, \ x \in \mathbb{R}, \ x \geq 1.$$

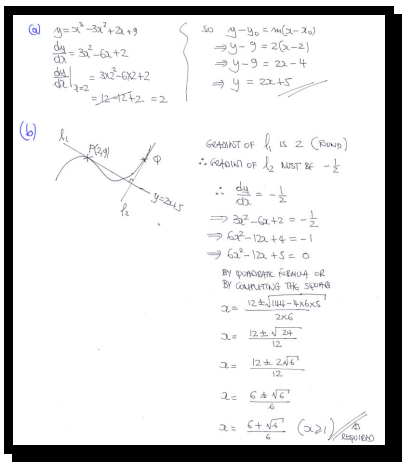
a) Find an equation of the tangent to C at P, giving the answer in the form $y = mx + c$, where m and c are constants.

The point Q also lies on C so that the tangent to C at Q is perpendicular to the tangent to C at P.

b) Show that the x coordinate of Q is

$$\frac{6+\sqrt{6}}{6}.$$

$\boxed{y = 2x + 5}$



Question 33

The volume, V cm^3, of a soap bubble is modelled by the formula

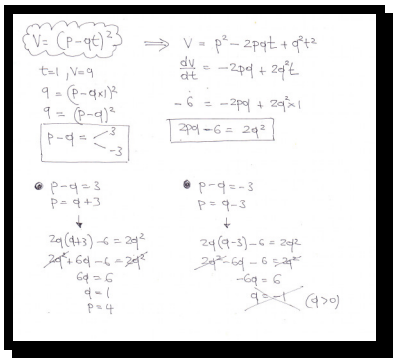
$$V=\left(p-qt\right)^2,\ t\geq 0,$$

where p and q are positive constants, and t is the time in seconds, measured after a certain instant.

When $t=1$ the volume of a soap bubble is 9 cm^3 and at that instant its volume is decreasing at the rate of 6 cm^3 per second.

Determine the value of p and the value of q.

$$\boxed{p=4,\ \ q=1}$$



Question 34

A curve C has equation

$$y = 2x^3 - 5x^2 + a,\ x \in \mathbb{R},$$

where a is a constant.

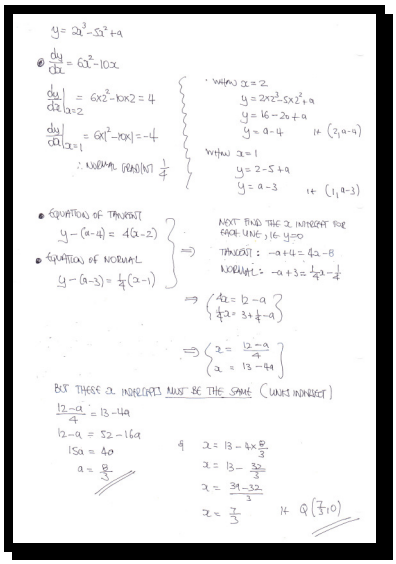
The tangent to C at the point where $x = 2$ and the normal to C at the point where $x = 1$, meet at the point Q.

Given that Q lies on the x axis, determine in any order …

a) … the value of a.

b) … the coordinates of Q.

$$\boxed{a = \frac{8}{3}},\ \boxed{Q\left(\frac{7}{3}, 0\right)}$$



Question 35

The curve C has equation

$$y=\frac{x^3\left(5x\sqrt{x}-128\right)}{\sqrt{x}},\ x\in\mathbb{R},\ \ x>0.$$

a) Determine expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

b) Show that the y coordinate of the stationary point of C is $-k\sqrt[3]{4}$, where k is a positive integer.

c) Evaluate $\frac{d^2y}{dx^2}$ at the stationary point of C.

Give the answer in terms of $\sqrt[3]{2}$.

d) Find the value of $\frac{d^3y}{dx^3}$ at the point on C, where $\frac{d^2y}{dx^2}=0$.

$\boxed{}$, $\boxed{\frac{dy}{dx}=20x^3-320x^{\frac{3}{2}}}$, $\boxed{\frac{d^2y}{dx^2}=60x^2-480x^{\frac{1}{2}}}$, $\boxed{\frac{d^3y}{dx^3}=120x-240x^{-\frac{1}{2}}}$,

$\boxed{k=3072}$, $\boxed{960\sqrt[3]{2}}$, $\boxed{360}$

