CALCULUS

INTRODUCTION

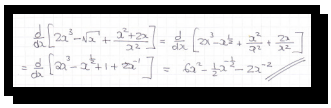
Exam Questions

Question 1 (**)

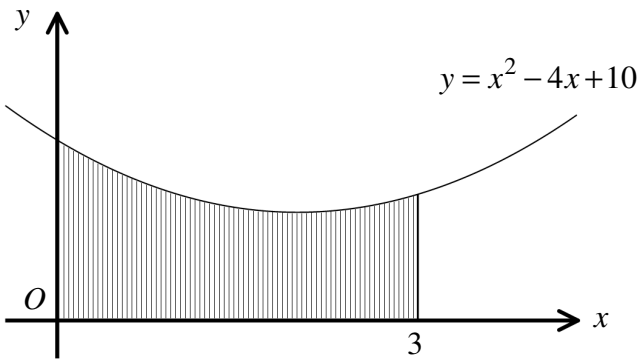
Differentiate the following expression with respect to x

$$2x^3-\sqrt{x}+\frac{x^2+2x}{x^2},\ x>0.$$

$\square$, $\boxed{6x^2+\frac{1}{2\sqrt{x}}-\frac{2}{x^2}}$



Question 2 (**)

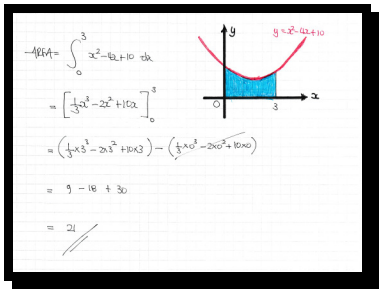


The figure above shows the curve with equation

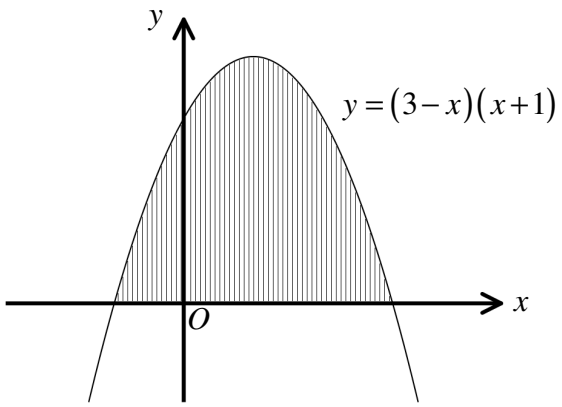
$$y = x^2 - 4x + 10 , \ x \in \mathbb{R} .$$

Find the area of the region, bounded by the curve the coordinate axes and the straight line with equation $x = 3$.

$\square$, $\boxed{\text{area} = 21}$



Question 3 (**)

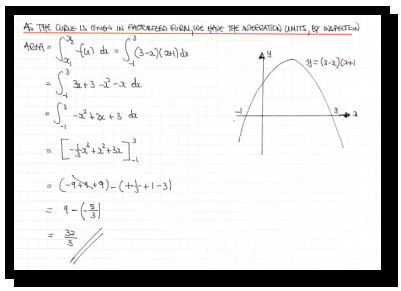


The figure above shows the curve with equation

$$y=(3-x)(x+1),\ x\in\mathbb{R}.$$

Find the exact area of the region, bounded by the curve and the x axis, shown shaded in the figure above.

$\boxed{}$, $\boxed{\text{area}=\frac{32}{3}}$



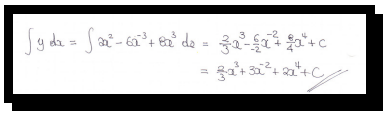
Question 4 (**)

$$y = 2x^2 - \frac{6}{x^3} + 8x^3 , \ x \neq 0 .$$

Find a fully simplified expression for

$$\int y \, dx .$$

$\boxed{}$, $\boxed{\frac{2}{3}x^3 + 3x^{-2} + 2x^4 + C}$

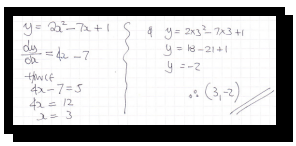


Question 5 (**)

$$f(x) = 2x^2 - 7x + 1, \ x \in \mathbb{R} .$$

Find the coordinates of the point on the curve with equation $y = f(x)$, whose gradient is 5.

$\boxed{(3,-2)}$

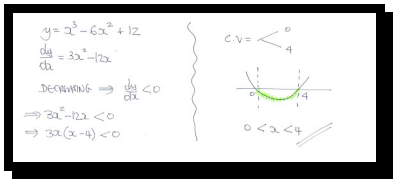


Question 6 **(**)**

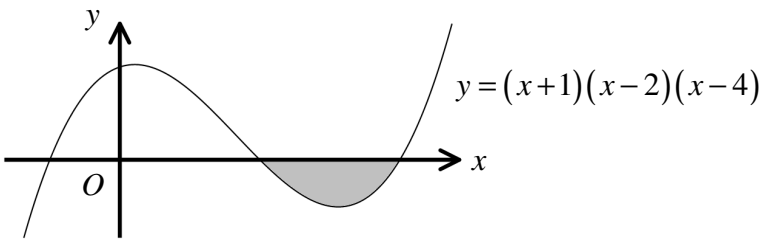
$$y = x^3 - 6x^2 + 12\,,\ x \in \mathbb{R}\,.$$

Find the range of values of x for which y is decreasing.

$\boxed{}$, $\boxed{0 < x < 4}$



Question 7 **(**)**



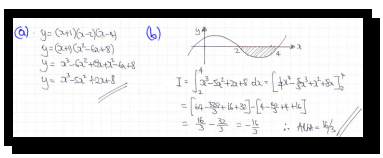
The figure above shows the curve with equation

$$y = (x+1)(x-2)(x-4)\,,\ x \in \mathbb{R}\,.$$

a) Write the equation of the curve in the form $y = x^3 + ax^2 + bx + c$, where a, b and c are constants.

b) Find the exact area the shaded region.

$\boxed{}$, $\boxed{y = x^3 - 5x^2 + 2x + 8}$, $\boxed{\text{area} = \frac{16}{3}}$



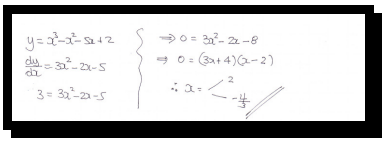
Question 8 (**)

The curve C has equation

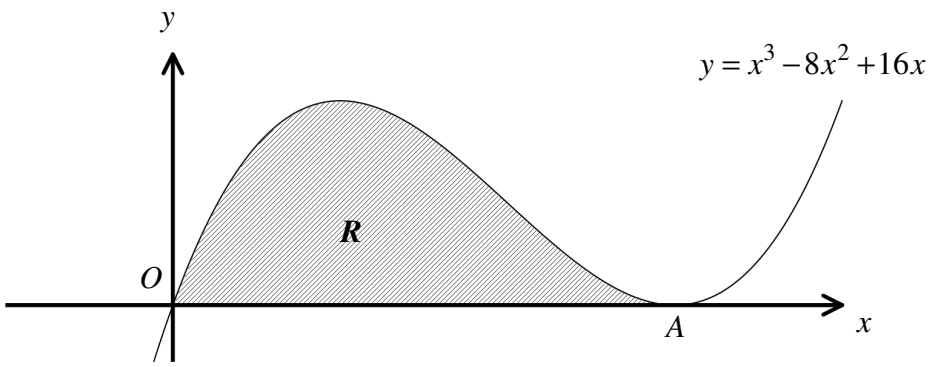
$$y = x^3 - x^2 - 5x + 2 .$$

Find the x coordinates of the points on C with gradient 3.

$$\boxed{x = -\frac{4}{3}, 2}$$



Question 9 (**)



The figure above shows the cubic curve with equation

$$y = x^3 - 8x^2 + 16x\,,\ x \in \mathbb{R}\,.$$

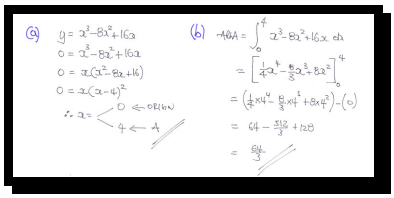
The curve meets the x axis at the origin O and at the point A.

a) Show clearly that $x = 4$ at A.

The finite region R is bounded by the curve and the x axis.

b) Find the exact area of R.

$\square$, $\boxed{\frac{64}{3}}$



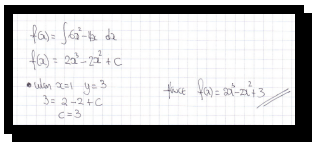
Question 10 (**)

The point $P(1,3)$ lies on the curve with equation $y=f(x)$, whose gradient function is given by

$$f'(x)=6x^2-4x,\ x\in\mathbb{R}.$$

Find an equation for $f(x)$.

$\square$, $\boxed{f(x)=2x^3-2x^2+3}$



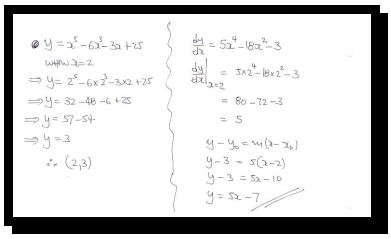
Question 11 (**)

The curve C has equation

$$y=x^5-6x^3-3x+25.$$

Find an equation of the tangent to C at the point where $x=2$.

$\boxed{y=5x-7}$



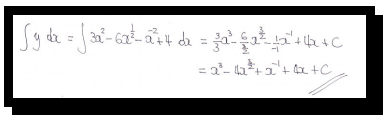
Question 12 **(**)**

$$y=3x^2-6\sqrt{x}-\frac{1}{x^2}+4\,,\ x>0\,.$$

Find a fully simplified expression for

$$\int y\,dx\,.$$

$\boxed{}$, $\boxed{4x+x^3-4x^{\frac{3}{2}}+x^{-1}+C}$



Question 13 **(**)**

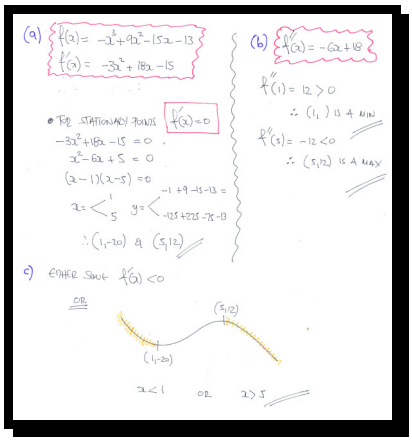
$$f(x)=-x^3+9x^2-15x-13\,,\ x\in\mathbb{R}\,.$$

a) Find the coordinates of the stationary points of $f(x)$.

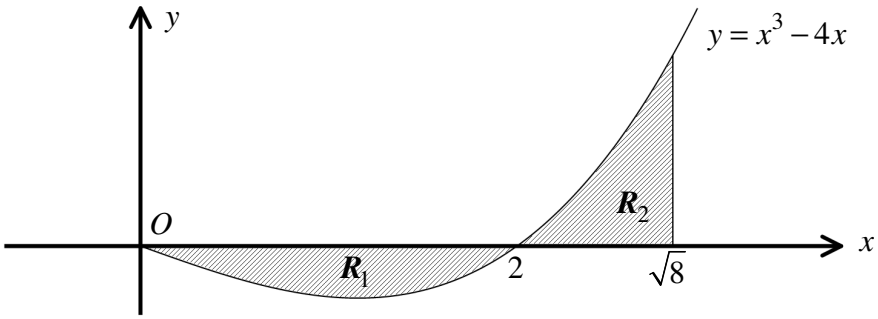
b) Determine the nature of each of the two stationary points found in part (a).

c) Hence find the range of values of x for which $f(x)$ is decreasing.

$\boxed{}$, $\boxed{\text{min at } (1,-20),\ \ \text{max at } (5,12)}$, $\boxed{x<1\cup x>5}$



Question 14 (**)



The figure above shows the cubic curve with equation

$$y = x^3 - 4x , \ x \geq 0 .$$

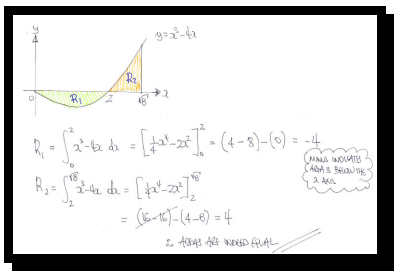
The curve meets the x axis at the origin O and at the point where $x = 2$.

The finite region R_1 is bounded by the curve and the x axis, for $0 \leq x \leq 2$.

The region R_2 is bounded by the curve and the x axis, for $2 \leq x \leq \sqrt{8}$.

Show that the area of R_1 is equal to the area of R_2.

$\boxed{}$, proof



Question 15 (**)

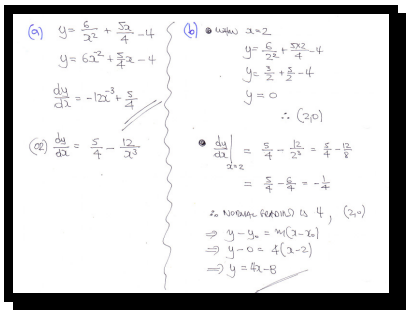
The curve C has equation

$$y=\frac{6}{x^2}+\frac{5x}{4}-4,\ x\neq 0.$$

a) Find an expression for $\dfrac{dy}{dx}$.

b) Determine an equation of the normal to the curve at the point where $x=2$.

$$\boxed{\frac{dy}{dx}=\frac{5}{4}-\frac{12}{x^3}},\ \boxed{y=4x-8}$$



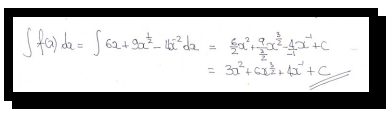
Question 16 (**)

$$f(x)=6x+9\sqrt{x}-\frac{4}{x^2},\ x>0.$$

Find a fully simplified expression for

$$\int f(x)\,dx.$$

$$\boxed{\ \ },\ \boxed{3x^2+6x^{\frac{3}{2}}+4x^{-1}+C}$$



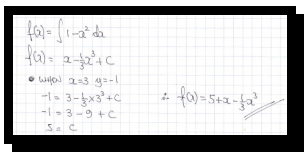
Question 17 (**)

The point $P(3,-1)$ lies on the curve with equation $y=f(x)$, whose gradient function is given by

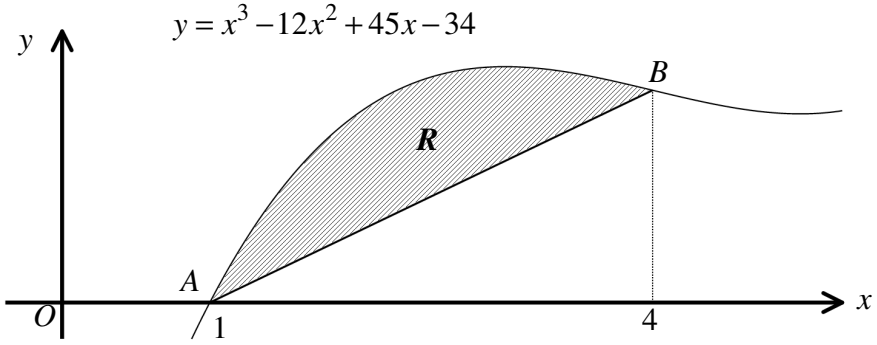
$$f'(x)=1-x^2,\ x\in\mathbb{R}.$$

Find an equation for $f(x)$.

$\square$, $\boxed{f(x)=5+x-\frac{1}{3}x^3}$



Question 18 (**)



The figure above shows the curve with equation

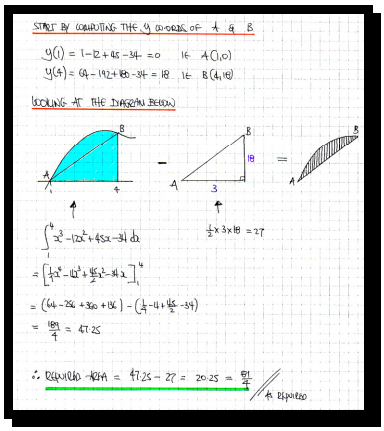
$$y = x^3 - 12x^2 + 45x - 34 .$$

The points A and B lie on the curve, where $x = 1$ and $x = 4$, respectively.

The finite region R is bounded by the curve and the straight line segment AB.

Show that the area of R, shown shaded in the figure, is exactly $\frac{81}{4}$.

[ ], [proof]

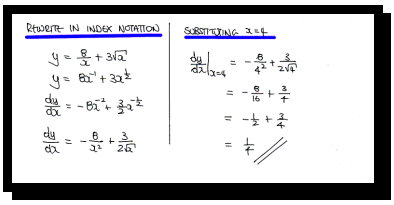


Question 19 (**)

$$y=\frac{8}{x}+3\sqrt{x}\,,\ x>0\,.$$

Find the value of $\dfrac{dy}{dx}$ at the point where $x=4$.

$\square$, $\left.\dfrac{dy}{dx}\right|_{x=4}=\dfrac{1}{4}$



Question 20 (+)**

The curve C has equation

$$y=-x^2(x+1),\ x\in\mathbb{R}.$$

The curve meets the coordinate axes at the origin O and at the point A.

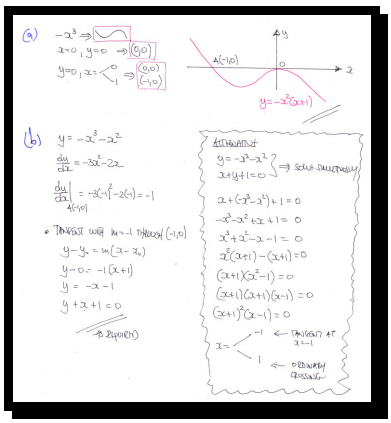
a) Sketch the graph of C, indicating clearly the coordinates of A.

b) Show that the straight line with equation

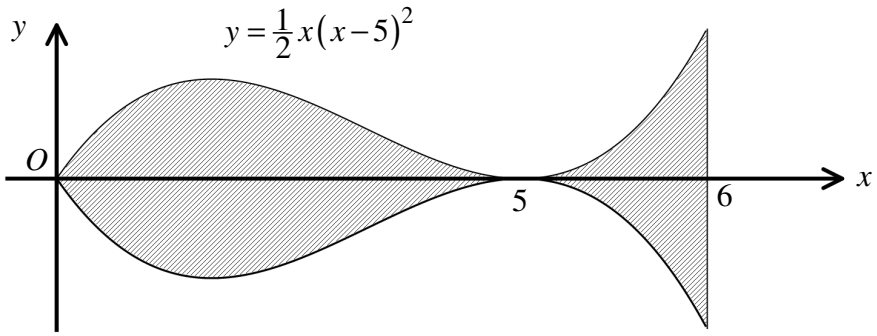
$$x+y+1=0,$$

is a tangent to C at A.

$\square$, $\boxed{A(-1,0)}$



Question 21 (+)**



A fish logo is generated by the curve C with equation

$$y=\tfrac{1}{2}x(x-5)^2,\ 0\le x\le 6,$$

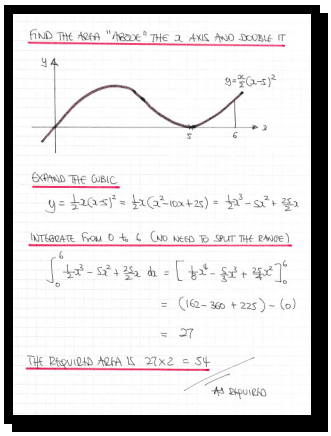
and its reflection in the x axis.

The curve C meets the x axis at the origin O and at the point $(5,0)$.

The finite region R is bounded by C, its reflection in the x axis and the straight line with equation $x=6$.

Show that the area of R, shown shaded in the figure, is 54 square units.

[ ], [proof]


FIND THE AREA "ABOVE" THE x AXIS AND DOUBLE IT

EXPAND THE CUBIC

$y=\frac{1}{2}x(x-5)^2=\frac{1}{2}x(x^2-10x+25)=\frac{1}{2}x^3-5x^2+\frac{25}{2}x$

INTEGRATE FROM 0 TO 6 (NO NEED TO SPLIT THE RANGE)

$\int_0^6 \frac{1}{2}x^3-5x^2+\frac{25}{2}x\,dx=\left[\frac{1}{8}x^4-\frac{5}{3}x^3+\frac{25}{4}x^2\right]_0^6$

$=(162-360+225)-(0)$

$=27$

THE REQUIRED AREA IS $27\times 2=54$

AS REQUIRED

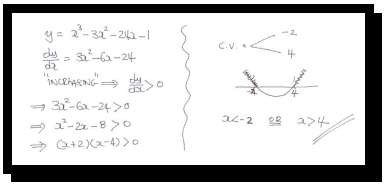
Question 22 (+)**

A curve C has equation

$$y = x^3 - 3x^2 - 24x - 1,\ x \in \mathbb{R}.$$

Find the range of values of x, for which y is increasing.

$\boxed{\quad}$, $\boxed{x < -2 \ \cup \ x > 4}$



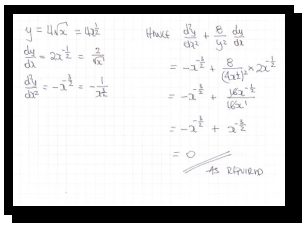
Question 23 (+)**

$$y = 4\sqrt{x},\ x \in \mathbb{R},\ x > 0.$$

Show clearly that

$$\frac{d^2y}{dx^2} + \frac{8}{y^2}\frac{dy}{dx} = 0.$$

$\boxed{\quad}$, $\boxed{\text{proof}}$



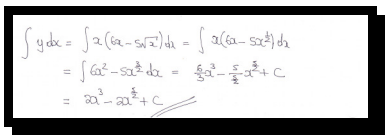
Question 24 (+)**

$$y = x\left(6x - 5\sqrt{x}\right),\ x \geq 0 .$$

By showing all steps in the workings, find an expression for

$$\int y\ dx .$$

$\square$, $\boxed{2x^3 - 2x^{\frac{5}{2}} + C}$

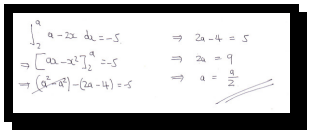


Question 25 (+)**

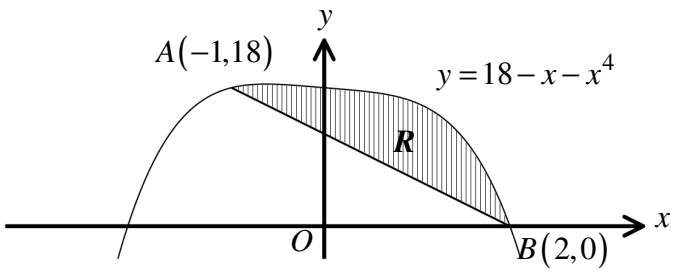
Find the value of the constant a if

$$\int_2^a a - 2x\ dx = -5 .$$

$\square$, $\boxed{a = \frac{9}{2}}$



Question 26 (+)**



The figure above shows the curve C with equation

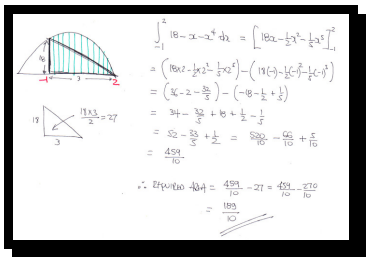
$$y = 18 - x - x^4 .$$

The curve crosses the x axis at $B(2,0)$ and the point $A(-1,18)$ lies on C.

The shaded region R is bounded by the curve and the straight line segment AB.

Find the area of the shaded region.

$\square$, $\boxed{\text{area} = 18.9}$

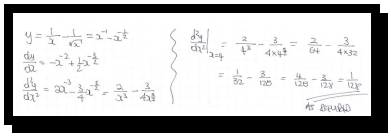


Question 27 (+)**

$$y=\frac{1}{x}-\frac{1}{\sqrt{x}},\ x\in\mathbb{R},\ x>0.$$

Show that the value of $\dfrac{d^2y}{dx^2}$ where $x=4$, is $\dfrac{1}{128}$.

proof

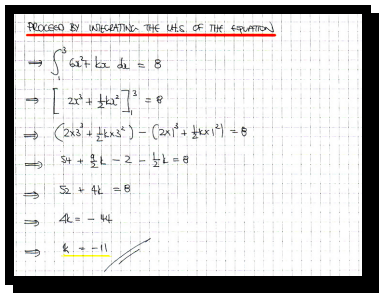


Question 28 (+)**

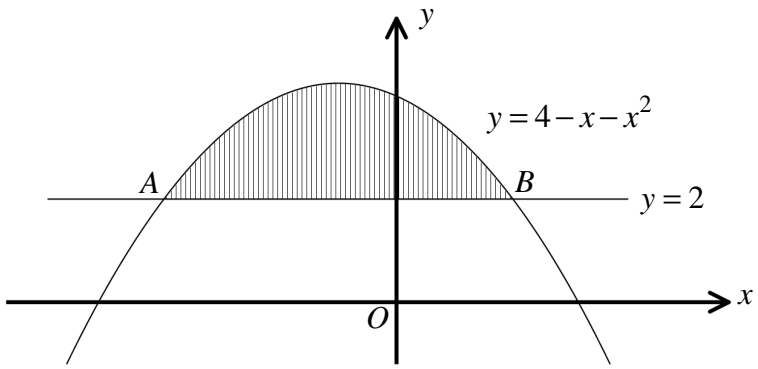
Find the value of the constant k if

$$\int_1^3 6x^2+kx\ dx=8.$$

$\boxed{}$, $\boxed{k=-11}$



Question 29 (+)**



The figure above shows a quadratic curve and a straight line with respective equations

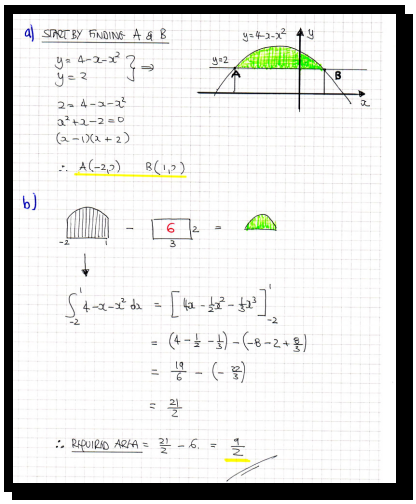
$$y = 4 - x - x^2 \quad \text{and} \quad y = 2 \, .$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{}$, $\boxed{A(-2,2), \ B(1,2)}$, $\boxed{\text{area} = \frac{9}{2}}$

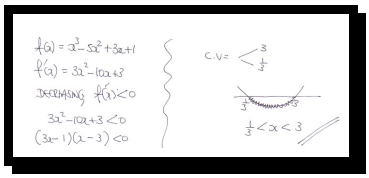


Question 30 (+)**

$$f(x) \equiv x^3 - 5x^2 + 3x + 1,\ x \in \mathbb{R}.$$

Find the range of values of x, for which $f(x)$ is decreasing.

$\square$, $\boxed{\frac{1}{3} < x < 3}$

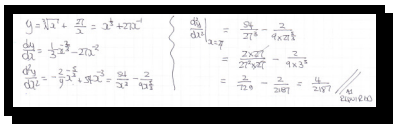


Question 31 (+)**

$$y = \sqrt[3]{x} + \frac{27}{x},\ x \in \mathbb{R},\ x > 0.$$

Show clearly that the value of $\dfrac{d^2y}{dx^2}$ where $x = 27$, is $\dfrac{4}{2187}$.

$\boxed{\text{proof}}$



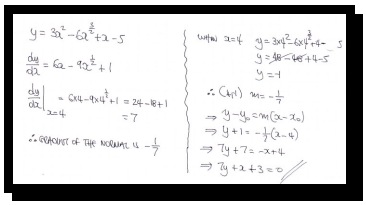
Question 32 (+)**

The curve C has equation

$$y=3x^2-6x^{\frac{3}{2}}+x-5,\ x\geq 0.$$

Find an equation of the normal to C at the point where $x=4$.

$$\boxed{7y+x+3=0}$$

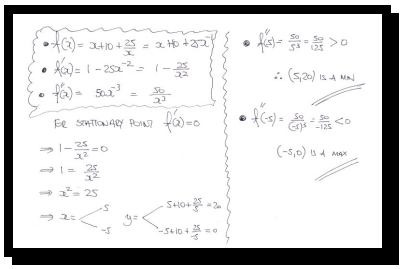


Question 33 (+)**

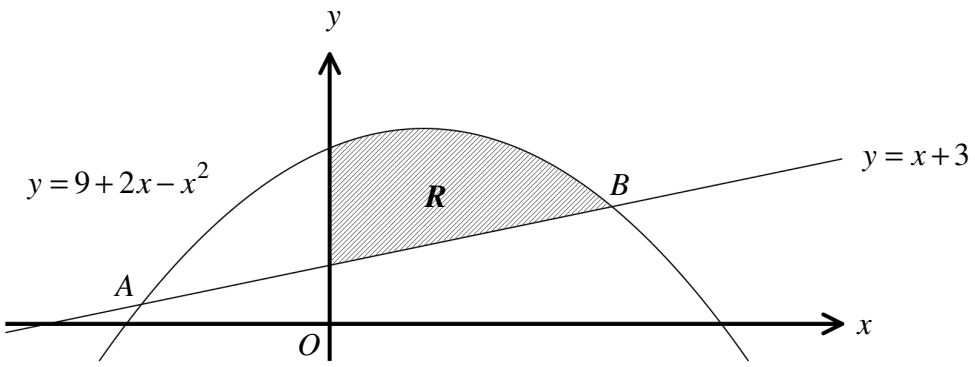
$$f(x)\equiv x+10+\frac{25}{x},\ x\neq 0.$$

Find the coordinates of the two stationary points of $f(x)$ and use $f''(x)$ to determine their nature.

$$\boxed{\ \ \ \ }, \boxed{\min(5,20)}, \boxed{\max(-5,0)}$$



Question 34 **(**+)**



The figure above shows the graph of the curve C with equation

$$y = 9 + 2x - x^2 ,$$

and the straight line L with equation

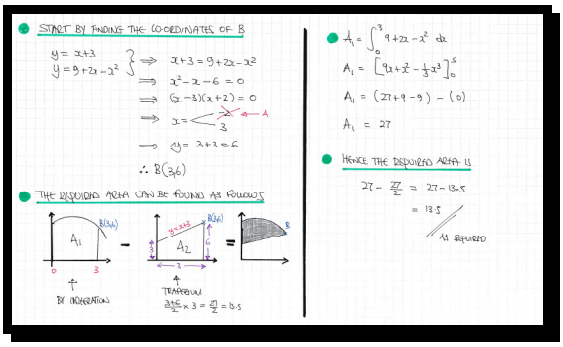
$$y = x + 3 .$$

The curve meets the straight line at the points A and B.

The finite region R, shown shaded in the figure, is bounded by the curve C, the straight line L and the coordinate axes.

Show that the area of R is 13.5 square units.

[ ], proof



Question 35 (+)**

A curve has equation

$$y = x^3 - 10x + 2, \ x \in \mathbb{R}.$$

The point $P(2,-10)$ lies on the curve.

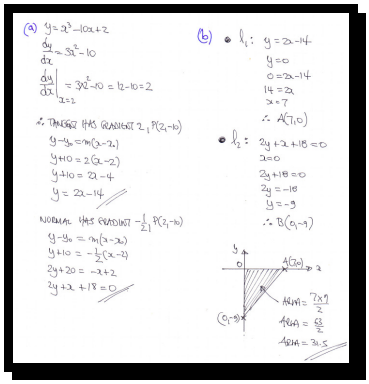
The straight lines l_1 and l_2 are the tangent and the normal to the curve at P, respectively.

a) Find an equation for l_1 and an equation for l_2.

l_1 crosses the x axis at A and l_2 crosses the y axis at B.

b) Find the area of the triangle OAB where O is the origin.

$\boxed{}$, $\boxed{y = 2x - 14}$, $\boxed{2y + x + 18 = 0}$, $\boxed{\text{area} = 31.5}$

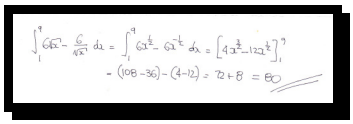


Question 36 (+)**

By showing clear workings, find the value of

$$\int_1^9 6\sqrt{x}-\frac{6}{\sqrt{x}}\,dx\,.$$

$\boxed{}$, $\boxed{80}$

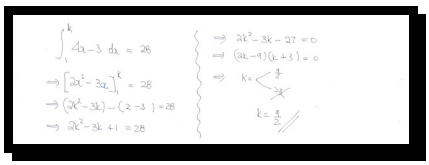


Question 37 (+)**

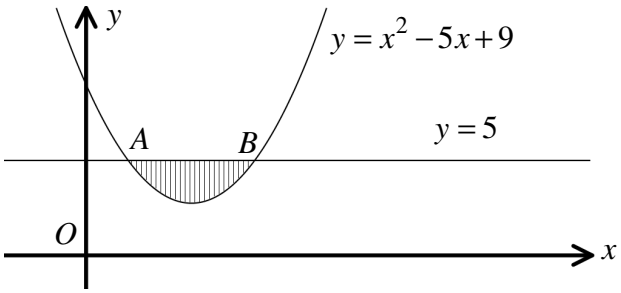
Find the value of the constant k, $k>1$, given that

$$\int_1^k 4x-3\;\;dx=28\,.$$

$\boxed{k=\frac{9}{2}}$



Question 38 (+)**



The figure above shows a quadratic curve and a straight line with respective equations

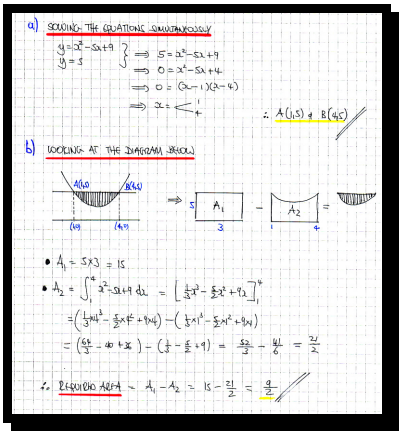
$$y = x^2 - 5x + 9 \qquad \text{and} \qquad y = 5 .$$

The points A and B are the points of intersection between the straight line and the quadratic curve.

a) Find the coordinates of A and B.

b) Calculate the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{}$, $\boxed{A(1,5), B(4,5)}$, $\boxed{\text{area} = \frac{9}{2}}$



Question 39 (+)**

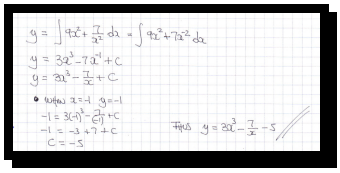
The curve C with equation $y=f(x)$ has gradient function

$$\frac{dy}{dx}=9x^2+\frac{7}{x^2},\ x\neq 0.$$

The point $A(-1,-1)$ lies on C.

Find an equation for C.

$$\boxed{y=3x^3-\frac{7}{x}-5}$$

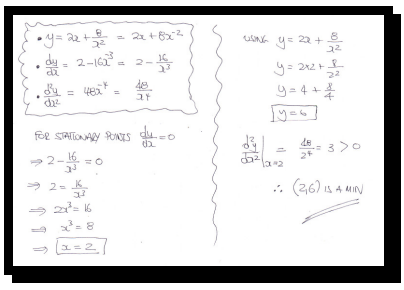


Question 40 (+)**

$$y=2x+\frac{8}{x^2},\ x\neq 0.$$

Find the coordinates of the stationary point of y and determine its nature.

$$\boxed{\ \ \ \ },\ \boxed{\min(2,6)}$$



Question 41 (***)

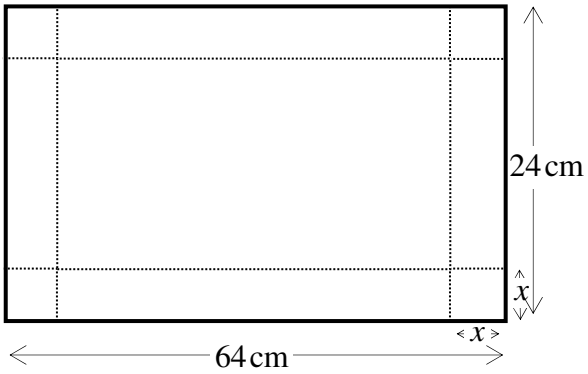


figure 1

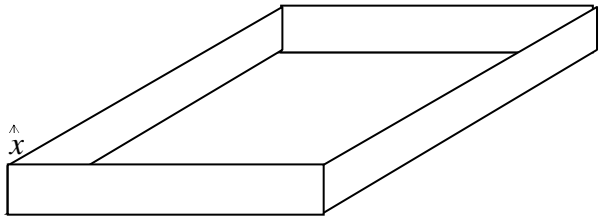


figure 2

An **open** box is to be made out of a rectangular piece of card measuring 64 cm by 24 cm. Figure 1 shows how a square of side length x cm is to be cut out of each corner so that the box can be made by folding, as shown in figure 2.

a) Show that the volume of the box, V cm^3, is given by

$$V = 4x^3 - 176x^2 + 1536x .$$

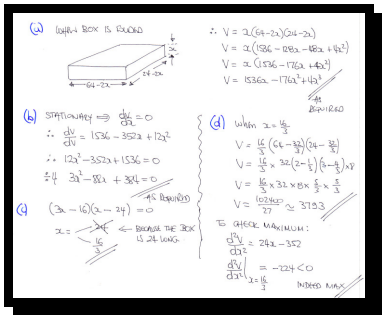
b) Show further that the stationary points of V occur when

$$3x^2 - 88x + 384 = 0 .$$

c) Find the value of x for which V is stationary.
(You may find the fact $24 \times 16 = 384$ useful.)

d) Find, to the nearest cm^3, the maximum value for V, justifying that it is indeed the maximum value.

$$\boxed{x = \frac{16}{3}} \ , \quad \boxed{V_{\max} \approx 3793}$$



Question 42 (***)

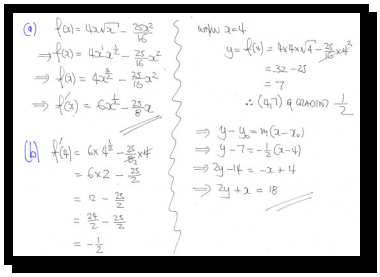
A curve C has the following equation

$$f(x)=4x\sqrt{x}-\frac{25x^2}{16},\ x\geq 0.$$

a) Find a simplified expression for $f'(x)$.

b) Determine an equation of the tangent to the curve at the point where $x=4$, giving the answer in the form $ax+by=c$, where a, b and c are integers.

$\square$, $\boxed{f'(x)=6x^{\frac{1}{2}}-\frac{25}{8}x}$, $\boxed{x+2y=18}$



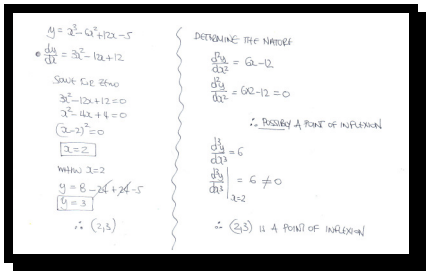
Question 43 (***)

The curve C has equation

$$y=x^3-6x^2+12x-5.$$

Find the coordinates of the stationary point of C and use a clear method to determine its nature.

$\square$, $\boxed{\text{point of inflexion at } (2,3)}$



Question 44 **(***)**

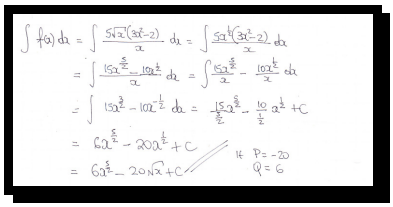
$$f(x)=\frac{5\sqrt{x}\left(3x^2-2\right)}{x},\ x>0.$$

Show clearly that

$$\int f(x)\,dx=P\sqrt{x}+Qx^{\frac{5}{2}}+C,$$

where P and Q are integers to be found, and C is an arbitrary constant.

$\boxed{P=-20}$, $\boxed{Q=6}$



Question 45 **(***)**

The point $P(3,0)$ lies on the curve C whose gradient function is given by

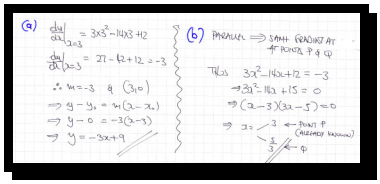
$$\frac{dy}{dx}=3x^2-14x+12.$$

a) Find an equation of the tangent to C at the point P.

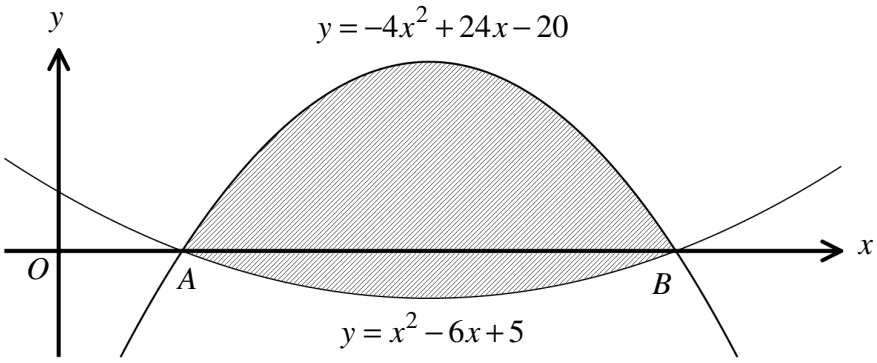
The point Q lies on C, so that the tangent at Q is parallel to the tangent at P.

b) Find the x coordinate of Q.

$\boxed{y=9-3x}$, $\boxed{x=\frac{5}{3}}$



Question 46 (***)



The figure above shows the graph of the curves with equations

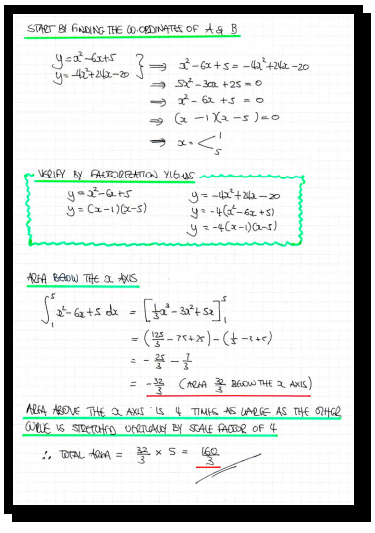
$$y=-4x^2+24x-20 \quad \text{and} \quad y=x^2-6x+5 .$$

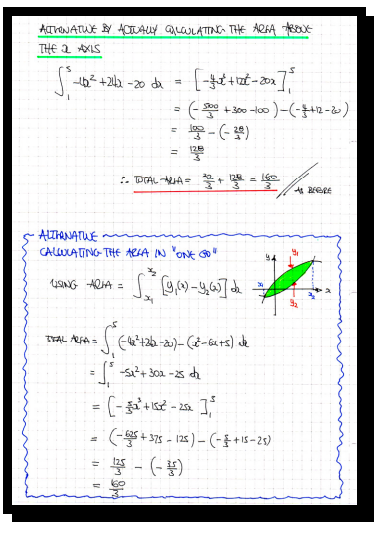
The two curves intersect each other at the points A and B.

The finite region R bounded by the two curves is shown shaded in the figure.

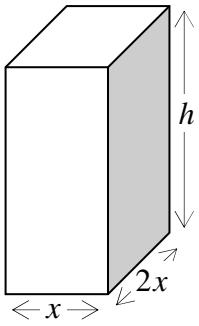
Find the exact area of R.

$\square$, $\frac{160}{3}$





Question 47 (***)



The figure above shows the design of a fruit juice carton with capacity of 1000 cm^3.

The design of the carton is that of a closed cuboid whose base measures x cm by $2x$ cm, and its height is h cm.

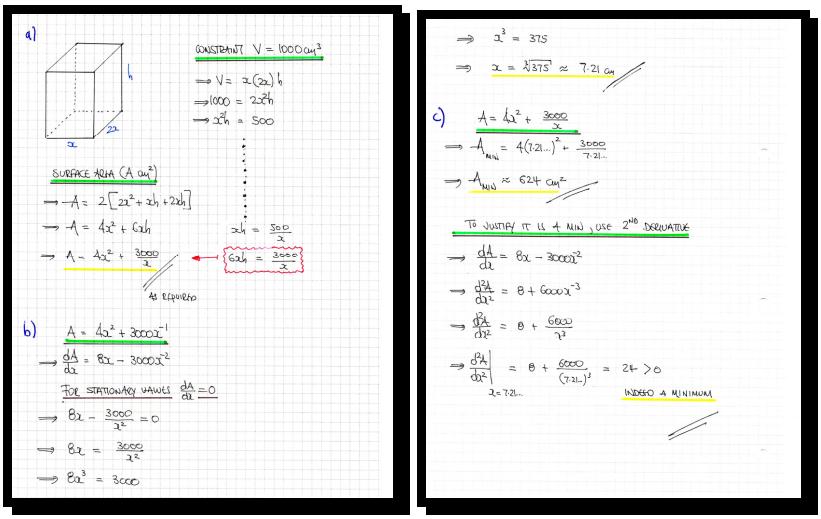
a) Show that the surface area of the carton, $A\text{ cm}^2$, is given by

$$A = 4x^2 + \frac{3000}{x}.$$

b) Find the value of x for which A is stationary.

c) Calculate the minimum value for A, justifying fully the fact that it is indeed the minimum value of A.

$\boxed{}$, $\boxed{x = \sqrt[3]{375} \approx 7.21}$, $\boxed{A_{\min} \approx 624}$



Question 48 (***)

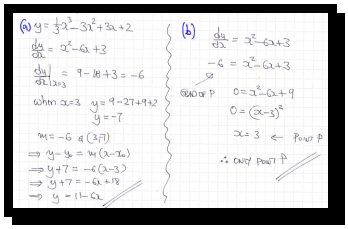
The point P, where $x=3$, lies on the curve C whose equation is

$$y=\frac{1}{3}x^3-3x^2+3x+2.$$

a) Find an equation of the tangent to C at the point P.

b) Show that there is no other point on C where the gradient is the same as the gradient at P.

$$\boxed{y=11-6x}$$



Question 49 (***)

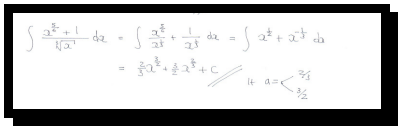
$$y=\frac{x^{\frac{5}{6}}+1}{\sqrt[3]{x}},\ x>0.$$

Show clearly that

$$\int y\,dx=ax^{\frac{1}{a}}+\frac{1}{a}x^a+C,$$

where a is a rational constant to be found, and C is an arbitrary constant.

$$\boxed{a=\frac{2}{3}\ \text{ or }\ \frac{3}{2}}$$



Question 50 (***)

The cubic equation C passes through the origin O and its gradient function is

$$\frac{dy}{dx}=6x^2-6x-20\,.$$

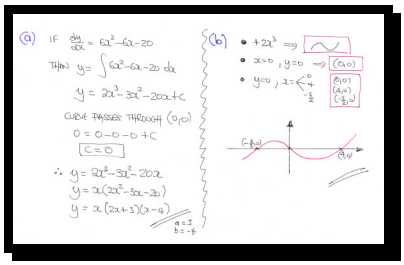
a) Show clearly that the equation of C can be written as

$$y=x\left(2x+a\right)\left(x+b\right),$$

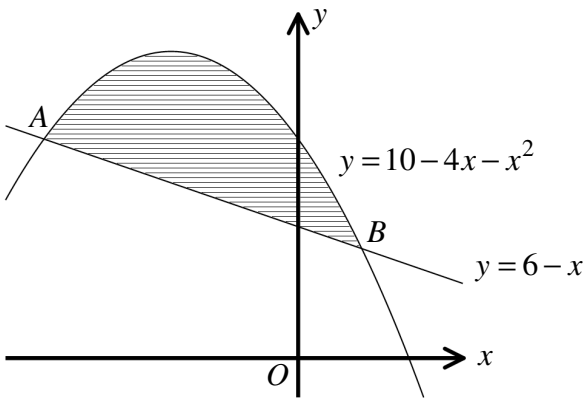
where a and b are constants.

b) Sketch the graph of C, indicating clearly the coordinates of the points where the graph meets the coordinate axes.

$\boxed{}$, $\boxed{a=5}$, $\boxed{b=-4}$



Question 51 (***)



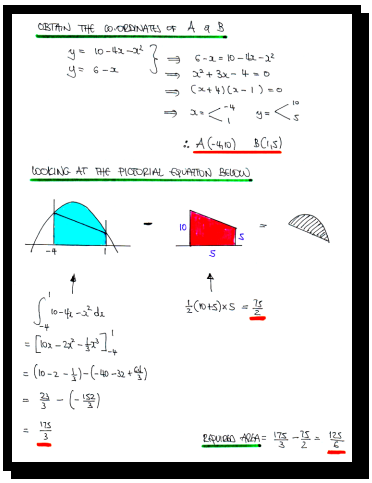
The figure above shows a quadratic curve and a straight line with respective equations

$$y = 10 - 4x - x^2 \qquad \text{and} \qquad y = 6 - x \,.$$

The points A and B, are the points of intersection between the quadratic curve and the straight line.

Calculate the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\square$, $\boxed{\text{area} = \frac{125}{6}}$



Question 52 **(***)**

The gradient function of the curve with equation

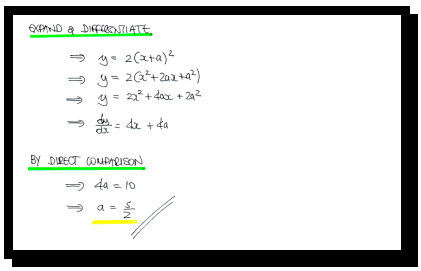
$$y=2\left(x+a\right)^2,$$

where a is a non zero constant, is given by

$$\frac{dy}{dx}=4x+10.$$

Determine the value of a.

$\boxed{\phantom{a=\frac{5}{2}}}$, $\boxed{a=\frac{5}{2}}$



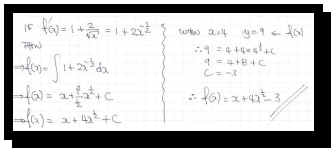
Question 53 **(***)**

The point $P\left(4,9\right)$ lies on the curve with equation $y=f\left(x\right)$, whose gradient function is given by

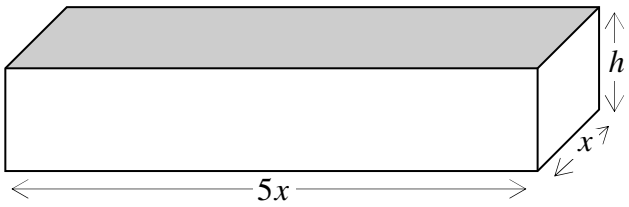
$$f'\left(x\right)=1+\frac{2}{\sqrt{x}},\ x>0.$$

Find an equation for $f\left(x\right)$.

$\boxed{f\left(x\right)=x+4\sqrt{x}-3}$



Question 54 **(***)**



The figure above shows a **solid** brick, in the shape of a cuboid, measuring $5x$ cm by x cm by h cm. The total surface area of the brick is 720 cm^2.

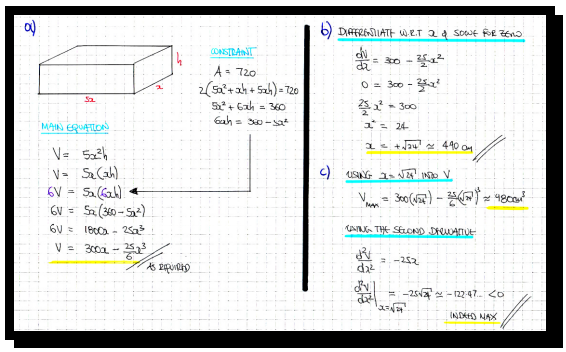
a) Show that the volume of the brick, V cm^3, is given by

$$V = 300x - \frac{25}{6}x^3 .$$

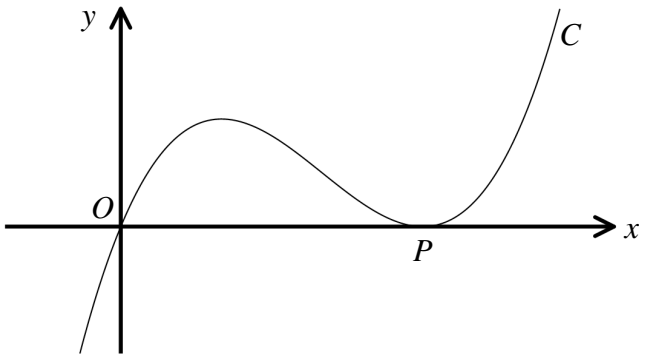
b) Find the value of x for which V is stationary.

c) Calculate the maximum value for V, fully justifying the fact that it is indeed the maximum value.

$\square$, $\boxed{x = 2\sqrt{6} \approx 4.90}$, $\boxed{V_{\max} = 400\sqrt{6} \approx 980}$



Question 55 (***)



The figure above shows the cubic curve C which meets the coordinates axes at the origin O and at the point P.

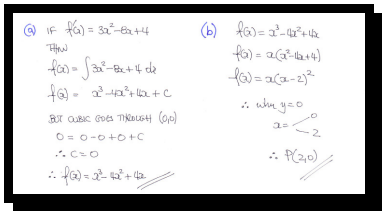
The gradient function of C is given by

$$f'(x)=3x^2-8x+4\,.$$

a) Find an equation for C.

b) Determine the coordinates of P.

$\square$, $\boxed{f(x)=x^3-4x^2+4x}$, $\boxed{P(2,0)}$



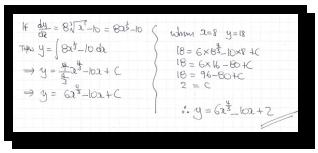
Question 56 (***)

The point $P(8,18)$ lies on the curve C, whose gradient function is given by

$$\frac{dy}{dx}=8\sqrt[3]{x}-10,\ x\geq 0.$$

Find an equation for C.

$\square$, $\boxed{y=6x^{\frac{4}{3}}-10x+2}$



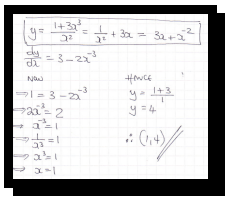
Question 57 (***)

A curve C has equation given by

$$y=\frac{1+3x^3}{x^2},\ x\neq 0$$

Find the coordinates of the point on C where the gradient is 1.

$\boxed{(1,4)}$



Question 58 (***)

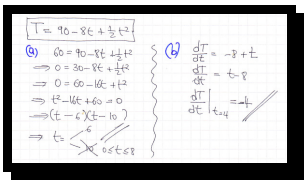
The temperature, T in °C, of a hot drink t minutes after it was made is given by

$$T = 90 - 8t + \frac{1}{2}t^2 \ , \ 0 \le t \le 8 .$$

a) Calculate after how many minutes the drink has a temperature of 60 °C.

b) Find the rate of change of temperature of the drink 4 minutes after it was made.

$\boxed{t = 6}$, $\boxed{-4 \text{ °C/min}}$



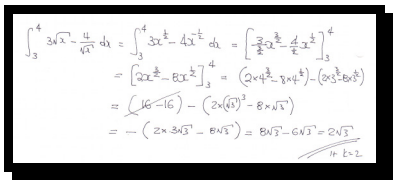
Question 59 (***)

Show clearly that

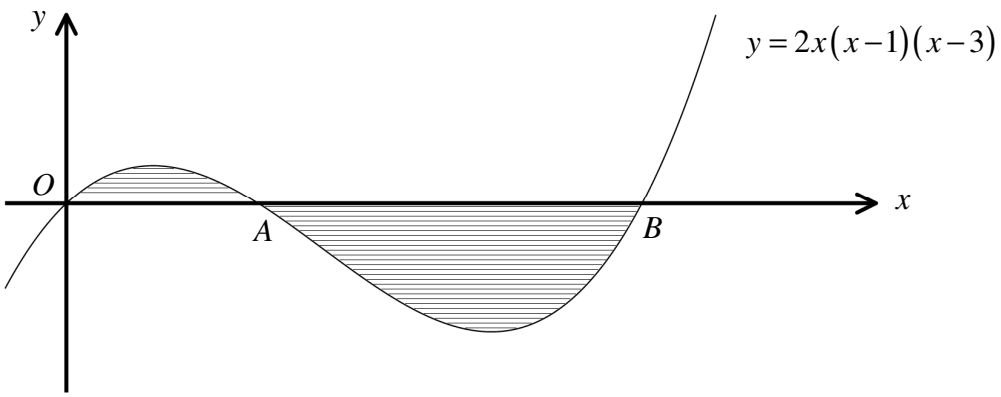
$$\int_3^4 3\sqrt{x} - \frac{4}{\sqrt{x}} \ dx = k\sqrt{3} ,$$

where k is an integer to be found.

$\boxed{k = 2}$



Question 60 (***)



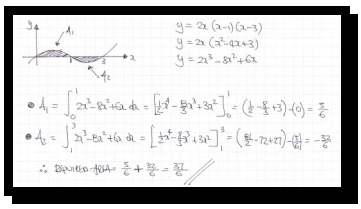
The figure above shows part of the curve with equation

$$y = 2x(x-1)(x-3), \ x \in \mathbb{R}.$$

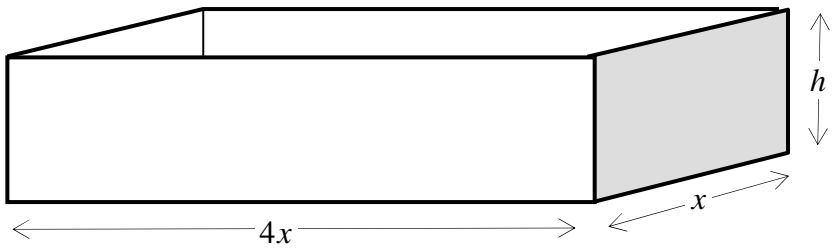
The curve meets the x axis at the origin and at the points A and B.

Determine the exact area of the finite region bounded by the curve and the x axis, shown shaded in the figure above.

$\square$, $\boxed{\text{area} = \frac{37}{6}}$



Question 61 (***)



The figure above shows a box in the shape of a cuboid with a rectangular base x cm by $4x$ cm and **no top**. The height of the box is h cm.

It is given that the surface area of the box is 1728 cm^2.

a) Show clearly that

$$h=\frac{864-2x^2}{5x}.$$

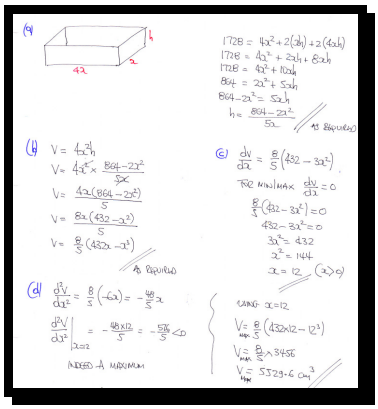
b) Use part (a) to show that the volume of the box , V cm^3, is given by

$$V=\frac{8}{5}\left(432x-x^3\right).$$

c) Find the value of x for which V is stationary.

d) Find the maximum value for V, justifying the fact that it is the maximum.

$\boxed{x=12}$, $\boxed{V_{\max}=5529.6}$



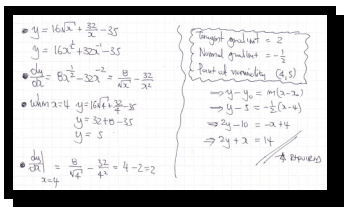
Question 62 (***)

The curve C and the line L have equations

$$C:\ y=16\sqrt{x}+\frac{32}{x}-35 \quad \text{and} \quad L:\ 2y+x=14\,.$$

Show that L is a normal to C at the point where $x=4$.

proof



Question 63 (***)

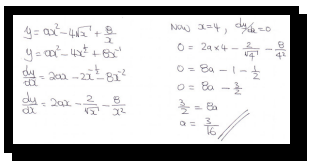
The curve C has equation

$$y=ax^2-4\sqrt{x}+\frac{8}{x}\ ,\ x>0\,,$$

where a is a non zero constant.

Given that $\dfrac{dy}{dx}=0$ at the point on C where $x=4$, find the value of a.

, $a=\dfrac{3}{16}$



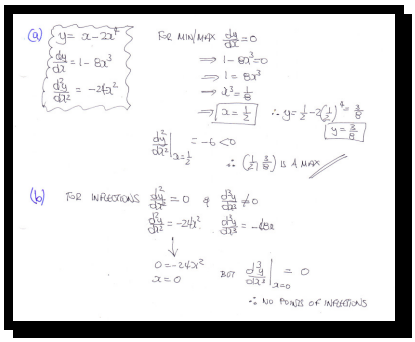
Question 64 **(***)**

$$y = x - 2x^4 , \ x \in \mathbb{R} .$$

a) Find the coordinates of the stationary point of y and determine its nature.

b) Show clearly that y has no points of inflection.

$\square$, $\boxed{\max\left(\frac{1}{2}, \frac{3}{8}\right)}$

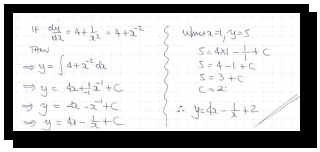


Question 65 **(***)**

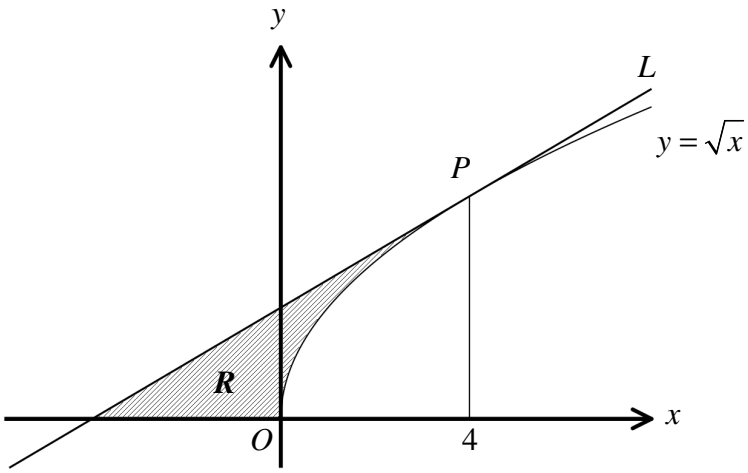
$$\frac{dy}{dx} = 4 + \frac{1}{x^2} , \ x \neq 0 .$$

Given that $y = 5$ when $x = 1$, express y in terms of x.

$\square$, $\boxed{y = 4x - \frac{1}{x} + 2}$



Question 66 **(***)**



The figure above shows the graph of the curve C with equation

$$y=\sqrt{x}\,,\ x\geq 0\,.$$

The point P lies on C where $x=4$.

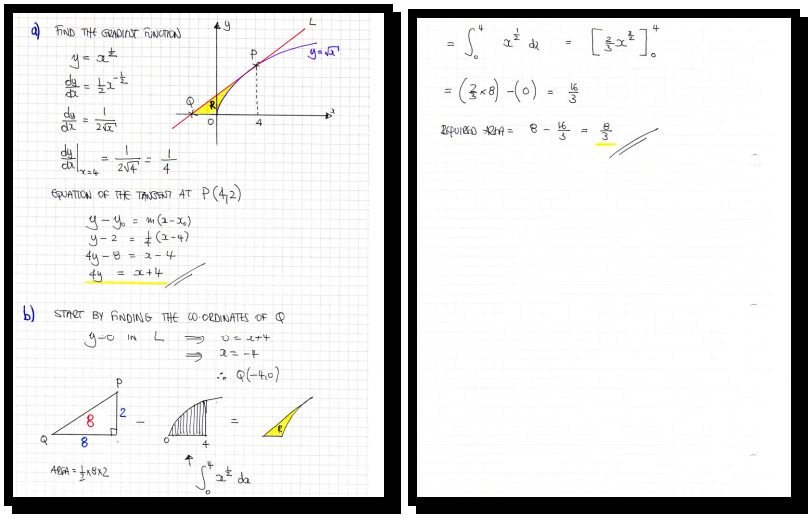
The straight line L is the tangent to C at P.

a) Find an equation of L.

The finite region R, shown shaded in the figure, is bounded by C, L and the x axis.

b) Find the exact area of R.

$\square$, $\boxed{y=\frac{1}{4}x+1}$, $\boxed{\frac{8}{3}}$



Question 67 (***)

The curve C has equation

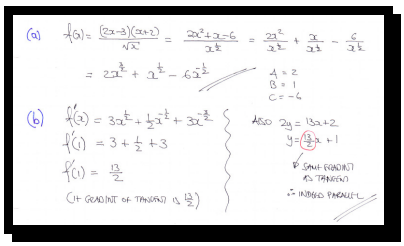
$$f(x)=\frac{(2x-3)(x+2)}{\sqrt{x}},\ x>0.$$

a) Express $f(x)$ in the form $Ax^{\frac{3}{2}}+Bx^{\frac{1}{2}}+Cx^{-\frac{1}{2}}$, where A, B and C are constants to be found.

b) Show that the tangent to C at the point where $x=1$ is parallel to the line with equation

$$2y=13x+2.$$

$\boxed{A=2}$, $\boxed{B=1}$, $\boxed{C=-6}$



Question 68 (***)

A curve with equation $y=f(x)$ passes through the point $(2,3)$.

The gradient function of the curve is given by

$$f'(x)=(x-3)(3x-1).$$

a) Find an equation of the curve, giving the answer as a polynomial in its simplest form.

b) Show clearly that

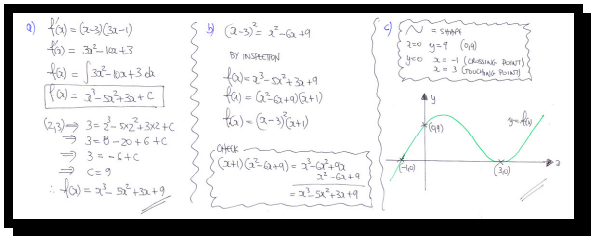
$$f(x)\equiv(x+k)(x-3)^2,$$

where k is a constant to be found.

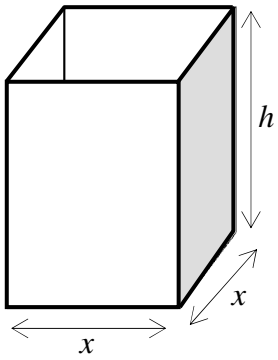
c) Sketch the graph of $f(x)$.

The sketch must show the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

$\boxed{}$, $\boxed{f(x)\equiv x^3-5x^2+3x+9}$, $\boxed{k=1}$



Question 69 (***)



The figure above shows the design of a large water tank in the shape of a cuboid with a square base and **no top**.

The square base is of length x metres and its height is h metres.

It is given that the volume of the tank is 500 m^3.

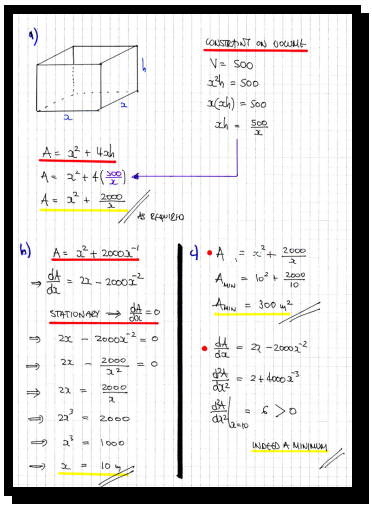
a) Show that the surface area of the tank, $A\text{ m}^2$, is given by

$$A = x^2 + \frac{2000}{x}.$$

b) Find the value of x for which A is stationary.

c) Find the minimum value for A, fully justifying the fact that it is the minimum.

$\boxed{}$, $\boxed{x = 10}$, $\boxed{A_{\min} = 300}$



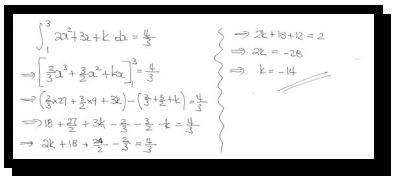
Question 70 (***)

$f(x)=2x^2+3x+k$, where k is a constant.

Find the value of k, given that

$$\int_1^3 f(x)\ dx=\frac{4}{3}.$$

$\boxed{k=-14}$



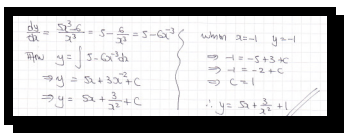
Question 71 (***)

The point $P(-1,-1)$ lies on the curve C, whose gradient function is given by

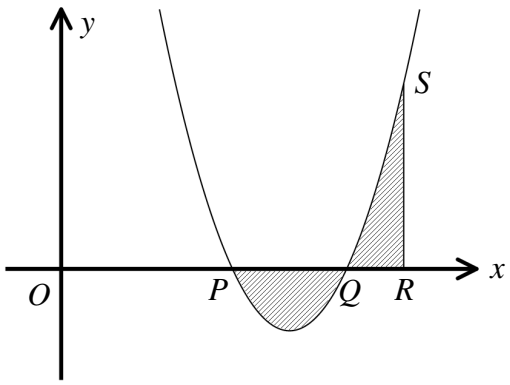
$$\frac{dy}{dx}=\frac{5x^3-6}{x^3},\ x\neq 0.$$

Find an equation for C.

$\boxed{y=5x+\frac{3}{x^2}+1}$



Question 72 (***)



The figure above shows the graph of the quadratic curve C with equation

$$y = 3x^2 - 8x + 5 .$$

The curve crosses the x axis at the points P and Q.

a) Find the coordinates of P and Q.

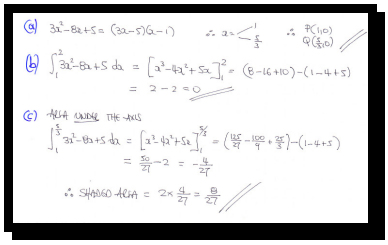
b) Show clearly that

$$\int_1^2 3x^2 - 8x + 5 \ dx = 0 .$$

The point R lies on the x axis where $x = 2$ and the point S lies on C so that RS is parallel to the y axis.

c) Find the area of the shaded region.

$$\boxed{P(1,0),\ Q\left(\frac{5}{3},0\right)} \qquad \boxed{\frac{8}{27}}$$



Question 73 (***)

A cubic curve has equation

$$f(x)=2x^3-7x^2+6x+1.$$

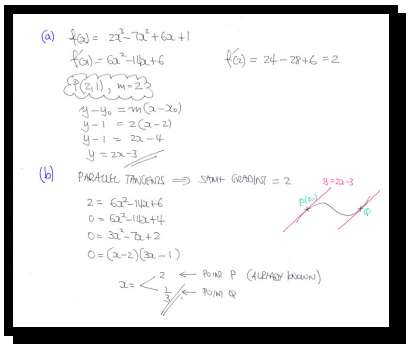
The point $P(2,1)$ lies on the curve.

a) Find an equation of the tangent to the curve at P.

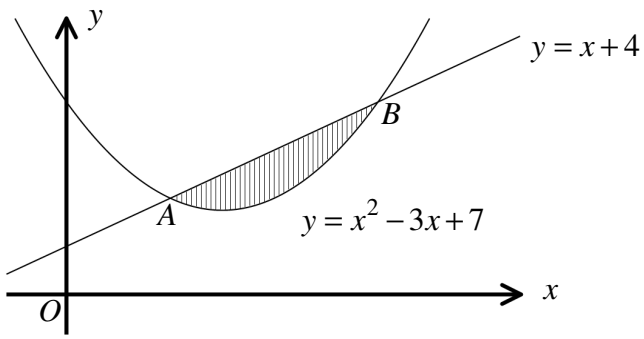
The point Q lies on the curve so that the tangent to the curve at Q is parallel to the tangent to the curve at P.

b) Determine the x coordinate of Q.

$$\boxed{y=2x-3}\,,\quad \boxed{x_Q=\frac{1}{3}}$$



Question 74 (***)



The figure above shows a curve C and a straight line L with respective equations

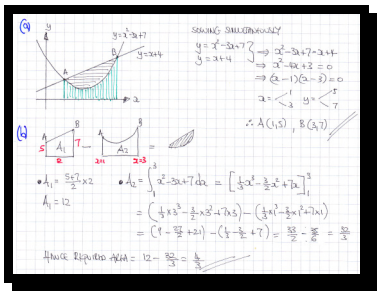
$$y = x^2 - 3x + 7 \quad \text{and} \quad y = x + 4 .$$

The curve and the straight line meet at the points A and B.

a) Find the coordinates of A and B.

b) Find the exact area of the finite region bounded by C and L, shown shaded in the figure above.

$\boxed{A(1,5), B(3,7)}$, $\boxed{\text{area} = \frac{4}{3}}$



Question 75 (***)

The curve C has equation

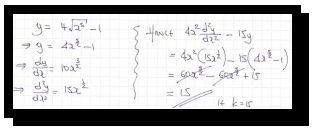
$$y = 4\sqrt{x^5} - 1, \ x \geq 0$$

Show clearly that

$$4x^2 \frac{d^2y}{dx^2} - 15y = k,$$

where k is an integer to be found.

$\square$, $\boxed{k = 15}$

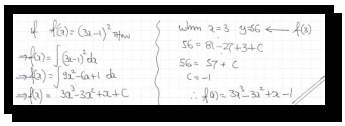


Question 76 (***)

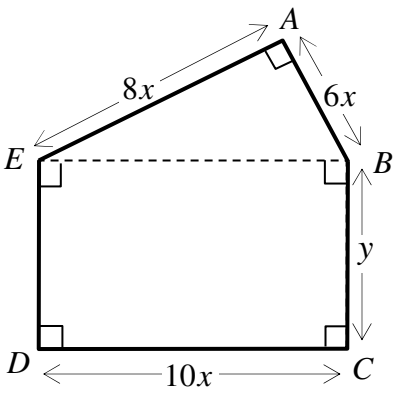
$$f'(x) = (3x-1)^2.$$

Given that $f(3) = 56$, find an expression for $f(x)$.

$\square$, $\boxed{f(x) = 3x^3 - 3x^2 + x - 1}$



Question 77 **(***)**



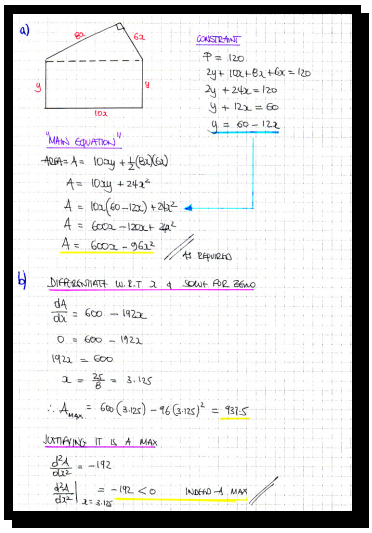
The figure above shows a pentagon $ABCDE$ whose measurements, in cm, are given in terms of x and y.

a) If the perimeter of the pentagon is 120 cm, show clearly that its area, A cm^2, is given by

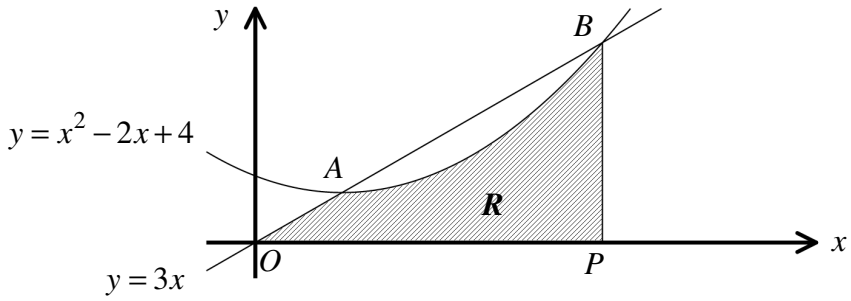
$$A = 600x - 96x^2 .$$

b) Use a method based on differentiation to calculate the maximum value for A, fully justifying the fact that it is indeed the maximum value.

$\boxed{}$, $\boxed{A_{\max} = 937.5}$



Question 78 **(***)**



The figure above shows the graph of the curve C with equation

$$y = x^2 - 2x + 4, \ x \in \mathbb{R}$$

intersected by the straight line L with equation

$$y = 3x, \ x \in \mathbb{R}.$$

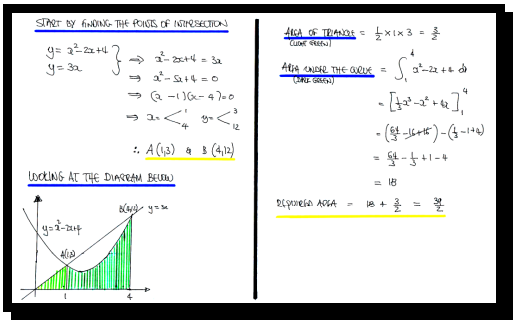
The curve meets the straight line at the points A and B.

The point P is located on the x axis so that the straight line segment BP is parallel to the y axis.

The finite region R is bounded by C, L, BP and the x axis.

Show that the area of R, shown shaded in the figure, is $\frac{39}{2}$.

[ ], [proof]



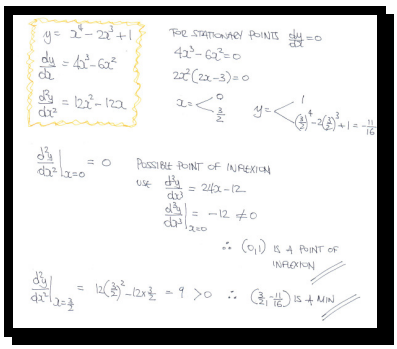
Question 79 (***)

The curve C has equation

$$y = x^4 - 2x^3 + 1, \ x \in \mathbb{R}.$$

Find the coordinates of the stationary points of C and determine their nature.

$\square$, $\boxed{\text{min at } \left(\frac{3}{2}, -\frac{11}{16}\right), \ \text{ point of inflexion } (0,1)}$



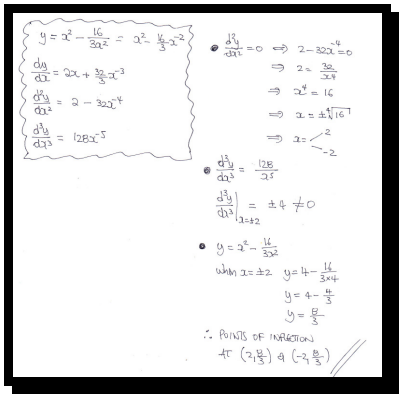
Question 80 (***) non calculator

The curve C has equation

$$y = x^2 - \frac{16}{3x^2}, \ x \neq 0 .$$

Show that C has two points of inflection and determine their coordinates.

$$\boxed{\left(2, \frac{8}{3}\right), \ \left(-2, \frac{8}{3}\right)}$$



Question 81 **(***)**

$$y=\left(\frac{2x+1}{3x^2}\right)^2,\ x>0.$$

a) Express y in the form $Ax^{-2}+Bx^{-3}+Cx^{-4}$, where A, B and C are fractions to be found.

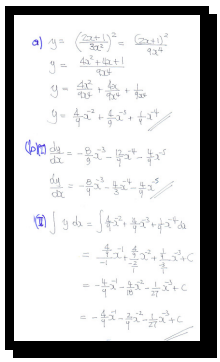
b) Hence determine simplified expressions for …

i. $\ldots\dfrac{dy}{dx}$.

ii. $\ldots\displaystyle\int y\,dx$

$\square$, $\boxed{y=\frac{4}{9}x^{-2}+\frac{4}{9}x^{-3}+\frac{1}{9}x^{-4}}$, $\boxed{\frac{dy}{dx}=-\frac{8}{9}x^{-3}-\frac{4}{3}x^{-4}-\frac{4}{9}x^{-5}}$,

$$\boxed{\int y\,dx=-\frac{4}{9}x^{-1}-\frac{2}{9}x^{-2}-\frac{1}{27}x^{-3}+C}$$



Question 82 (***)

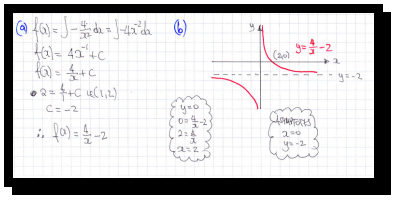
The curve C with equation $y=f(x)$ satisfies

$$f'(x)=-\frac{4}{x^2},\ x\neq 0.$$

a) Given that $f(1)=2$, find an expression for $f(x)$.

b) Sketch the graph of $f(x)$, indicating clearly the asymptotes of the curve and the coordinates of any points where the curve crosses the coordinate axes.

$\square$, $\boxed{f(x)=\frac{4}{x}-2}$, $\boxed{(2,0)}$



Question 83 (***)

The total cost C, in £, for a certain car journey, is modelled by

$$C=\frac{200}{V}+\frac{2V}{25},\ V>30,$$

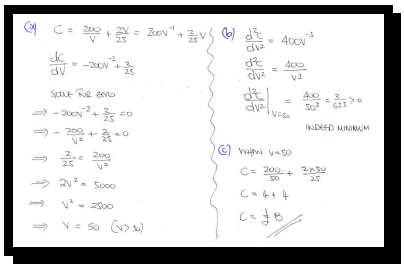
where V is the average speed in miles per hour.

a) Find the value of V for which C is stationary.

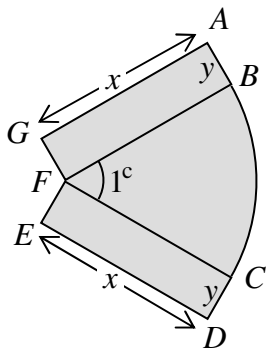
b) Justify that this value of V minimizes C.

c) Hence determine the minimum total cost of the journey.

$\boxed{}$, $\boxed{V=50}$, $\boxed{£8}$



Question 84 **(***)**



The figure above shows a clothes design consisting of two identical rectangles attached to each of the straight sides of a circular sector of radius x cm.

The rectangles measure x cm by y cm and the circular sector subtends an angle of one radian at the centre.

The perimeter of the design is 40 cm.

a) Show that the area of the design, A cm^2, is given by

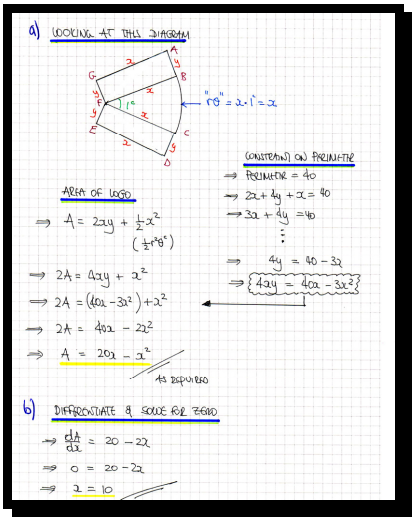
$$A = 20x - x^2 .$$

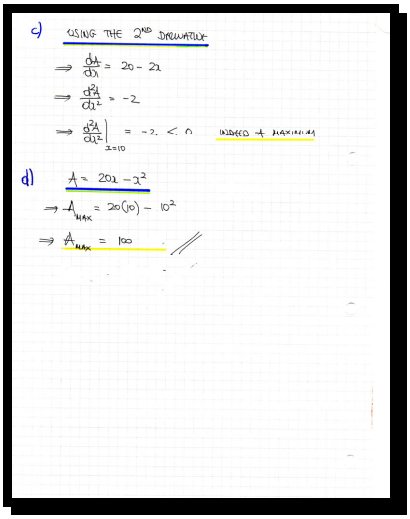
b) Determine **by differentiation** the value of x for which A is stationary.

c) Show that the value of x found in part **(b)** gives the maximum value for A.

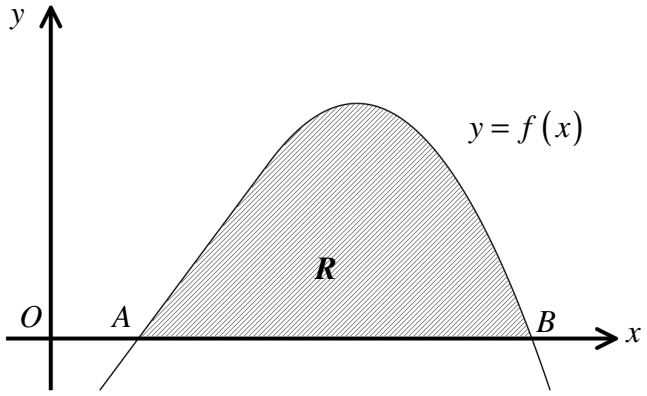
d) Find the maximum area of the design.

$\boxed{}$, $\boxed{x = 10}$, $\boxed{A_{\max} = 100}$





Question 85 **(***+)**



The figure above shows the graph of the curve with equation

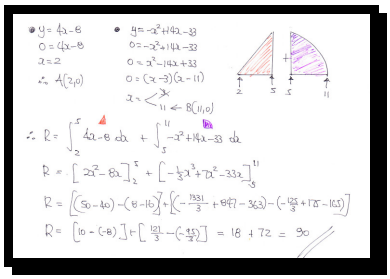
$$f(x)=\begin{cases} 4x-8 & x<5 \\ -x^2+14x-33 & x\geq 5 \end{cases}.$$

The curve meets the x axis at the points A and B.

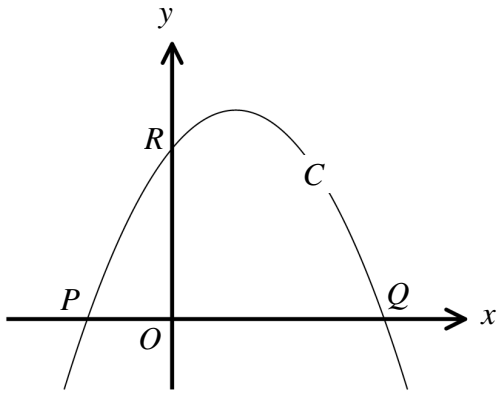
The finite region R, shown shaded in the figure above, is bounded by the curve and the x axis.

Find the area of R.

$\square$, 90



Question 86 (*+)**



The figure above shows the curve C which meets the coordinates axes at the points P, Q and R.

Given the gradient function of C is given by

$$f'(x)=3-4x,$$

and that $f(1)=2f(2)$, determine the coordinates of P, Q and R.

$\square$, $\boxed{P(-1,0)}$, $\boxed{P\left(\frac{5}{2},0\right)}$, $\boxed{R(0,5)}$

$f'(x)=3-4x$

$f(x)=\int 3-4x\,dx = 3x-2x^2+k$

$\therefore \boxed{f(x)=-2x^2+3x+k}$

$f(1)=2f(2)$

$-2+3+k=2(-8+6+k)$

$1+k=-4+2k$

$5=k$

$\therefore \boxed{f(x)=-2x^2+3x+5}$

when $x=0$ $f(0)=5$ $\therefore R(0,5)$

when $y=0$ $0=-2x^2+3x+5$

$2x^2-3x-5=0$

$(2x-5)(x+1)=0$

$\therefore x=-1$ $P(-1,0)$

$\frac{5}{2}$ $Q(\frac{5}{2},0)$

Question 87 (*+)**

The curve C has equation

$$y=(x-1)\left(x^2+4x+5\right),\ x\in\mathbb{R}.$$

a) Show that C meets the x axis at only one point.

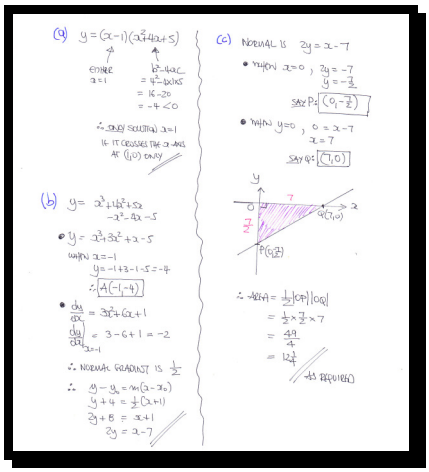
The point A, where $x=-1$, lies on C.

b) Find an equation of the normal to C at A.

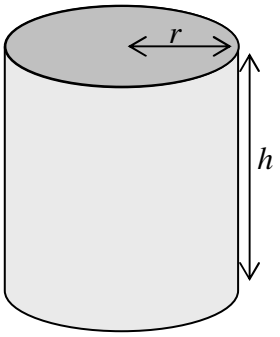
The normal to C at A meets the coordinate axes at the points P and Q.

c) Show further that the area of the triangle OPQ, where O is the origin, is $12\frac{1}{4}$ square units.

$\square$, $\boxed{2y=x-7}$



Question 88 (*+)**



The figure above shows a **closed** cylindrical can of radius r cm and height h cm.

a) Given that the surface area of the can is 192π cm^2, show that the volume of the can, V cm^3, is given by

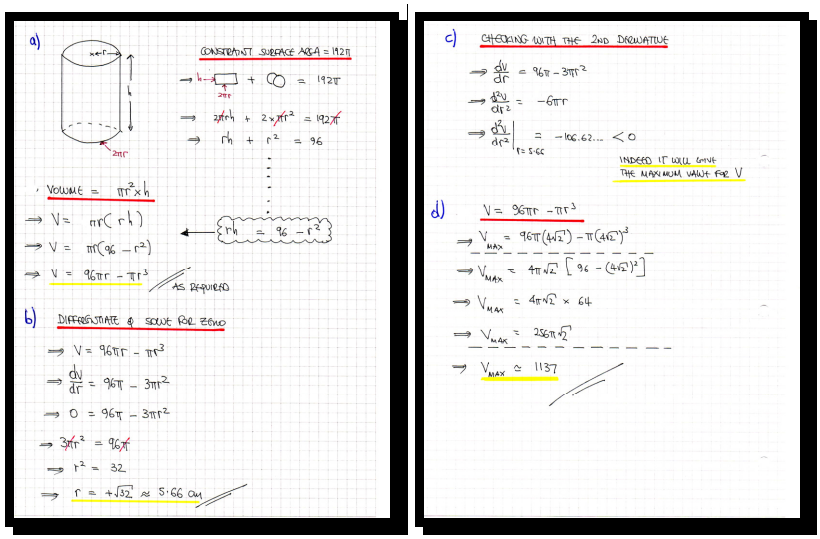
$$V = 96\pi r - \pi r^3 .$$

b) Find the value of r for which V is stationary.

c) Justify that the value of r found in part **(b)** gives the maximum value for V.

d) Calculate the maximum value of V.

$\square$, $\boxed{r = 4\sqrt{2} \approx 5.66}$, $\boxed{V_{\max} = 256\pi\sqrt{2} \approx 1137}$



Question 89 (*+)**

The curve C has equation

$$y = 2x^3 - 9x^2 + 12x - 10 .$$

a) Find the coordinates of the two points on the curve where the gradient is zero.

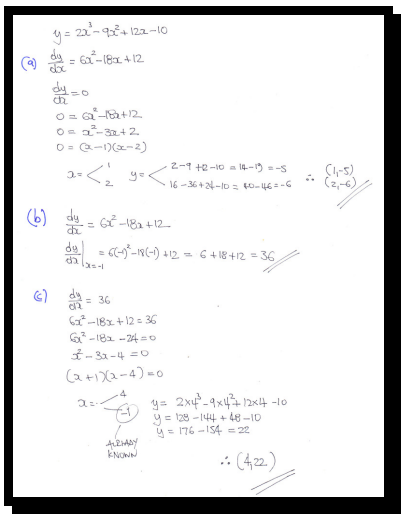
The point P lies on C and its x coordinate is -1.

b) Determine the gradient of C at the point P.

The point Q lies on C so the gradient at Q is the same as the gradient at P.

c) Find the coordinates of Q.

$\boxed{}$, $\boxed{(1,-5),(2,-6)}$, $\boxed{36}$, $\boxed{Q(4,22)}$



Question 90 (*+)**

The gradient function at every point on a curve C is given by

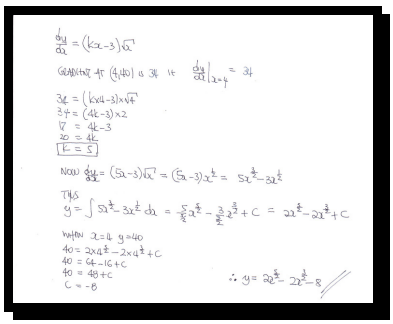
$$\frac{dy}{dx}=(kx-3)\sqrt{x},$$

where k is a non zero constant.

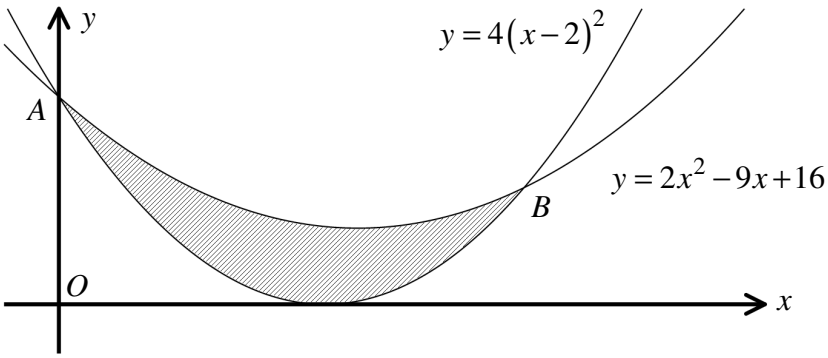
The point $P(4,40)$ lies on C and the gradient at P is 34.

Determine an equation of C.

$\boxed{}$, $\boxed{y=2x^{\frac{5}{2}}-2x^{\frac{3}{2}}-8}$



Question 91 (*+)**



The figure above shows the graph of the curves with equations

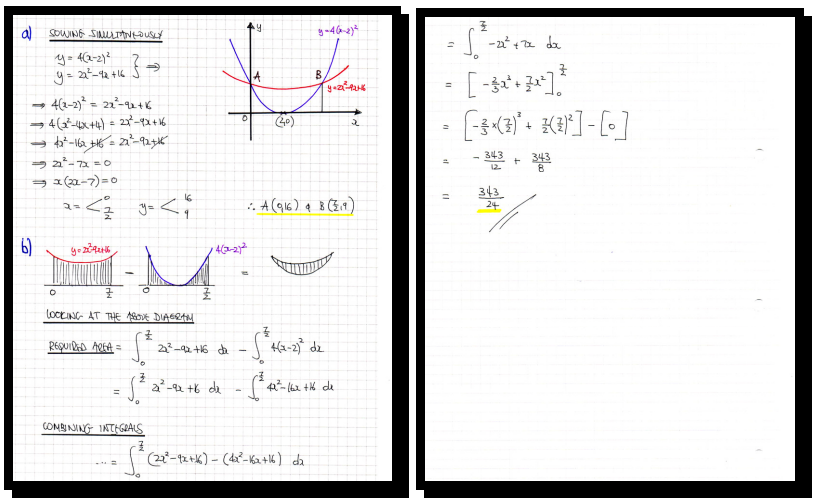
$$y=4(x-2)^2 \quad \text{and} \quad y=2x^2-9x+16\,.$$

The curves meet each other at the points A and B.

a) Determine the coordinates of A and B.

b) Find the exact area of the finite region bounded by the two curves, shown shaded in the above figure.

$\square$, $\boxed{A(0,16)\ B\left(\frac{7}{2},9\right)}$, $\boxed{\frac{343}{24}}$



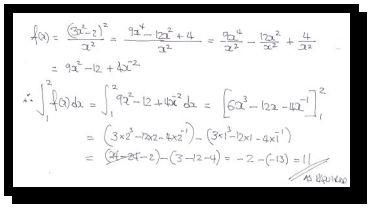
Question 92 (*+)**

$$f(x)=\frac{\left(3x^2-2\right)^2}{x^2},\ x\neq 0.$$

Show clearly that

$$\int_1^2 f(x)\ dx=11.$$

proof



Question 93 (*+)**

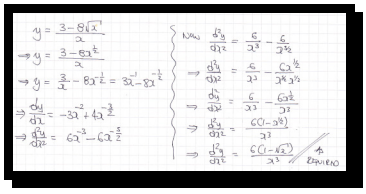
The curve C has equation

$$y=\frac{3-8\sqrt{x}}{x},\ x>0$$

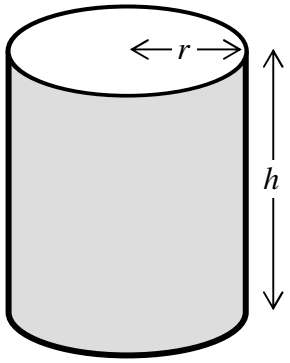
Show clearly that

$$\frac{d^2y}{dx^2}=\frac{6\left(1-\sqrt{x}\right)}{x^3}.$$

proof



Question 94 **(***+)**



A pencil holder is in the shape of a right circular cylinder, which is **open** at one of its circular ends.

The cylinder has radius r cm and height h cm and the total surface area of the cylinder, including its base, is 360 cm^2.

a) Show that the volume, $V\text{ cm}^3$, of the cylinder is given by

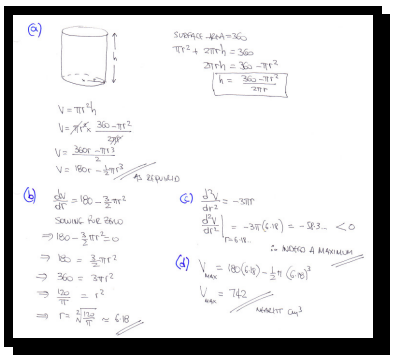
$$V=180r-\frac{1}{2}\pi r^3 .$$

b) Determine by differentiation the value of r for which V has a stationary value.

c) Show that the value of r found in part (b) gives the maximum value for V.

d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

$$\boxed{r=\sqrt{\frac{120}{\pi}}\approx 6.18}\,,\qquad \boxed{V_{\max}\approx 742}$$



Question 95 **(***+)**

A curve has equation

$$y = x - 8\sqrt{x}\,,\ x \in \mathbb{R}\,,\ x \geq 0\,.$$

The curve meets the x axis at the origin and at the point P.

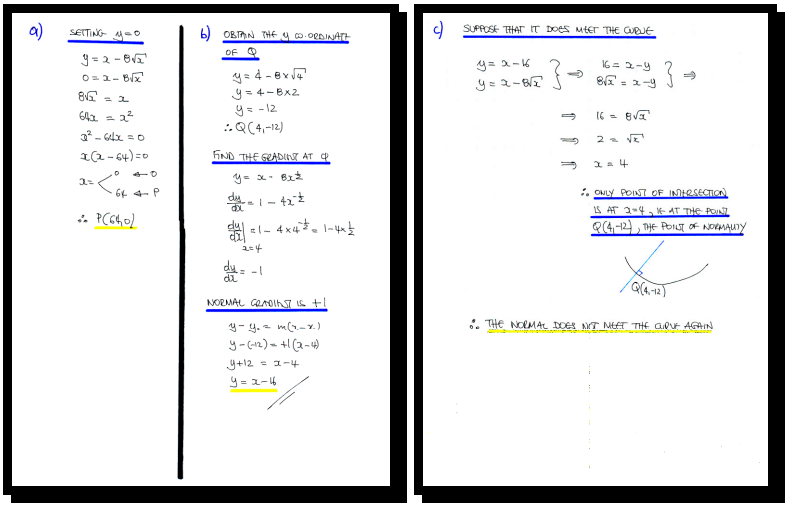
a) Determine the coordinates of P.

The point Q, where $x = 4$, lies on the curve.

b) Find an equation of the normal to curve at Q.

c) Show that the normal to the curve at Q does not meet the curve again.

$\boxed{}$, $\boxed{P(64,0)}$, $\boxed{y = x - 16}$

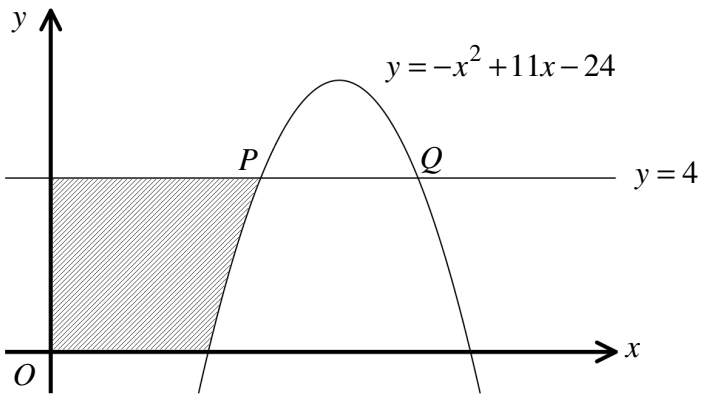


Question 96 (*+)**

The diagram below shows a parabola and a straight line with respective equations

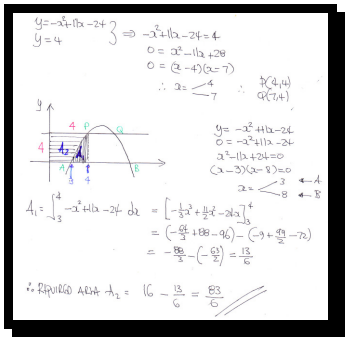
$$y=-x^2+11x-24 \quad \text{and} \quad y=4\,.$$

The points P and Q are the intersections between the parabola and the straight line.



Find the exact area of the shaded region, bounded by the curve, the coordinate axes and the straight line with equation $y=4$.

$\square$, $\boxed{\text{area}=\frac{83}{6}}$

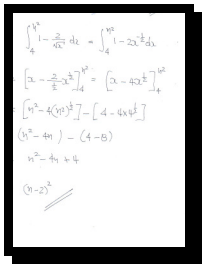


Question 97 **(***+)**

Given that n is a positive integer greater than 2, show clearly that

$$\int_4^{n^2} 1-\frac{2}{\sqrt{x}}\,dx=(n-2)^2\,,\ x>0\,.$$

proof



Question 98 **(***+)**

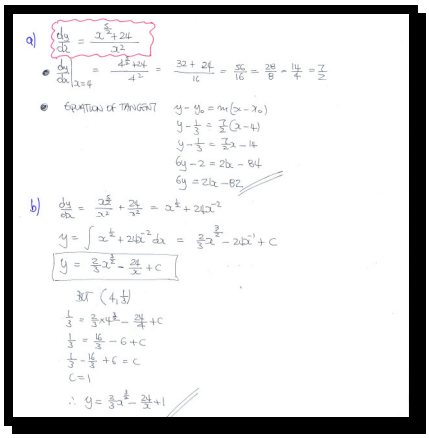
The point $P\left(4,\frac{1}{3}\right)$ lies on the curve C whose gradient function is given by

$$\frac{dy}{dx}=\frac{x^{\frac{5}{2}}+24}{x^2}\,,\ x\neq 0\,.$$

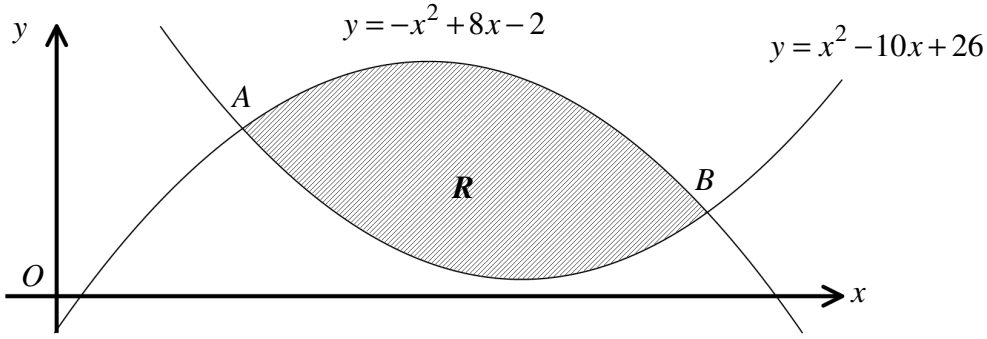
a) Determine an equation of the tangent to C at P.

b) Find an equation of C.

$\square$, $6y=21x-82$, $y=\frac{2}{3}x^{\frac{3}{2}}-\frac{24}{x}+1$



Question 99 (*+)**



The figure above shows the graphs of the curves with equations

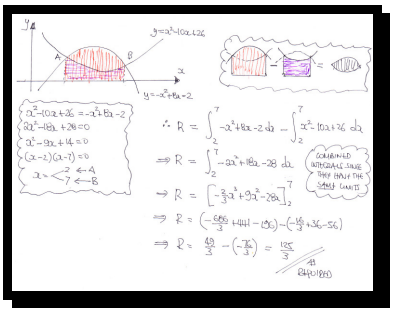
$$y=-x^2+8x-2 \quad \text{and} \quad y=x^2-10x+26 .$$

The two curves intersect each other at the points A and B.

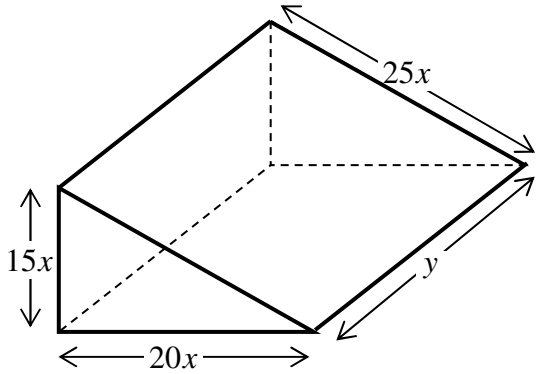
The finite region R bounded by the two curves is shown shaded in the figure above.

Show that the area of R is exactly $\frac{125}{3}$.

[ ], [proof]



Question 100 **(***+)**



The figure above shows a solid triangular prism with a **total** surface area of 3600 cm^2.

The triangular faces of the prism are right angled with a base of $20x$ cm and a height of $15x$ cm. The length of the prism is y cm.

a) Show that the volume of the prism, $V\text{ cm}^3$, is given by

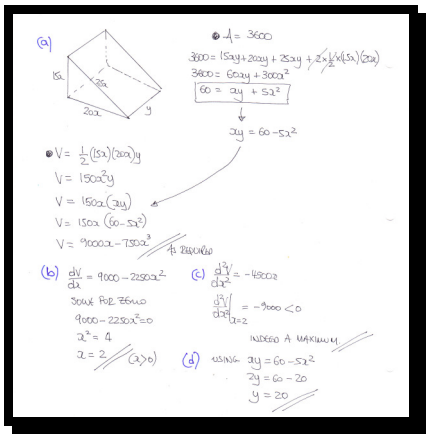
$$V = 9000x - 750x^3 .$$

b) Find the value of x for which V is stationary.

c) Show that the value of x found in part (b) gives the maximum value for V.

d) Determine the value of y when V is maximum.

$\boxed{}$, $\boxed{x=2}$, $\boxed{y=20}$



Question 101 (*+)**

The curve C has equation

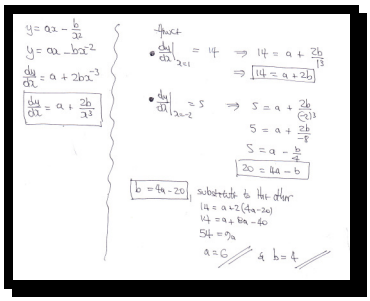
$$y = ax - \frac{b}{x^2}, \ x \neq 0,$$

where a and b are non zero constants.

The gradient of C at the points where $x = 1$ and $x = -2$ is 14 and 5, respectively.

Find the value of a and the value of b.

$\boxed{a = 6, \ b = 4}$

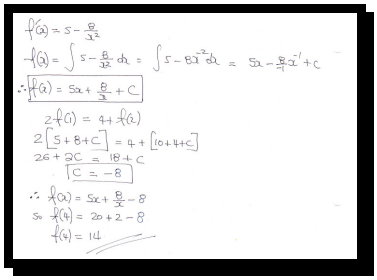


Question 102 (*+)**

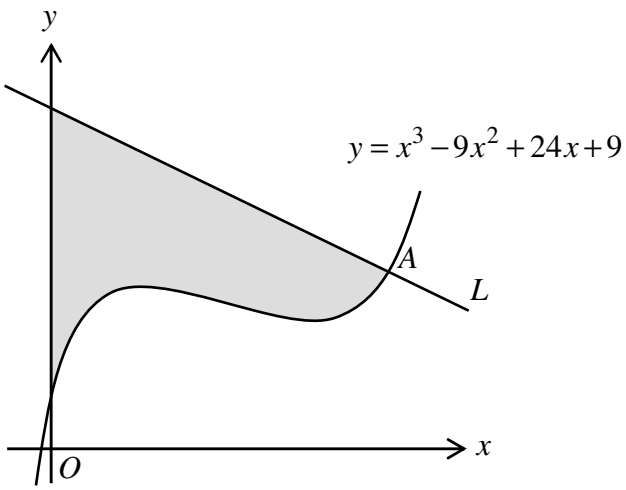
$$f'(x) = 5 - \frac{8}{x^2}, \ x \neq 0.$$

Find the value of $f(4)$, given that $2f(1) = 4 + f(2)$.

$\boxed{}$, $\boxed{f(4) = 14}$



Question 103 **(***+)**



The figure above shows the graph of the curve C with equation

$$y = x^3 - 9x^2 + 24x + 9 .$$

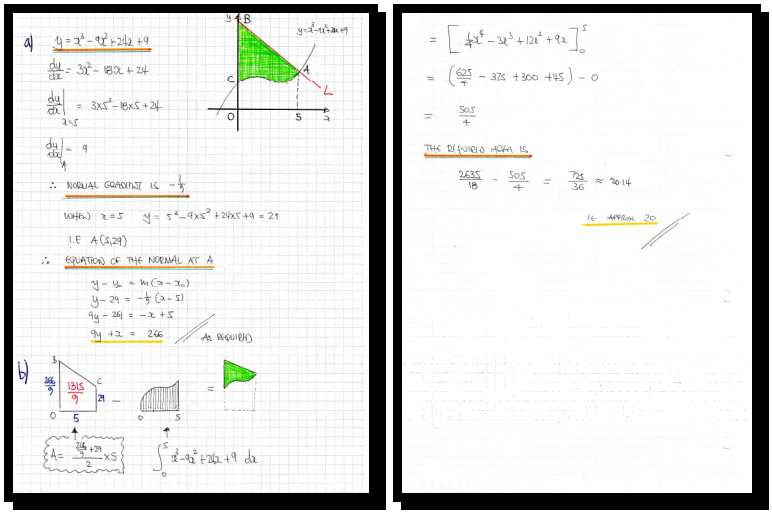
The straight line L is the normal to C at the point A, whose x coordinate is 5.

a) Show that an equation of L is

$$x + 9y = 266 .$$

b) Show further that the area of the finite region bounded by C, L and the y axis is approximately 20 square units.

[ ], [proof]



Question 104 **(***+)**

The curve C has equation

$$y=\left(1+\sqrt{x}\right)^2 \ , \ x\geq 0 \, .$$

a) Find an expression for $\dfrac{dy}{dx}$.

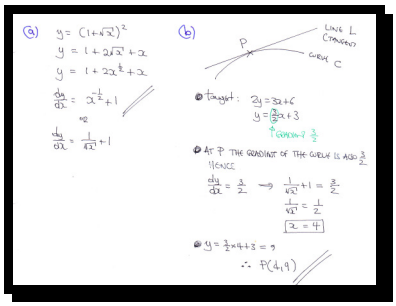
The straight line L with equation

$$2y=3x+6$$

is a tangent to C at the point P.

b) Use a calculus method to determine the coordinates of P.

$\square$, $\boxed{\dfrac{dy}{dx}=1+\dfrac{1}{\sqrt{x}}}$, $\boxed{P(4,9)}$

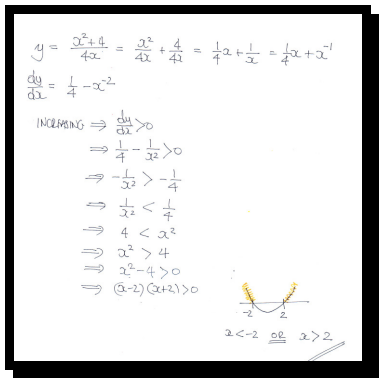


Question 105 (*+)**

$$y=\frac{x^2+4}{4x},\ x\neq 0.$$

Find the range of values of x for which y is increasing.

$\boxed{}$, $\boxed{x<-2\cup x>2}$



Question 106 (*+)**

The curve C has equation

$$y = x^3 - 9x^2 + 24x - 19\,,\ x \in \mathbb{R}\,.$$

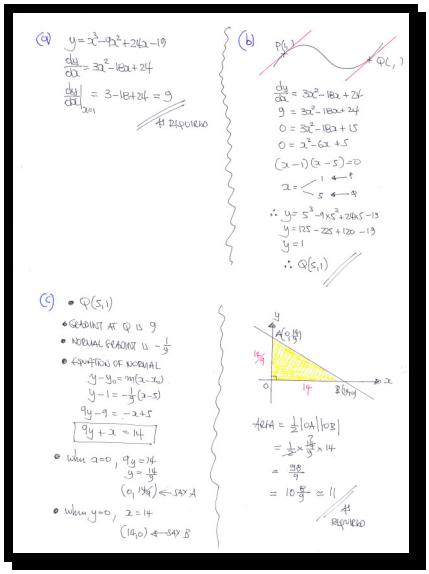
a) Show that the tangent to C at the point P, where $x = 1$, has gradient 9.

b) Find the coordinates of another point Q on C at which the tangent also has gradient 9.

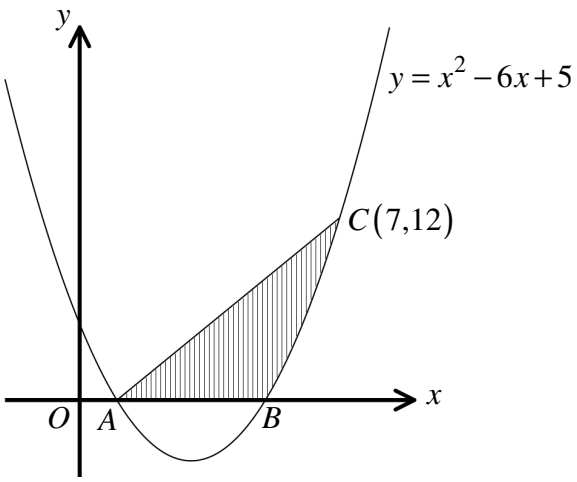
The normal to C at Q meets the coordinate axes at the points A and B.

c) Show further that the **approximate** area of the triangle OAB, where O is the origin, is 11 square units.

$\boxed{Q(5,1)}$



Question 107 (*+)**



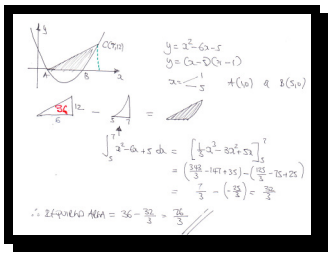
The diagram above shows the curve with equation

$$y = x^2 - 6x + 5 .$$

The point $C(7,12)$ lies on the curve while A and B are the points of intersection of the curve and the x axis.

Find the exact area of the shaded region, bounded by the curve, the straight line segment AC and the x axis.

$\square$, $\boxed{\text{area} = \frac{76}{3}}$



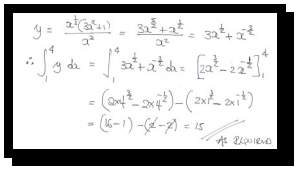
Question 108 (*+)**

$$y=\frac{x^{\frac{1}{2}}(3x^2+1)}{x^2},\ x>0.$$

Show clearly that

$$\int_1^4 y\ dx=15.$$

proof



Question 109 (*+)**

The gradient of every point on the curve C, with equation $y=f(x)$ satisfies

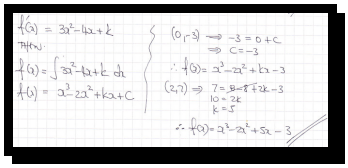
$$f'(x)=3x^2-4x+k,$$

where k is a non zero constant.

The points $P(0,-3)$ and $Q(2,7)$ both lie on C.

Find an equation for C.

$$y=x^3-2x^2+5x-3$$



Question 110 (*+)**

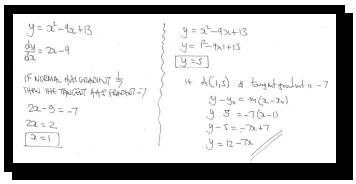
The point A lies on the curve with equation

$$y = x^2 - 9x + 13 .$$

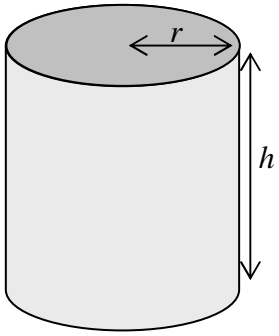
The gradient of the normal to the curve at the point A is $\frac{1}{7}$.

Find an equation of the tangent to the curve at A.

$$\boxed{y = 12 - 7x}$$



Question 111 (*+)**



The figure above shows a **closed** cylindrical can, of radius r cm and height h cm.

a) If the volume of the can is 330 cm^3, show that surface area of the can, $A\text{ cm}^2$, is given by

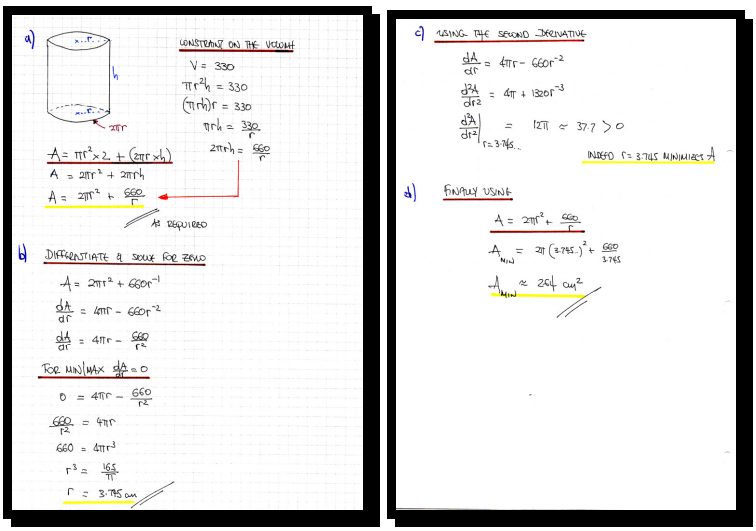
$$A=2\pi r^2+\frac{660}{r}.$$

b) Find the value of r for which A is stationary.

c) Justify that the value of r found in part **(b)** gives the minimum value for A.

d) Hence calculate the minimum value of A.

$\boxed{}$, $\boxed{r\approx 3.745}$, $\boxed{A_{\text{min}}\approx 264}$



Question 112 (*+)**

The point $A(2,1)$ lies on the curve with equation

$$y=\frac{(x-1)(x+2)}{2x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Find the gradient of the curve at A.

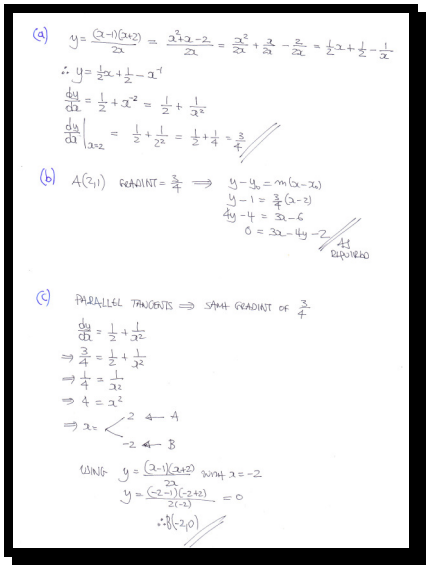
b) Show that the tangent to the curve at A has equation

$$3x-4y-2=0.$$

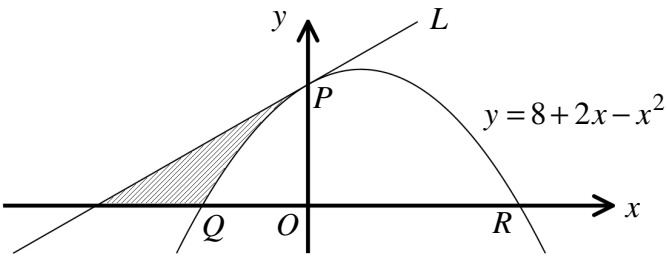
The tangent to the curve at the point B is parallel to the tangent to the curve at A.

c) Determine the coordinates of B.

$\boxed{\text{gradient at } A=\frac{3}{4}}$, $\boxed{B(-2,0)}$



Question 113 **(***+)**



The figure above shows the graph of a curve with equation

$$y = 8 + 2x - x^2 .$$

The curve meets the y axis at the point P, and the x axis at the points Q and R.

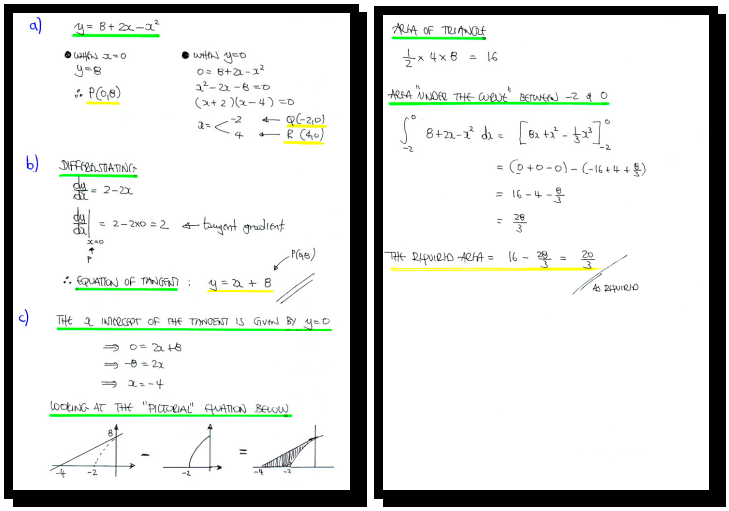
a) Determine the coordinates of P, Q and R.

The straight line L is the tangent to the curve at P.

b) Find an equation of L.

c) Show that the area of the finite region bounded by the curve, the tangent L and the x axis is $\frac{20}{3}$.

$\boxed{}$, $\boxed{P(0,8)}$, $\boxed{Q(-2,0)}$, $\boxed{R(4,0)}$, $\boxed{y = 2x + 8}$



Question 114 (*+)**

On a given lorry journey the cost C, in pence per mile, is modelled by

$$C=\frac{192}{V}+\frac{V^2}{144},\ V>6,$$

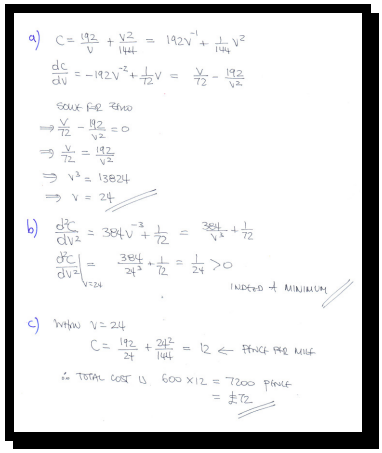
where V is the lorry's average speed in metres per second.

a) Find the speed, in metres per second, for which the cost, in pence per mile, is stationary.

b) Justify that this value of the speed minimizes the cost.

c) Hence determine the minimum cost of a 600 mile journey.

$\boxed{}$, $\boxed{V=24}$, $\boxed{£72}$



Question 115 (*+)**

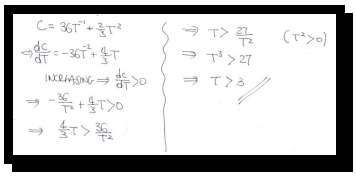
A certain chemical industrial process is carried out at low temperatures.

The wastage cost £C during this chemical process and the average temperature T °C are related by the equation

$$C=\frac{36}{T}+\frac{2T^2}{3}\text{, } T>0\text{.}$$

Find the average temperature during which the wastage cost is increasing.

$\boxed{}$, $\boxed{T>3}$



Question 116 **(***+)**

The curve C has equation $y=f(x)$ given by

$$f(x)=2(x-2)^3,\quad x\in\mathbb{R}.$$

a) Sketch the graph of $f(x)$.

b) Find an expression for $f'(x)$.

The point $P(3,2)$ lies on C and the straight line l_1 is the tangent to C at P.

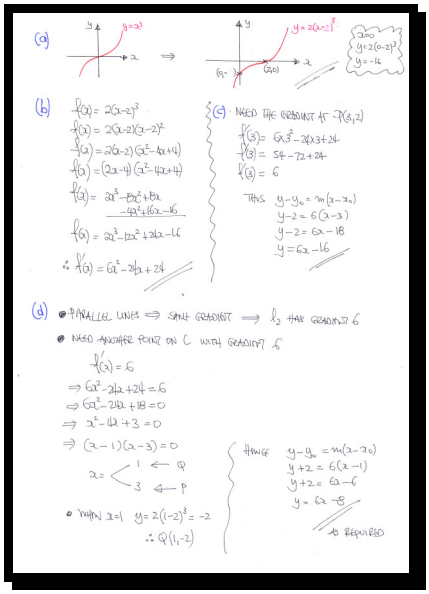
c) Find an equation of l_1.

The straight line l_2 is another tangent at a different point Q on C.

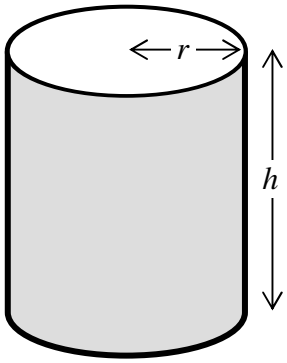
d) Given that l_1 is parallel to l_2 show that an equation of l_2 is

$$y=6x-8.$$

$\square$, $\boxed{f'(x)=6x^2-24x+24}$, $\boxed{y=6x-16}$



Question 117 **(***+)**



A hollow container, made of thin sheet metal, is in the shape of a right circular cylinder, which is **open** at one of its circular ends.

The container has radius r cm, height h cm and a capacity of 1500 cm^3.

a) Show that the surface area, $A\text{ cm}^2$, of the container is given by

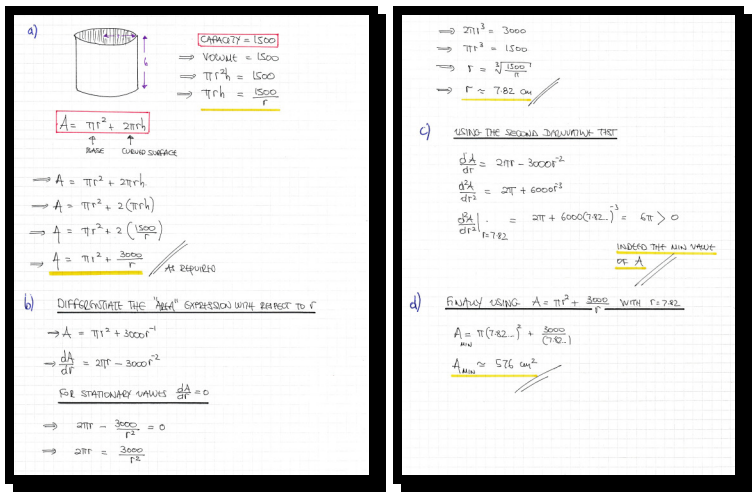
$$A=\pi r^2+\frac{3000}{r}.$$

b) Determine the value of r for which A has a stationary value.

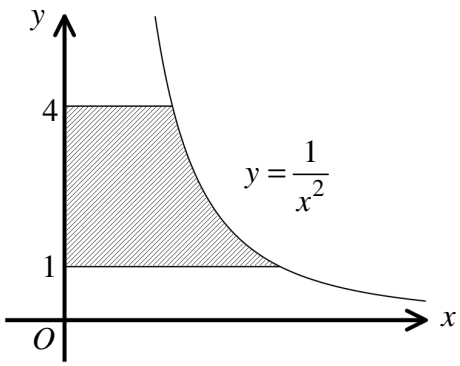
c) Show that the value of r found in part **(b)** gives the minimum value for A.

d) Calculate, to the nearest cm^2, the minimum surface area of the container.

$\boxed{}$, $\boxed{r\approx 7.816}$, $\boxed{A_{\min}\approx 576}$



Question 118 (*+)**

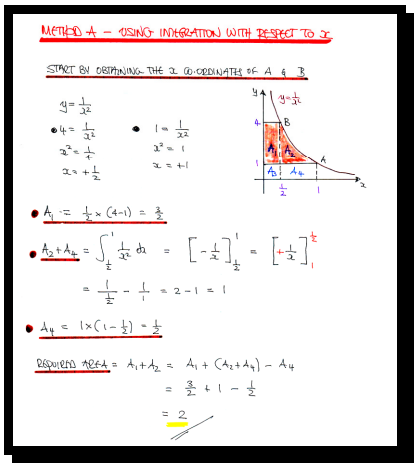


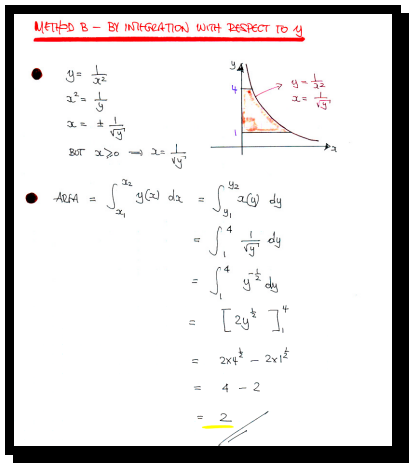
The figure above shows part of the graph of the curve with equation

$$y=\frac{1}{x^2}\,,\ x\in\mathbb{R}\,,\ x>0\,.$$

Find the area bounded by the curve, the y axis, and the straight lines with equations $y=1$ and $y=4$.

$\square$, 2





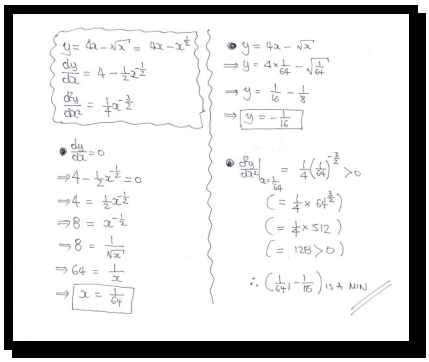
Question 119 (*+) non calculator**

The curve C has equation

$$y = 4x - \sqrt{x}\,,\ x \geq 0\,.$$

Find the coordinates of the stationary point of C, and determine its nature.

$$\boxed{\min\left(\frac{1}{64}, -\frac{1}{16}\right)}$$



Question 120 (*+)**

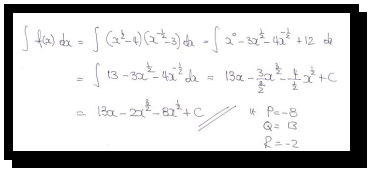
$$f(x) = \left(x^{\frac{1}{2}} - 4\right)\left(x^{-\frac{1}{2}} - 3\right),\ x > 0\,.$$

Show clearly that

$$\int f(x)\,dx = P\sqrt{x} + Qx + Rx^{\frac{3}{2}} + C\,,$$

where P, Q and R are integers to be found, and C is an arbitrary constant.

$$\boxed{P = -8},\ \boxed{Q = 13},\ \boxed{R = -2}$$



Question 121 **(***+)**

$$f(x) \equiv x^3 - 2x^2 - x - 6\,,\ x \in \mathbb{R}\,.$$

a) Use the factor theorem to show that $(x-3)$ is a factor of $f(x)$.

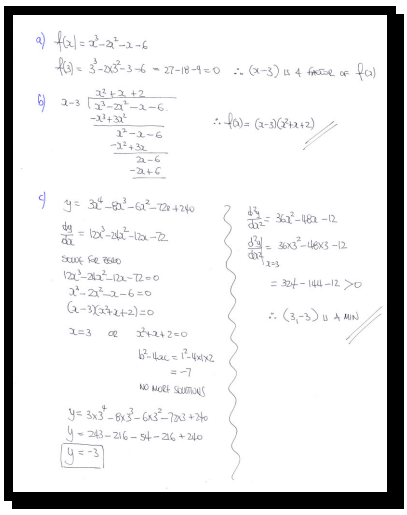
b) Hence express $f(x)$ as the product of a linear and a quadratic factor.

The curve C has equation

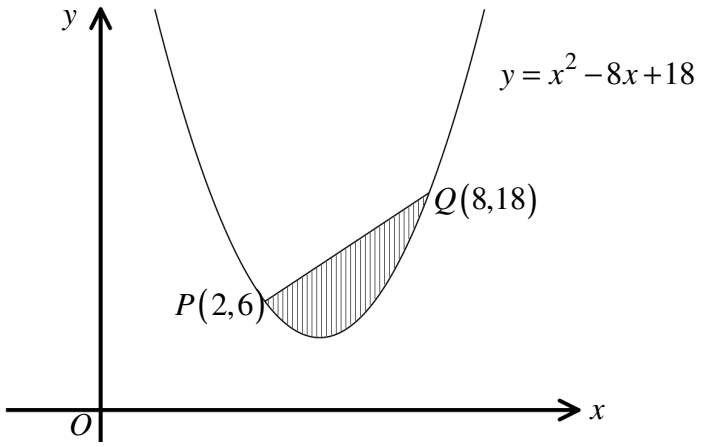
$$y = 3x^4 - 8x^3 - 6x^2 - 72x + 240\,.$$

c) Show that C has a single stationary point, and determine its coordinates and its nature.

$\square$, $\boxed{f(x) \equiv (x-3)\left(x^2+x+2\right)}$, $\boxed{\min(3,-3)}$



Question 122 (*+)**



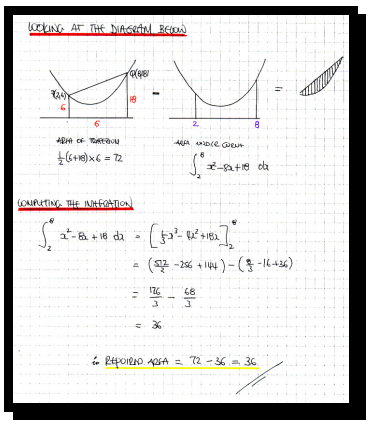
The figure above shows the parabola with equation

$$y=x^2-8x+18\,,\ x\in\mathbb{R}\,.$$

The points $P(3,3)$ and $Q(6,6)$ both lie on the parabola.

Find the exact of the shaded region, bounded by the curve and the straight line segment between P and Q.

$\boxed{}$, $\boxed{\text{area}=36}$

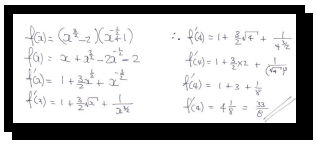


Question 123 (*+)**

$$f(x)=\left(x^{\frac{3}{2}}-2\right)\left(x^{-\frac{1}{2}}+1\right),\ x>0$$

Find the exact value of $f'(4)$.

$$\boxed{f'(4)=\frac{33}{8}}$$



Question 124 (*+)**

The curve C has equation

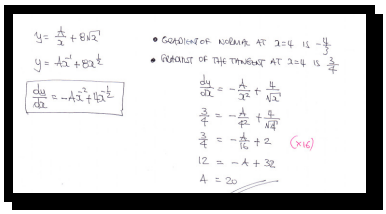
$$y=\frac{A}{x}+8\sqrt{x},\ x>0,$$

where A is a non zero constant.

The normal to the curve at C, at the point where $x=4$, has gradient $-\frac{4}{3}$.

Find the value of A.

$$\boxed{A=20}$$



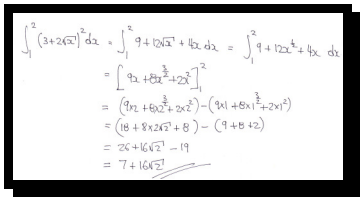
Question 125 (*+)**

Find the exact value of

$$\int_1^2 \left(3+2\sqrt{x}\right)^2 \, dx ,$$

giving the answer in the form $a+b\sqrt{2}$, where a and b are integers.

$\boxed{7+16\sqrt{2}}$



Question 126 (*+)**

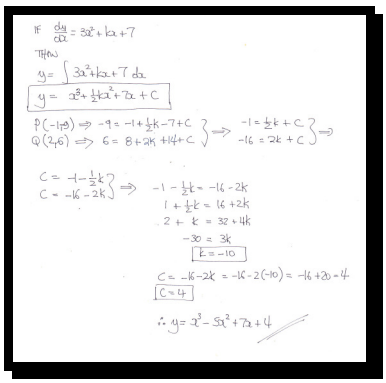
A cubic curve C passes through the points $P(-1,-9)$ and $Q(2,6)$ and its gradient function is given by

$$\frac{dy}{dx} = 3x^2 + kx + 7 ,$$

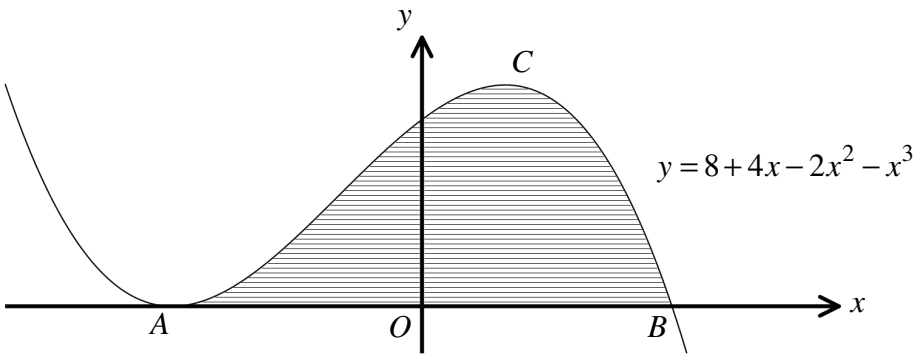
where k is a non zero constant.

Find an equation for C.

$\boxed{}$, $\boxed{y = x^3 - 5x^2 + 7x + 4}$



Question 127 **(***+)**



The figure above shows part of the curve with equation

$$y = 8 + 4x - 2x^2 - x^3 .$$

The curve meets the x axis at A and B.

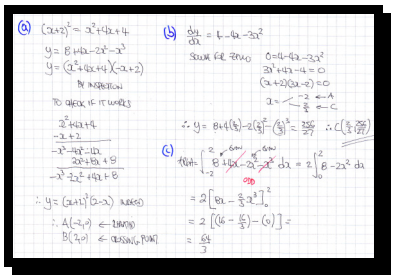
a) Verify that the coordinates of A are $(-2,0)$ and hence use algebra to show that the coordinates of B are $(2,0)$.

The point C is a stationary point of the curve.

b) Use calculus to determine the exact coordinates of C.

c) Find the exact area of the finite region bounded by the curve and the x axis.

$\square$, $\boxed{C\left(\frac{2}{3},\frac{256}{27}\right)}$, $\boxed{\text{area} = \frac{64}{3}}$



Question 128 (*+)**

The curve C has equation

$$y=(x-2)\left(x^2-2x-3\right),\ x\in\mathbb{R}.$$

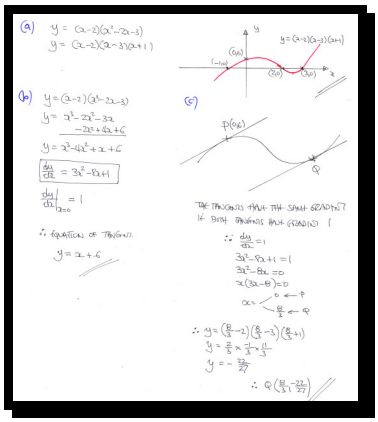
a) Sketch the graph of C, indicating the coordinates of any points where the curve meets the coordinate axes.

b) Find an equation of the tangent to C at the point P, where P is the point where C crosses the y axis.

The point Q lies on C so that the tangent to the curve at Q is parallel to the tangent to the curve at P.

c) Determine the exact coordinates of Q.

$$\boxed{(-1,0),(2,0),(3,0),(0,6)},\ \boxed{y=x+6},\ \boxed{Q\left(\frac{8}{3},-\frac{22}{27}\right)}$$



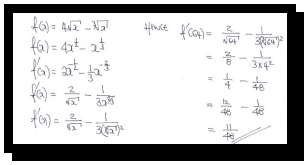
Question 129 (*+) non calculator**

$$f(x)=4\sqrt{x}-\sqrt[3]{x}\,,\ x\geq 0\,.$$

Show clearly that

$$f'(64)=\frac{11}{48}\,.$$

proof



Question 130 (*+)**

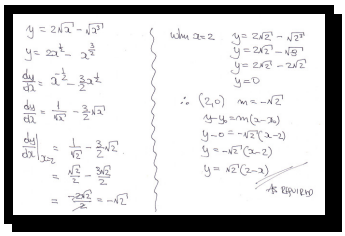
The curve C has equation

$$y=2\sqrt{x}-\sqrt{x^3}\,,\ x\geq 0\,.$$

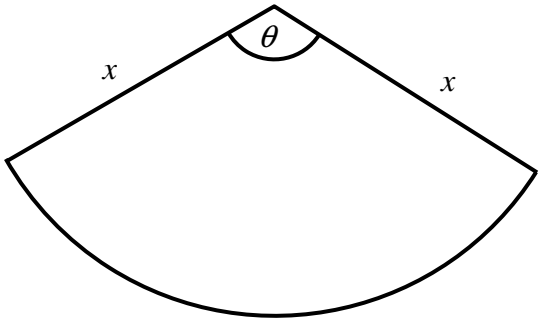
Show that the equation of the tangent to C at the point where $x=2$, can be written as

$$y=\sqrt{2}\,(2-x)$$

proof



Question 131 **(***+)**



A circular sector of radius x cm subtends an angle of θ radians at the centre.

The area of the sector is 36 cm^2 and its perimeter is P cm.

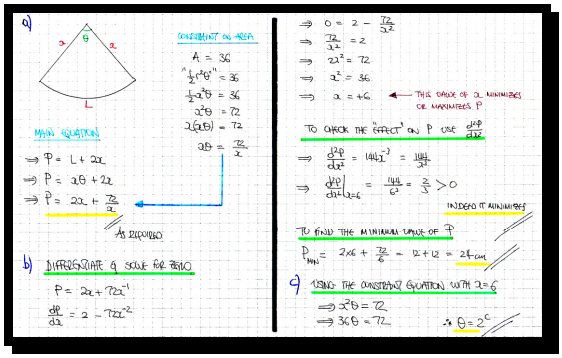
a) Show clearly that

$$P=2x+\frac{72}{x}.$$

b) Find the minimum value of P, fully justifying the fact that it is a minimum.

c) Deduce the value of θ when P is minimum.

$\boxed{}$, $\boxed{P_{\min}=24}$, $\boxed{\theta=2^{\text{c}}}$



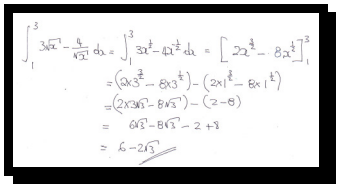
Question 132 (*+)**

Find the exact value of

$$\int_1^3 3\sqrt{x}-\frac{4}{\sqrt{x}}\,dx\,,$$

giving the answer in the form $p+q\sqrt{3}$, where p and q are integers.

$\boxed{6-2\sqrt{3}}$



$$\int_1^3 3\sqrt{x}-\frac{4}{\sqrt{x}}\,dx=\int_1^3 3x^{\frac{1}{2}}-4x^{-\frac{1}{2}}\,dx=\left[2x^{\frac{3}{2}}-8x^{\frac{1}{2}}\right]_1^3$$

$$=\left(2\times 3^{\frac{3}{2}}-8\times 3^{\frac{1}{2}}\right)-\left(2\times 1^{\frac{3}{2}}-8\times 1^{\frac{1}{2}}\right)$$

$$=\left(2\times 3\sqrt{3}-8\sqrt{3}\right)-(2-8)$$

$$=6\sqrt{3}-8\sqrt{3}-2+8$$

$$=6-2\sqrt{3}$$

Question 133 (*+)**

A curve C has equation

$$y=4x^3+7x^2+x+11,\quad x\in\mathbb{R}.$$

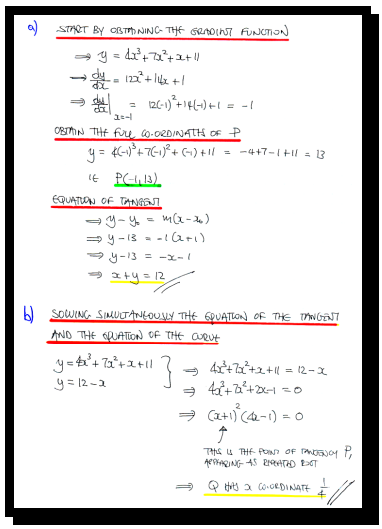
The point P lies on C, where $x=-1$.

a) Find an equation of the tangent to C at P.

The tangent to C at P meets C again at the point Q.

b) Determine the x coordinate of Q.

$\boxed{}$, $\boxed{y=12-x}$, $\boxed{x_Q=\frac{1}{4}}$



Question 134 **(***+)**

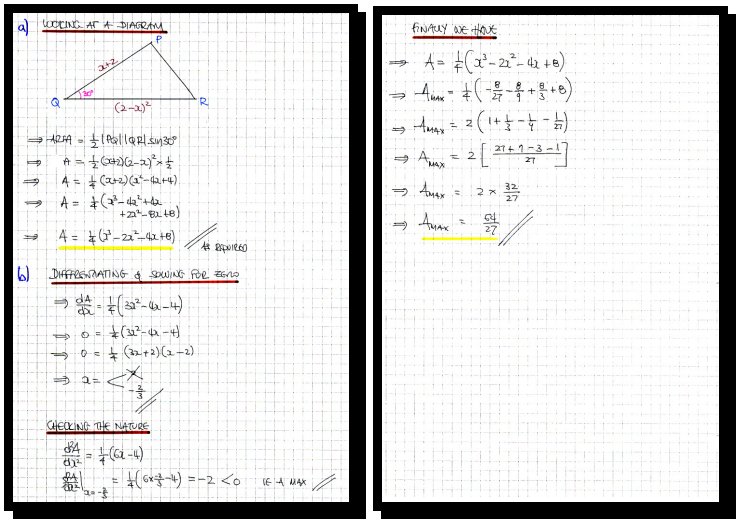
A triangle PQR has $|PQ| = x+2$ cm, $|QR| = (2-x)^2$ cm and $\angle PQR = 30°$.

a) Show that the area of the triangle, A cm^2, is given by

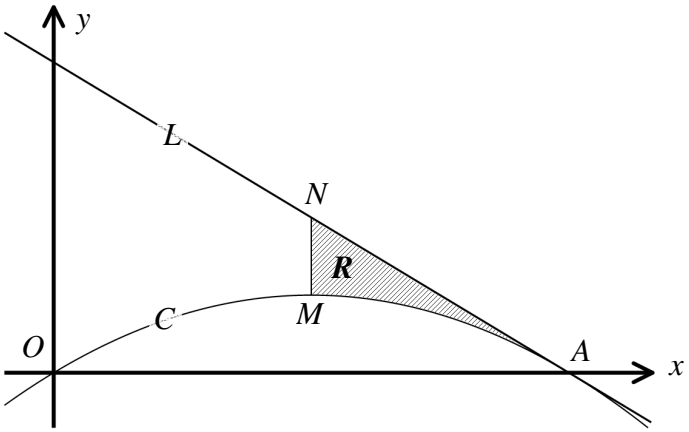
$$A = \frac{1}{4}\left(x^3 - 2x^2 - 4x + 8\right).$$

b) Determine the value of x for which A is stationary and hence find, with justification, the greatest value of A.

$\boxed{}$, $\boxed{x = -\frac{2}{3}}$, $\boxed{A_{\max} = \frac{64}{27} \approx 2.37}$



Question 135 **(***+)**



The figure above shows the graph of the curve C with equation

$$y=6x-x^2,\ x\in\mathbb{R}.$$

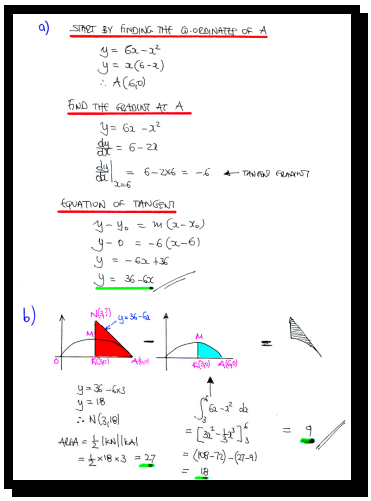
The curve meets the x axis at the origin O and at the point A. The straight line L is the tangent to C at A.

a) Find an equation of L.

The point M is the maximum point of C. The point N lies on L so that MN is parallel to the y axis. The finite region R, shown shaded in the figure above, is bounded by C, L and the straight line segment MN.

b) Determine the area of R.

$\boxed{}$, $\boxed{y=36-6x}$, $\boxed{9}$



Question 136 (*+)**

The curve C has equation

$$y=\frac{(x+4)^2}{\sqrt{x}},\ x>0.$$

a) Show that the gradient function of C is

$$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+4x^{-\frac{1}{2}}-8x^{-\frac{3}{2}}.$$

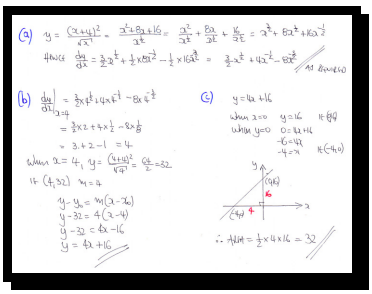
The point P lies on C where $x=4$.

The straight line L is the tangent to C at the point P.

b) Find an equation of L.

c) Find the area of the triangle OAB, where A and B are the points where L crosses the coordinate axes, and O is the origin.

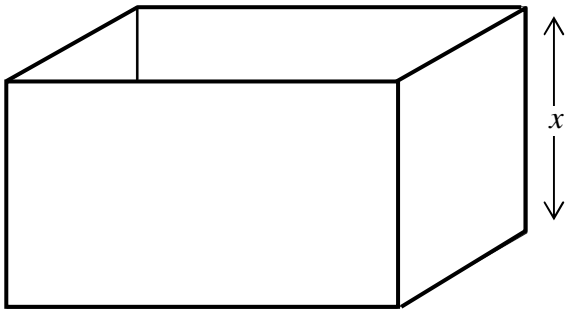
$\boxed{y=4x+16}$, $\boxed{\text{area}=32}$



Question 137 (*+)**

The figure below shows a large tank in the shape of a cuboid with a **rectangular** base and **no top**.

Two of the vertical opposite faces of the cuboid are square and the height of the cuboid is x metres.

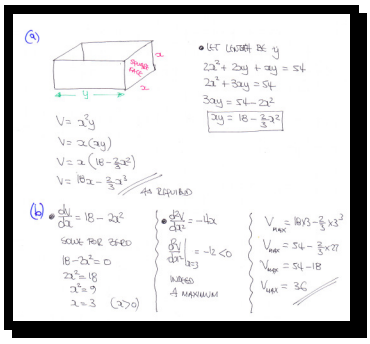


a) Given that the surface area of the tank is 54 m^2, show that the capacity of the tank, $V\text{ m}^3$, is given by

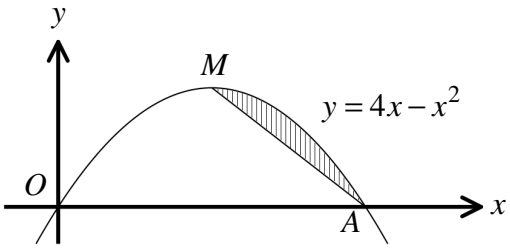
$$V=18x-\frac{2}{3}x^3.$$

b) Find the maximum value for V, fully justifying the fact that it is indeed the maximum value.

$$\boxed{V_{\max}=36}$$



Question 138 **(***+)**



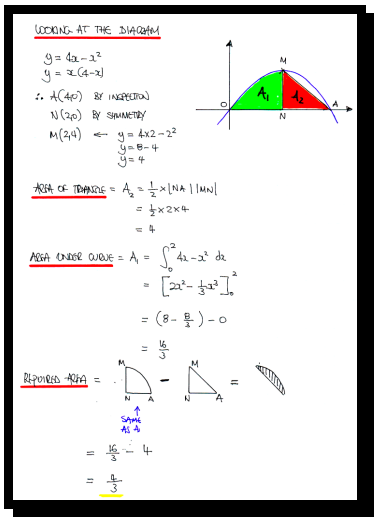
The figure above shows the curve with equation

$$y = 4x - x^2 , \ x \in \mathbb{R} .$$

The point M is the maximum point of the curve and the point A is one of the x intercepts of the curve.

Find the exact area of the shaded region, bounded by the curve and the straight line segment joining A and M.

$\square$, $\boxed{\text{area} = \frac{4}{3}}$



Question 139 (*+)**

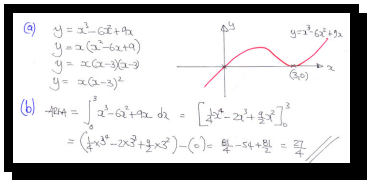
The curve C has equation

$$y = x^3 - 6x^2 + 9x .$$

a) Sketch the graph of C, indicating clearly the coordinates of the points where the graph meets the coordinate axes.

b) Determine the exact value of the area of the finite region bounded by the curve and the x axis .

$\boxed{(0,0)\ \&\ (3,0)}$, $\boxed{\text{area} = \frac{27}{4}}$



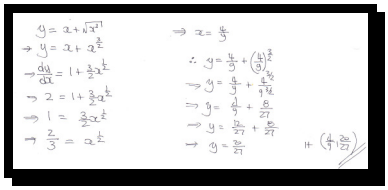
Question 140 (*+) non calculator**

The curve C has equation

$$y = x + \sqrt{x^3}\ , \quad x \neq 0$$

Find the exact coordinates of the point on the curve where the gradient is 2.

$\boxed{\left(\frac{4}{9}, \frac{20}{27}\right)}$



Question 141 (*+)**

The curve C has equation

$$y = 4x^3 - 7x - 1, \ x \in \mathbb{R}.$$

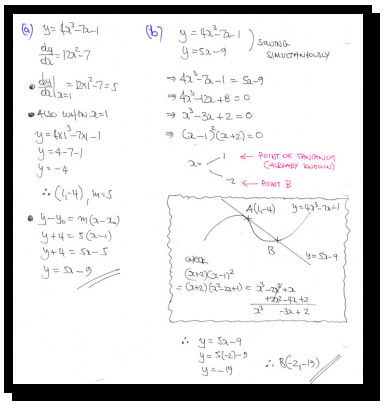
The point A lies on C where $x = 1$.

a) Find an equation of the tangent to C at A.

The tangent to C at A meets C again at the point B.

b) Find the coordinates of B.

$\square$, $\boxed{y = 5x - 9}$, $\boxed{B(-2,-19)}$



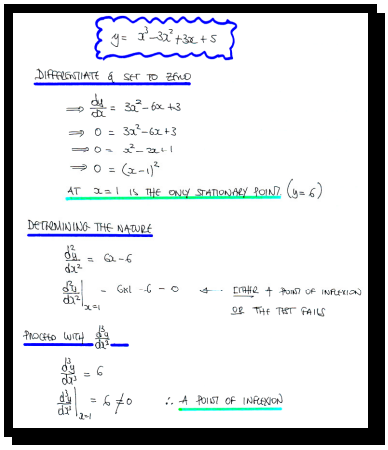
Question 142 (*+)**

The curve C has equation

$$y = x^3 - 3x^2 + 3x + 5 .$$

Show that C has only one stationary point and determine its nature.

$\boxed{}$, $\boxed{\text{point of inflection at } (1,6)}$

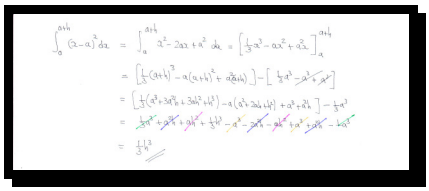


Question 143 (*+)**

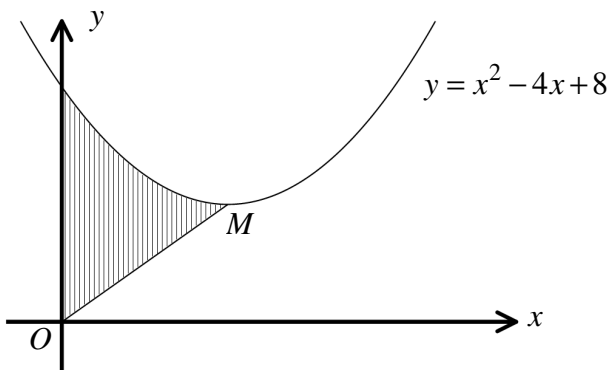
Prove by showing all the parts in the calculation that

$$\int_a^{a+h} (x-a)^2 \, dx = \frac{1}{3}h^3 .$$

$\boxed{\text{proof}}$



Question 144 **(***+)**



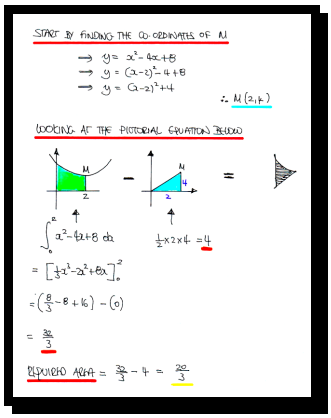
The figure above shows the curve with equation

$$y = x^2 - 4x + 8, \ x \in \mathbb{R}.$$

The point M is the minimum point of the curve.

Find the area of the shaded region, bounded by the curve, the y axis and the straight line segment from O to M.

$\square$, $\boxed{\text{area} = \frac{20}{3}}$



Question 145 **(***+)**

The curve C has equation

$$y = x^3 + 2x^2 - 7, \ x \in \mathbb{R}.$$

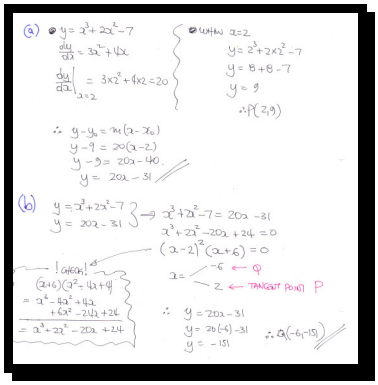
The point P lies on C where $x = 2$.

a) Find an equation of the tangent to C at P.

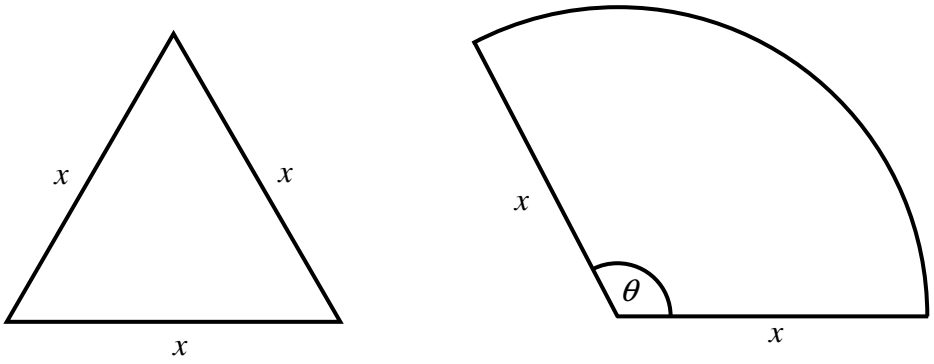
The tangent to C at P meets C again at the point Q.

b) Show that the coordinates of Q are $(-6, -151)$.

$$\boxed{y = 20x - 31}$$



Question 146 **(***+)**



A wire of total length 60 cm is to be cut into two pieces. The first piece is bent to form an equilateral triangle of side length x cm and the second piece is bent to form a circular sector of radius x cm. The circular sector subtends an angle of θ radians at the centre.

a) Show that

$$x\theta = 60 - 5x \, .$$

The **total** area of the **two** shapes is A cm^2.

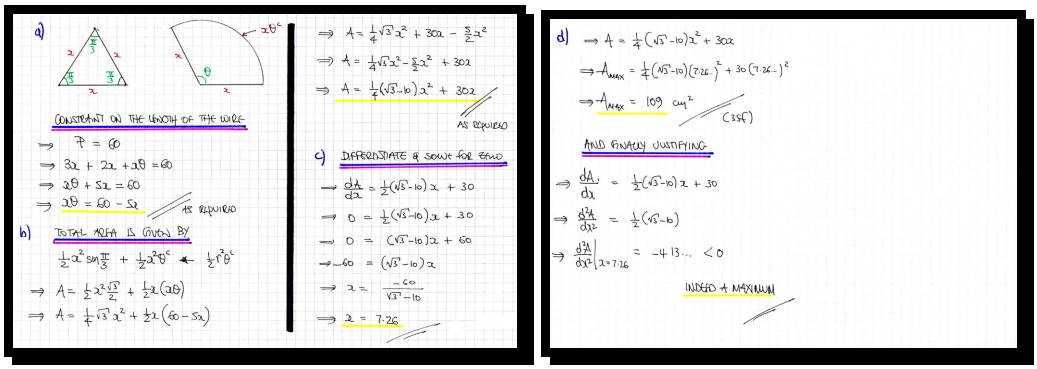
b) Show clearly that

$$A = \frac{1}{4}\left(\sqrt{3} - 10\right)x^2 + 30x \, .$$

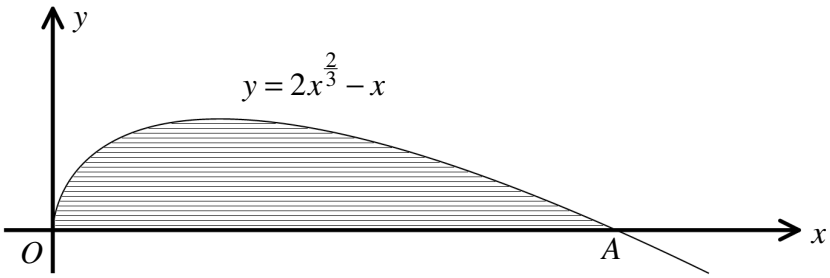
c) Use differentiation to find the value of x for which A is stationary.

d) Find, correct to three significant figures, the maximum value of A, justifying the fact that it is indeed the maximum value of A.

$\boxed{}$, $\boxed{x \approx 7.26}$, $\boxed{A_{\max} \approx 109}$



Question 147 (*+)**



The figure above shows part of the curve with equation

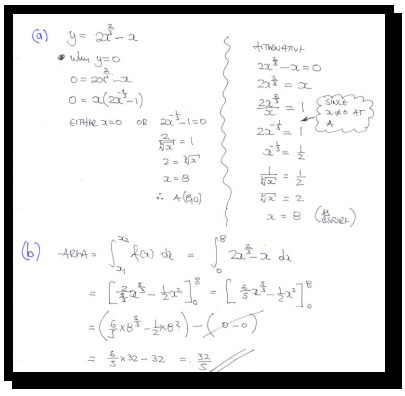
$$y = 2x^{\frac{2}{3}} - x\,,\ x \geq 0\,.$$

The curve meets the x axis at the point A.

a) Show that the coordinates of A are $(8,0)$.

b) Find the exact area of the finite region bounded by the curve and the x axis .

$$\boxed{\text{area} = \frac{32}{5}}$$



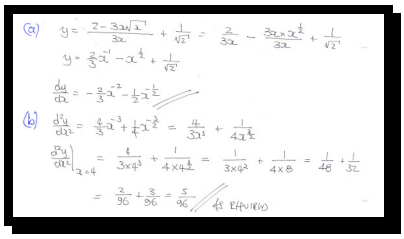
Question 148 (*+) non calculator**

$$y=\frac{2-3x\sqrt{x}}{3x}+\frac{1}{\sqrt{2}},\ x>0.$$

a) Find an expression for $\dfrac{dy}{dx}$.

b) Show that the value of $\dfrac{d^2y}{dx^2}$ where $x=4$, is $\dfrac{5}{96}$.

$$\boxed{\frac{dy}{dx}=-\frac{2}{3}x^{-2}-\frac{1}{2}x^{-\frac{1}{2}}}$$



Question 149 (*+)**

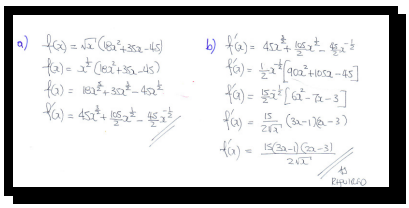
$$f(x)=\sqrt{x}\left(18x^2+35x-45\right),\ x\geq 0\,.$$

a) Find a simplified expression for $f'(x)$.

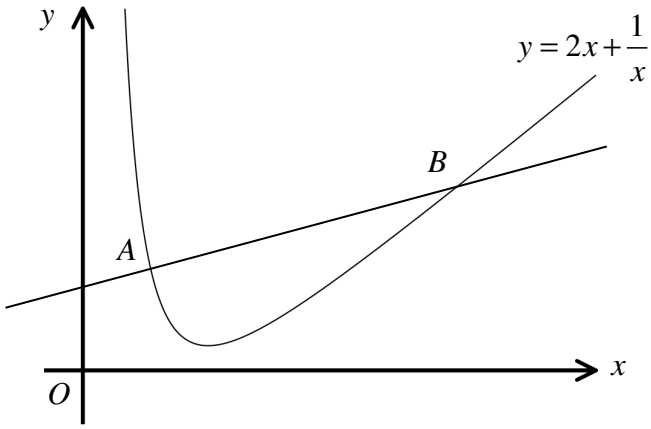
b) Show clearly that

$$f'(x)=\frac{15(3x-1)(2x-3)}{2\sqrt{x}}\,.$$

$$\boxed{f'(x)=45x^{\frac{3}{2}}+\frac{105}{2}x^{\frac{1}{2}}-\frac{45}{2}x^{-\frac{1}{2}}}$$



Question 150 **(***+)**



The figure above shows part of the curve C, with equation

$$y = 2x + \frac{1}{x}, \quad x \neq 0$$

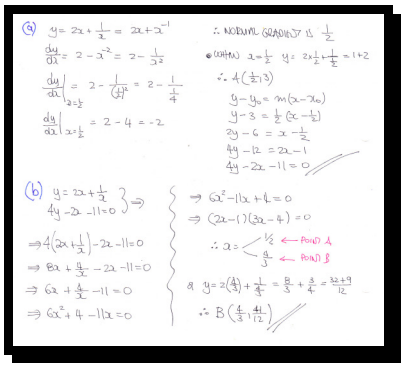
The point A lies on C where $x = \frac{1}{2}$.

a) Find an equation of the normal to C at A.

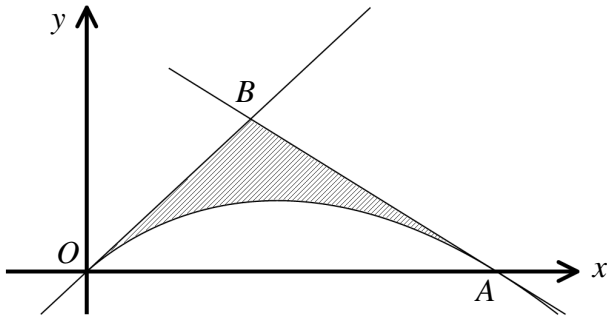
The normal meets the curve again at the point B.

b) Determine the exact coordinates of B.

$\square$, $\boxed{2x - 4y + 11 = 0}$, $\boxed{B\left(\frac{4}{3}, \frac{41}{12}\right)}$



Question 151 **(***+)**



The figure above show the graph of the curve C with equation

$$y=x\left(1-x^{\frac{2}{3}}\right),\ x\in\mathbb{R},\ x\geq 0.$$

The curve meets the coordinate axes at the origin and at the point $A(1,0)$.

The two tangents to C at the origin O and at the point A, meet at the point B.

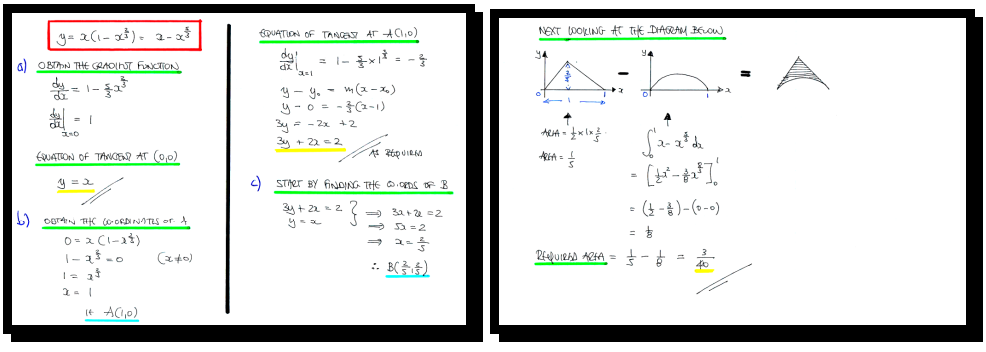
a) Calculate the value of $\dfrac{dy}{dx}$ at O, and hence write down an equation of the tangent to C at O.

b) Show that an equation of the tangent to C at A is

$$2x+3y=2$$

c) Determine the area of the finite region bounded by C and the tangents to C at O and at A.

$\boxed{}$, $\boxed{y=x}$, $\boxed{\text{area}=\dfrac{3}{40}}$



Question 152 (*+)**

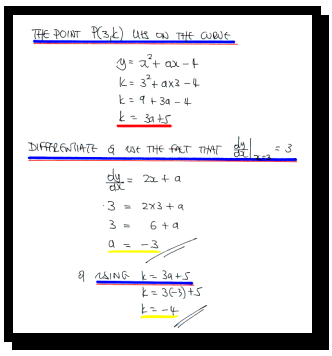
The point $P(3,k)$ lies on the curve with equation

$$y = x^2 + ax - 4,$$

where a and k are constants.

Given that the gradient at P is 3 determine the value of a and the value of k.

$\boxed{}$, $\boxed{a=-3,\ \ k=-4}$



Question 153 **(***+)**

The curve C has equation

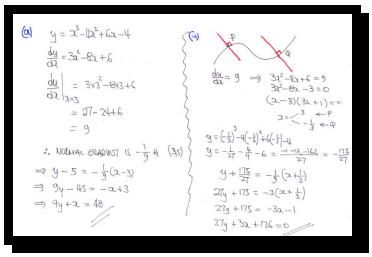
$$y = x^3 - 4x^2 + 6x - 4 .$$

a) Find an equation of the normal to C at the point $P(3,5)$.

The point Q lies on C so that the normal to the curve at Q is parallel to the normal to the curve at P.

b) Find an equation of the normal to the curve at Q.

$\boxed{x+9y=48}$, $\boxed{3x+27y+176=0}$



Question 154 **(***+)**

The point $P(1,0)$ lies on the curve C with equation

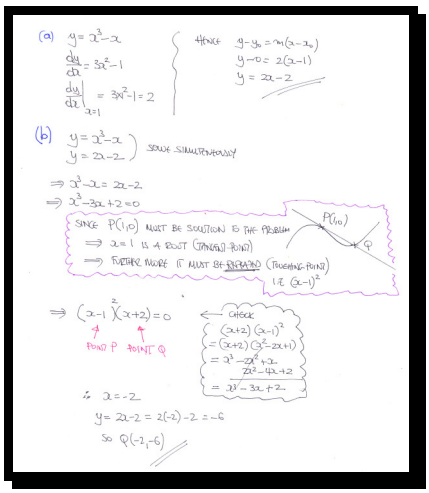
$$y = x^3 - x, \quad x \in \mathbb{R}.$$

a) Find an equation of the tangent to C at P, giving the answer in the form $y = mx + c$, where m and c are constants.

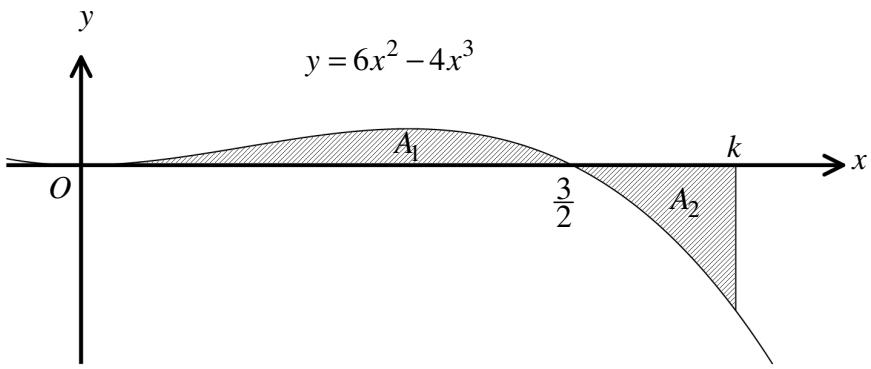
The tangent to C at P meets C again at the point Q.

b) Find the coordinates of Q.

$$\boxed{y = 2x - 2}, \quad \boxed{Q(-2,-6)}$$



Question 155 (*+)**



The figure above shows the graph of the curve with equation

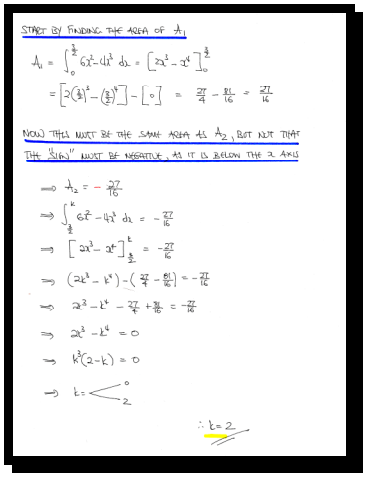
$$y=6x^2-4x^3,\ x\in\mathbb{R}.$$

The curve meets x axis at the origin O and at the point $\left(\frac{3}{2},0\right)$.

The point $(k,0)$, $k>\frac{3}{2}$ is such so that, the area A_1 of the region between the curve and the x axis for which $0\le x\le\frac{3}{2}$, **is equal** to the area A_2 of the region between the curve and the x axis for which $\frac{3}{2}\le x\le k$.

Determine the value of k.

$\boxed{}$, $\boxed{k=2}$


START BY FINDING THE AREA OF A_1

$A_1=\int_0^{\frac{3}{2}}6x^2-4x^3\,dx=\left[2x^3-x^4\right]_0^{\frac{3}{2}}$

$=\left[2\left(\frac{3}{2}\right)^3-\left(\frac{3}{2}\right)^4\right]-\left[0\right]=\frac{27}{4}-\frac{81}{16}=\frac{27}{16}$

NOW THIS MUST BE THE SAME AREA AS A_2, BUT NOT THAT THE $\int(\ldots)\,dx$ MUST BE NEGATIVE, AS IT IS BELOW THE x AXIS

$\Rightarrow A_2=-\frac{27}{16}$

$\Rightarrow\int_{\frac{3}{2}}^{k}6x^2-4x^3\,dx=-\frac{27}{16}$

$\Rightarrow\left[2x^3-x^4\right]_{\frac{3}{2}}^{k}=-\frac{27}{16}$

$\Rightarrow\left(2k^3-k^4\right)-\left(\frac{27}{4}-\frac{81}{16}\right)=-\frac{27}{16}$

$\Rightarrow 2k^3-k^4-\frac{27}{4}+\frac{81}{16}=-\frac{27}{16}$

$\Rightarrow 2k^3-k^4=0$

$\Rightarrow k^3(2-k)=0$

$\Rightarrow k=\begin{cases}0\\2\end{cases}$

$\therefore k=2$

Question 156 (*+)**

The curve C has equation

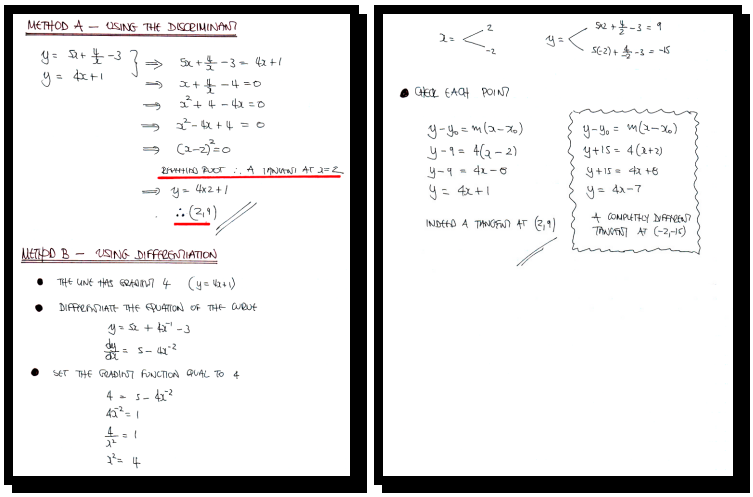
$$y=5x+\frac{4}{x}-3,\ x\neq 0$$

Show that the straight line with equation

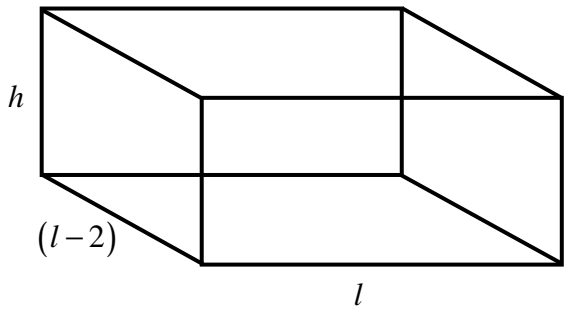
$$y=4x+1$$

is a tangent to C, and find the coordinates of the point of tangency.

$\boxed{}$, $\boxed{(2,9)}$



Question 157 **(***+)**



The figure above shows 12 rigid rods, joined together to form the framework of a storage container, which in the shape of a cuboid.

Each of the four upright rods has height h m. Each of the longer horizontal rods has length l m and each of the shorter horizontal rods have length $(l-2)$ m.

a) Given that the total length of the 12 rods is 36 m show that the volume, V m^3, of the container satisfies

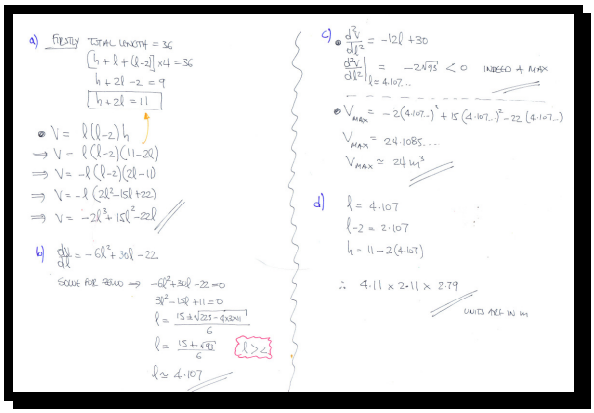
$$V=-2l^3+15l^2-22l\,.$$

b) Find, correct to 3 decimal places, the value of l which make V stationary.

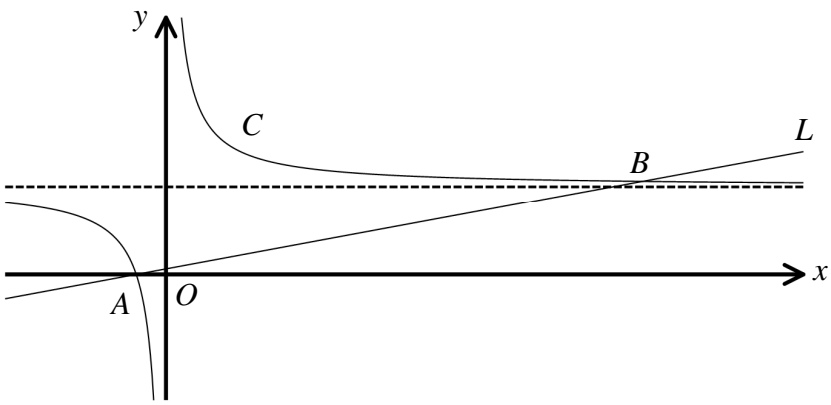
c) Justify that the value of l found in part (b) maximizes the value of V, and find this maximum value of V, correct to the nearest m^3.

d) State the three measurements of the container when its volume is maximum.

$\boxed{l=4.107}$, $\boxed{V_{\max}\approx 24}$, $\boxed{4.11\times 2.11\times 2.79}$



Question 158 **(***+)**



The figure above shows a sketch of the curve C with equation

$$y=2+\frac{1}{x},\ x\neq 0.$$

The dotted line represents an asymptote to C and the point A is the point where C crosses the x axis.

a) State the equations of the two asymptotes to the C.

b) Find the coordinates of A.

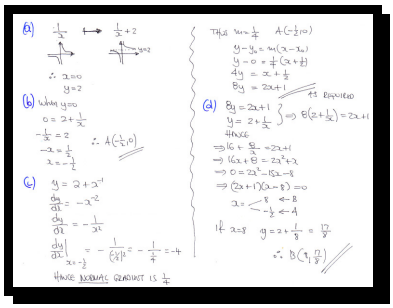
The straight line L is the normal to C at A, and B is the point where L meets C again.

c) Show that an equation for L is

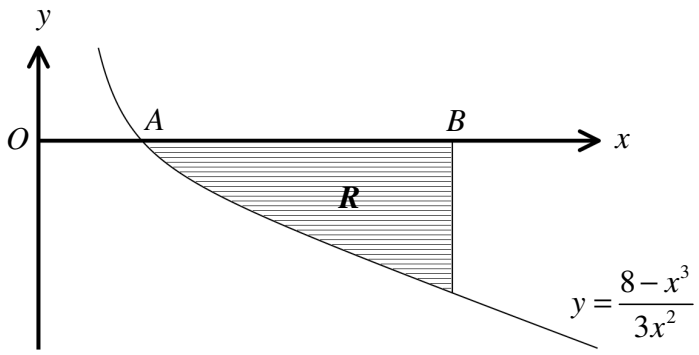
$$8y=2x+1.$$

d) Determine the coordinates of B.

$\square$, $\boxed{x=0,\ y=2}$, $\boxed{A\left(-\frac{1}{2},0\right)}$



Question 159 **(***+)**



The figure below shows part of the curve with equation

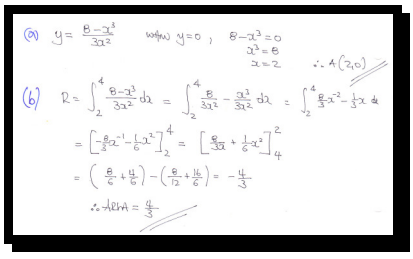
$$y=\frac{8-x^3}{3x^2},\ x>0.$$

The curve meets the x axis at the point A and B has coordinates $(4,0)$.

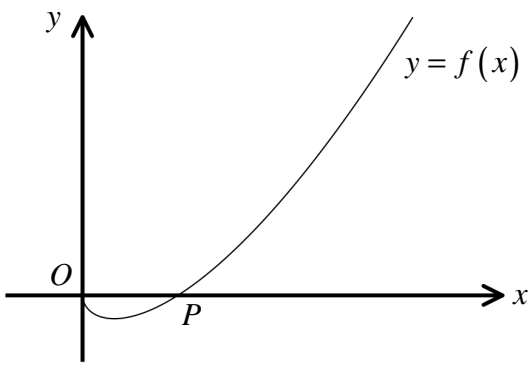
a) Find the coordinates of A.

b) Determine the exact area of the finite region R, bounded by the curve , the vertical line through B and the x axis .

$\boxed{C(4,0)}$, $\boxed{\text{area}=\frac{4}{3}}$



Question 160 (*+)**



The figure above shows a curve with equation $y=f\left(x\right)$ which meets the x axis at the origin O and at the point P.

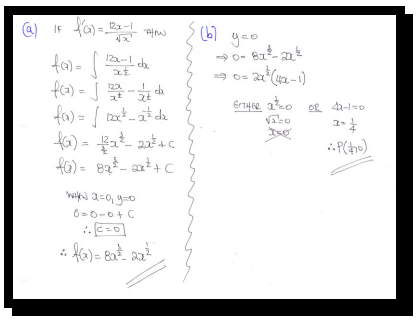
The gradient function of the curve is given by

$$f'\left(x\right)=\frac{12x-1}{\sqrt{x}},\ x>0.$$

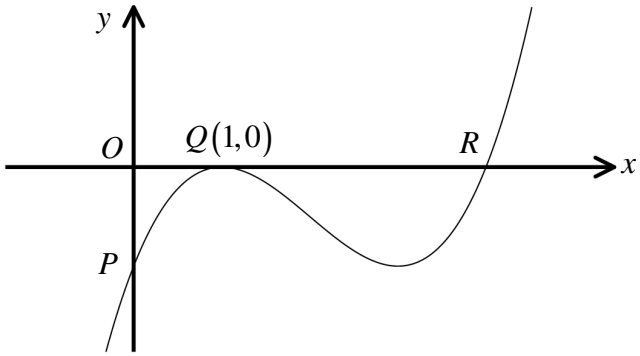
a) Find an equation of the curve.

b) Determine the coordinates of P.

$\square$, $\boxed{f\left(x\right)=8x^{\frac{3}{2}}-2\sqrt{x}}$, $\boxed{P\left(\frac{1}{4},0\right)}$



Question 161 **(***+)**



The figure above shows the graph of a cubic curve, which touches the x axis at the point $Q(1,0)$.

a) Determine an equation for the cubic curve , given its gradient function is

$$\frac{dy}{dx}=3x^2-12x+9\,.$$

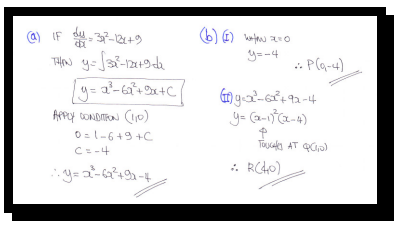
The cubic curve crosses the x axis and the y axis at the points R and P, respectively.

b) Determine the coordinates …

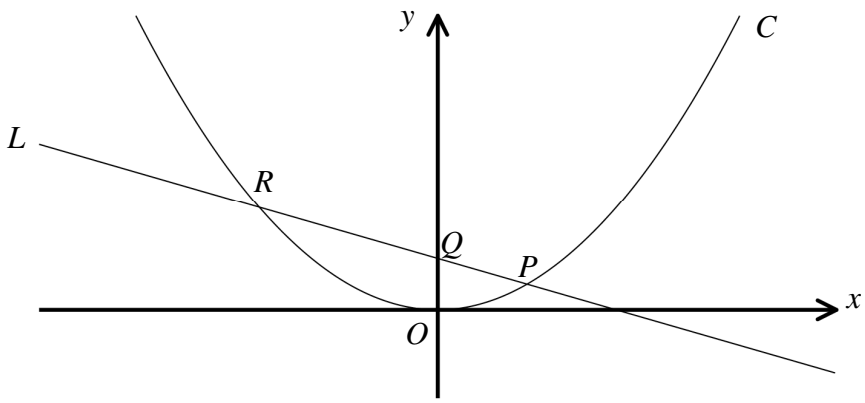
i. … of the point P.

ii. … of the point R.

$\square$, $\boxed{y=x^3-6x^2+9x-4}$, $\boxed{P(0,-4)}$, $\boxed{R(4,0)}$



Question 162 (*+)**



The figure above shows a sketch of the curve C with equation

$$y = 2x^2, \ x \in \mathbb{R}.$$

The straight line L passes through the points $P\left(\frac{1}{2},\frac{1}{2}\right)$ and $Q(0,1)$, where the point P lies on C.

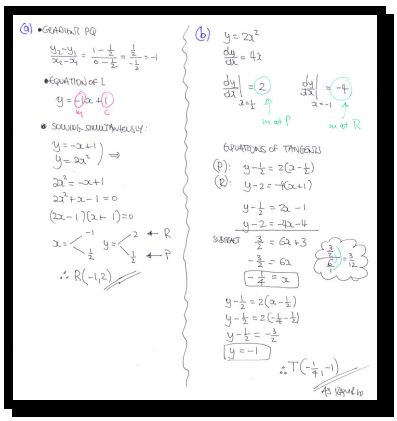
The straight line L meets C again, at the point R.

a) Find the coordinates of R.

The tangents to C at the points P and R meet at the point T.

b) Show that the coordinates of T are $\left(-\frac{1}{4},-1\right)$

$$\boxed{R(-1,2)}$$



Question 163 (*+)**

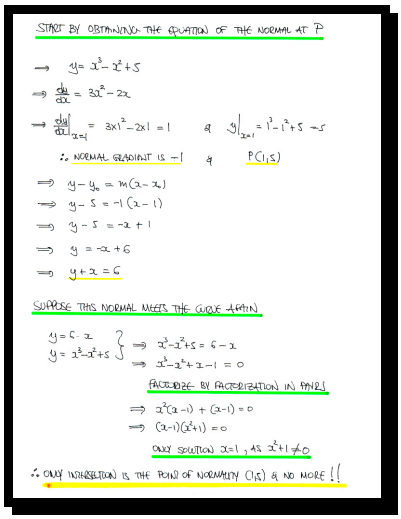
A cubic curve has equation

$$y = x^3 - x^2 + 5\,,\ x \in \mathbb{R}\,.$$

The point P lies on the curve where $x = 1$.

Show that the normal to the curve at P does not meet the curve again.

$\boxed{}$, proof



Question 164 (*+)**

The curve C has equation

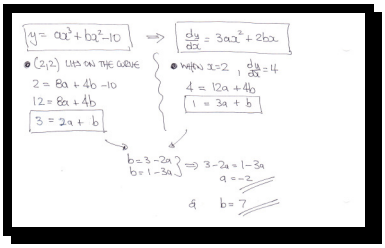
$$y = ax^3 + bx^2 - 10,$$

where a and b are constants.

The point $A(2,2)$ lies on C.

Given that the gradient at A is 4, determine the value of a and the value of b.

$\boxed{a = -2}$, $\boxed{b = 7}$



Question 165 (*+)**

The curve C has equation

$$y = 2x^3 - 6x^2 + 3x + 5 .$$

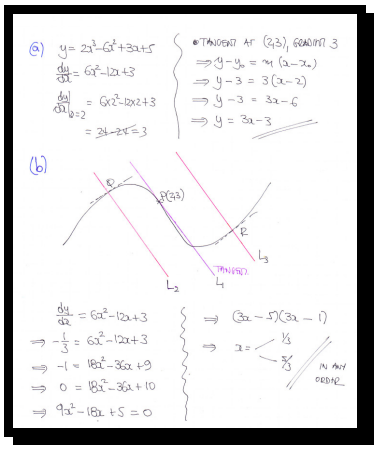
The point $P(2,3)$ lies on C and the straight line L_1 is the tangent to C at P.

a) Find an equation of L_1.

The straight lines L_2 and L_3 are parallel to L_1, and they are the respective normals to C at the points Q and R.

b) Determine the x coordinate of Q and the x coordinate of R.

$\boxed{}$, $\boxed{y = 3x - 3}$, $\boxed{x = \frac{1}{3}, \frac{5}{3}}$



Question 166 (*+)**

The point $P(2,9)$ lies on the curve C with equation

$$y = x^3 - 3x^2 + 2x + 9, \ x \in \mathbb{R}, \ x \geq 1.$$

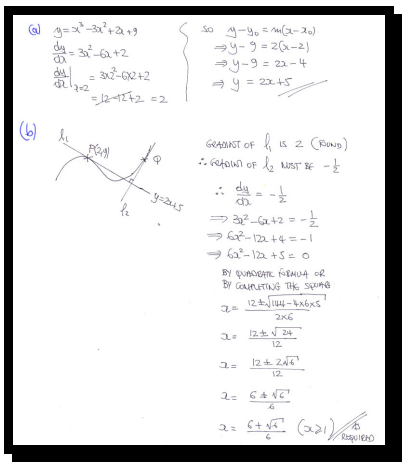
a) Find an equation of the tangent to C at P, giving the answer in the form $y = mx + c$, where m and c are constants.

The point Q also lies on C so that the tangent to C at Q is perpendicular to the tangent to C at P.

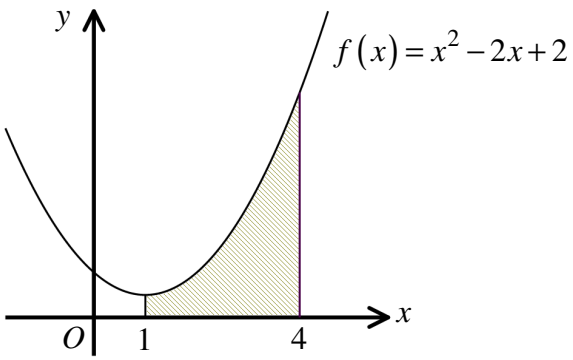
b) Show that the x coordinate of Q is

$$\frac{6+\sqrt{6}}{6}.$$

$\square$, $\boxed{y = 2x + 5}$



Question 167 **(***+)**



The curve C has equation

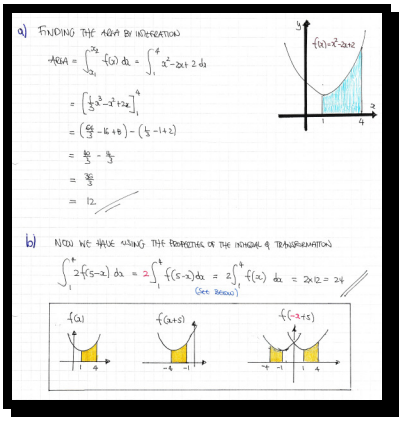
$$f(x)=x^2-2x+2, \quad x\in\mathbb{R}.$$

a) Find the area of the finite region bounded by C, the x axis and the straight lines with equations $x=1$ and $x=4$, shown shaded in the figure above.

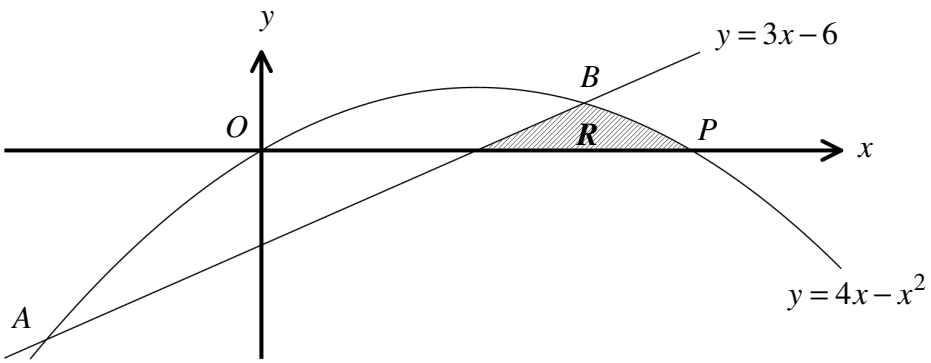
b) Hence evaluate

$$\int_1^4 2f(5-x)\ dx.$$

[ ], [12], [24]



Question 168 (****)



The figure above shows the graph of the curve C with equation

$$y = 4x - x^2 , \ x \in \mathbb{R} ,$$

intersected by the straight line L with equation

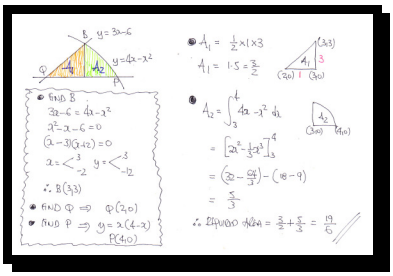
$$y = 3x - 6 , \ x \in \mathbb{R} .$$

As shown in the above figure, C meets L at the points A and B, and crosses the x axis at the origin O and at the point P.

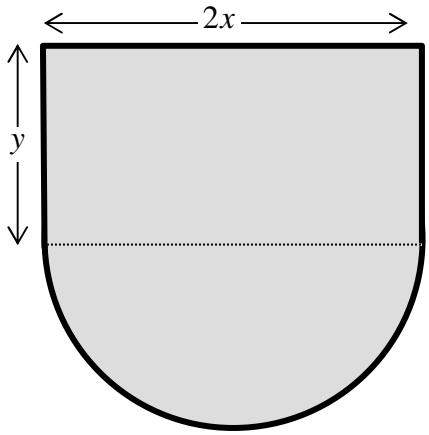
The finite region R is bounded by C, L and the x axis.

Show that the area of R, shown shaded in the figure, is $\frac{19}{6}$.

[ ], [proof]



Question 169 **(****)**



The figure above shows the design of a theatre stage which is the shape of a semicircle attached to rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

The perimeter of the stage is 60 m.

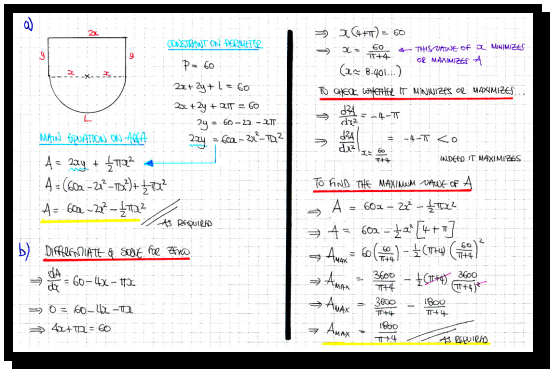
a) Show that the total area of the stage, A m^2, is given by

$$A = 60x - 2x^2 - \frac{1}{2}\pi x^2 .$$

b) Show further, by using a **differentiation** method, that the maximum area of the stage is

$$\frac{1800}{\pi + 4} \text{ m}^2 .$$

[ ], [proof]



Question 170 (****)

The curve C and the straight line L have respective equations

$$y = 2x^2 - 6x + 5 \quad \text{and} \quad 2y + x = 4 .$$

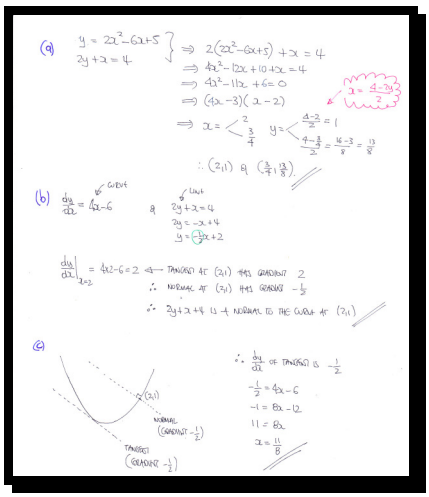
a) Find the coordinates of the points of intersection between C and L.

b) Show that L is a normal to C.

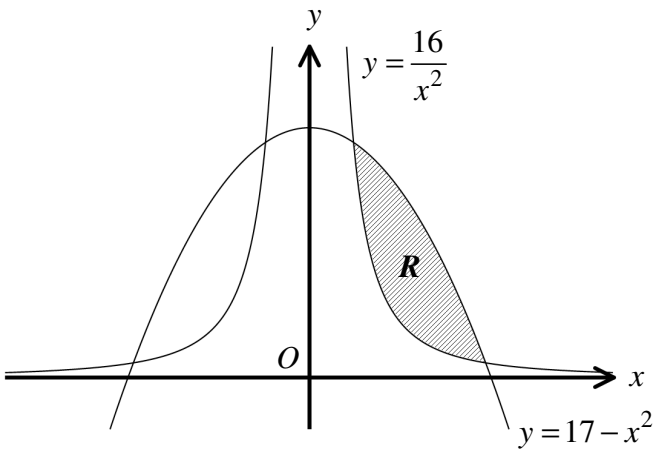
The tangent to C at the point P is parallel to L.

c) Determine the x coordinate of P.

$\square$, $\boxed{(2,1), \left(\frac{3}{4}, \frac{13}{8}\right)}$, $\boxed{x_P = \frac{11}{8}}$



Question 171 (****)



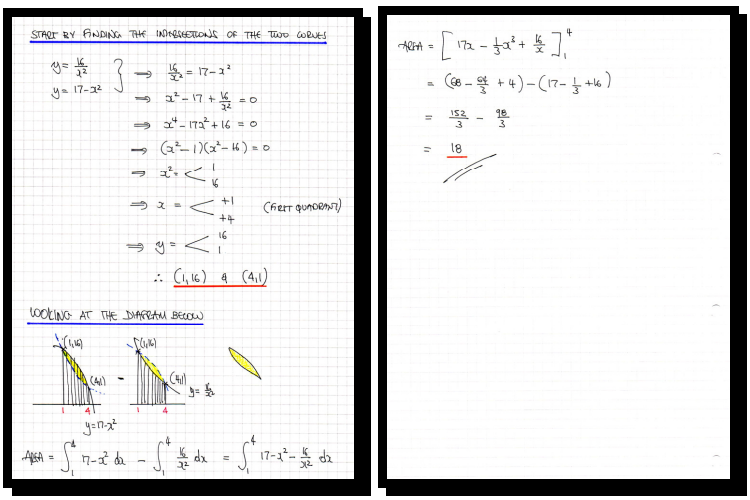
The figure above shows the graphs of the curves with equations

$$y=\frac{16}{x^2} \qquad \text{and} \qquad y=17-x^2 .$$

The finite region R, shown shaded in the figure above, is bounded by the two curves in the first quadrant.

Find the area of R.

, 18



Question 172 (****)

The curve C and the straight line L have equations

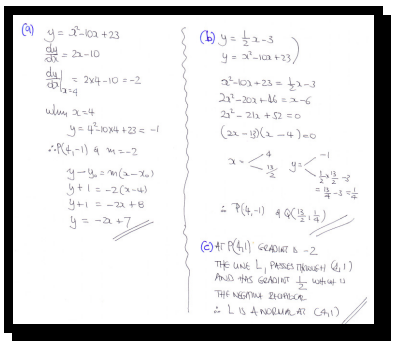
$$C:\ y=x^2-10x+23 \quad \text{and} \quad L:\ y=\frac{1}{2}x-3\,.$$

a) Find an equation of the tangent to C at the point P, where $x=4$.

b) Determine the coordinates of the points of intersection between L and C.

c) Show that L is a normal to C.

$\boxed{}$, $\boxed{y=-2x+7}$, $\boxed{P(4,-1),\ Q\left(\frac{13}{2},\frac{1}{4}\right)}$

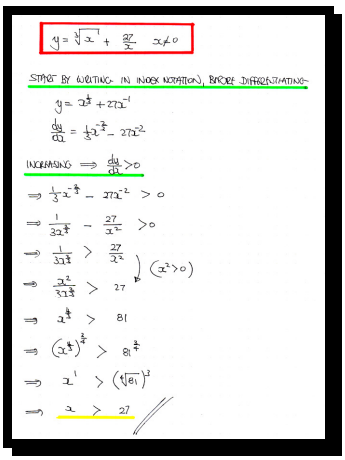


Question 173 (****)

$$y=\sqrt[3]{x}+\frac{27}{x},\ x\neq 0.$$

Find the range of values of x for which y is increasing.

$\boxed{}$, $\boxed{x>27}$



Question 174 (****)

The points $(-2,-1)$ and $(1,-4)$ lie on the curve C with equation $y=f(x)$.

The gradient function of C is given by

$$\frac{dy}{dx}=3x^2+4x+k\ ,$$

where k is a constant.

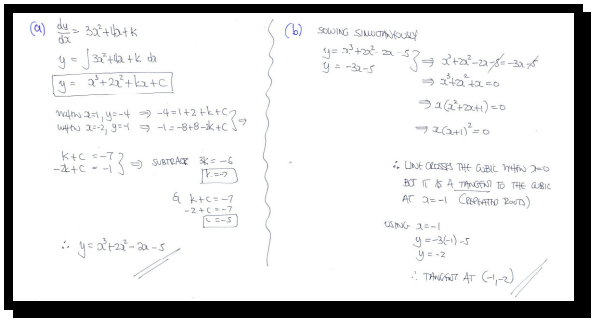
a) Find an equation of C, in the form $y=f(x)$.

The straight line L has equation

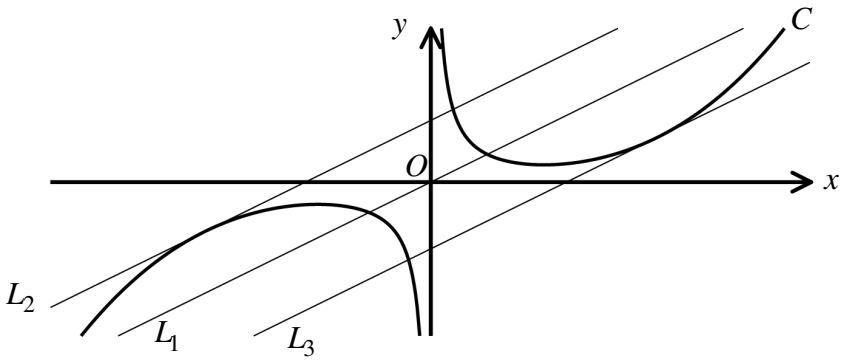
$$y=-3x-5\,.$$

b) Show that L is a tangent to C and determine further the coordinates of the point of tangency.

$\square$, $\boxed{y=x^3+2x^2-2x-5}$, $\boxed{(-1,-2)}$



Question 175 (****)



The figure above shows the curve C with equation

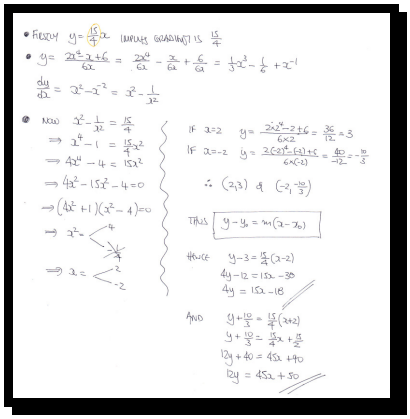
$$y=\frac{2x^4-x+6}{6x},\ x\neq 0.$$

The straight line L_1 has equation $4y=15x$.

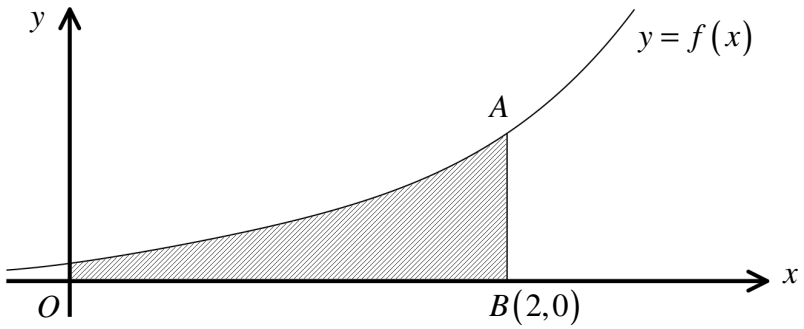
The straight lines L_2 and L_3 are tangents to C.

Given that L_1, L_2 and L_3 are parallel to one another, determine an equation of L_2 and an equation of L_3.

$\square$, $\boxed{4y=15x-18 \text{ or } y=\frac{15}{4}x-\frac{9}{2}}$, $\boxed{12y=45x+50 \text{ or } y=\frac{15}{4}x+\frac{25}{6}}$



Question 176 (****)



The figure above shows part the curve C with equation $y=f\left(x\right)$.

The gradient function of this curve is given by

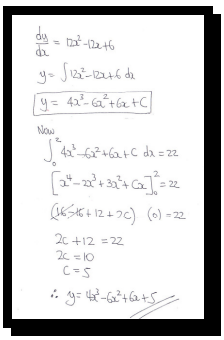
$$\frac{dy}{dx}=12x^2-12x+6.$$

The point A lies on C and the point $B\left(2,0\right)$ lies on the x axis, so that the straight line segment AB is parallel to the y axis.

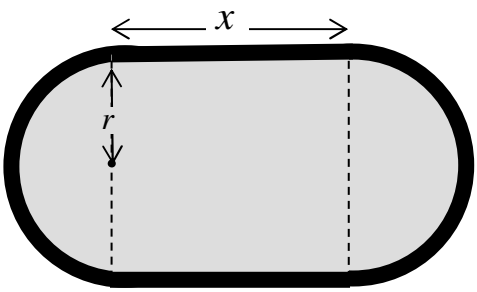
The area of the finite region bounded by C, the coordinate axes and the straight line segment AB, shown shaded in the figure, is 22 square units.

Find an equation of C.

$\square$, $\boxed{y=x^4-2x^3+3x^2+5x+5}$



Question 177 (****)



The figure above shows the design of an athletics track inside a stadium.

The track consists of two semicircles, each of radius r m, joined up to a rectangular section of length x metres.

The total length of the track is 400 m and encloses an area of A m^2.

a) By obtaining and manipulating expressions for the total length of the track and the area enclosed by the track, show that

$$A = 400r - \pi r^2 .$$

In order to hold field events safely, it is required for the area enclosed by the track to be as large as possible.

b) Determine by **differentiation** an exact value of r for which A is stationary.

c) Show that the value of r found in part **(b)** gives the maximum value for A.

d) Show further that the maximum area the area enclosed by the track is

$$\frac{40000}{\pi} \text{ m}^2 .$$

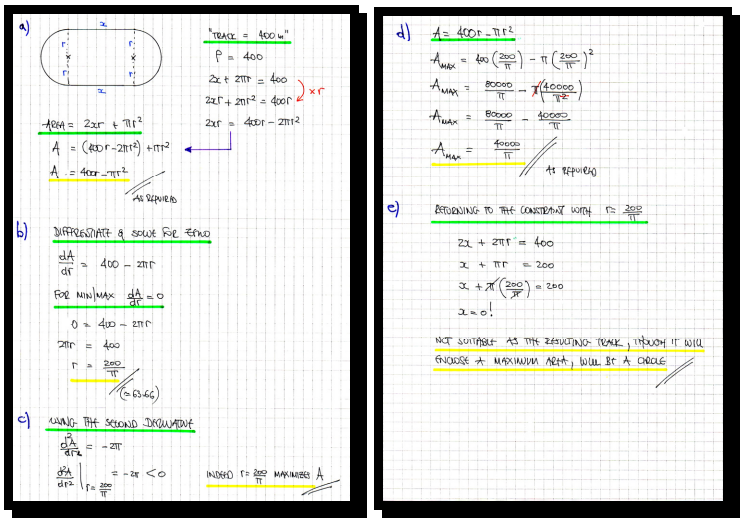
[continues overleaf]

[continued from overleaf]

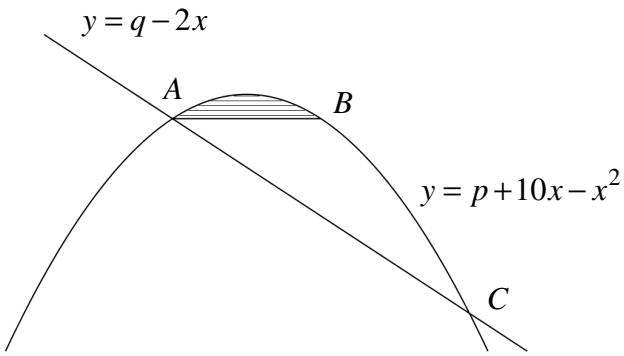
The calculations for maximizing the area of the field within the track are shown to a mathematician. The mathematician agrees that the calculations are correct but he feels the resulting shape of the track might not be suitable.

e) Explain, by calculations, the mathematician's reasoning.

$\square$, $\boxed{r = \dfrac{200}{\pi} \approx 63.66}$



Question 178 **(****)**



The figure above shows a curve and a straight line with respective equations

$$y=p+10x-x^2 \quad \text{and} \quad y=q-2x,$$

where p and q are constants.

The curve meets the straight line at the points A and C, and the point B lies on the curve so that AB is parallel to the x axis.

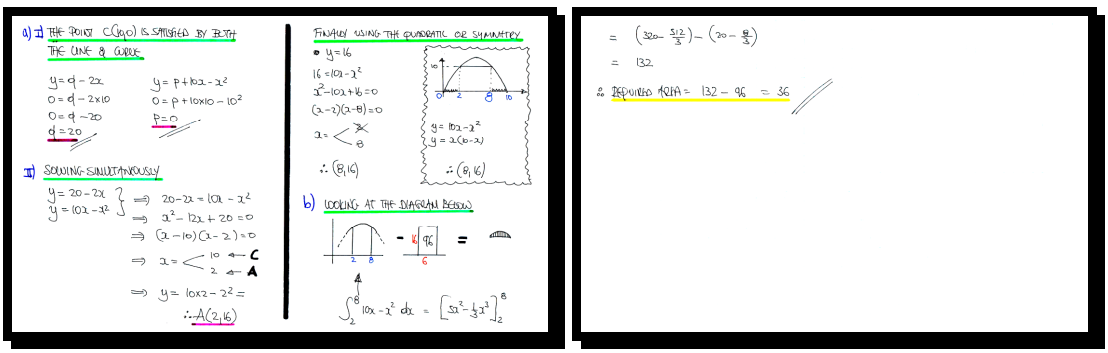
a) Given the coordinates of C are $(10,0)$ find …

i. … the value of p and the value of q.

ii. … the coordinates of A and B.

a) Determine the value of the area of the finite region bounded by the curve and the straight line segment AB.

$\boxed{}$, $\boxed{p=0}$, $\boxed{q=20}$, $\boxed{A(2,16)}$, $\boxed{B(8,16)}$ $\boxed{\text{area}=36}$



Question 179 (****)

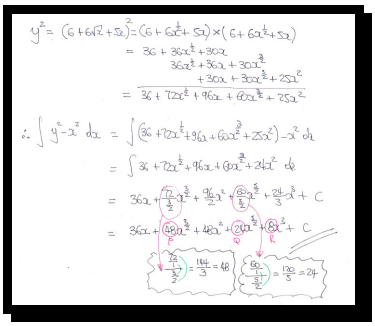
$$y = 6 + 6\sqrt{x} + 5x\,,\ x \geq 0\,.$$

Show clearly that

$$\int \left(y^2 - x^2 \right) dx = 36x + Px^{\frac{3}{2}} + 48x^2\sqrt{x} + Qx^{\frac{5}{2}} + Rx^3 + C\,,$$

where P, Q and R are constants to be found, and C is an arbitrary constant.

$$\boxed{P = 48}\,,\ \boxed{Q = 24}\,,\ \boxed{R = 8}$$

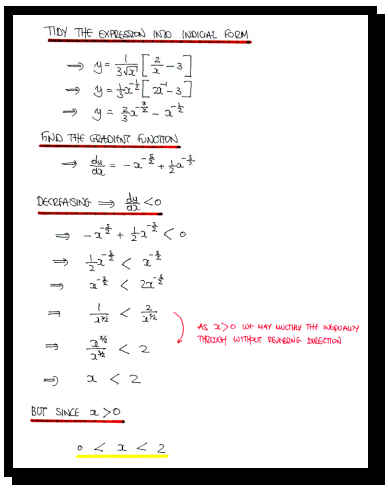


Question 180 (****)

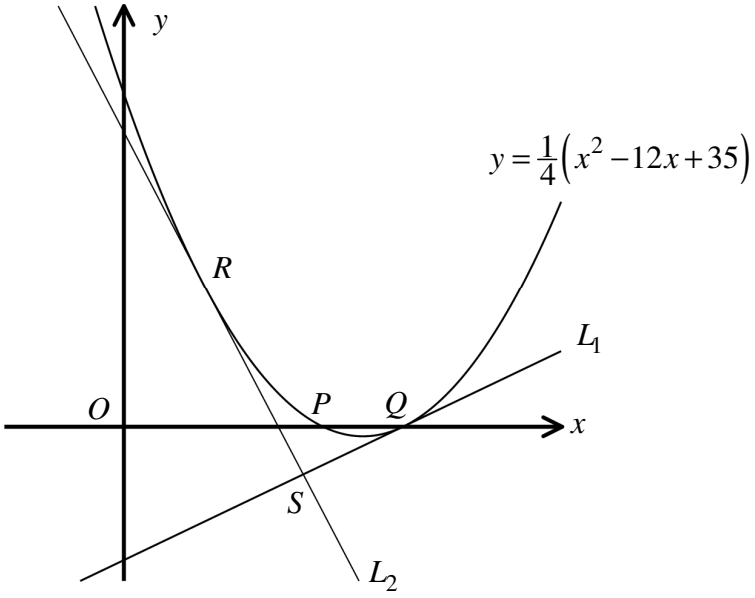
$$y=\frac{1}{3\sqrt{x}}\left[\frac{2}{x}-3\right],\ x>0\,.$$

Find the range of values of x for which y is decreasing.

$\boxed{\phantom{0<x<2}}$, $\boxed{0<x<2}$



Question 181 (****)



The figure above shows the curve with equation

$$y=\frac{1}{4}\left(x^2-12x+35\right).$$

The curve crosses the x axis at the points $P\left(x_1,0\right)$ and $Q\left(x_2,0\right)$, where $x_2>x_1$.

The tangent to the curve at Q is the straight line L_1.

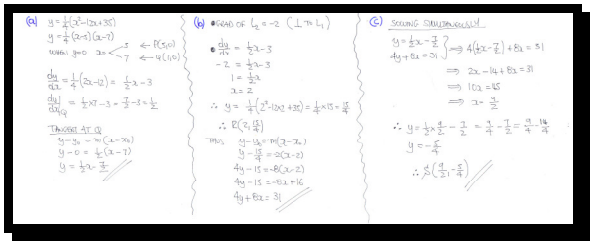
a) Find an equation of L_1.

The tangent to the curve at the point R is denoted by L_2. It is further given that L_2 meets L_1 at right angles, at the point S.

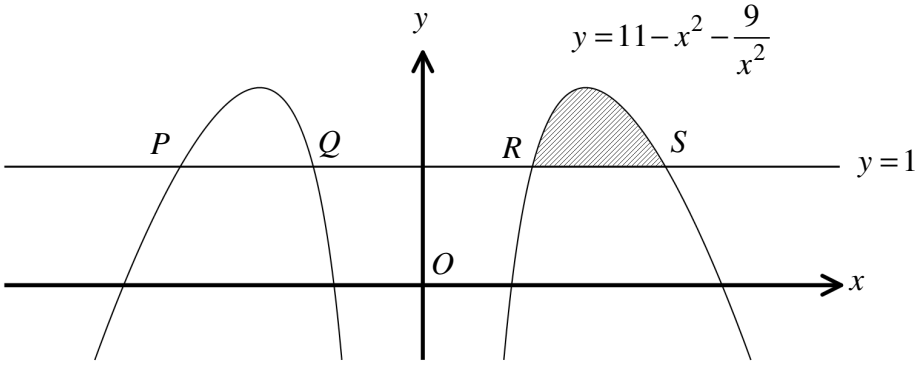
b) Find an equation of L_2.

c) Determine the exact coordinates of S.

$\square$, $\boxed{y=\frac{1}{2}x-\frac{7}{2}}$, $\boxed{4y+8x=31}$, $\boxed{S\left(\frac{9}{2},-\frac{5}{4}\right)}$



Question 182 (****)



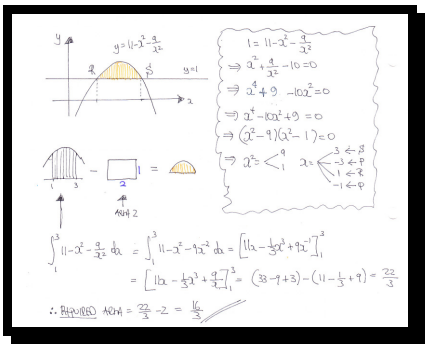
The figure above shows the curve C with equation

$$y=11-x^2-\frac{9}{x^2},\ x\neq 0.$$

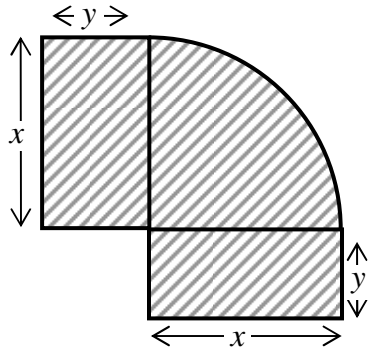
The straight line with equation $y=1$ meets C at the points P, Q, R and S, where R and S have positive x coordinates, as shown in the figure.

Find the area of the finite region bounded by C and the line segment RS.

$$\frac{16}{3}$$



Question 183 **(****)**



The figure above shows the design for an earring consisting of a quarter circle with two identical rectangles attached to either straight edge of the quarter circle. The quarter circle has radius x cm and the each of the rectangles measure x cm by y cm.

The earring is assumed to have negligible thickness and treated as a two dimensional object with area 12.25 cm^2.

a) Show that the perimeter, P cm, of the earring is given by

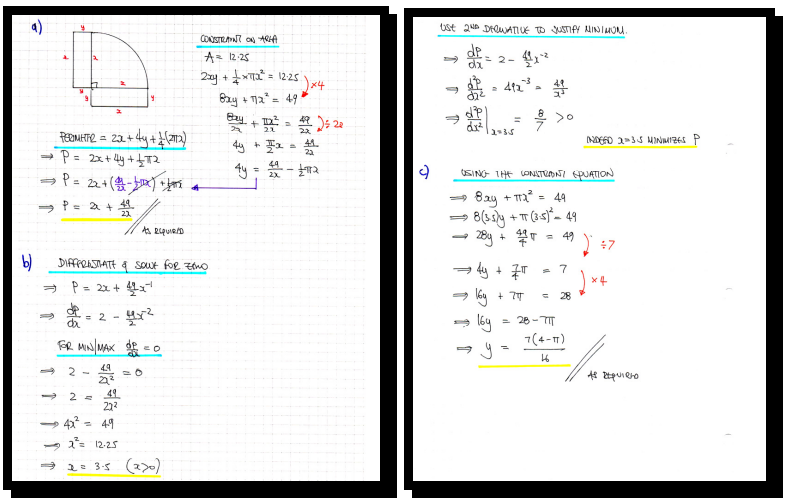
$$P = 2x + \frac{49}{2x}.$$

b) Find the value of x that makes the perimeter of the earring minimum, fully justifying that this value of x produces a minimum perimeter.

c) Show that for the value of x found in part **(b)**, the corresponding value of y is

$$\frac{7}{16}(4-\pi).$$

$\boxed{}$, $\boxed{x = 3.5}$



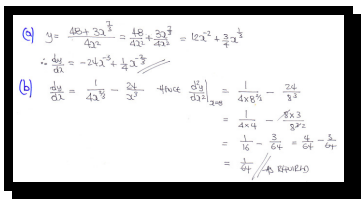
Question 184 (****) non calculator

$$y=\frac{48+3x^{\frac{7}{3}}}{4x^2},\ x>0.$$

a) Find an expression for $\dfrac{dy}{dx}$.

b) Show that the value of $\dfrac{dy}{dx}$ where $x=8$, is $\dfrac{1}{64}$.

$$\boxed{\frac{dy}{dx}=\frac{1}{4}x^{-\frac{2}{3}}-24x^{-3}}$$

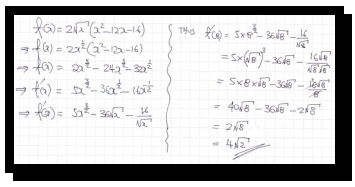


Question 185 (****) non calculator

$$f(x)=2\sqrt{x}\left(x^2-12x-16\right),\ x>0$$

Find the exact value of $f'(8)$.

$$\boxed{f'(8)=4\sqrt{2}}$$



Question 186 (****)

A curve C has equation

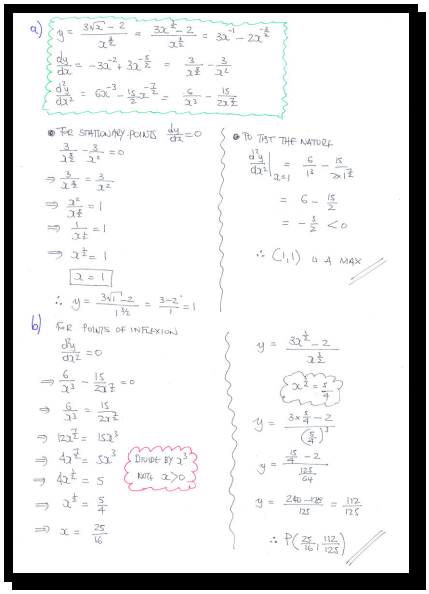
$$y=\frac{3\sqrt{x}-2}{x^{\frac{3}{2}}},\ x>0.$$

a) Find the coordinates of the stationary point of C, and determine its nature.

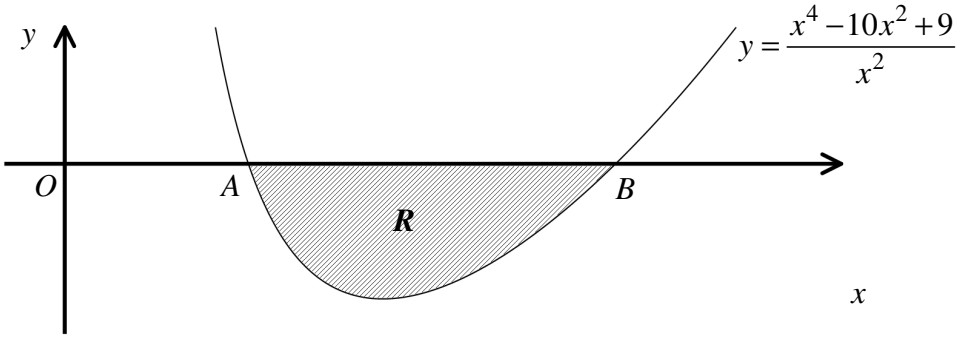
The curve has a non stationary turning point, P.

b) Determine the coordinates of P.

$\boxed{\text{max at } (1,1)}$, $\boxed{P\left(\frac{25}{16},\frac{112}{125}\right)}$



Question 187 (****)



The figure above shows the graph of the curve with equation

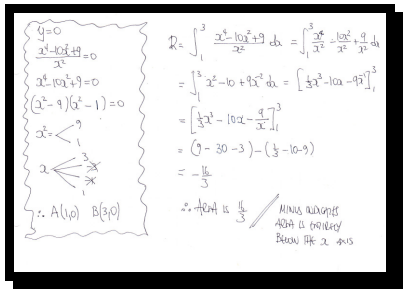
$$y = \frac{x^4 - 10x^2 + 9}{x^2}, \ x > 0 .$$

The curve meets the x axis at the points A and B.

The finite region R, shown shaded in the figure above, is bounded by the curve and the x axis.

Find the exact area of R.

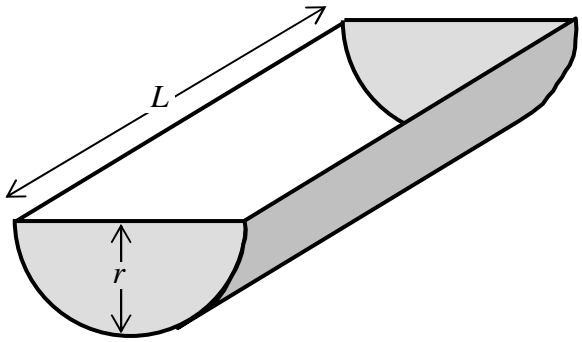
$$\frac{16}{3}$$



Question 188 (****)

The figure below shows the design of an animal feeder which in the shape of a hollow, open topped half cylinder, made of thin sheet metal. The radius of the semicircular ends is r cm and the length of the feeder is L cm.

The metal used in the construction of the feeder is 600π cm^2.



a) Show that the capacity, V cm^3, of the feeder is given by

$$V = 300\pi r - \frac{1}{2}\pi r^3 .$$

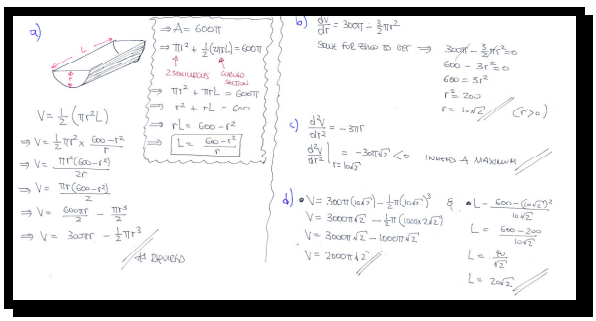
The design of the feeder is such so its capacity is maximum.

b) Determine the exact value of r for which V is stationary.

c) Show that the value of r found in part (b) gives the maximum value for V.

d) Find, in exact form, the capacity and the length of the feeder.

$$\boxed{r = 10\sqrt{2} \approx 14.14}, \quad \boxed{L = 20\sqrt{2} \approx 28.28}, \quad \boxed{V_{\max} = 2000\pi\sqrt{2} \approx 8886}$$



Question 189 (****) non calculator

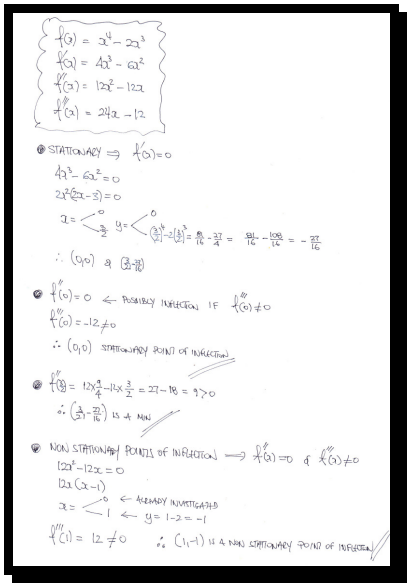
A quartic curve has equation

$$f(x)=x^4-2x^3.$$

This curve has three turning points of whom two are stationary.

Find the coordinates of these three turning points and determine their nature.

$\boxed{\min\left(\frac{3}{2},-\frac{27}{16}\right)}$, $\boxed{\text{point of inflection } (0,0)}$, $\boxed{\text{point of inflection } (1,-1)}$



Question 190 (****) non calculator

A cubic curve has equation

$$y=-x^3+7x^2-8x-10\,.$$

a) Find the range of values of x, for which y is increasing.

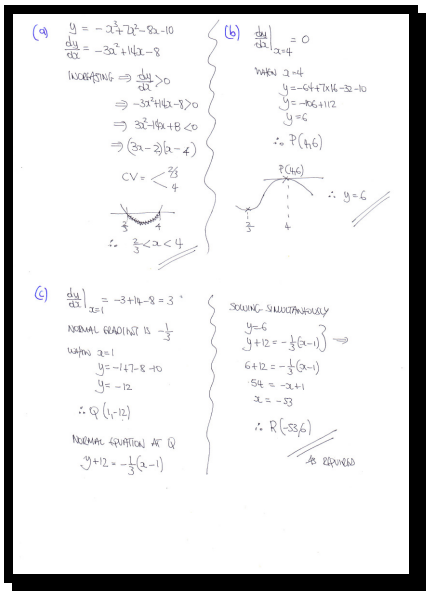
b) Find an equation of the tangent to the curve at the point P, where $x=4$.

The point Q also lies on the curve, where $x=1$.

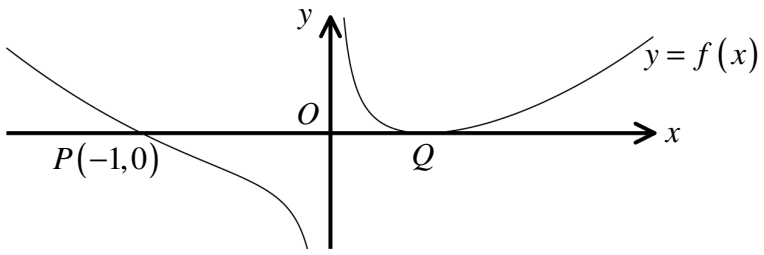
The normal to the curve at Q meets the tangent to the curve at P, at the point R.

c) Show that the coordinates of R are $(-53,6)$.

$\boxed{\frac{2}{3}<x<4}$, $\boxed{y=6}$



Question 191 **(****)**



The figure above shows a curve with equation $y=f(x)$.

The curve meets the x axis at the points $P(-1,0)$ and Q, and its gradient function is given by

$$f'(x)=\frac{8x^3-1}{x^2},\ x\neq 0.$$

a) Find an equation of the tangent to the curve at P.

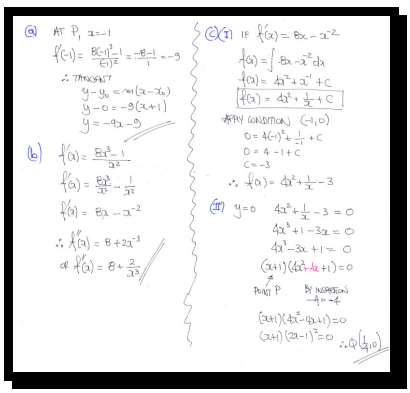
b) Find an expression for $f''(x)$.

c) Determine …

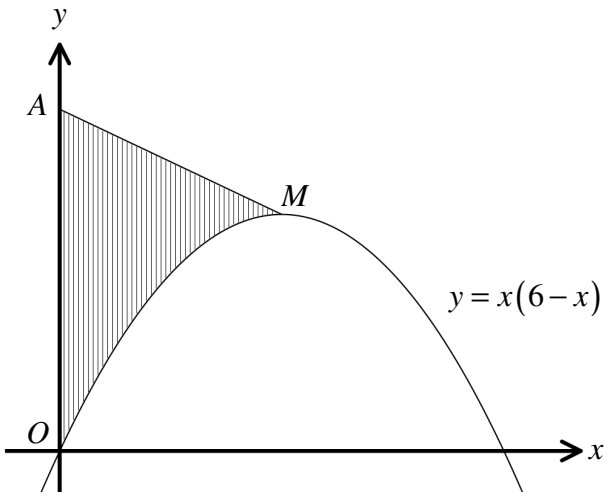
i. … an equation of the curve.

ii. … the coordinates of Q.

$\square$, $\boxed{y=-9x-9}$, $\boxed{f''(x)=8+2x^{-3}}$, $\boxed{y=4x^2+\frac{1}{x}-3}$, $\boxed{Q\left(\frac{1}{2},0\right)}$



Question 192 **(****)**



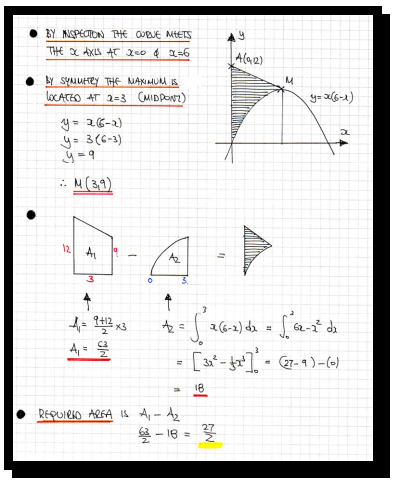
The figure above shows the curve C with equation

$$y = x(6-x), \ x \in \mathbb{R}.$$

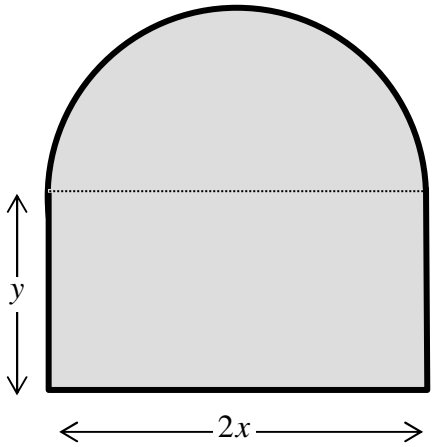
The point M is the maximum point of C and the point A has coordinates $(0,12)$.

Find the exact area of the shaded region, bounded by the curve, the y axis and the straight line segment from A to M.

$\square$, $\boxed{\text{area} = \frac{27}{2}}$



Question 193 (****)



The figure above shows the design of a window which is the shape of a semicircle attached to rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

The **perimeter** of the window is 6 m.

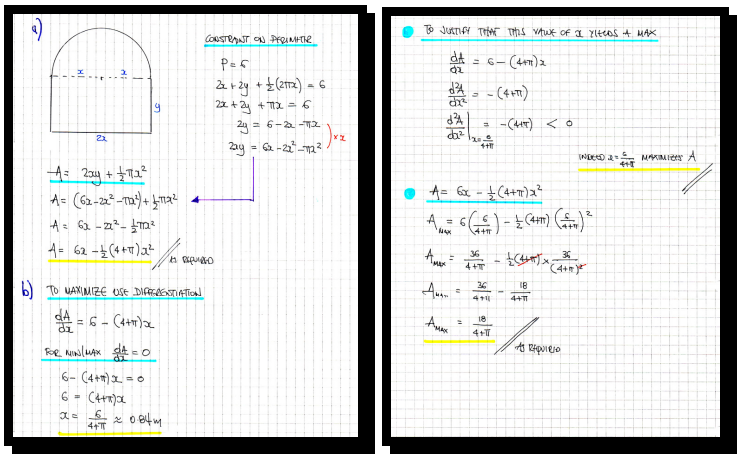
a) Show that the total area of the window, A m^2, is given by

$$A = 6x - \frac{1}{2}(4+\pi)x^2 .$$

b) Given that the measurements of the window are such so that A is maximum, show by a method involving differentiation that this maximum value of A is

$$\frac{18}{4+\pi} .$$

[ ], proof



Question 194 (****)

The point $A(p,q)$ lies on the curve with equation

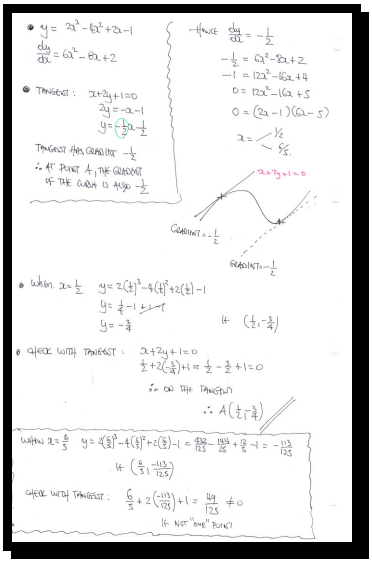
$$y=2x^3-4x^2+2x-1.$$

The tangent to the curve at A has equation

$$x+2y+1=0.$$

Find the coordinates of A.

$\square$, $\boxed{A\left(\frac{1}{2},-\frac{3}{4}\right)}$



Question 195 (****) non calculator

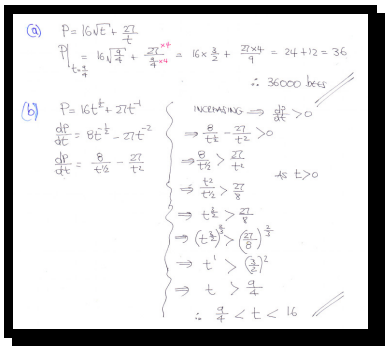
The population, P in thousands, of a colony of wild bees, t weeks after a certain instant, is given by the formula

$$P=16\sqrt{t}+\frac{27}{t}\ ,\ 0<t<16\ .$$

a) Calculate the number of bees in the colony after $2\frac{1}{4}$ weeks.

b) Find the range of values of t during which the bees' population is increasing.

$\boxed{}$, $\boxed{P=36000}$, $\boxed{\frac{9}{4}<t<16}$



Question 196 (****)

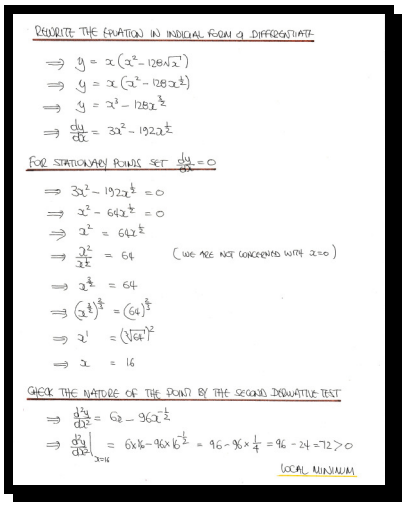
A curve has equation

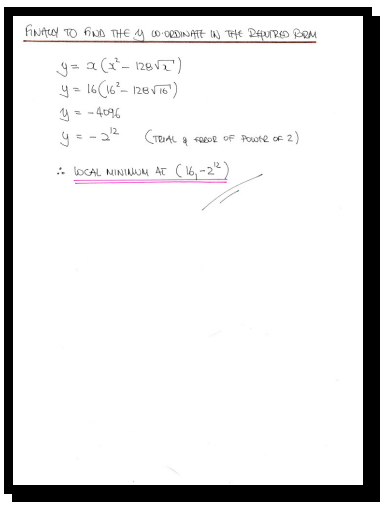
$$y = x\left(x^2 - 128\sqrt{x}\right), \qquad x \in \mathbb{R}, \qquad x > 0 .$$

The curve has a single stationary point with coordinates $\left(2^{\alpha},\ -2^{\beta}\right)$, where α and β are positive integers.

Find the value of β and justify that the stationary point is a local minimum.

$\boxed{}$, $\boxed{\beta = 12}$





Question 197 (****)

The cubic curve with equation

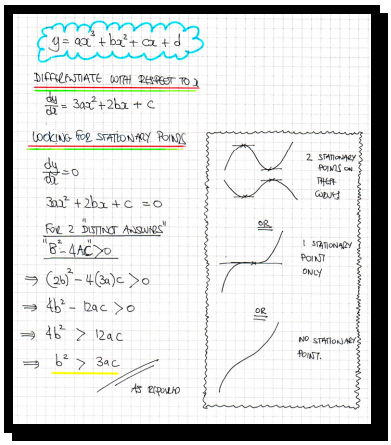
$$y = ax^3 + bx^2 + cx + d \ ,$$

where a , b , c are non zero constants and d is a constant, has one local maximum and one local minimum.

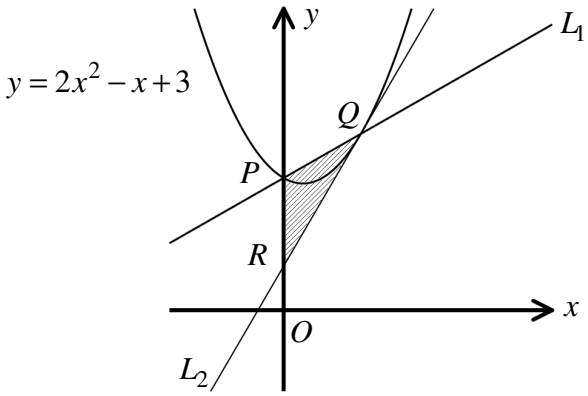
Show clearly that

$$b^2 > 3ac$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 198 **(****)**



The figure above shows the curve C with equation

$$y = 2x^2 - x + 3 .$$

C crosses the y axis at the point P. The normal to C at P is the straight line L_1.

a) Find an equation of L_1.

L_1 meets the curve again at the point Q.

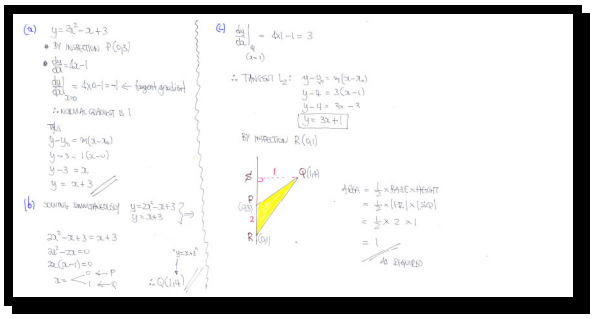
b) Determine the coordinates of Q.

The tangent to C at Q is the straight line L_2.

L_2 meets the y axis at the point R.

c) Show that the area of the triangle PQR is one square unit.

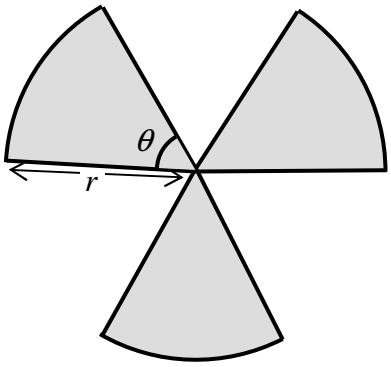
$\boxed{}$, $\boxed{y = x + 3}$, $\boxed{Q(1,4)}$



Question 199 (****)

The figure below shows the design of a hazard warning logo which consists of three identical sectors of radius r cm, joined together at the centre.

Each sector subtends an angle θ radians at the centre and the sectors are equally spaced so that the logo has rotational symmetry of order 3.



The area of the logo is 75 cm^2.

a) Show that the perimeter P cm of the logo is given by

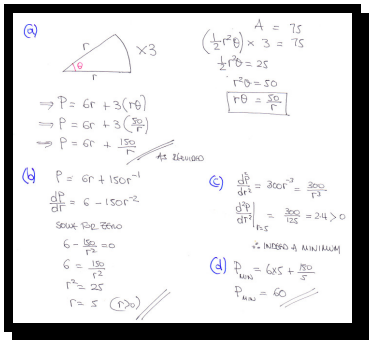
$$P = 6r + \frac{150}{r}.$$

b) Determine by differentiation the value of r for which P is stationary.

c) Show that the value of r found in part (b) gives the minimum value for P.

d) Find the minimum perimeter of the feeder.

$\boxed{r = 5}$, $\boxed{P_{\min} = 60}$



Question 200 (****)

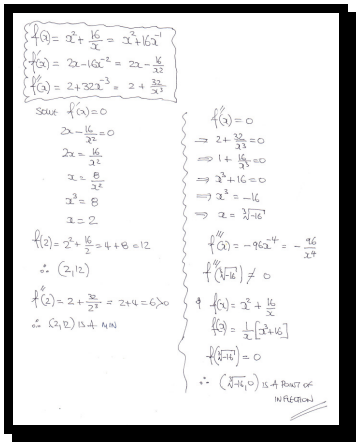
$$f(x)=x^2+\frac{16}{x},\ x\neq 0.$$

The curve C has equation $y=f(x)$.

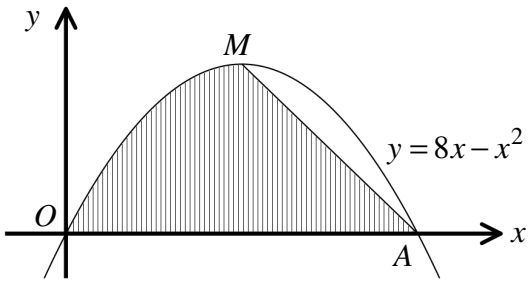
Show that C has two turning points, of which one is stationary, and the other is a non stationary point of inflection.

Determine the exact coordinates of each point.

$$\boxed{\text{point of inflection at } \left(\sqrt[3]{-16},0\right),\min(2,12)}$$



Question 201 (****)



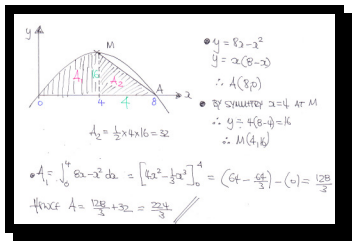
The figure above shows the quadratic curve with equation

$$y = 8x - x^2 , \ x \in \mathbb{R} .$$

The point M is the maximum point of the curve and the point A is one of the curve's x intercepts.

Find the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment from A to M.

$\square$, $\boxed{\text{area} = \frac{224}{3}}$



Question 202 (****)

The point P, whose x coordinate is $\frac{1}{4}$, lies on the curve with equation

$$y=\frac{k+4x\sqrt{x}}{7x},\ x\in\mathbb{R},\ x>0,$$

where k is a non zero constant.

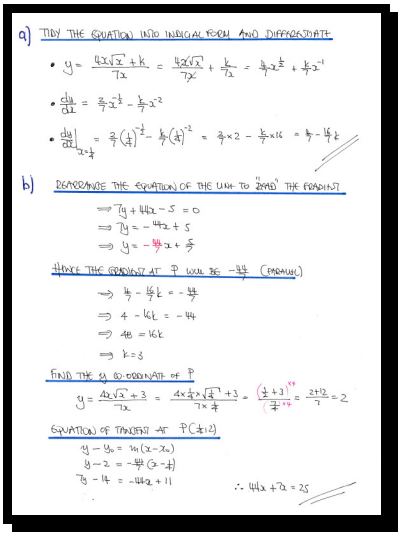
a) Determine, in terms of k, the gradient of the curve at P.

The tangent to the curve at P is parallel to the straight line with equation

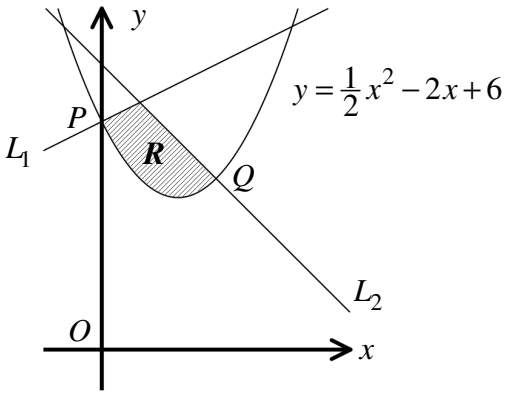
$$44x+7y-5=0.$$

b) Find an equation of the tangent to the curve at P.

$\square$, $\boxed{\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}}=\frac{4-16k}{7}}$, $\boxed{44x+7y=25}$



Question 203 (****)



The figure above shows the graph of the curve C with equation

$$y=\frac{1}{2}x^2-2x+6\,.$$

The point P is the point where C meets the y axis so that the straight line L_1 is the normal to C at P.

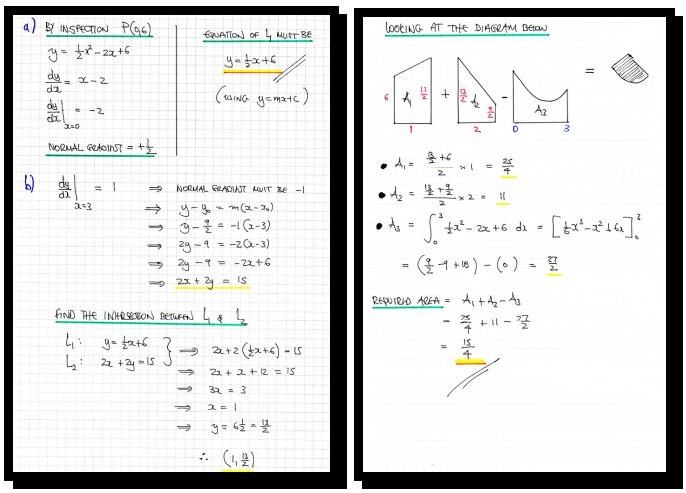
a) Find an equation for L_1.

The point $Q\left(3,\frac{9}{2}\right)$ lies on C and the straight line L_2 is the normal to C at Q.

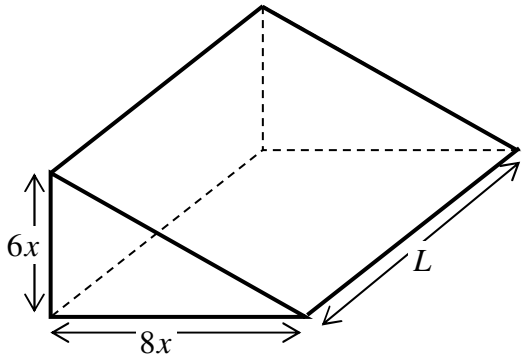
The finite region R, shown shaded in the figure above, is bounded by L_1, L_2 and C.

b) Find the area of R.

$\square$, $\boxed{y=\frac{1}{2}x+6}$, $\boxed{\frac{15}{4}}$



Question 204 **(****)**



The figure above shows a triangular prism with a volume of 960 cm^3.

The triangular faces of the prism are right angled with a base $8x$ cm and a height of $6x$ cm. The length of the prism is L cm.

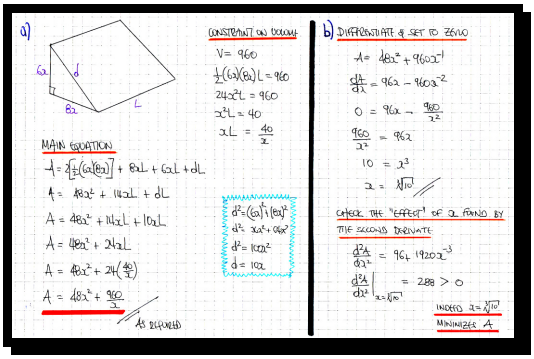
a) Show that the surface area of the prism, $A\text{ cm}^2$, is given by

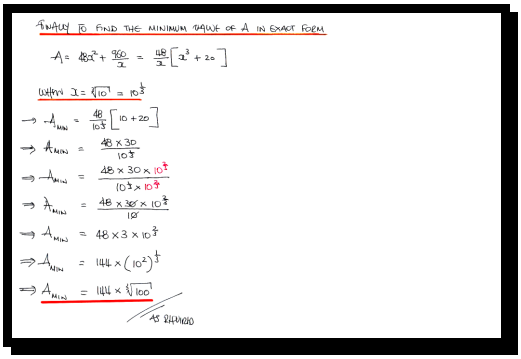
$$A = 48x^2 + \frac{960}{x}.$$

b) Determine an exact value of x for which A is stationary and show that this value of x minimizes A.

c) Show further that the minimum surface area of the prism is $144\sqrt[3]{100}\text{ cm}^2$.

$\boxed{}$, $\boxed{x = \sqrt[3]{10} \approx 2.15}$



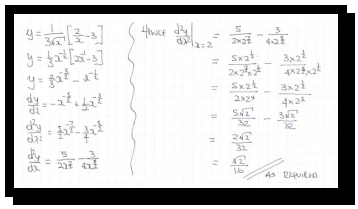


Question 205 (****) non calculator

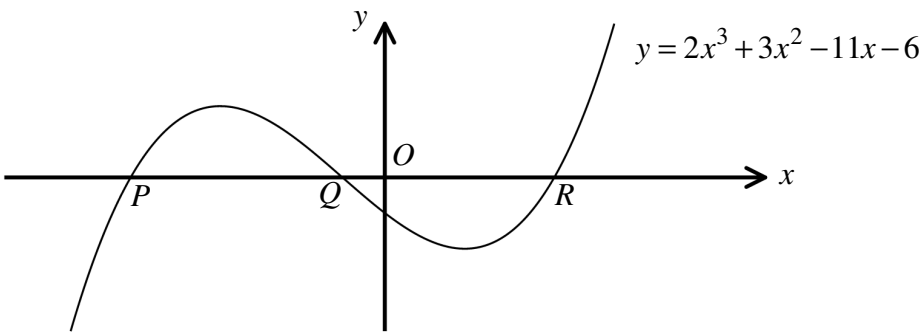
$$y=\frac{1}{3\sqrt{x}}\left(\frac{2}{x}-3\right),\ x\in\mathbb{R},\ x>0.$$

Show that the value of $\dfrac{d^2y}{dx^2}$ where $x=2$, is $\dfrac{\sqrt{2}}{16}$.

proof



Question 206 (****)



The figure above shows the curve with equation

$$y = 2x^3 + 3x^2 - 11x - 6 .$$

The curve crosses the x axis at the points P, Q and $R(2,0)$.

The tangent to the curve at R is the straight line L_1.

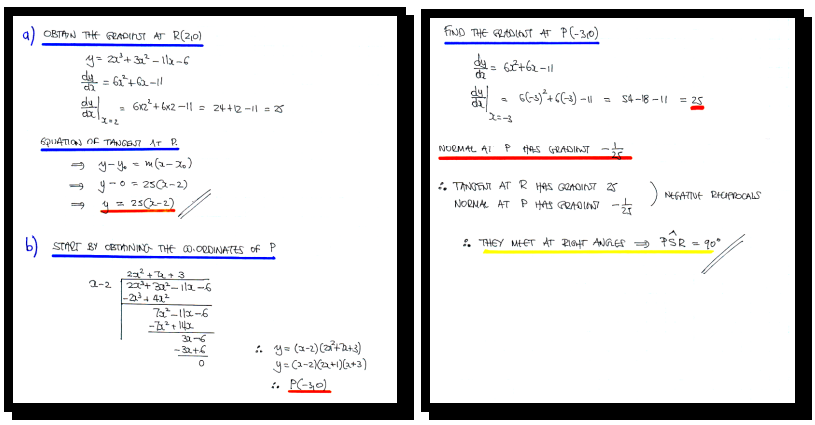
a) Find an equation of L_1.

The normal to the curve at P is the straight line L_2.

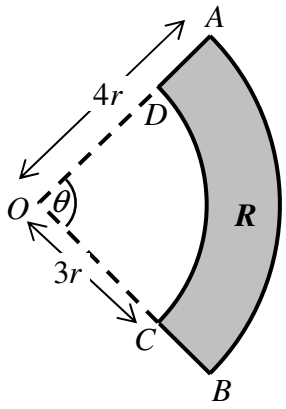
The point S is the point of intersection between L_1 and L_2.

b) Show that $\angle PSR = 90°$.

$\square$, $\boxed{y = 25x - 50}$



Question 207 **(****)**



The figure above shows a circular sector OAB of radius $4r$ subtending an angle θ radians at the centre O. Another circular sector OCD of radius $3r$ also subtending an angle θ radians at the centre O is removed from the first sector leaving the shaded region R.

It is given that R has an area of 50 square units.

a) Show that the perimeter P, of the region R, is given by

$$P = 2r + \frac{100}{r}.$$

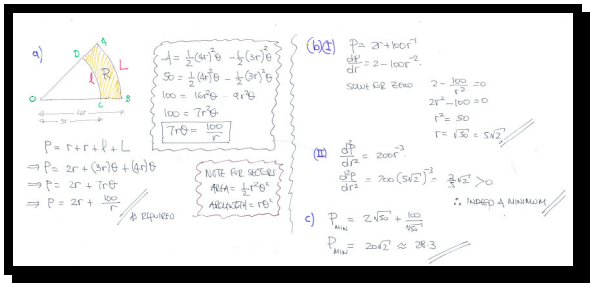
b) Given that the value of r can vary, …

i. … find an exact value of r for which P is stationary.

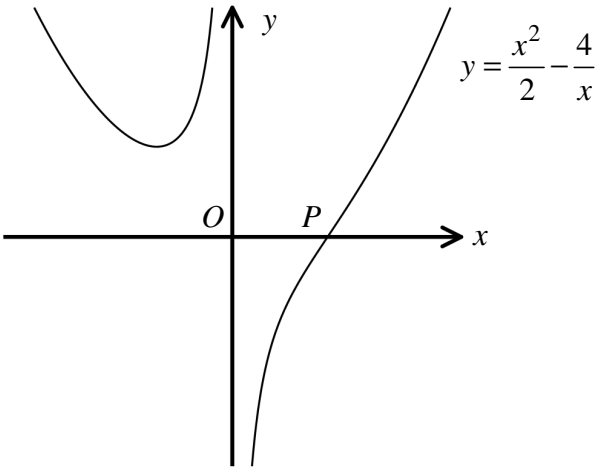
ii. … show clearly that the value of r found above gives the minimum value for P.

c) Calculate the minimum value of P.

$\boxed{}$, $\boxed{r = 5\sqrt{2} \approx 7.07}$, $\boxed{P_{\min} = 20\sqrt{2} \approx 28.28}$



Question 208 **(****)**



The figure above shows the curve C with equation

$$y=\frac{x^2}{2}-\frac{4}{x},\ x\neq 0.$$

The curve crosses the x axis at the point P.

The straight line L is the normal to C at P.

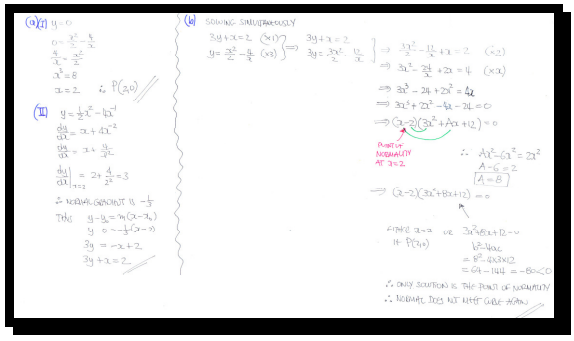
a) Find …

i. … the coordinates of P.

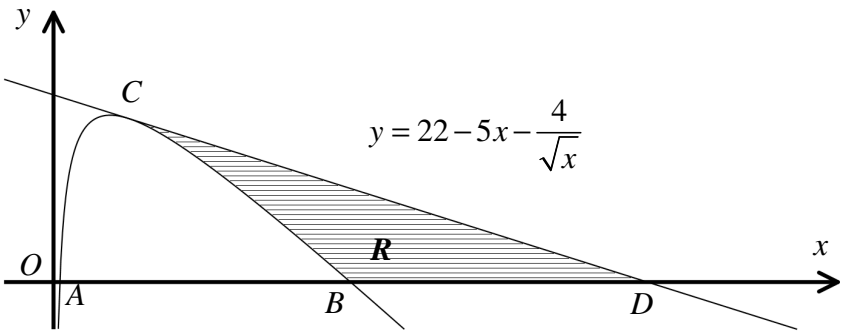
ii. … an equation of L.

b) Show that L does not meet C again.

$\boxed{}$, $\boxed{P(2,0)}$, $\boxed{x+3y=2}$



Question 209 **(****)**



The figure above shows part of the curve with equation

$$y=22-5x-\frac{4}{\sqrt{x}},\ x>0$$

The curve meets the x axis at A and B. The point C lies on the curve where $x=1$.

a) Verify that the coordinates of B are $(4,0)$.

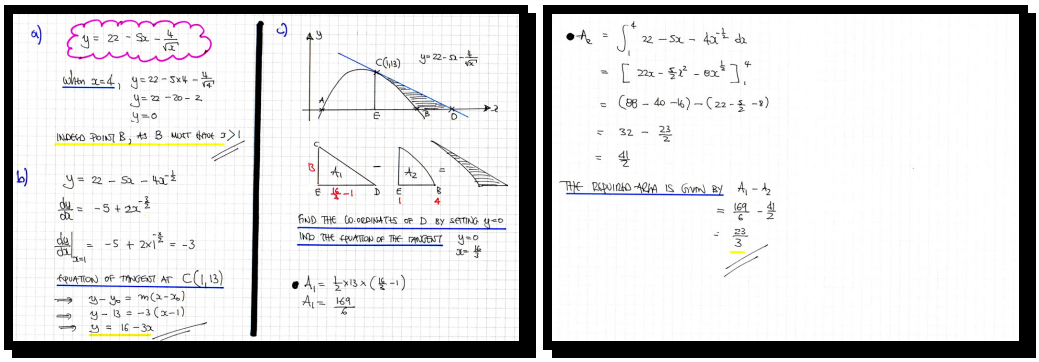
b) Find an equation of the tangent to the curve at C.

The tangent to the curve at C crosses the x axis at the point D.

The finite region R, shown shaded in the above figure, is bounded by the curve, the tangent to the curve at C and the x axis.

c) Determine the exact area of R.

$\boxed{}$, $\boxed{y+3x=16}$, $\boxed{D\left(\frac{16}{3},0\right)}$, $\boxed{\text{area}=\frac{23}{3}}$



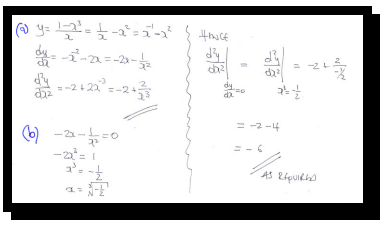
Question 210 (****)

$$y=\frac{1-x^3}{x},\ x\neq 0\,.$$

a) Find an expression for $\dfrac{d^2y}{dx^2}$.

b) Show that the value of $\dfrac{d^2y}{dx^2}$ where $\dfrac{dy}{dx}=0$, is -6.

$$\boxed{\frac{d^2y}{dx^2}=2x^{-3}-2}$$



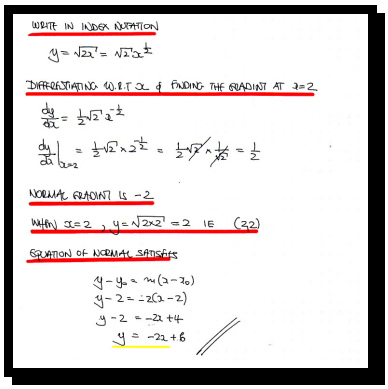
Question 211 (****)

The curve C has equation

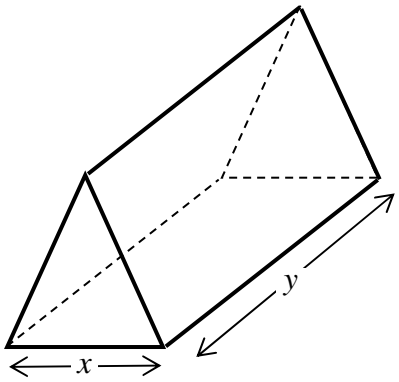
$$y=\sqrt{2x}\,,\ x\geq 0$$

Find an equation of the normal to C at the point where $x=2$, giving the answer in the form $y=mx+c$, where m and c are constants.

$\boxed{}$, $\boxed{y=6-2x}$



Question 212 (****)



The figure above shows a triangular prism whose triangular faces are parallel to each other and are in the shape of **equilateral** triangles of side length x cm.

The length of the prism is y.

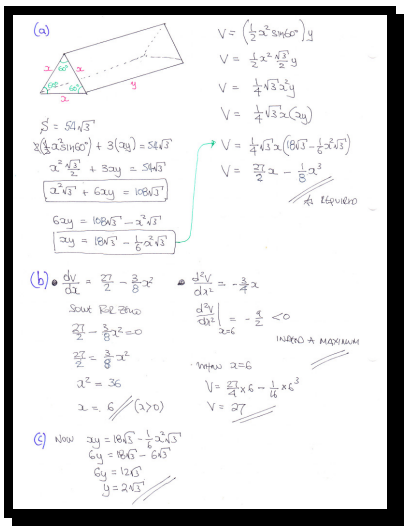
a) Given that total surface area of the prism is exactly $54\sqrt{3}$ cm^2, show clearly that the volume of the prism, V cm^3, is given by

$$V=\frac{27}{2}x-\frac{1}{8}x^3 .$$

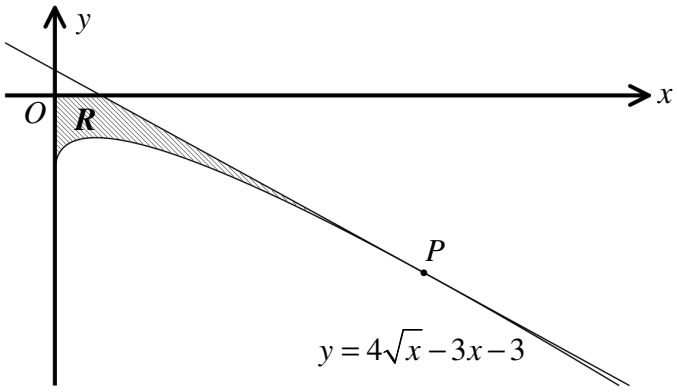
b) Find the maximum value of V, fully justifying the fact that it is indeed the maximum value.

c) Determine the value of y when V takes this maximum value.

$\boxed{}$, $\boxed{V_{\max}=27}$, $\boxed{y=2\sqrt{3}}$



Question 213 **(****)**



The figure above shows part of the curve with equation

$$y=4\sqrt{x}-3x-3,\ x>0$$

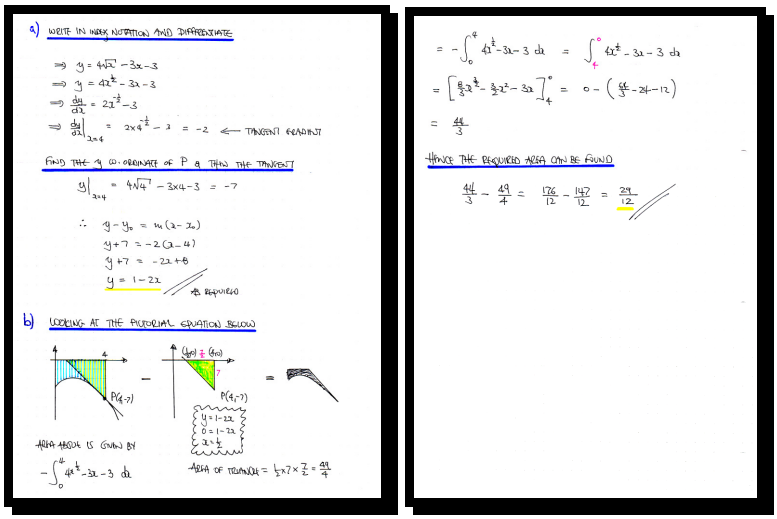
a) Show that an equation of the tangent to the curve at the point P, where $x=4$, is given by

$$y=1-2x.$$

The finite region R is bounded by the curve, the tangent to the curve at P and the coordinate axes.

b) Determine the exact area of R.

$\square$, $\boxed{\text{area}=\frac{29}{12}}$



Question 214 (****)

The straight line with equation

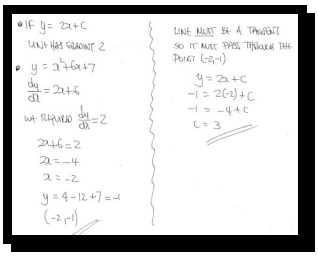
$$y = 2x + c$$

is a tangent to the curve with equation

$$y = x^2 + 6x + 7 .$$

Without using the discriminant, determine the value of the constant c and find the point of contact between the tangent and the curve.

$\boxed{c = 3}$, $\boxed{(-2, -1)}$



Question 215 (****)

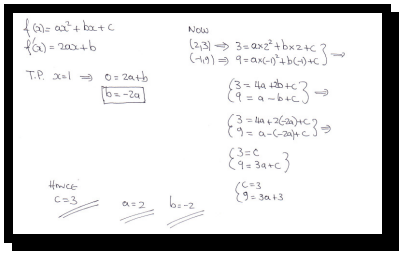
$$f(x)=ax^2+bx+c,\ x\in\mathbb{R},$$

where a, b and c are constants.

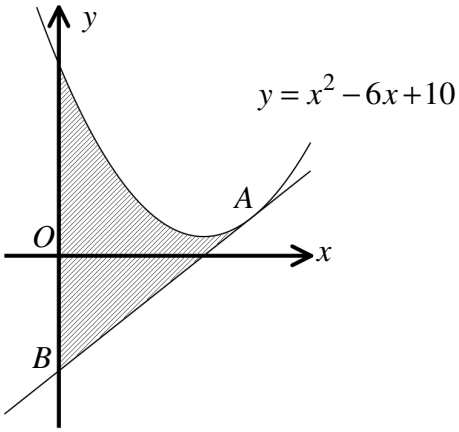
It is given that $f(x)$ passes through the points $(2,3)$ and $(-1,9)$, and the curve has a stationary point where $x=1$.

Determine the value of a, b and c,

$$\boxed{a=2},\ \boxed{b=-4},\ \boxed{c=3}$$



Question 216 (****)



The figure above shows the curve with equation

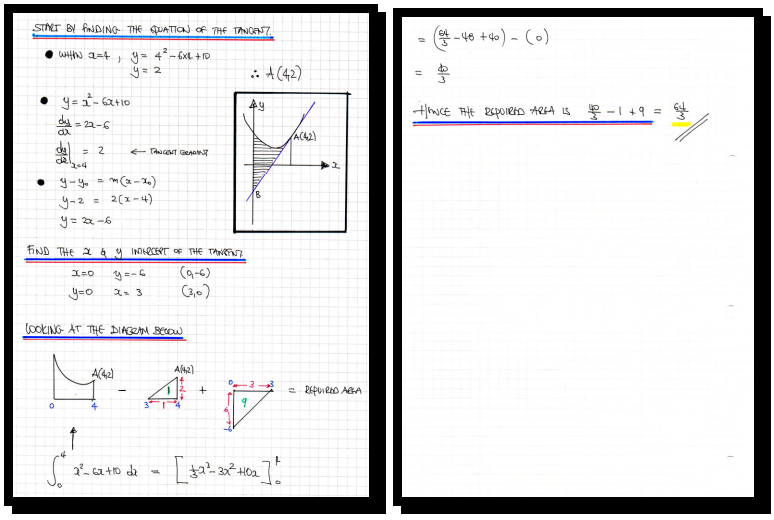
$$y = x^2 - 6x + 10 , \ x \in \mathbb{R} .$$

The point A, where $x = 4$, lies on the curve.

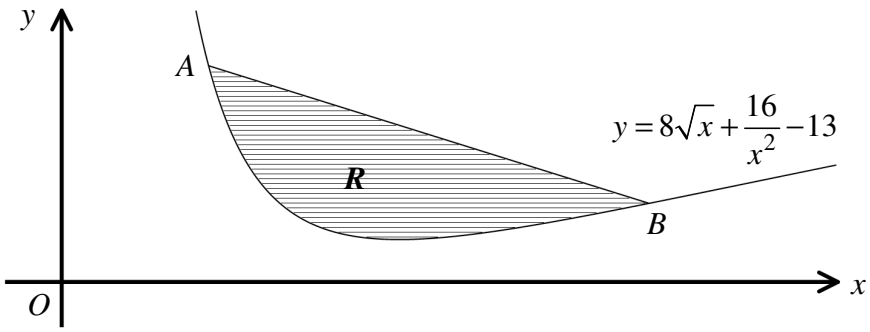
The tangent to the curve at A, meets the y axis at B.

Determine the area of the finite region bounded by the curve, the tangent to the curve at A and the y axis.

$\boxed{}$, $\boxed{\frac{64}{3}}$



Question 217 (****)



The figure above shows part of the curve with equation

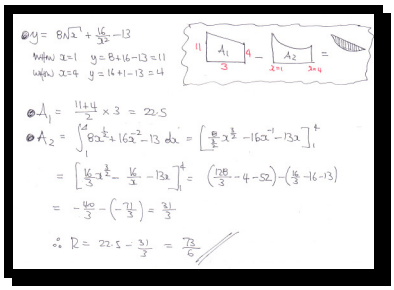
$$y=8\sqrt{x}+\frac{16}{x^2}-13\,,\ x>0\,.$$

The points A and B lie on the curve where $x=1$ and $x=4$, respectively.

The finite region R is bounded by the curve, the straight line segment AB.

Determine the exact area of R.

$\square$, $\boxed{\text{area}=\frac{73}{6}}$



Question 218 (****)

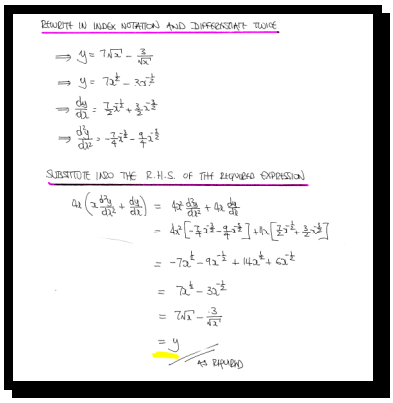
The curve C has equation

$$y = 7\sqrt{x} - \frac{3}{\sqrt{x}}, \ x > 0$$

Show clearly that

$$y = 4x\left(x\frac{d^2y}{dx^2} + \frac{dy}{dx} \right).$$

[ ], proof



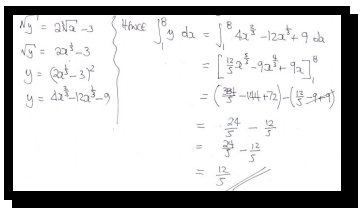
Question 219 (****) non calculator

$$\sqrt{y}=2\sqrt[3]{x}-3\,,\ x>0\,.$$

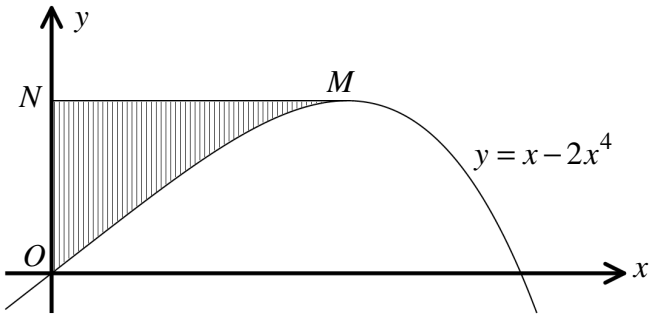
Show clearly that

$$\int_1^8 y\ dx=\frac{12}{5}\,.$$

proof



Question 220 (****)



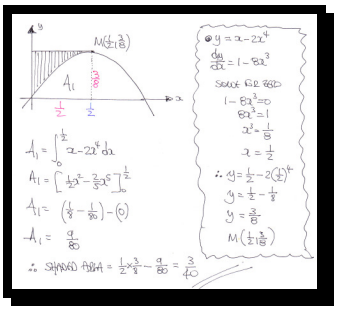
The diagram below shows the quartic curve with equation

$$y = x - 2x^4 , \ x \in \mathbb{R} .$$

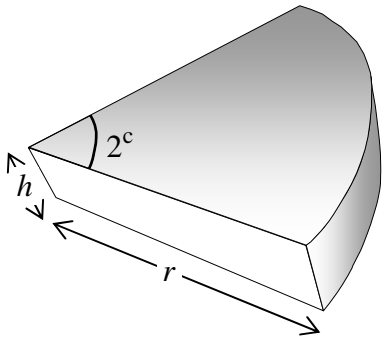
The point M is the maximum point on the curve and the point N lies on the y axis so that the straight line segment MN is parallel to the x axis.

Find the exact area of the shaded region, bounded by the curve, the y axis and the straight line segment from M to N.

$\square$, $\boxed{\text{area} = \frac{3}{40}}$



Question 221 (****)



The figure above shows solid right prism of height h cm.

The cross section of the prism is a circular sector of radius r cm, subtending an angle of 2 radians at the centre.

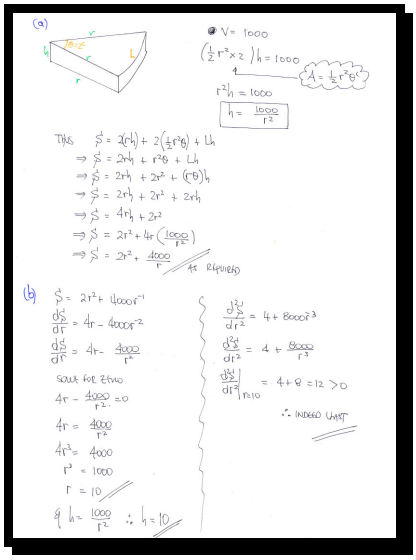
a) Given that the volume of the prism is 1000 cm^3, show clearly that

$$S = 2r^2 + \frac{4000}{r},$$

where S cm^2 is the total surface area of the prism.

b) Hence determine the value of r and the value of h which make S **least**, fully justifying your answer.

$\boxed{}$, $\boxed{r = 10}$, $\boxed{h = 10}$



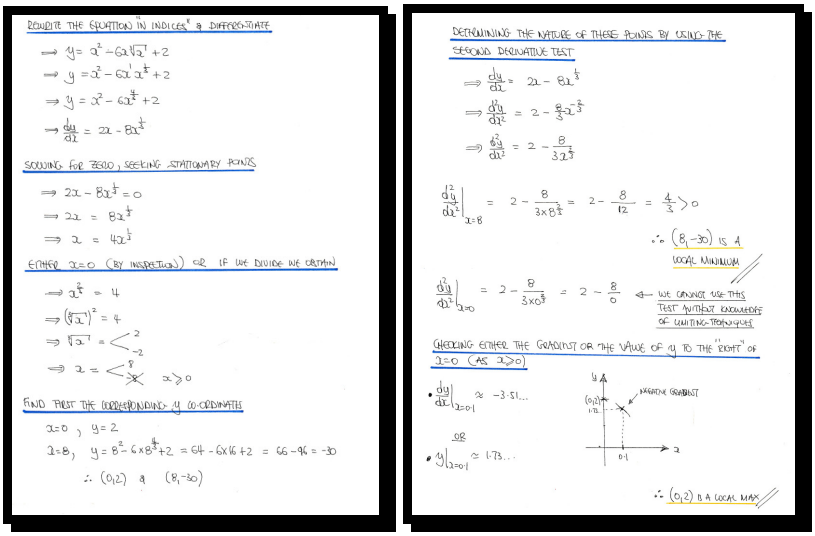
Question 222 (****)

A curve has equation

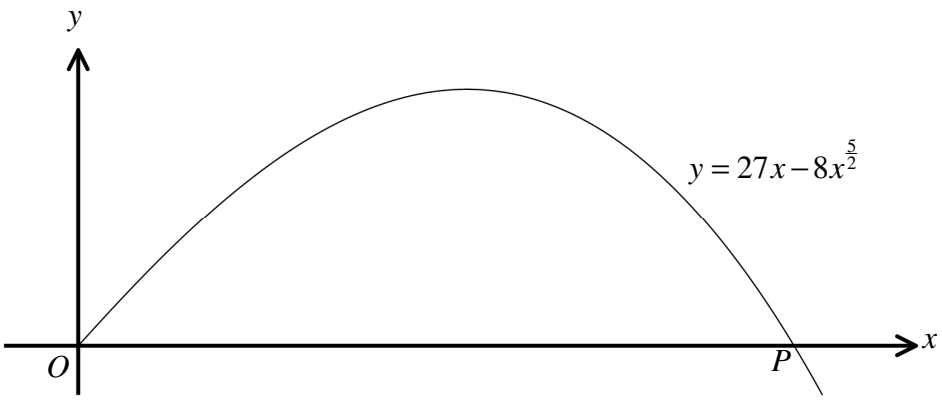
$$y = x^2 - 6x\sqrt[3]{x} + 2, \qquad x \in \mathbb{R}, \qquad x \geq 0.$$

Find the coordinates of the stationary points of the curve and classify them as local maxima, local minima or a points of inflexion.

$\boxed{}$, $\boxed{\text{local minimum at } (8,-30)}$, $\boxed{\text{local maximum at } (0,2)}$



Question 223 (****)



The figure above shows the curve C with equation

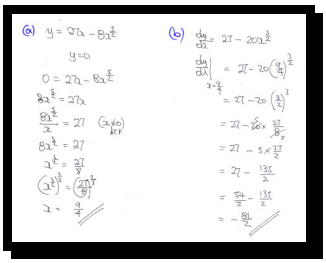
$$y = 27x - 8x^{\frac{5}{2}}, \ x \geq 0$$

The curve meets the x axis at the origin O and at the point P.

a) Find the exact x coordinate of P.

b) Calculate the gradient of C at the point P.

$$\boxed{x = \frac{9}{4}}, \quad \boxed{\frac{dy}{dx} = -\frac{81}{2}}$$

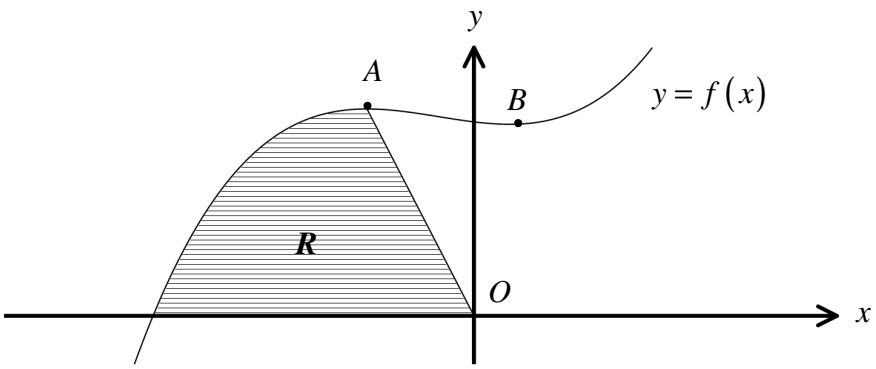


Question 224 (****)

$$f(x)=x^3+x^2-x+15\,,\ x\in\mathbb{R}$$

a) Show that $(x+3)$ is a factor of $f(x)$.

The figure below shows the curve C with equation $y=f(x)$.



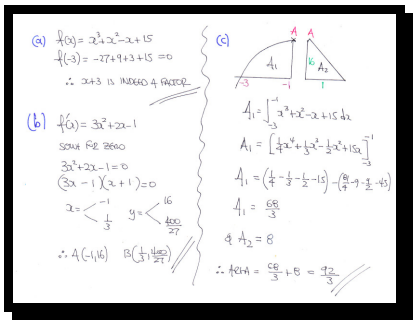
The points A and B are stationary points of C.

b) Find the exact coordinates of A and B.

The finite region R is bounded by the curve, the x axis and the straight line segment OA, where O is the origin.

c) Determine the exact area R.

$\square$, $\boxed{A(-1,16),\ B\left(\frac{1}{3},\frac{400}{27}\right)}$, $\boxed{\text{area}=\frac{92}{3}}$



Question 225 (****)

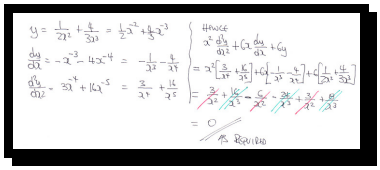
The curve C has equation

$$y=\frac{1}{2x^2}+\frac{4}{3x^3},\ x>0.$$

Show clearly that

$$x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}+6y=0.$$

$\boxed{}$, $\boxed{\text{proof}}$

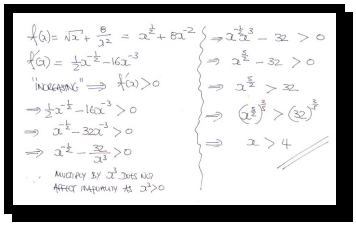


Question 226 (****)

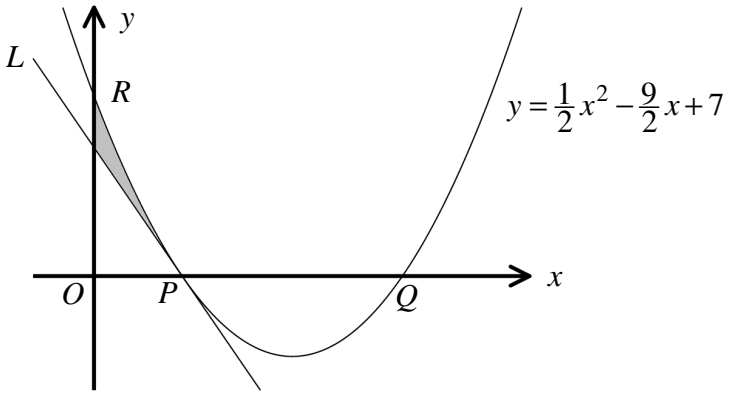
$$f(x)\equiv\sqrt{x}+\frac{8}{x^2},\ x>0.$$

Find the range of values of x, for which $f(x)$ is increasing.

$\boxed{x>4}$



Question 227 **(****)**



The diagram above shows the quadratic curve C with equation

$$y=\frac{1}{2}x^2-\frac{9}{2}x+7\,.$$

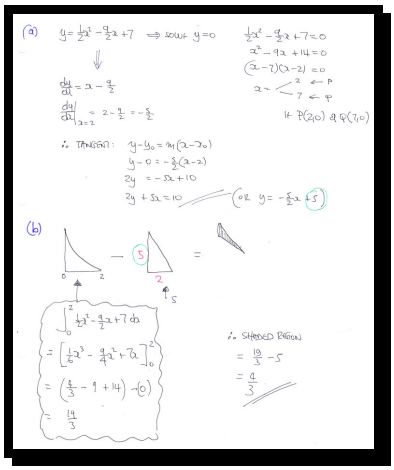
The curve crosses the x axis at the points P and Q, and the y axis at the point R.

The line L is the tangent to C at the point P.

a) Find an equation of L.

b) Find the exact area of the shaded region bounded by the tangent at P, the curve and the y axis.

$\boxed{2y+5x=10}$, $\boxed{\text{area}=\frac{4}{3}}$



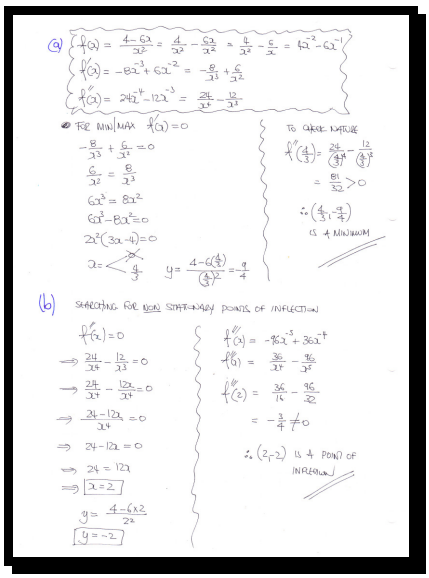
Question 228 (****)

$$f(x)=\frac{4-6x}{x^2},\ x\neq 0.$$

a) Find the coordinates of the stationary point of $f(x)$ and determine its nature.

b) Show clearly that $f(x)$ has a point of inflection and determine its coordinates.

$\boxed{\min\left(\frac{4}{3},-\frac{9}{4}\right)}$, $\boxed{(2,-2)}$



Question 229 (****) non calculator

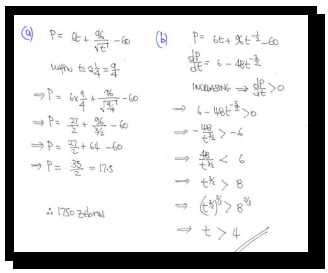
The population, P in hundreds, of a herd of zebra t years after a certain instant is given by the formula

$$P = 6t + \frac{96}{\sqrt{t}} - 60 \ , \ t > 0 .$$

a) Find the number of zebra in the herd after $2\frac{1}{4}$ years.

b) Find the time during which the herd's population is increasing.

$\boxed{P = 1750}$, $\boxed{t > 4}$



Question 230 (****)

$$f\left(x\right)=\frac{\left(x^2-3\right)^2}{x^3},\ x\neq 0\,.$$

a) Show clearly that

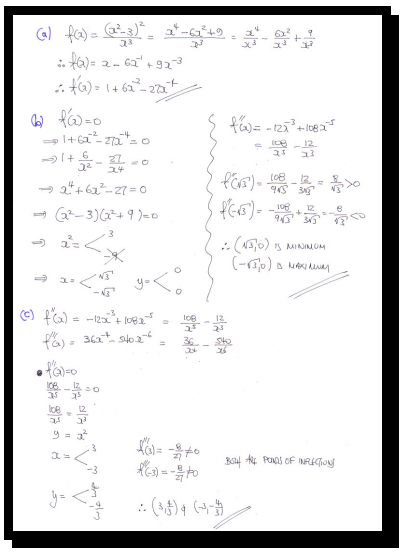
$$f'\left(x\right)=A+Bx^{-2}+Cx^{-4},$$

where A, B and C are constants to be found.

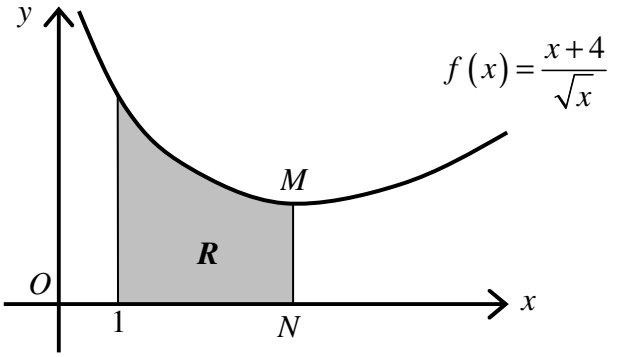
b) Find the exact coordinates of the stationary points of $f\left(x\right)$ and use $f''\left(x\right)$ to determine their nature.

c) Show that $f\left(x\right)$ has two points of inflection and determine their coordinates.

$$\boxed{f'\left(x\right)=1+6x^{-2}-27x^{-4}},\ \boxed{\min\left(\sqrt{3},0\right)},\ \boxed{\max\left(-\sqrt{3},0\right)},\ \boxed{\left(3,\frac{4}{3}\right),\left(-3,-\frac{4}{3}\right)}$$



Question 231 (****)



The figure above shows the curve C with equation

$$f(x)=\frac{x+4}{\sqrt{x}},\ x>0.$$

a) Determine the coordinates of the minimum point of C, labelled as M.

The point N lies on the x axis so that MN is parallel to the y axis. The finite region R is bounded by C, the x axis, the straight line segment MN and the straight line with equation $x=1$.

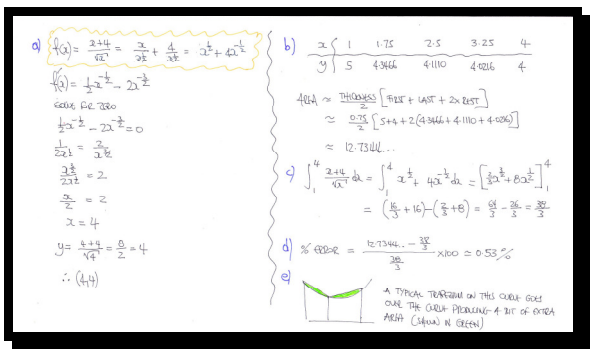
b) Use the trapezium rule with 4 strips of equal width to estimate the area of R.

c) Use integration to find the exact area of R.

d) Calculate the percentage error in using the trapezium rule to find the area of R.

e) Explain with the aid of a diagram why the trapezium rule overestimates the area of R.

$\boxed{}$, $\boxed{M(4,4)}$, $\boxed{\text{area}\approx 12.7344...}$, $\boxed{\text{area}=\frac{38}{3}}$, $\boxed{0.53\%}$



Question 232 (****)

The curve C with equation $y=f(x)$ passes through the point $P(16,-5)$, and its gradient function $f'(x)$ is given by

$$f'(x)=\frac{x-6}{\sqrt{x}},\ x>0.$$

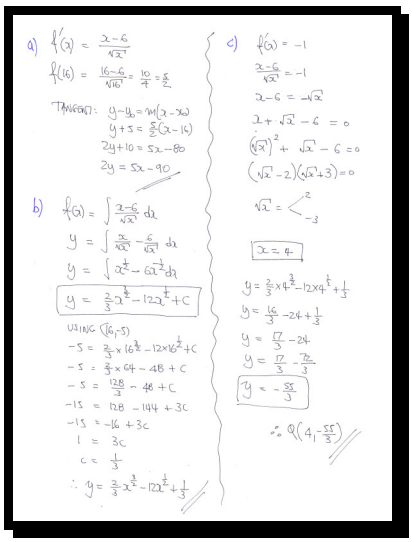
a) Find an equation of the tangent to C at P.

b) Determine an equation of C.

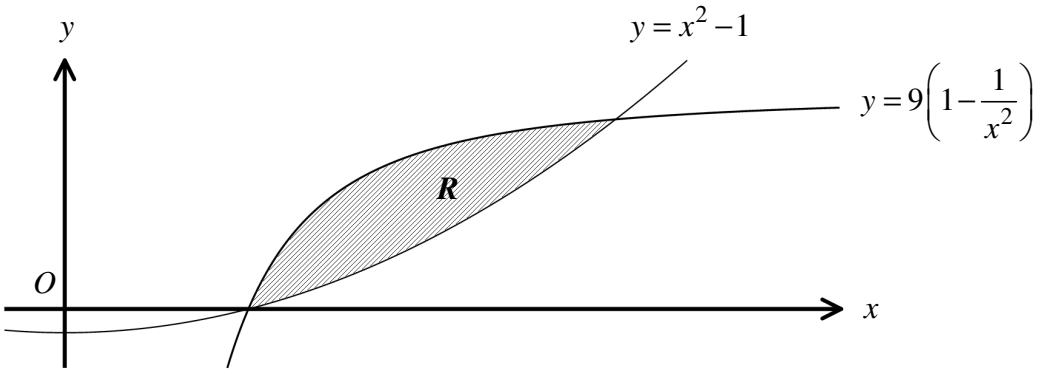
The point Q lies on C and the gradient of C at that point is -1.

c) Find the coordinates of Q.

$\boxed{}$, $\boxed{2y=5x-90}$, $\boxed{y=\frac{2}{3}x^{\frac{3}{2}}-12x^{\frac{1}{2}}+\frac{1}{3}}$, $\boxed{Q\left(4,-\frac{55}{3}\right)}$



Question 233 (****)



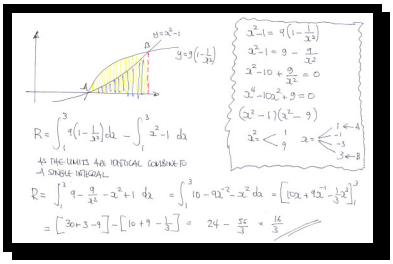
The figure above shows the graphs of the curves with equations

$$y = x^2 - 1 \quad \text{and} \quad y = 9\left(1 - \frac{1}{x^2}\right).$$

The finite region R is bounded by the two curves in the 1st quadrant, and is shown shaded in the figure above.

Determine the exact area of R.

$\square$, $\boxed{\frac{16}{3}}$



Question 234 **(****)**

The curve C has equation

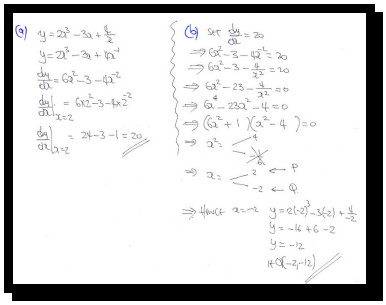
$$y = 2x^3 - 3x + \frac{4}{x}, \ x \neq 0$$

The point $P(2,12)$ lies on C.

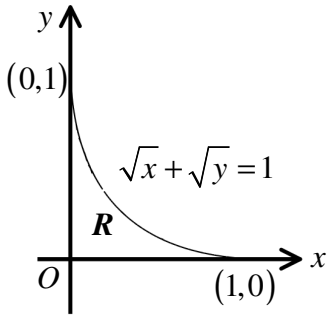
a) Find the gradient at P.

b) Determine the coordinates of another point Q that lies on C, so that the gradient at Q is the same gradient as P.

$\boxed{}$, $\boxed{\text{gradient} = 20}$, $\boxed{Q(-2,-12)}$



Question 235 (****)



The figure above shows the curve with equation

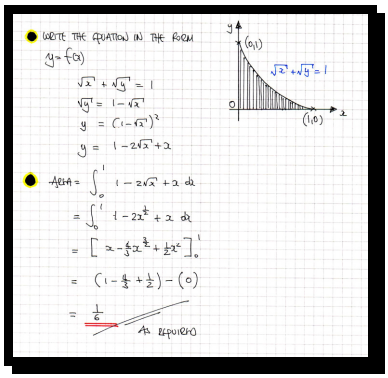
$$\sqrt{x}+\sqrt{y}=1\ ,\ x\in\mathbb{R},\ 0\le x\le 1.$$

The curve meets the coordinate axes at the points $(1,0)$ and $(0,1)$.

The finite region R is bounded by the curve and the coordinate axes.

Show that the area of R is $\frac{1}{6}$.

$\boxed{}$, proof



Question 236 (****) non calculator

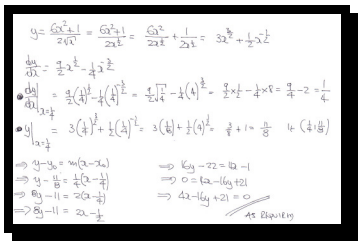
The curve C has equation

$$y=\frac{6x^2+1}{2\sqrt{x}},\ x>0$$

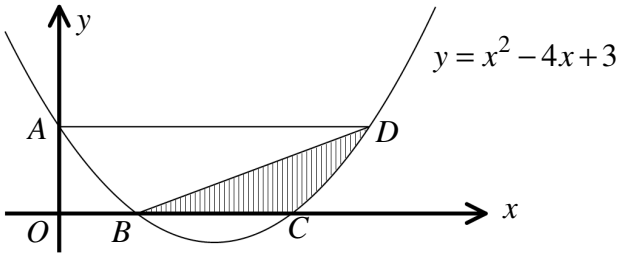
Show that an equation of the tangent to C at the point where $x=\frac{1}{4}$ is

$$4x-16y+21=0.$$

proof



Question 237 (****)



The figure above shows a quadratic curve with equation

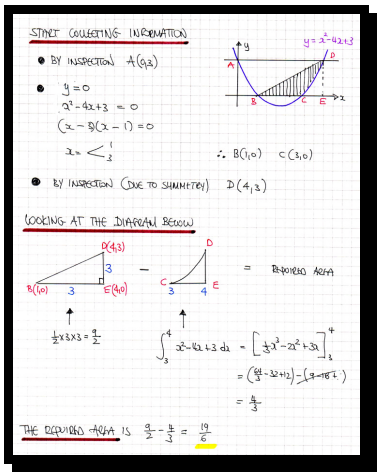
$$y = x^2 - 4x + 3 .$$

The points A, B and C are the points where the curve meets the coordinate axes.

The point D lies on the curve so that AD is parallel to the x axis.

Calculate the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment BD.

$\square$, $\boxed{\text{area} = \frac{19}{6}}$



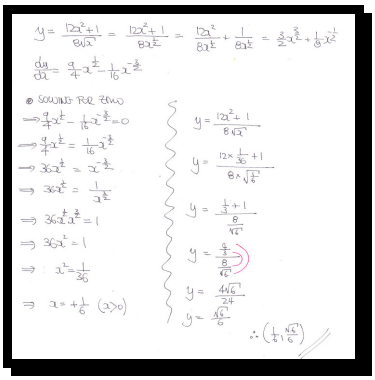
Question 238 (****) non calculator

The curve C has equation

$$y=\frac{12x^2+1}{8\sqrt{x}},\ x>0$$

Find the exact coordinates of the turning point of C.

$$\left(\frac{1}{6},\frac{\sqrt{6}}{6}\right)$$



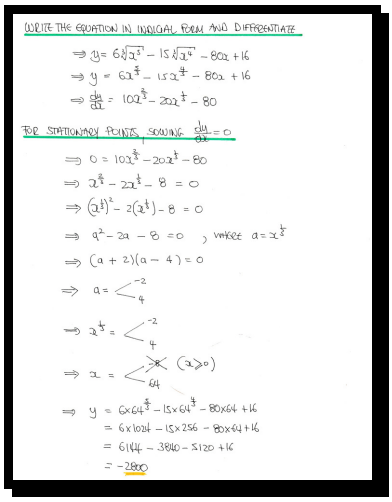
Question 239 (****)

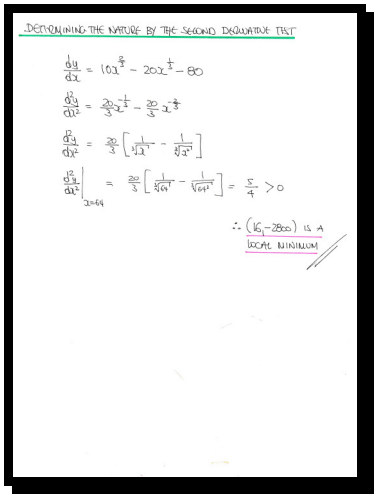
A curve has equation

$$y=6\sqrt[3]{x^5}-15\sqrt[3]{x^4}-80x+16\,,\qquad x\in\mathbb{R}\,,\qquad x\geq 0\,.$$

Find the coordinates of the stationary point of the curve and determine whether it is a local maximum, a local minimum or a point of inflexion.

$\boxed{\phantom{\text{local minimum at } (16,-2800)}}$, $\boxed{\text{local minimum at } (16,-2800)}$





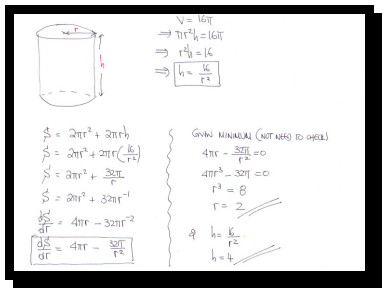
Question 240 (****)

A tank is in the shape of a closed right circular cylinder of radius r m and height h m.

The tank has a volume of 16π m^3 and is made of thin sheet metal.

Given the surface area of the tank is a minimum, determine the value of r and the value of h.

$\boxed{}$, $\boxed{r=2}$, $\boxed{h=4}$



Question 241 (**+)**

The curve C has equation

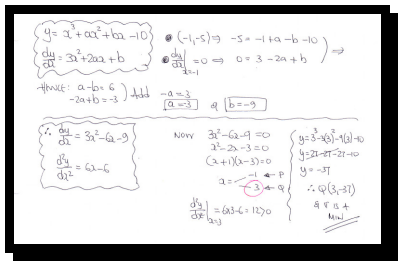
$$y=x^3+ax^2+bx-10,$$

where a and b are constants.

The curve has two stationary points P and Q.

Given the coordinates of P are $(-1,-5)$, find the coordinates of Q and use $\dfrac{d^2y}{dx^2}$ to determine its nature.

$\boxed{}$, $\boxed{Q(3,-37),\text{min}}$



Question 242 **(****+)**

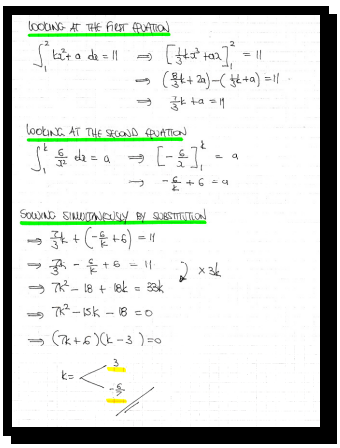
It is given that

$$\int_1^2 kx^2 + a \ dx = 11 \qquad \text{and} \qquad \int_1^k \frac{6}{x^2} \ dx = a ,$$

where a and k are constants.

Determine the possible values of k.

$\square$, $\boxed{k = 3 \cup k = -\frac{6}{7}}$



Question 243 **(****+)**

The profit of a small business, £ P is modelled by the equation

$$P=\frac{\left(54x+6y-xy-324\right)^2}{3x},$$

where x and y are positive variables associated with the running of the company.

It is further known that x and y constrained by the relation

$$3x+y=54.$$

a) Show clearly that

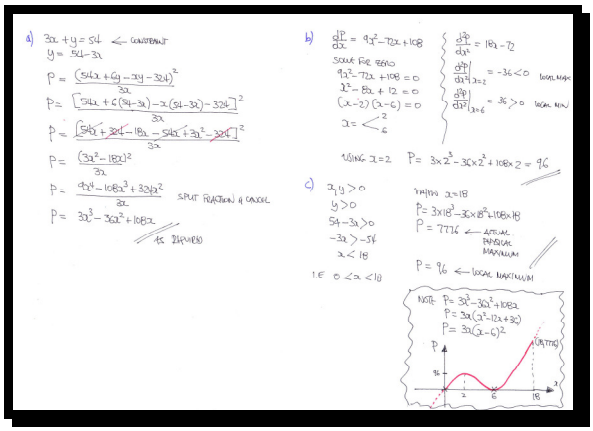
$$P=108x-36x^2+3x^3.$$

b) Hence show that the stationary value of P produces a maximum value of £96 .

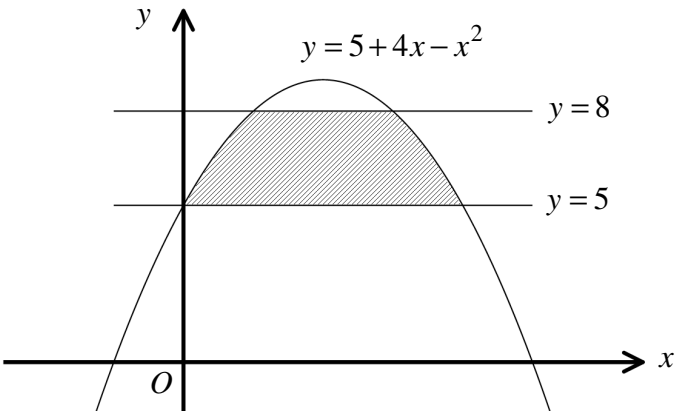
The owner is very concerned about the very small profit and shows the calculations to a mathematician. The mathematician agrees that the calculations are correct but he asserts that the profit is substantially higher.

c) Explain, by calculations, the mathematician's reasoning.

[ ] , [proof]



Question 244 (**+)**



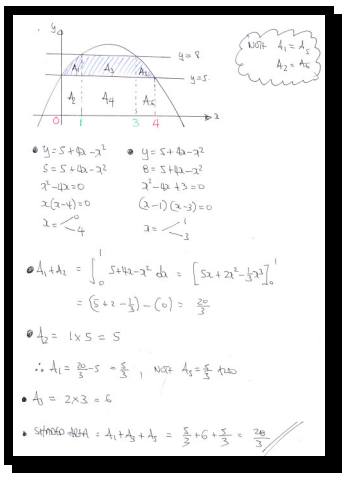
The figure above shows the curve C with equation

$$y = 5 + 4x - x^2 ,$$

intersected by the horizontal straight lines with equations $y = 5$ and $y = 8$.

Calculate the exact area of the shaded region, bounded by C and the two straight lines.

$\square$, $\boxed{\text{area} = \frac{28}{3}}$



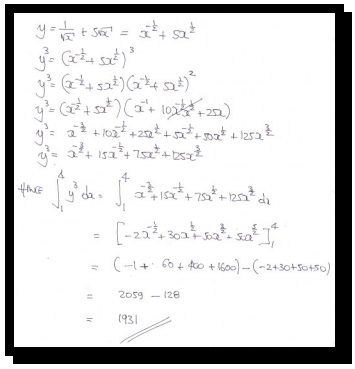
Question 245 (**+) non calculator**

$$y=\frac{1}{\sqrt{x}}+5\sqrt{x}\,,\ x>0\,.$$

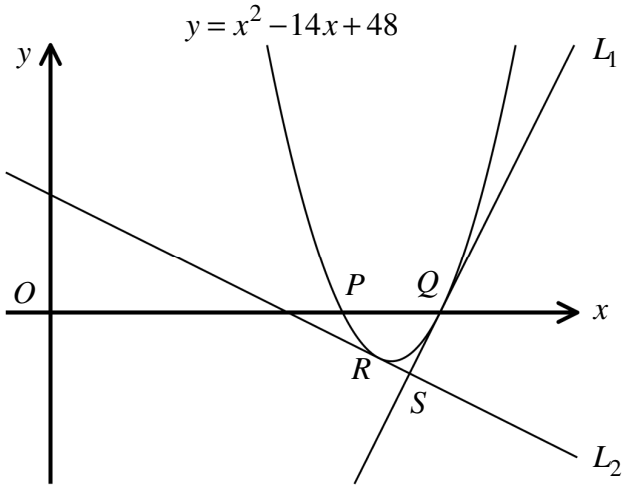
Show clearly that

$$\int_1^4 y^3\ dx=1931\,.$$

proof



Question 246 (**+)**



The figure above shows the curve with equation

$$y = x^2 - 14x + 48 .$$

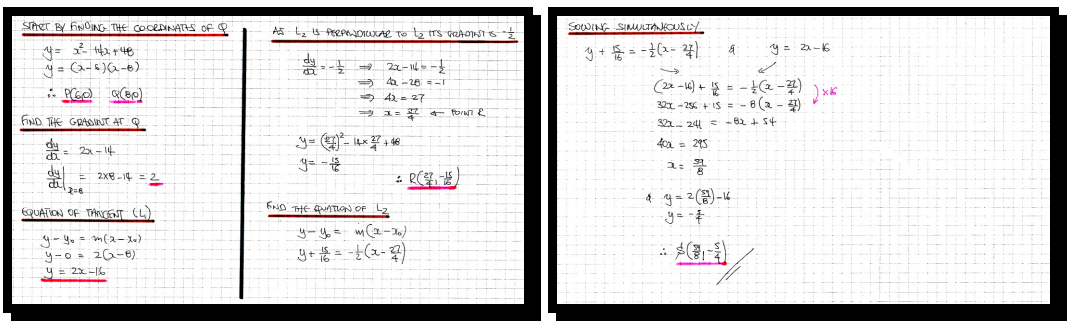
The curve crosses the x axis at the points $P(x_1, 0)$ and $Q(x_2, 0)$, where $x_2 > x_1$.

The tangent to the curve at Q is the straight line L_1.

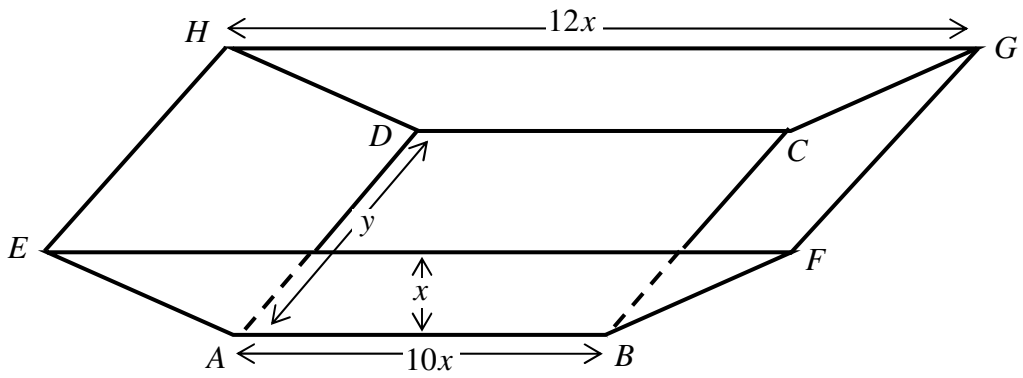
The tangent to the curve at some point R is denoted by L_2.

Given further that L_1 meets L_2 at right angles, find the coordinates of the point of intersection between L_1 meets L_2, denoted in the figure by S.

$\square$, $\boxed{S\left(\frac{59}{8}, -\frac{5}{4}\right)}$



Question 247 (**+)**



The figure above shows the design of a baking tray with a horizontal rectangular base $ABCD$, measuring $10x$ cm by y cm.

The faces $ABFE$ and $DCGH$ are isosceles trapeziums, parallel to each other.

The lengths of the edges EF and HG are $12x$ cm.

The faces $ADHE$ and $BCGF$ are identical rectangles.

The height of the tray is x cm.

The capacity of the tray is 1980 cm^3.

a) Show that the surface area, A cm^2, of the tray is given by

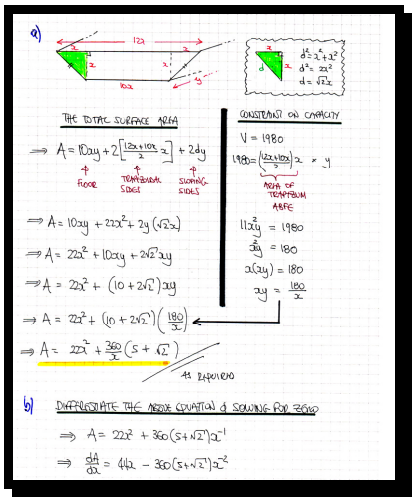
$$A = 22x^2 + \frac{360}{x}\left(5+\sqrt{2}\right).$$

b) Determine the value of x for which A is stationary, showing that this value of x minimizes the value for A.

c) Calculate the minimum surface area of the tray.

$\boxed{}$, $\boxed{x \approx 3.744}$, $\boxed{A_{\text{min}} \approx 925}$

[solution overleaf]



a)

THE TOTAL SURFACE AREA

$\Rightarrow A = 10xy + 2\left[\frac{12x \times 10x}{2}\right] + 2dy$

↑ FLOOR ↑ TRAPEZIUM SIDES ↑ SLOPING SIDES

$\Rightarrow A = 10xy + 22x^2 + 2y(\sqrt{2}x)$

$\Rightarrow A = 22x^2 + 10xy + 2\sqrt{2}xy$

$\Rightarrow A = 22x^2 + (10 + 2\sqrt{2})xy$

$\Rightarrow A = 22x^2 + (10 + 2\sqrt{2})\left(\frac{180}{x}\right)$

$\Rightarrow A = 22x^2 + \frac{360}{x}(5 + \sqrt{2})$ AS REQUIRED

CONSTRAINT ON CAPACITY

$V = 1980$

$1980 = \left(\frac{12x \times 10x}{2}\right) \times y$

↑ AREA OF TRAPEZIUM ABASE

$11x^2y = 1980$

$x^2y = 180$

$x(xy) = 180$

$xy = \frac{180}{x}$

b) DIFFERENTIATE THE ABOVE EQUATION & SOLVING FOR ZERO

$\Rightarrow A = 22x^2 + 360(5 + \sqrt{2})x^{-1}$

$\Rightarrow \frac{dA}{dx} = 44x - 360(5 + \sqrt{2})x^{-2}$

$\Rightarrow \frac{dA}{dx} = 44x - \frac{360(5 + \sqrt{2})}{x^2}$

SOLVING FOR ZERO

$\Rightarrow 0 = 44x - \frac{360(5 + \sqrt{2})}{x^2}$

$\Rightarrow 0 = 44x^3 - 360(5 + \sqrt{2})$

$\Rightarrow x^3 = \frac{90(5 + \sqrt{2})}{11}$

$\Rightarrow x \simeq 3.744$

USING THE SECOND DERIVATIVE

$\Rightarrow \frac{d^2A}{dx^2} = 44 + 720(5 + \sqrt{2})x^{-3}$

$\Rightarrow \frac{d^2A}{dx^2} = 132 > 0$

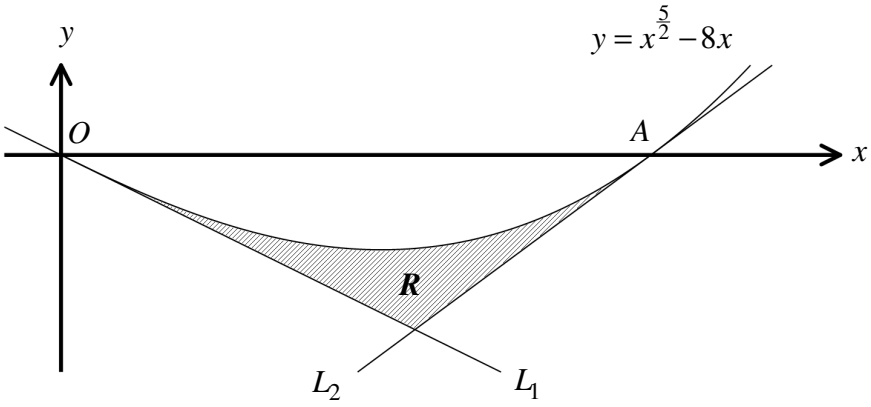
INDEED $x \simeq 3.744$ MINIMIZES A

c) $A = 22x^2 + \frac{360}{x}(5 + \sqrt{2})$

$\Rightarrow A_{MIN} = 22(3.744)^2 + \frac{360}{3.744}(5 + \sqrt{2})$

$\Rightarrow A_{MIN} \simeq 925 \text{ cm}^2$

Question 248 (**+)**



The figure above shows the graph of the curve C with equation

$$y = x^{\frac{5}{2}} - 8x, \ x \geq 0.$$

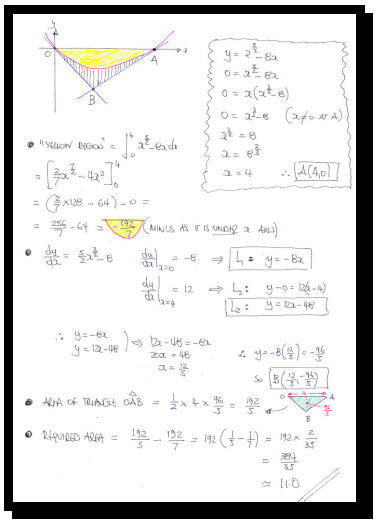
The curve meets the x axis at the origin O and at the point A.

The tangent to C at O is denoted by L_1 and the tangent to C at A is denoted by L_2.

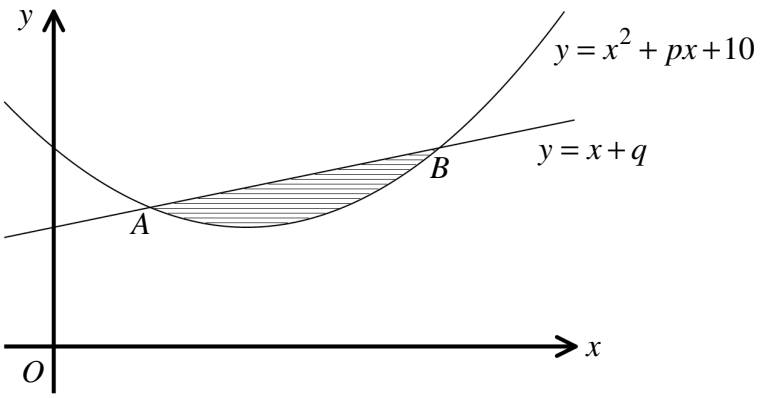
The finite region R, shown shaded in the figure above, is bounded by C, L_1 and L_2.

Determine the area of R.

$$\frac{384}{35} \approx 11.0$$



Question 249 (**+)**



The figure above shows the graphs of a quadratic curve and a straight line with respective equations

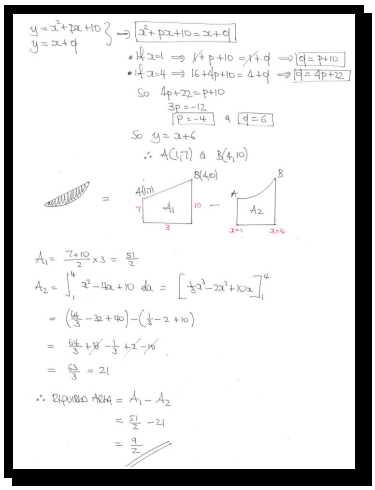
$$y = x^2 + px + 10 \quad \text{and} \quad y = x + q \,,$$

where p and q are constants.

The curve and the straight line intersect at the points A and B, whose x coordinates are 1 and 4, respectively.

Determine the area of the finite region bounded by the curve and the straight line, shown shaded in the figure.

$$\boxed{\text{area} = \frac{9}{2}}$$



Question 250 (**+)**

The volume, V cm^3, of a soap bubble is modelled by the formula

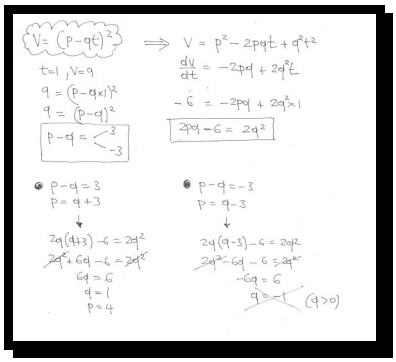
$$V=\left(p-qt\right)^2,\ t\geq 0,$$

where p and q are positive constants, and t is the time in seconds, measured after a certain instant.

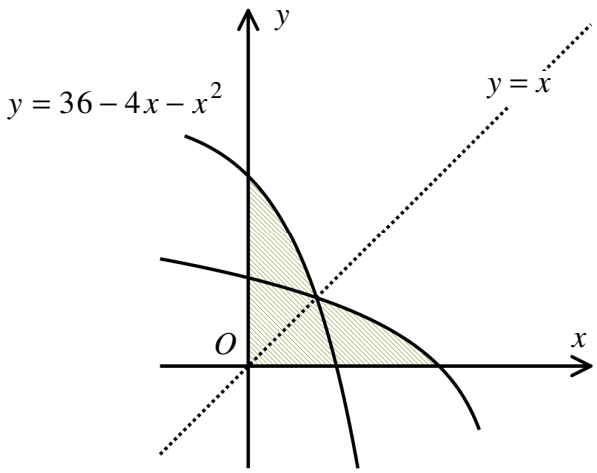
When $t=1$ the volume of a soap bubble is 9 cm^3 and at that instant its volume is decreasing at the rate of 6 cm^3 per second.

Determine the value of p and the value of q.

$\boxed{}$, $\boxed{p=4}$, $\boxed{q=1}$



Question 251 **(****+)**



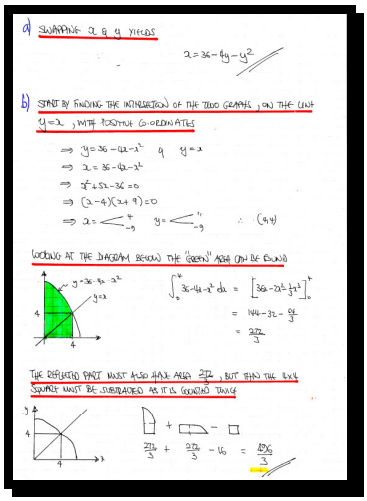
The figure above shows the graph of the curve with equation $y=36-4x-x^2$ and its reflection about the straight line with equation $y=x$.

a) Write down the equation of the curve which is the reflection of the curve with equation $y=36-4x-x^2$ about the straight line with equation $y=x$.

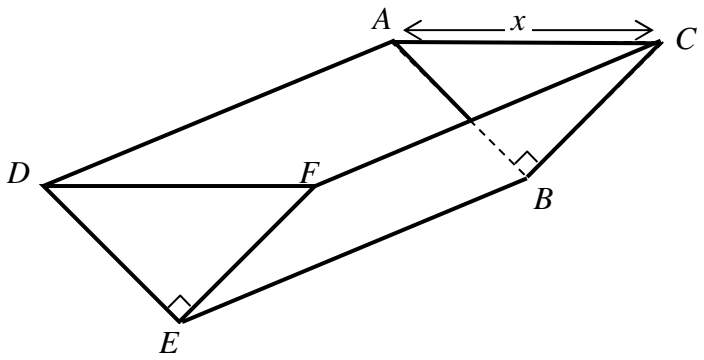
b) Determine an exact value for the area of the finite region bounded by the curve with equation $y=36-4x-x^2$, its reflection about the straight line with equation $y=x$, and the positive x and y axes.

This region is shown shaded in the figure above.

$\boxed{}$, $\boxed{x=36-4y-y^2}$, $\boxed{\text{AREA}=\frac{496}{3}=165\frac{1}{3}}$



Question 252 (**+)**



The figure above shows the design of a horse feeder which in the shape of a hollow, open topped triangular prism.

The triangular faces at the two ends of the feeder are isosceles and right angled, so that $AB = BC = DE = EF$ and $\widehat{ABC} = \widehat{DEF} = 90°$.

The triangular faces are vertical, and the edges AD, BE and CF are horizontal.

The capacity of the feeder is 4 m^3.

a) Show that the surface area, A m^2, of the feeder is given by

$$A = \frac{1}{2}x^2 + \frac{16\sqrt{2}}{x},$$

where x is the length of AC.

b) Determine by differentiation the value of x for which A is stationary, giving the answer in the form $k\sqrt{2}$, where k is an integer.

c) Show that the value of x found in part **(b)** gives the minimum value for A.

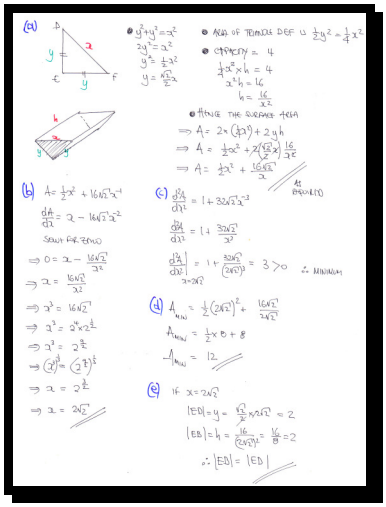
[continues overleaf]

[continued from overleaf]

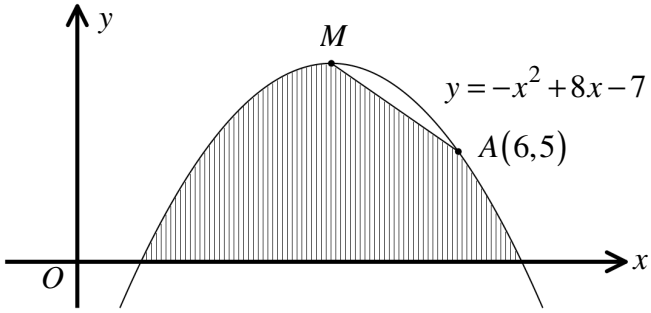
d) Show, by exact calculations, that the minimum surface area of the feeder is 12 m^2.

e) Show further that the length ED is equal to the length EB.

$\boxed{}$, $\boxed{x = 2\sqrt{2} \approx 2.82}$, $\boxed{ED = EB = 2}$



Question 253 **(****+)**



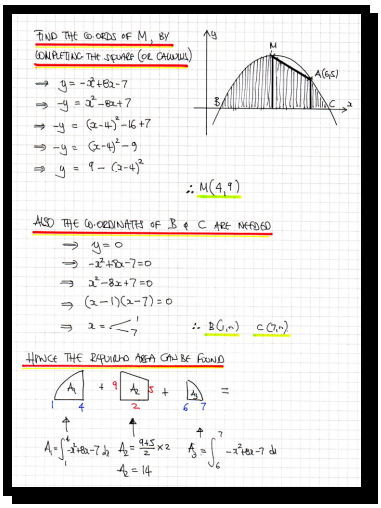
The figure above shows the quadratic curve with equation

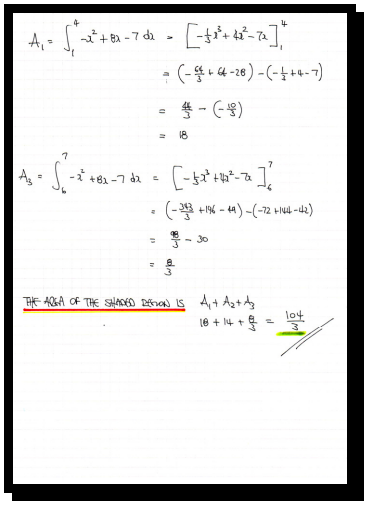
$$y=-x^2+8x-7\,.$$

The point M is the maximum point of the curve and A is another point on the curve whose coordinates are $(6,5)$.

Find the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment from A to M.

$\boxed{}$, $\boxed{\text{area}=\frac{104}{3}}$





Question 254 (**+)**

A curve C has equation

$$y = 2x^3 - 5x^2 + a, \ x \in \mathbb{R},$$

where a is a constant.

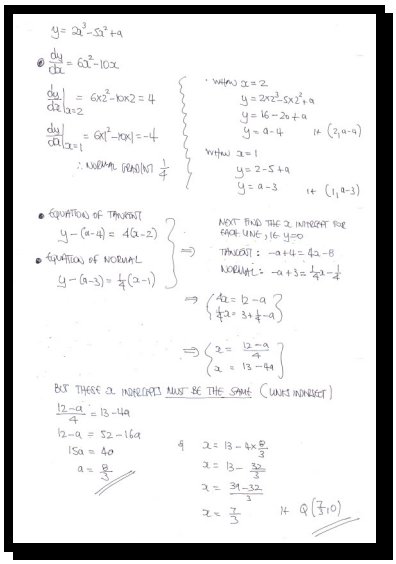
The tangent to C at the point where $x = 2$ and the normal to C at the point where $x = 1$, meet at the point Q.

Given that Q lies on the x axis, determine in any order …

a) … the value of a.

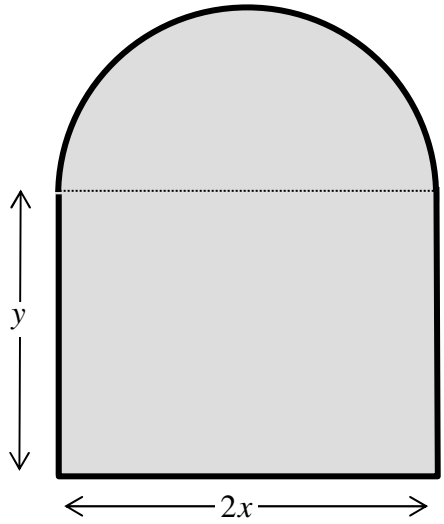
b) … the coordinates of Q.

$\square$, $\boxed{a = \frac{8}{3}}$, $\boxed{Q\left(\frac{7}{3}, 0\right)}$



Question 255 (**+)**

The figure below shows the design of a window which is the shape of a semicircle attached to rectangle.



The diameter of the semicircle is $2x$ metres and is attached to one side of the rectangle also measuring $2x$ meters. The other side of the rectangle is y metres.

The total area of the window is 2 m^2.

a) Show that perimeter, P m, is given by

$$P=\frac{1}{2}(4+\pi)x+\frac{2}{x}.$$

b) Determine by differentiation an exact value of x for which P is stationary.

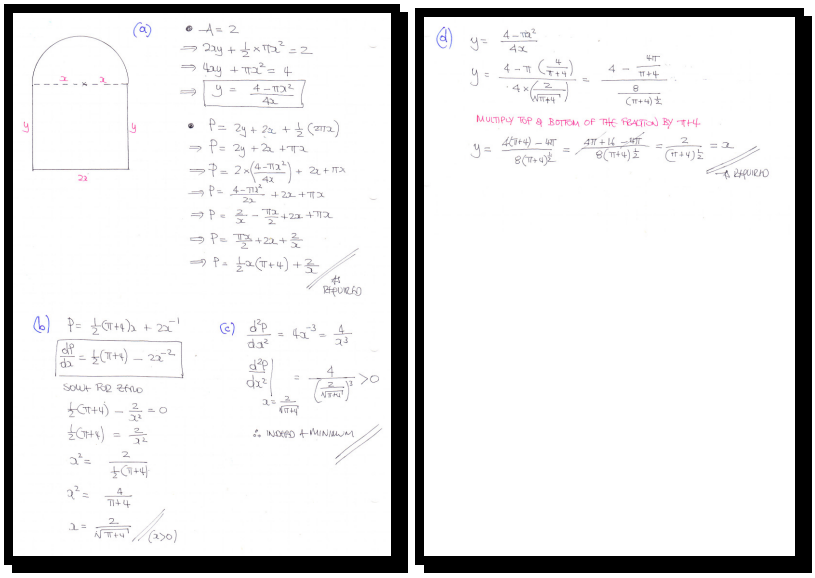
[continues overleaf]

[continued from overleaf]

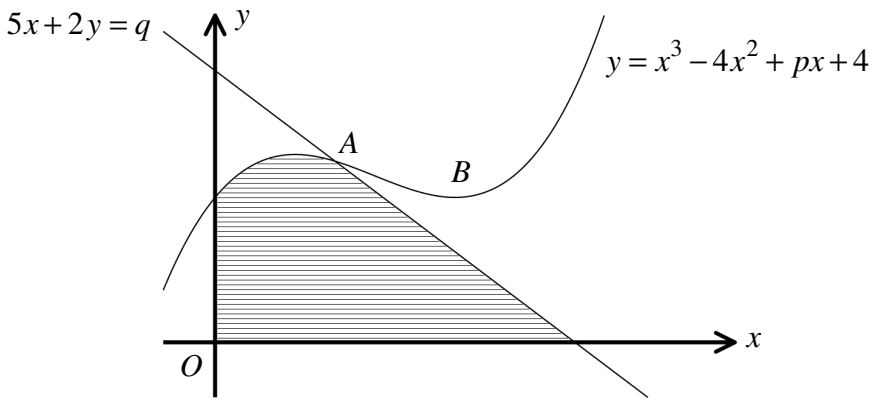
c) Show that the value of x found in part (b) gives the minimum value for P.

d) Show that when P takes a minimum value $x = y$.

$$\boxed{x = \frac{2}{\sqrt{\pi + 4}} \approx 0.748}$$



Question 256 (**+)**



The figure above shows a curve C and a straight line L, with respective equations

$$y = x^3 - 4x^2 + px + 4 \qquad \text{and} \qquad 5x + 2y = q,$$

where p and q are constants.

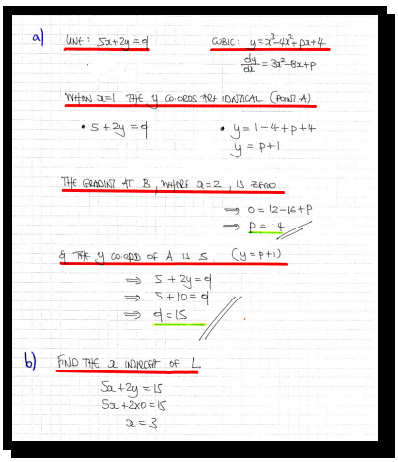
C and L intersect at the point A. The point B is a turning point C.

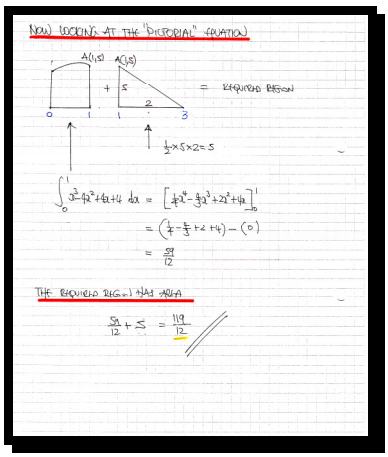
Given that the respective x coordinates of A and B are 1 and 2, determine …

a) … the value of p and the value of q

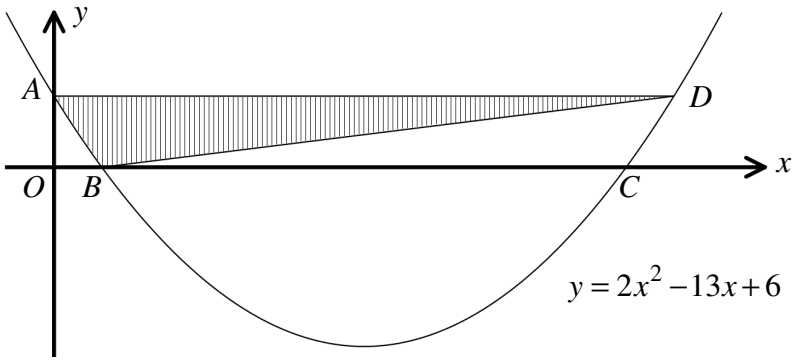
b) … the area of the shaded region bounded by C, L and the coordinate axes.

$\boxed{}$, $\boxed{p = 4,\ q = 15}$, $\boxed{\text{area} = \frac{119}{12}}$





Question 257 (**+)**



The figure above shows the curve with equation

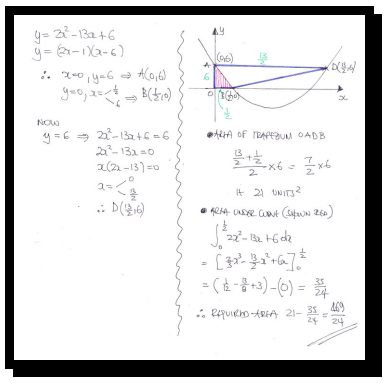
$$y = 2x^2 - 13x + 6 .$$

The points A, B and C are the points where the curve meets the coordinate axes.

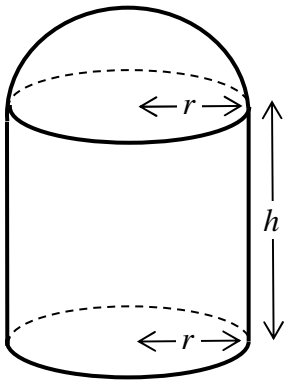
The point D is such so that the straight line segment AD is parallel to the x axis.

Find the exact area of the shaded region, bounded by the curve and the straight line segments BD and AD.

$\square$, $\boxed{\text{area} = \frac{469}{24}}$



Question 258 **(****+)**



The figure above shows a hollow container consisting of a right circular cylinder of radius r cm and of height h cm joined to a hemisphere of radius r cm.

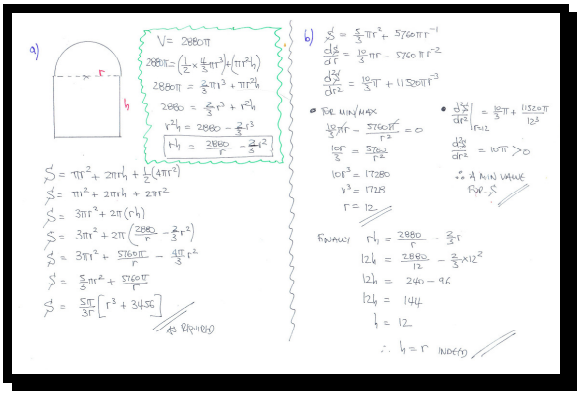
The cylinder is open on one of the circular ends and the hemisphere is also open on its circular base. The cylinder is joined to the hemisphere at their open ends so that the resulting object is completely sealed.

a) Given that volume of the container is exactly 2880π cm^3, show clearly that the total surface area of the container, S cm^2, is given by

$$S=\frac{5\pi}{3r}\left(r^3+3456\right).$$

b) Show further than when S is minimum, $r=h$.

proof



Question 259 (**+)**

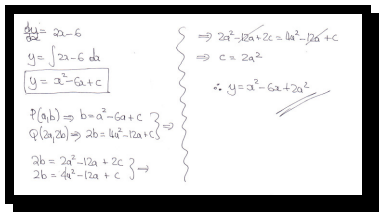
A quadratic curve C passes through the points $P(a,b)$ and $Q(2a,2b)$, where a and b are constants.

The gradient at any given point on C is given by

$$\frac{dy}{dx}=2x-6.$$

Find an equation for C, in terms of a.

$\boxed{}$, $\boxed{y=x^2-6x+2a^2}$



Question 260 **(****+)**

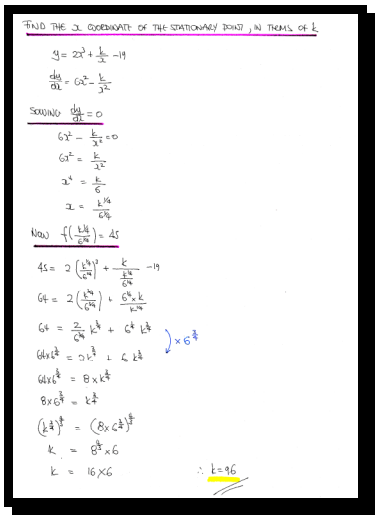
A curve has equation

$$y = 2x^3 + \frac{k}{x} - 19, \qquad x > 0,$$

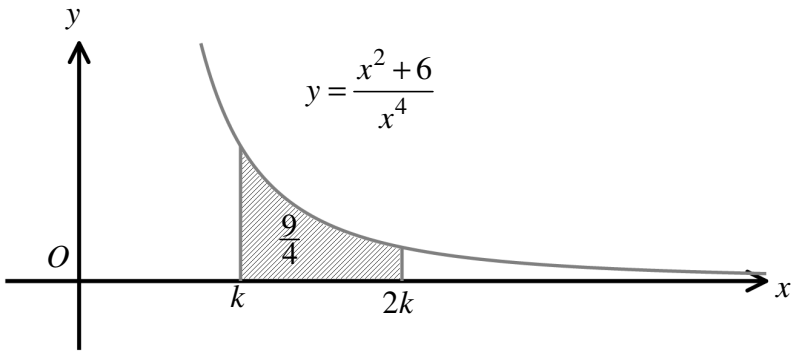
where k is a positive constant.

If the y coordinate of the stationary point of this curve is 45, find the value of k.

$\boxed{}$, $\boxed{k = 96}$



Question 261 (**+)**



The figure above shows the graph of the curve with equation

$$y=\frac{x^2+6}{x^4},\quad x>0.$$

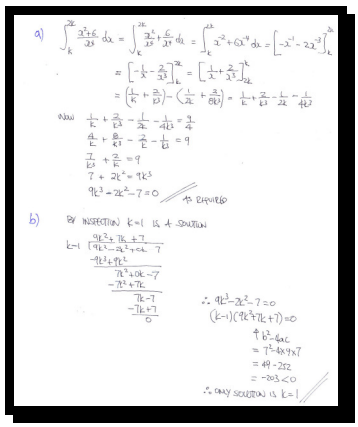
The area of the region between the curve and the x axis for $k\le x\le 2k$, where k is a positive constant, is exactly $\frac{9}{4}$.

a) Show clearly that

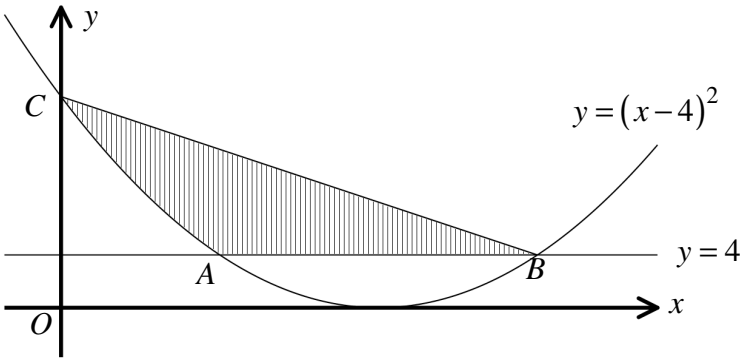
$$9k^3-2k^2-7=0.$$

b) Hence find the value of k, showing further that there is no other value of k which satisfies the equation of part **(b)**.

$\boxed{}$, $\boxed{k=1}$



Question 262 (**+)**



The diagram above shows the curve with equation

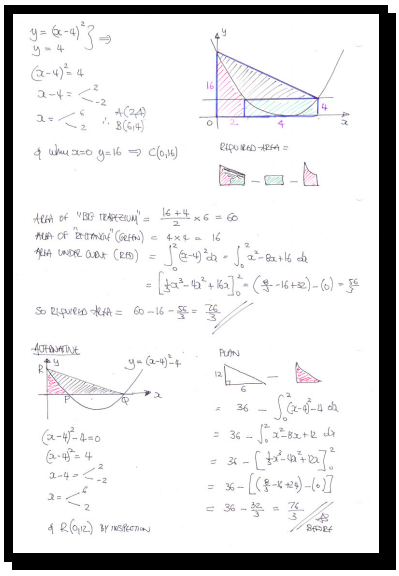
$$y=(x-4)^2,\ x\in\mathbb{R},$$

intersected by the straight line with equation $y=4$, at the points A and B.

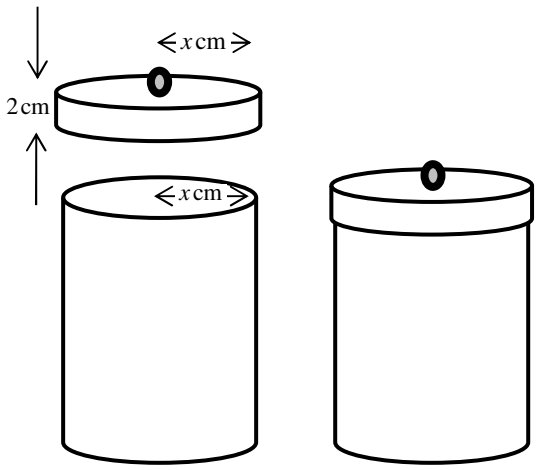
The curve meets the y axis at the point C.

Calculate the exact area of the shaded region, bounded by the curve and the straight line segments AB and BC.

$\square$, $\boxed{\text{area}=\frac{76}{3}}$



Question 263 (**+)**



The figure above shows the design of coffee jar with a "push on" lid.

The jar is in the shape of a right circular cylinder of radius x cm. It is fitted with a lid of width 2 cm, which fits tightly on the top of the jar, so it may be assumed that it has the same radius as the jar.

The jar and its lid is made of sheet metal and there is no wastage.

The total metal used to make the jar and its lid is 190π cm^2.

(*This figure does not include the handle of the lid which is made of different material.*)

a) Show that volume of the jar, V cm^3, is given by

$$V = \pi\left(95x - 2x^2 - x^3\right).$$

b) Determine by differentiation the value of x for which V is stationary.

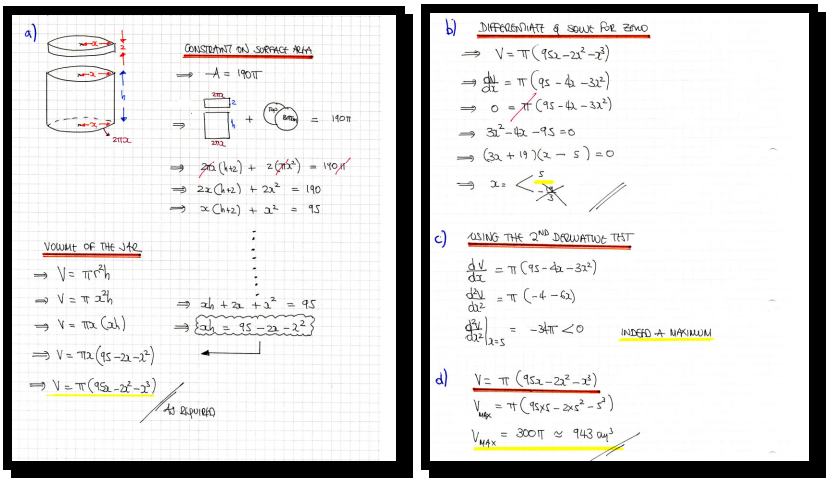
[continues overleaf]

[continued from overleaf]

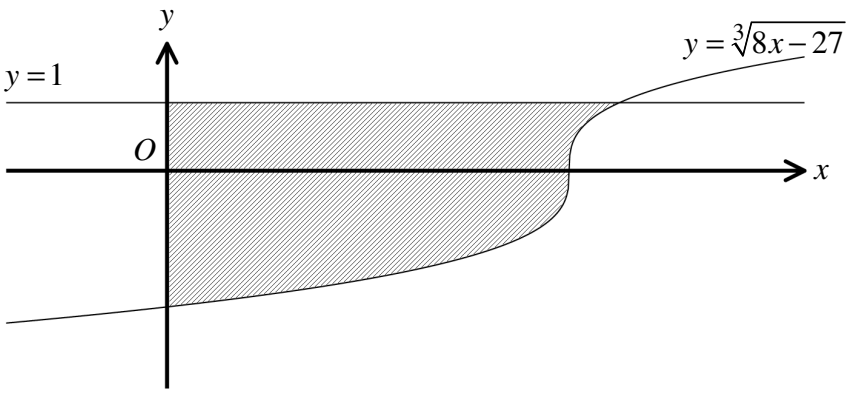
c) Show that the value of x found in part **(b)** gives the maximum value for V.

d) Hence determine the maximum volume of the jar.

$\boxed{}$, $\boxed{x = 5 \text{ cm}}$, $\boxed{V_{\max} = 300\pi \approx 942 \text{ cm}^3}$



Question 264 **(****+)**



The figure above shows the curve C with equation

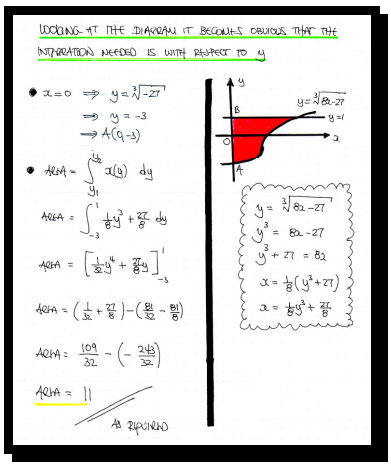
$$y=\sqrt[3]{8x-27}\,,\ x\in\mathbb{R}\,.$$

The finite region R, shown shaded in the figure, is bounded by C, the y axis and the straight line with equation $y=1$.

When the lengths are measured in m, R models the design of a yacht rudder.

Show that the area of the yacht rudder is 11 m^2.

, proof



Question 265 **(****+)**

The curve C has equation

$$y=\frac{x^3\left(5x\sqrt{x}-128\right)}{\sqrt{x}},\ x\in\mathbb{R},\ x>0.$$

a) Determine expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

b) Show that the y coordinate of the stationary point of C is $-k\sqrt[3]{4}$, where k is a positive integer.

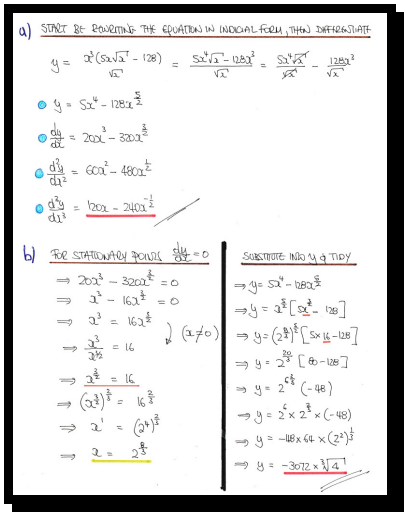
c) Evaluate $\frac{d^2y}{dx^2}$ at the stationary point of C.

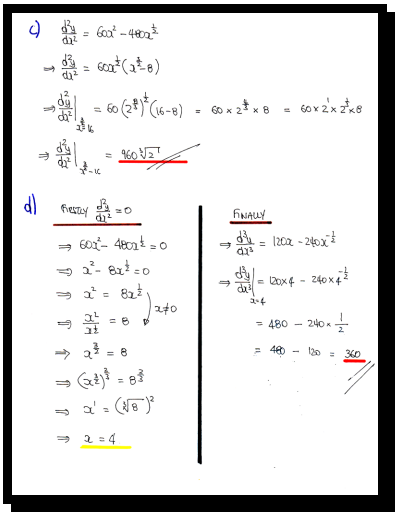
Give the answer in terms of $\sqrt[3]{2}$.

d) Find the value of $\frac{d^3y}{dx^3}$ at the point on C, where $\frac{d^2y}{dx^2}=0$.

$\boxed{}$, $\boxed{\frac{dy}{dx}=20x^3-320x^{\frac{3}{2}}}$, $\boxed{\frac{d^2y}{dx^2}=60x^2-480x^{\frac{1}{2}}}$, $\boxed{\frac{d^3y}{dx^3}=120x-240x^{-\frac{1}{2}}}$,

$\boxed{k=3072}$, $\boxed{960\sqrt[3]{2}}$, $\boxed{360}$





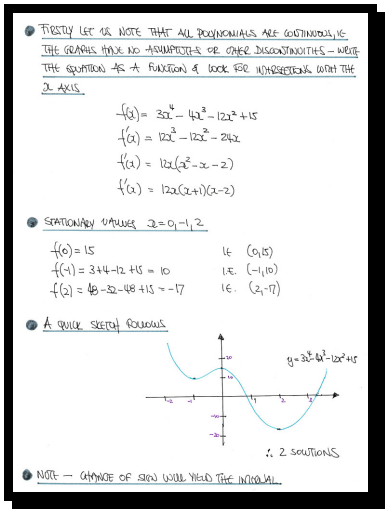
Question 266 (**+)**

Use differentiation to establish the number of real solutions of the equation

$$3x^4-4x^3-12x^2+15=0.$$

You are not expected to solve the equation.

[ ], [2 solutions]



Question 267 (**+)**

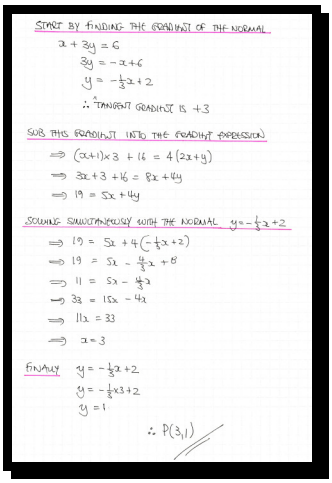
The gradient function of a curve satisfies the following relationship

$$(x+1)\frac{dy}{dx}+16=4(2x+y).$$

The normal to the curve at the point P has equation $x+3y=6$.

Determine the coordinates of P.

$\boxed{}$, $\boxed{P(3,1)}$


START BY FINDING THE GRADIENT OF THE NORMAL

$x+3y=6$

$3y=-x+6$

$y=-\frac{1}{3}x+2$

$\therefore$ TANGENT GRADIENT IS $+3$

SUB THIS GRADIENT INTO THE GRADIENT EXPRESSION

$\Rightarrow (x+1)\times 3+16=4(2x+y)$

$\Rightarrow 3x+3+16=8x+4y$

$\Rightarrow 19=5x+4y$

SOLVING SIMULTANEOUSLY WITH THE NORMAL $y=-\frac{1}{3}x+2$

$\Rightarrow 19=5x+4\left(-\frac{1}{3}x+2\right)$

$\Rightarrow 19=5x-\frac{4}{3}x+8$

$\Rightarrow 11=5x-\frac{4}{3}x$

$\Rightarrow 33=15x-4x$

$\Rightarrow 11x=33$

$\Rightarrow x=3$

FINALLY $y=-\frac{1}{3}x+2$

$y=-\frac{1}{3}\times 3+2$

$y=1$

$\therefore P(3,1)$

Question 268 (**+)**

A quartic curve C has equation

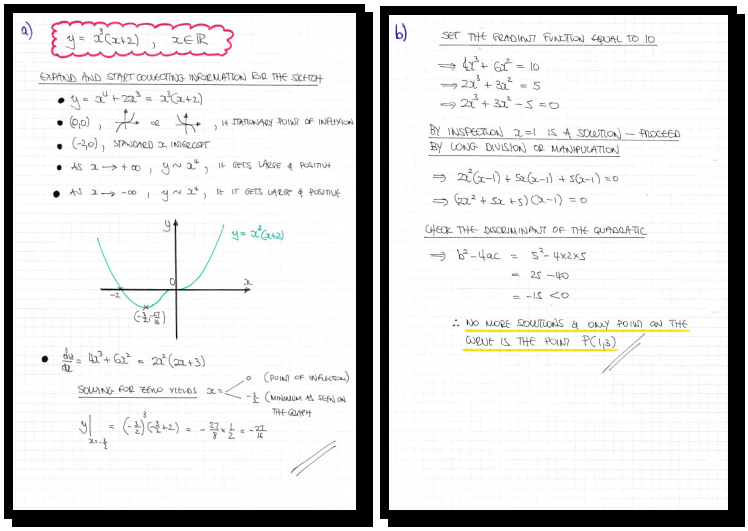
$$y = x^3(x+2), \ x \in \mathbb{R}.$$

a) Sketch the graph of C.

The sketch must contain the coordinates of any stationary points and the coordinates of the points of intersection with the coordinate axes.

b) Show that there is only one point on C where the gradient is 10.

[ ], [proof]



Question 269 (**+)**

The function $y=f\left(x\right)$ satisfies the following relationships

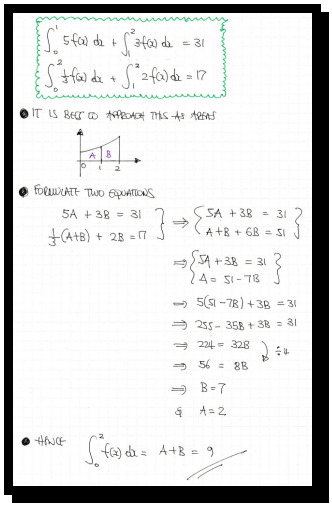
$$\int_0^1 5f\left(x\right)\,dx \;+\; \int_1^2 3f\left(x\right)\,dx \;=\; 31$$

$$\int_0^2 \frac{1}{3}f\left(x\right)\,dx \;+\; \int_1^2 2f\left(x\right)\,dx \;=\; 17\,.$$

Determine the value of

$$\int_0^2 f\left(x\right)\,dx\,.$$

$\square$, $\boxed{9}$



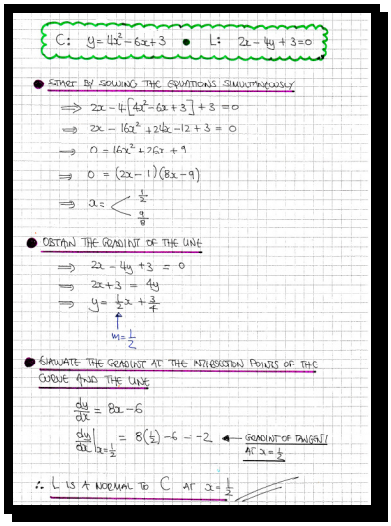
Question 270 (**+)**

A curve C and a straight L have respective equations

$$C:\ y=4x^2-6x+3 \qquad \text{and} \qquad L:\ 2x-4y+3=0.$$

Show that L is a normal to C, at some point.

$\boxed{}$, $\boxed{\text{proof}}$



Question 271 **(****+)**

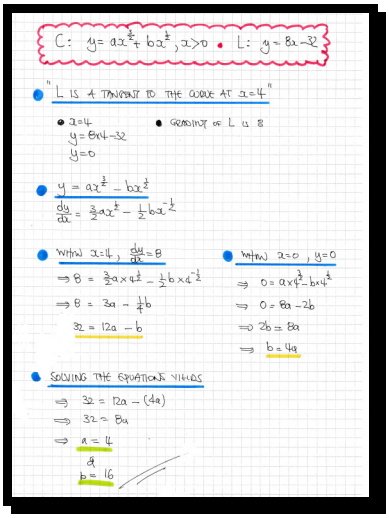
A curve C and a straight line L have respective equations

$$C:\ y=ax^{\frac{3}{2}}-bx^{\frac{1}{2}} \qquad \text{and} \qquad L:\ y=8x-32,$$

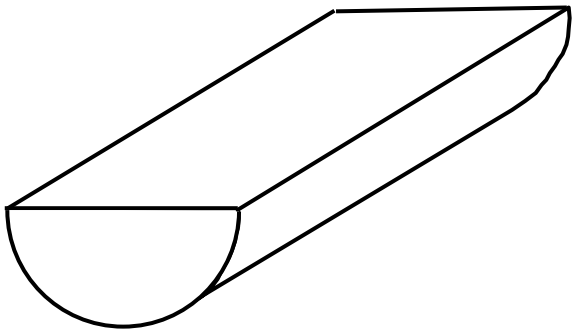
where a and b are non zero constants.

Given that L is a tangent to C at the point where $x=4$ is, determine the value of a and the value of b.

$\boxed{}$, $\boxed{a=4,\ b=16}$



Question 272 (**+)**

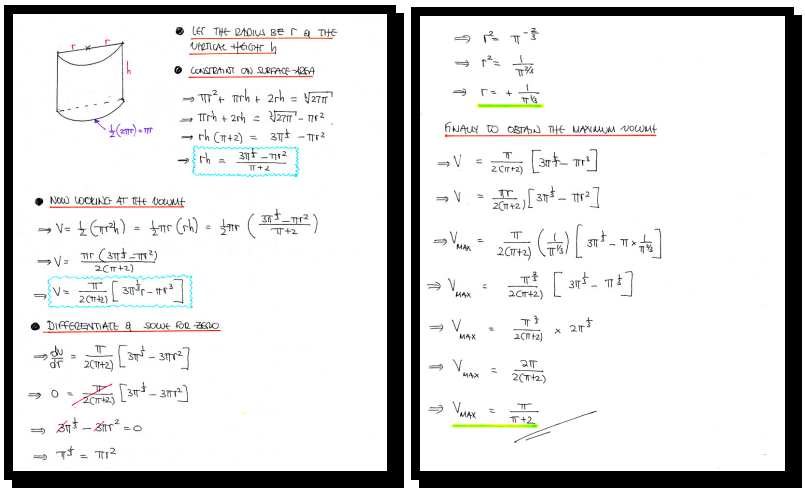


The figure above shows a solid prism, which is in the shape a right semi-circular cylinder.

The total surface area of the 4 faces of the prism is $\sqrt[3]{27\pi}$.

Given that the measurements of the prism are such so that its volume is maximized, find in exact simplified form the volume of the prism.

$\square$, $\boxed{V_{\max}=\dfrac{\pi}{\pi+2}}$



Question 273 (*****)

A curve C and a straight line L have respective equations

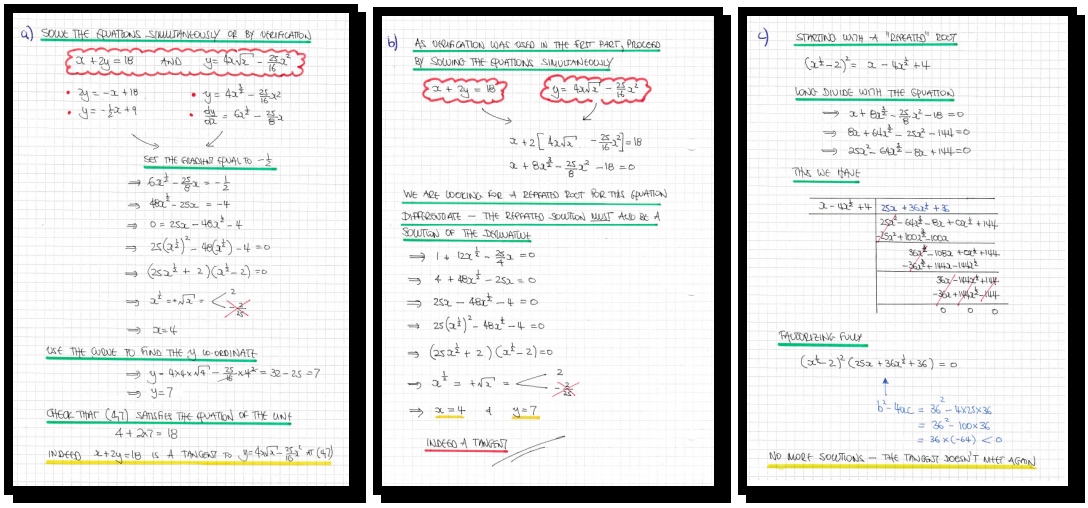
$$C:\ y=4x\sqrt{x}-\frac{25x^2}{16} \qquad \text{and} \qquad L:\ x+2y=18\,.$$

a) Show that L is a tangent to C, at some point to be found.

b) Verify the answer of part **(a)** by an alternative method.

c) Show further that L does not meet C again.

[ ], [proof]



Question 274 (*****)

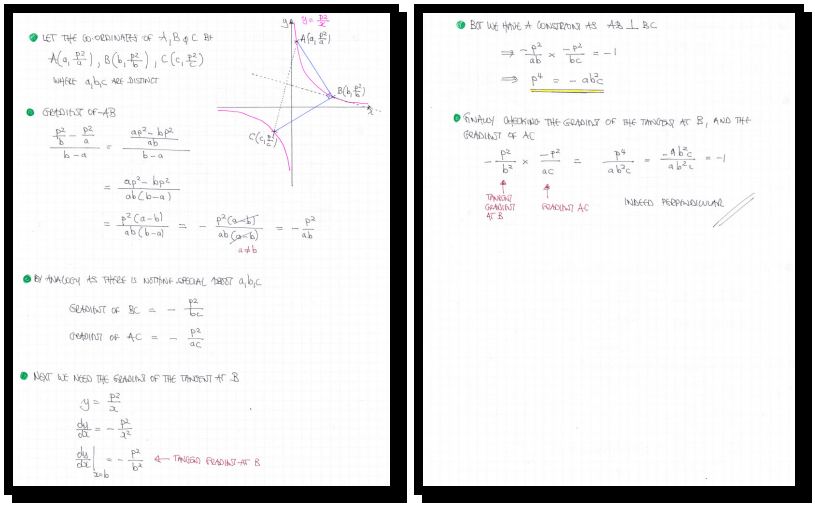
The distinct points A, B and C lie on the curve with equation

$$xy = p^2,$$

where p is a positive constant.

Given that ABC is a right angle, show that the tangent to the curve at B, is perpendicular to AC.

$\boxed{}$, proof



Question 275 (*****)

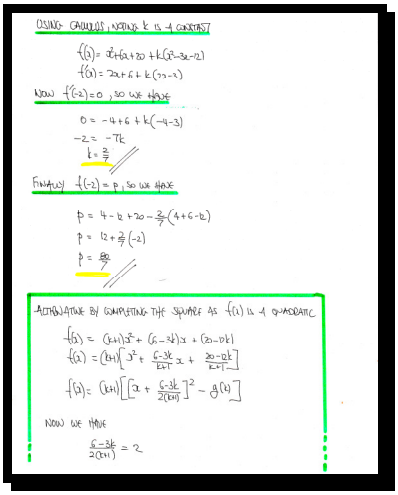
A quadratic curve has equation

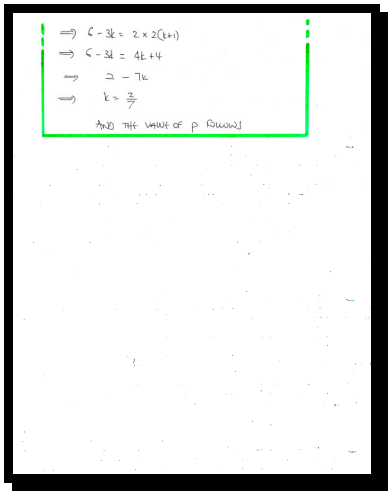
$$f(x) \equiv x^2 + 6x + 20 + k\left(x^2 - 3x - 12\right),$$

where k is a constant.

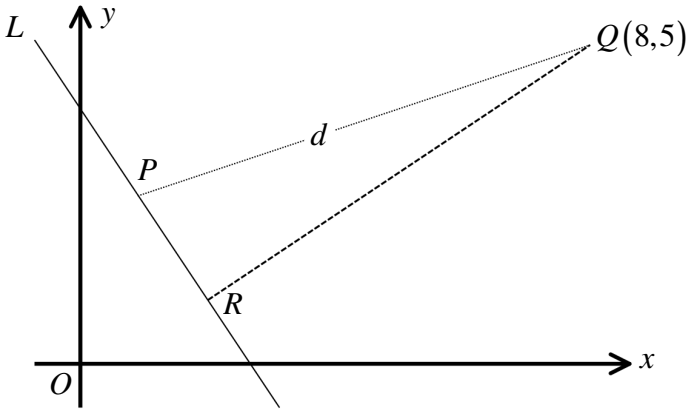
Given that the point $P(-2, p)$ is the minimum point of the curve, determine the value of each of the constants p and k.

$\boxed{}$, $\boxed{p = \frac{80}{7}, k = \frac{2}{7}}$





Question 276 **(*****)**



The straight line L has equation $3x+2y=8$.

The point $P(x,y)$ lies on L and the point $Q(8,5)$ lies outside L. The point R lies on L so that QR is perpendicular to L. The length PQ is denoted by d.

a) Show clearly that

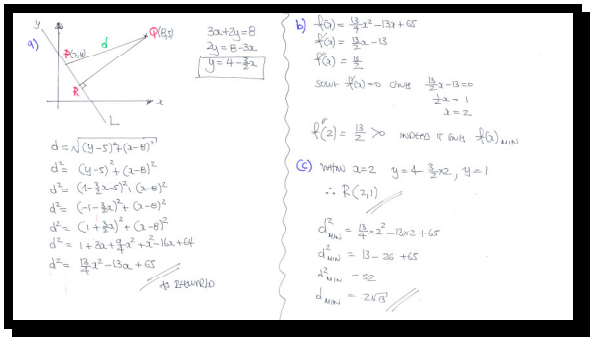
$$d^2=65-13x+\frac{13}{4}x^2.$$

Let $f(x)=65-13x+\frac{13}{4}x^2$.

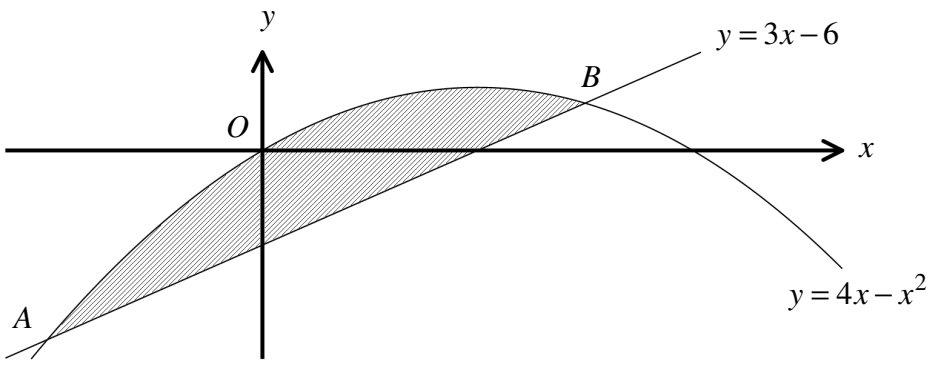
b) Use **differentiation** to find the stationary value of $f(x)$, fully justifying that this value of x minimizes the value of $f(x)$.

c) State the coordinates of R and find, as an exact surd, the shortest distance of the point Q from L

$\boxed{x=2}$, $\boxed{R(2,1)}$, $\boxed{\sqrt{52}}$



Question 277 (*****)



The figure above shows the graph of the curve C with equation

$$y = 4x - x^2 ,$$

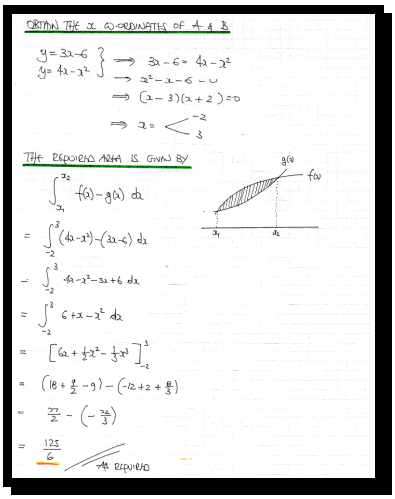
intersected by the straight line L with equation

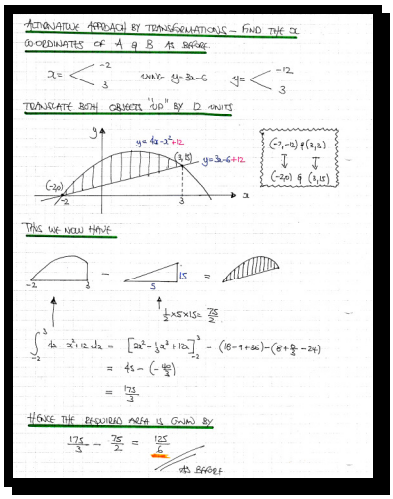
$$y = 3x - 6 .$$

The finite region R is bounded by C and L.

Show that the area of R, shown shaded in the above figure, is $\frac{125}{6}$.

[ ], [proof]





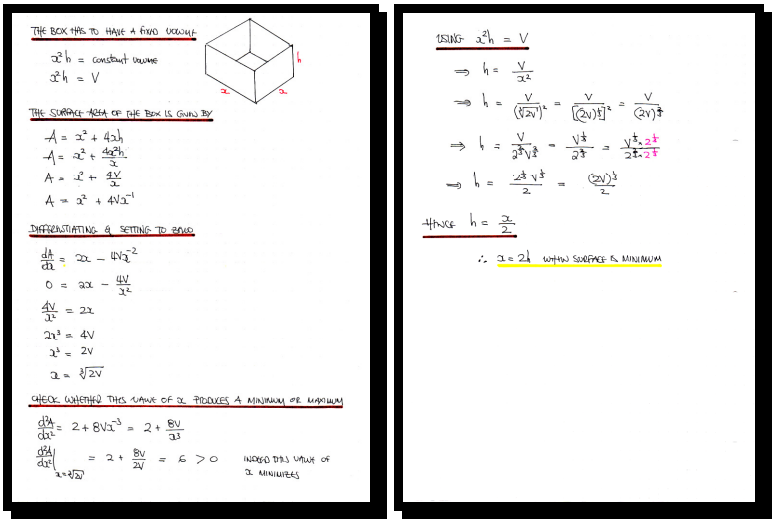
Question 278 (*****)

An **open** box is to be made of thin sheet metal, in the shape of a cuboid with a square base of length x and height h.

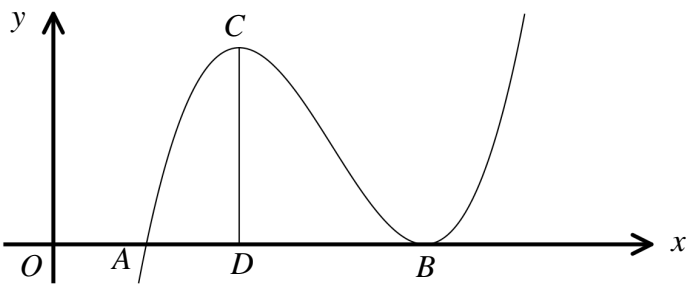
The box is to have a **fixed** volume.

Determine the value of x, in terms of h, when the surface area of the box is minimum.

$\boxed{}$, proof



Question 279 **(*****)**



The figure above shows a cubic curve whose coefficient of x^3 is 1.

The curve crosses the x axis at $A(a,0)$ and touches the x axis at $B(b,0)$, where a and b are positive constants.

The point C is a local maximum of the curve.

The point D lies on the x axis so that CD is parallel to the y axis.

Show, with a detailed method, that

$$|AB| = 3|AD|.$$

[ ], proof

START BY WRITING DOWN THE EQUATION OF THE CUBIC

$\Rightarrow y = (x-a)(x-b)^2$

$\Rightarrow y = (x-a)(x^2-2bx+b^2) = x^3-2bx^2+b^2x - ax^2+2abx-ab^2$

$\Rightarrow y = x^3-(a+2b)x^2+(2ab+b^2)x+ab^2$

DIFFERENTIATING & SETTING TO ZERO

$\Rightarrow \frac{dy}{dx} = 3x^2-2(a+2b)x+(2ab+b^2)$

$\Rightarrow 0 = 3x^2-2(a+2b)x+b(2a+b)$

AS $x=b$ IS A SOLUTION, BEING A LOCAL MINIMUM

$\Rightarrow (x-b)[3x-(2a+b)] = 0$

$\Rightarrow x = b \leftarrow$ POINT B, $\frac{2a+b}{3} \leftarrow$ POINT C

NOW WE HAVE $A(a,0)$, $B(b,0)$, $D\left(\frac{2a+b}{3},0\right)$

- $|AB| = b-a$
- $|AD| = \frac{2a+b}{3}-a = \frac{2a+b-3a}{3} = \frac{b-a}{3}$

$\therefore |AD| = \frac{b-a}{3}$

$|AD| = \frac{|AB|}{3}$

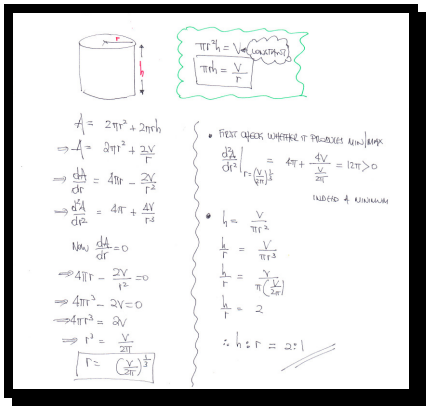
$|AB| = 3|AD|$ AS REQUIRED

Question 280 (*****)

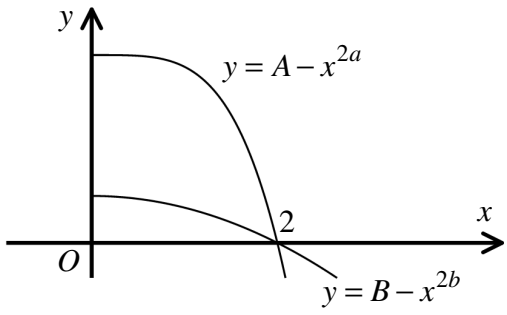
A solid right circular cylinder of fixed volume has radius r and height h.

Show clearly that when the surface area of the cylinder is minimum $h:r=2:1$.

proof



Question 281 (*****)



The figure above shows the curves with equations

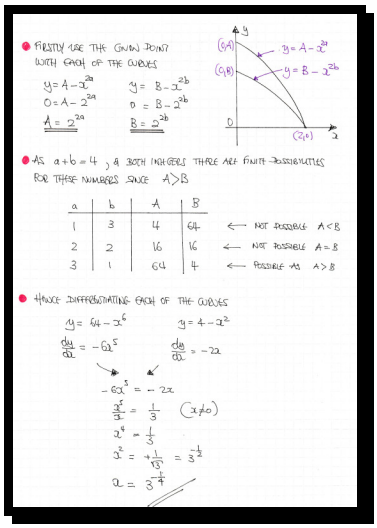
$$y = A - x^{2a} \quad \text{and} \quad y = B - x^{2b} \ , \quad x \geq 0 ,$$

where A and B are positive constants with $A > B$, and a and b are positive integers with $a + b = 4$.

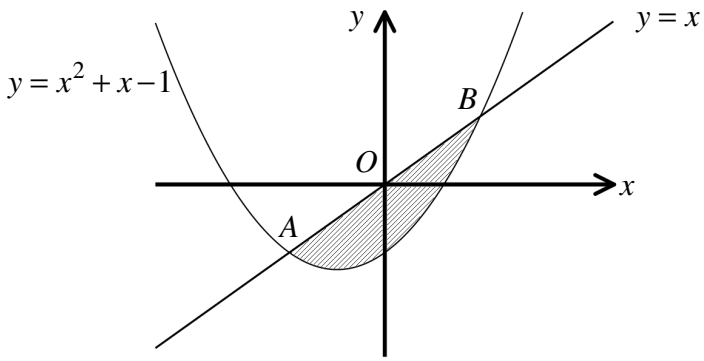
Both curves meet the x axis at the point $(2,0)$.

Find the exact value of x for which both curves have the same gradient.

$\square$, $\boxed{x = 3^{-\frac{1}{4}}}$



Question 282 **(*****)**



The figure above shows the graph of the curve C with equation

$$y = x^2 + x - 1,$$

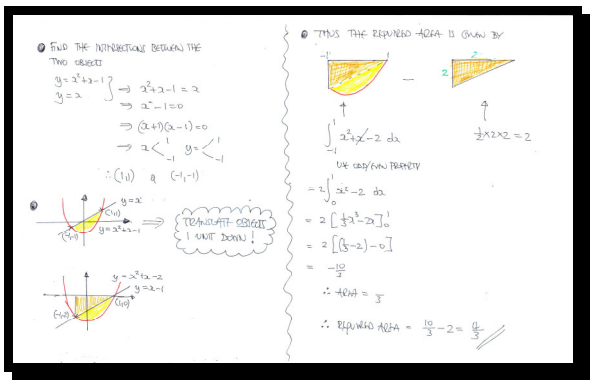
intersected by the straight line L with equation

$$y = x.$$

The points A and B, are the points of intersection between C and L, as shown in the above figure. The finite region R is bounded by C and L.

Show that the area of R, shown shaded in the above figure, is $\frac{4}{3}$.

proof



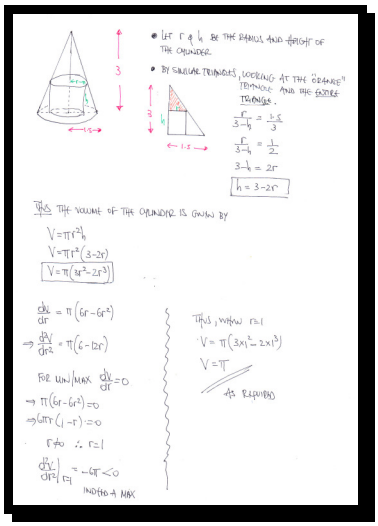
Question 283 (*****)

A solid right circular cylinder is to be cut out of a solid right circular cone, whose radius is 1.5 m and its height is 3 m.

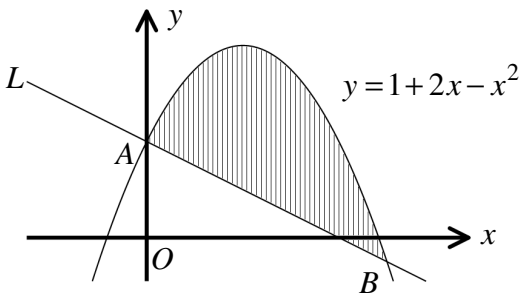
The axis of symmetry of the cone coincides with the axis of symmetry of the cylinder which passes though its circular ends. The circumference of one end of the cylinder is in contact with the curved surface of the cone and the other end of the cylinder lies on the base of the cone.

Show that the maximum volume of the cylinder to be cut out is π m^3.

$\boxed{}$, $\boxed{\text{proof}}$



Question 284 (*****)



The diagram above shows part of the curve C, with equation

$$y = 1 + 2x - x^2 .$$

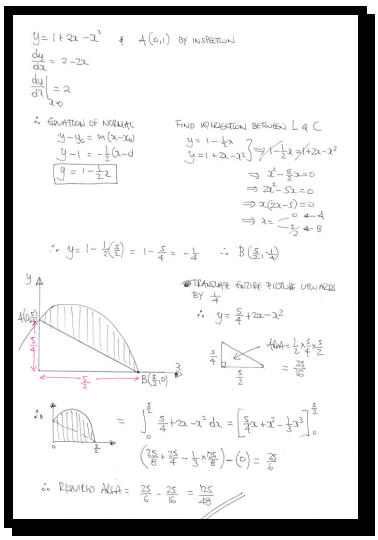
The curve crosses the y axis at the point A.

The straight line L is the normal to C at A.

The point B is a point of intersection between C and A.

Find the exact area of the finite region, bounded by C and L.

$\square$, $\boxed{\text{area} = \frac{125}{48}}$

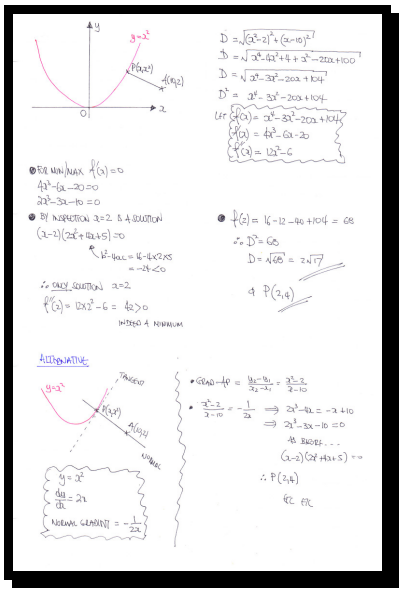


Question 285 (*****)

The point P lies on the curve with equation $y = x^2$, so that its distance from the point $A(10,2)$ is **least**.

Determine the coordinates of P and the distance AP.

$$\boxed{P(2,4)}, \quad \boxed{d_{\min} = 2\sqrt{17}}$$

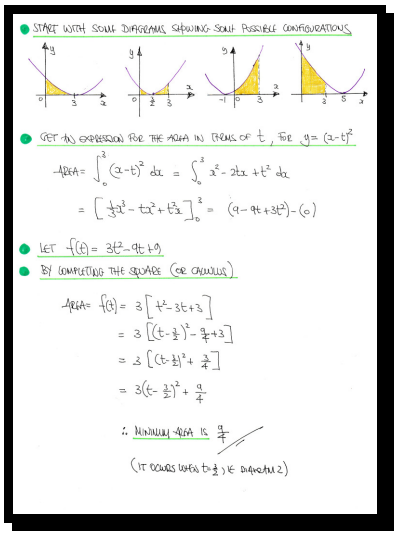


Question 286 (*****)

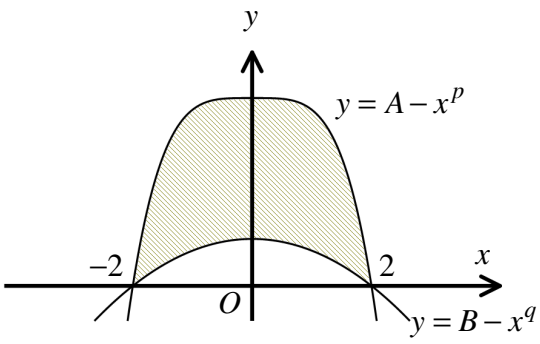
The curve with equation $y = x^2$, $x \in \mathbb{R}$, is translated parallel to the x axis, so that R denotes the area of the finite region bounded by the coordinated axes, the curve and the straight line with equation $x = 3$.

Determine the least value of R.

$\square$, $\boxed{\frac{9}{4}}$



Question 287 (*****)



The figure above shows the curves with equations

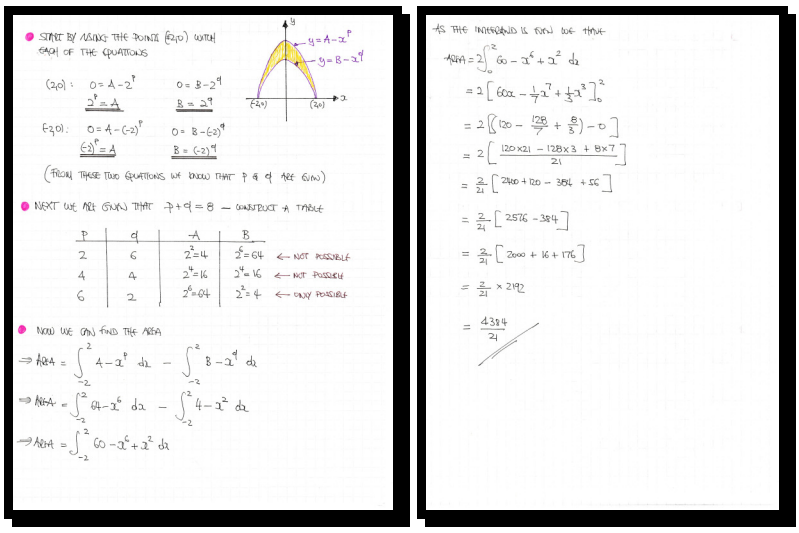
$$y = A - x^p \quad \text{and} \quad y = B - x^q \ ,$$

where A and B are positive constants with $A > B$, and p and q are positive integers with $p + q = 8$.

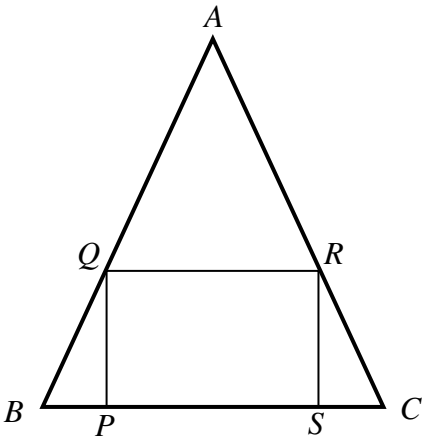
Both curves meet the x axis at the points $(-2,0)$ and $(2,0)$.

Find the exact value of the area bounded between the two curves.

$\square$, $\boxed{\text{area} = \dfrac{4384}{21}}$



Question 288 **(*****)**

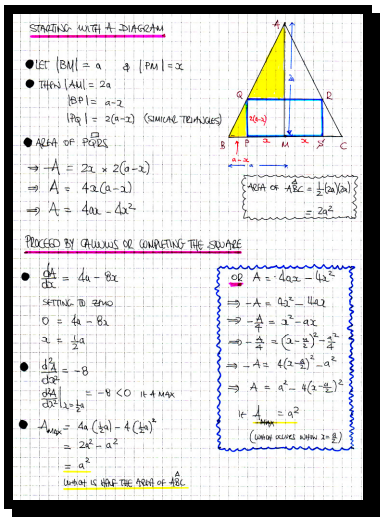


The figure above shows an isosceles triangle ABC, where $|AB|=|AC|$ and a rectangle $PQRS$ drawn inside the triangle.

The points P and S lie on BC, the point Q lies on AB and the point R lies on AC.

Given that the base of the triangle BC is equal in length to its height, show clearly that the largest area that the rectangle $PQRS$ can achieve is $\frac{1}{2}$ the area of the triangle ABC.

$\boxed{}$, proof



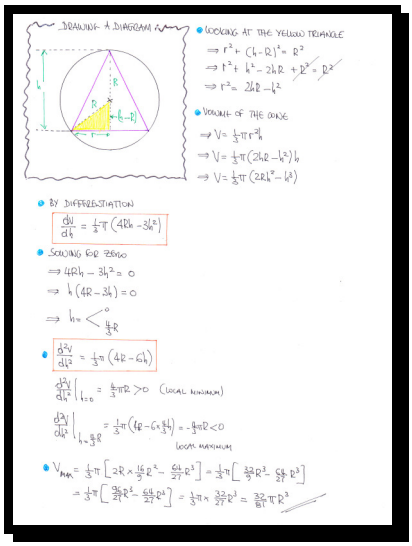
Question 289 (*****)

A right circular cone of radius r and height h is to be cut out of a sphere of radius R.

It is a requirement that the circumference of the base of the cone and its vertex lie on the surface of the sphere.

Determine, in exact form in terms of R, and with full justification, the maximum volume of the cone that can be cut out of this sphere.

$$\boxed{}\,, \qquad \boxed{V_{\max}=\frac{32\pi R^3}{81}}$$



Question 290 (*****)

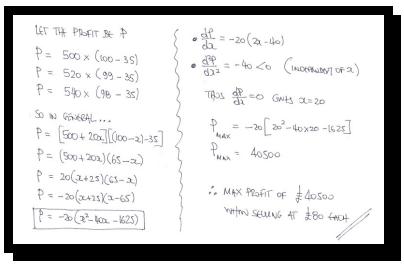
A mobile phone wholesaler buys a certain brand of phone for £35 a unit and sells it to shops for £100 a unit.

In a typical week the wholesaler expects to sell 500 of these phones.

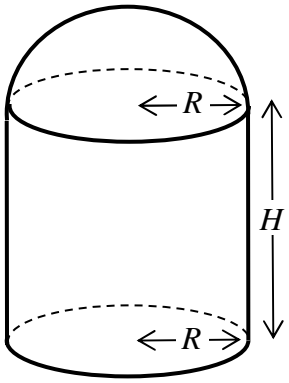
Research however showed that on a typical week for every £1 reduced of the selling price of this phone, an extra 20 sales can be achieved.

Determine the **selling** price for this phone if the weekly profit is to be maximized, and find this maximum weekly profit.

[ ], [£80, maximum profit £40500]



Question 291 (*****)



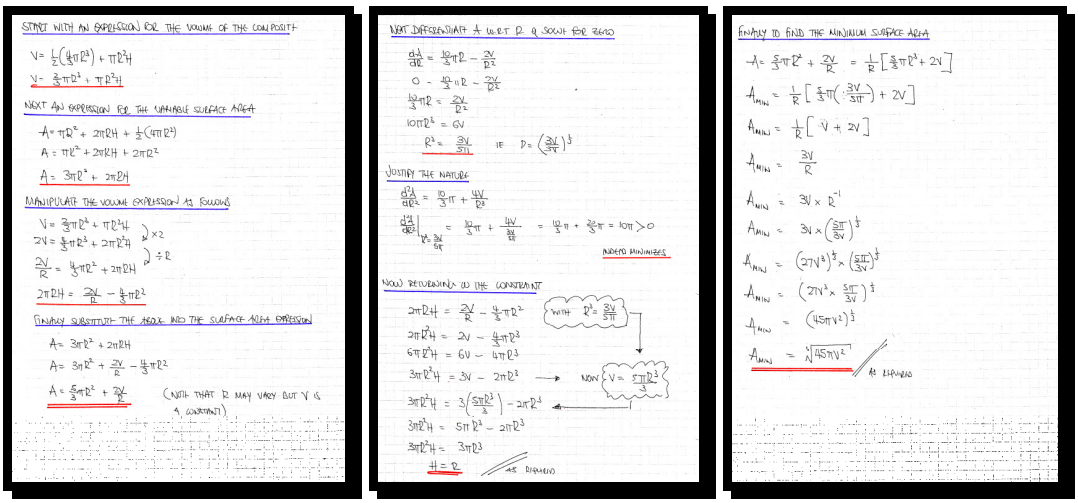
The figure above shows a hollow container consisting of a right circular cylinder of radius R and of height H joined to a hemisphere of radius R.

The cylinder is open on one of the circular ends and the hemisphere is also open on its circular base. The cylinder is joined to the hemisphere at their open ends so that the resulting object is completely sealed.

Given that volume of the container is V, show the surface area of the container is minimised when $R = H$, and hence show further that this minimum surface area is

$$\sqrt[3]{45\pi V^2}\,.$$

[ ], [proof]



Question 292 (*****)

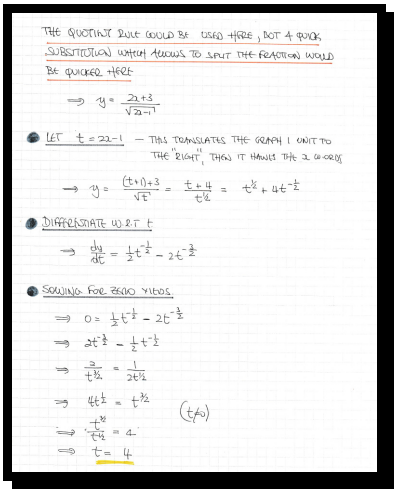
A curve C has equation

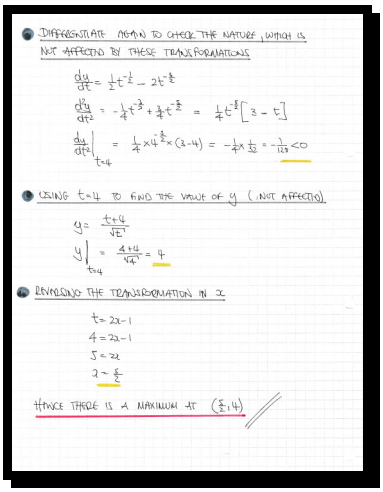
$$y=\frac{2x+3}{\sqrt{2x-1}},\ x\in\mathbb{R},\ x>\frac{1}{2}.$$

Find the coordinates of the stationary point of C, further determining the nature of this point.

You may not use the product rule, the quotient rule or logarithmic differentiation in this question.

$\boxed{\qquad}$, $\boxed{\max\left(\frac{5}{2},4\right)}$





Question 293 (*****)

The point P, whose y coordinate is 2, lies on the curve with equation

$$y=\frac{k+8x\sqrt{x}}{12x},\ x\in\mathbb{R},\ x>0,$$

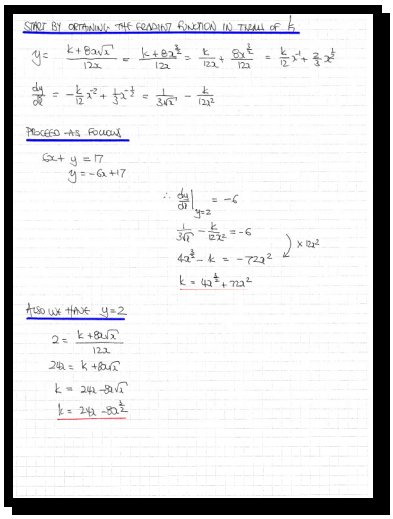
where k is a non zero constant.

The tangent to the curve at P is parallel to the straight line with equation

$$6x+y=17.$$

Determine the value of k.

$\boxed{}$, $\boxed{k=5}$



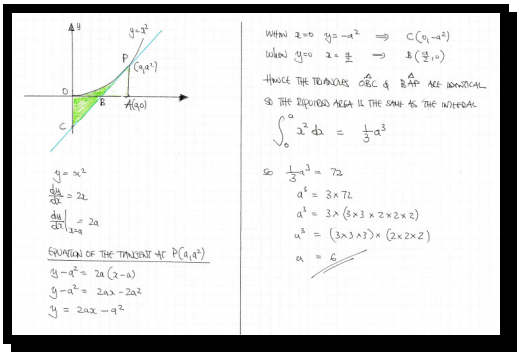
Question 294 (*****)

The point P lies on the curve with equation $y = x^2$, $x > 0$.

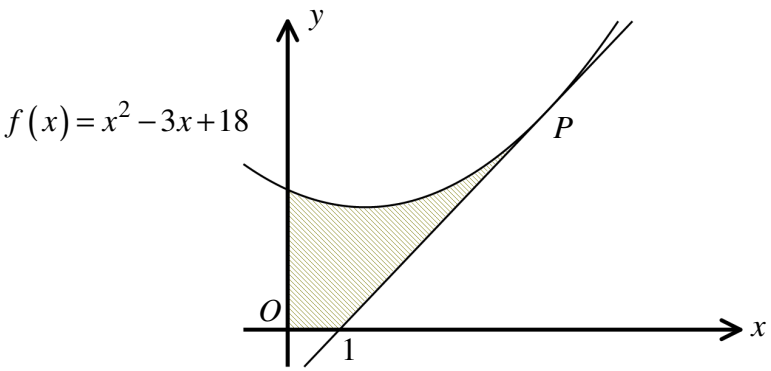
The finite region bounded by the curve, the tangent to the curve at P and the y axis has area of 72 square units.

Determine the x coordinate of P.

$\square$, 6



Question 295 (*****)



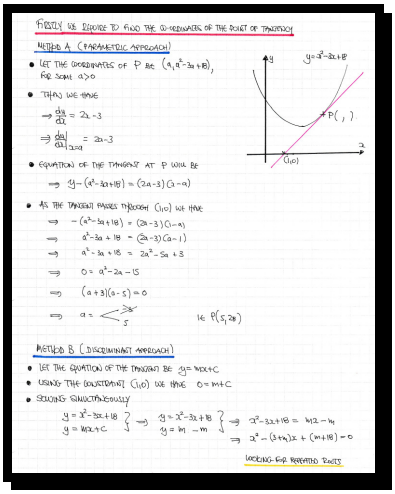
A quadratic curve has equation

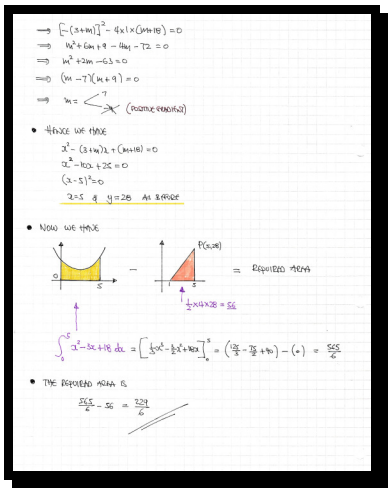
$$y=x^2-3x+18\,,\quad x\in\mathbb{R}\,.$$

The tangent to the curve at the point P meets the x axis at the point with coordinates $(1,0)$, as shown in the figure above.

Find the area of the finite region bounded by the curve, the coordinates axes and the tangent to the curve at P, shown shaded in figure above.

$\square$, $\boxed{\text{area}=\dfrac{229}{6}}$





Question 296 (*****)

The curve with equation $y=f(x)$, lies entirely in the first quadrant. The point P, whose x coordinate is a lies on this curve.

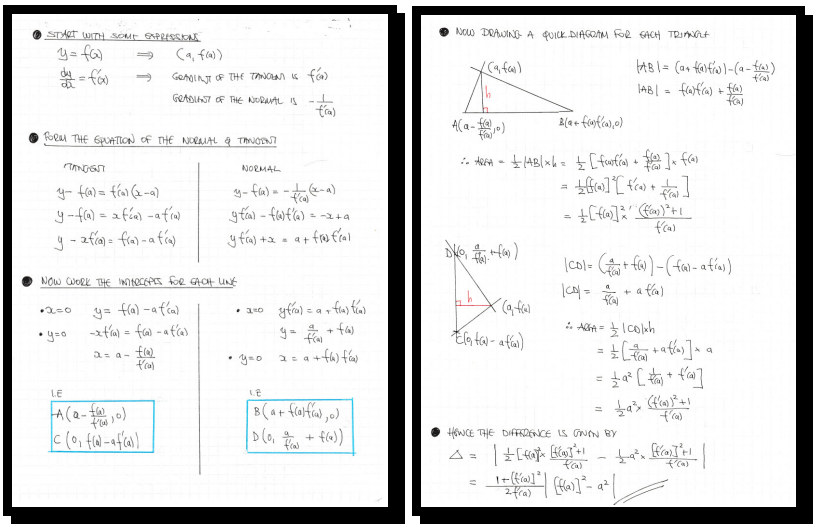
The tangent to the curve at P meets the x axis at the point A and the y axis at the point C.

The normal to the curve at P meets the x axis at the point B and the y axis at the point D.

Given further that the gradient at P is positive, show that the difference between the areas of the triangle PAB and the triangle PCD is given by

$$\frac{1+\left[f'(a)\right]^2}{2f'(a)}\left|\left[f(a)\right]^2-a^2\right|.$$

, proof



Question 297 (*****)

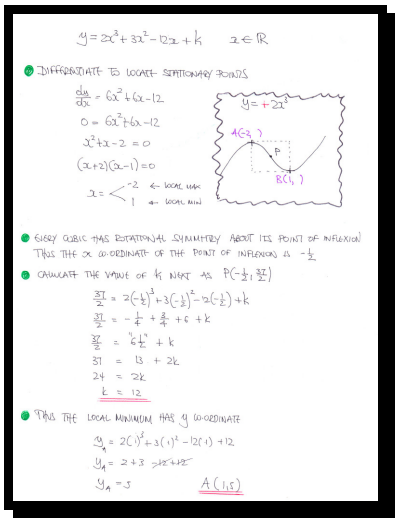
The points A and B are stationary points of the curve with equation

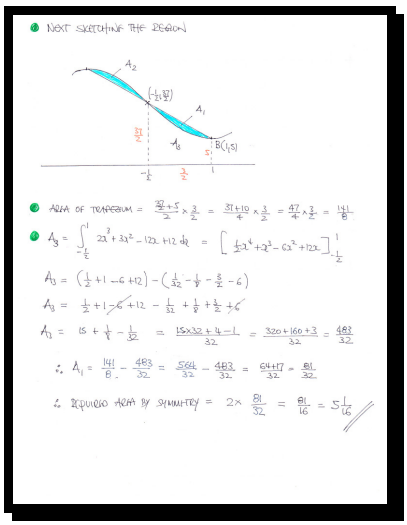
$$y = 2x^3 + 3x^2 - 12x + k \ , \ x \in \mathbb{R} \ ,$$

where k is a constant.

Given that $y = \frac{37}{2}$ at the point of inflexion of the curve, show that the area of the region bounded by the curve and the straight line through A and B is $5\frac{1}{16}$.

[ ] , [proof]





Question 298 (*****)

A quadratic curve has equation

$$y = ax^2 + bx + c\,,\ a \neq 0\,,$$

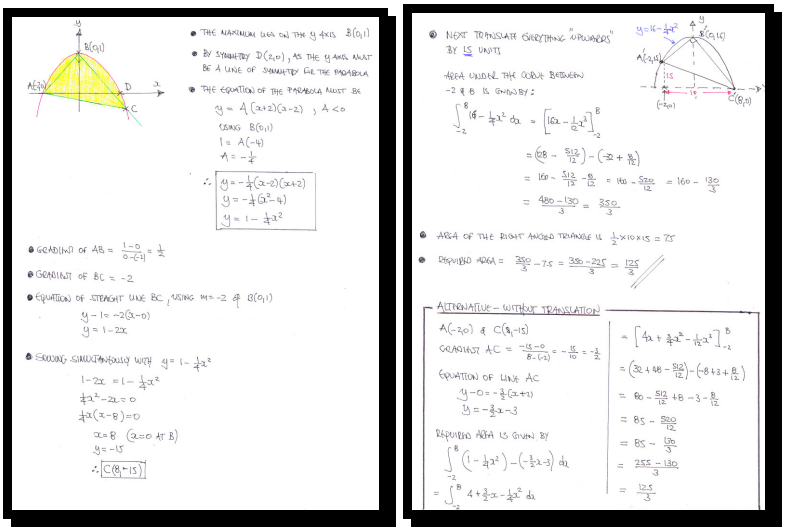
where a, b and c are constants.

The curve meets the x axis at $A(-2,0)$ and has a maximum point at $B(0,1)$.

The point C lies on the curve so that AB is perpendicular to BC.

Determine the area of the finite region bounded by the curve and the straight line segment AC.

$\boxed{}$, proof



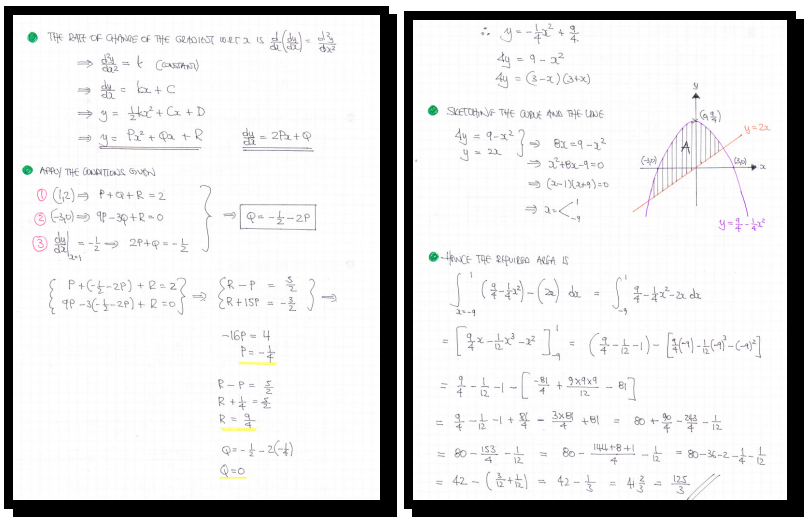
Question 299 **(*****)**

The rate of change, with respect to x, of the gradient of a curve is constant.

The curve passes through the points with coordinates $(1,2)$ and $(-3,0)$, the gradient at the former point being $-\frac{1}{2}$.

Show that the area of the finite region bounded between the curve and the straight line with equation $y=2x$ is $\frac{125}{3}$.

$\boxed{}$, $\boxed{\text{proof}}$



Question 300 (*****)

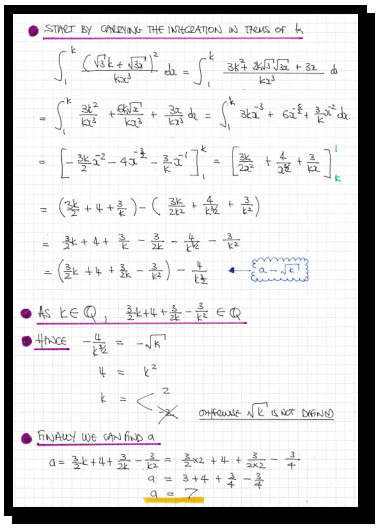
It is given that

$$\int_1^k \frac{\left(\sqrt{3}k+\sqrt{3x}\right)^2}{kx^3}\,dx = a-\sqrt{k}\,,$$

where a and k are integers.

Use algebra to determine the value of a.

$\boxed{}$, $\boxed{a=7}$



Question 301 **(*****)**

A cubic curve C has equation

$$y = ax^3 + bx, \quad x \in \mathbb{R},$$

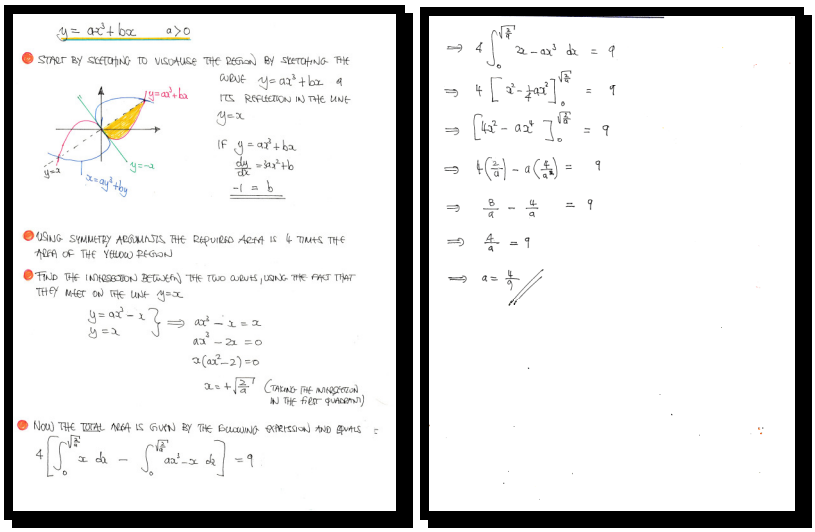
where a and b are non zero constants with $a > 0$.

The curve C' is the reflection of C about the straight line with equation $y = x$.

The straight line with equation $y = -x$ is a tangent to both C and C' at the origin O.

Given that the finite region bounded by C and C' has area 9, find the value of a.

$\square$, $\boxed{a = \frac{4}{9}}$



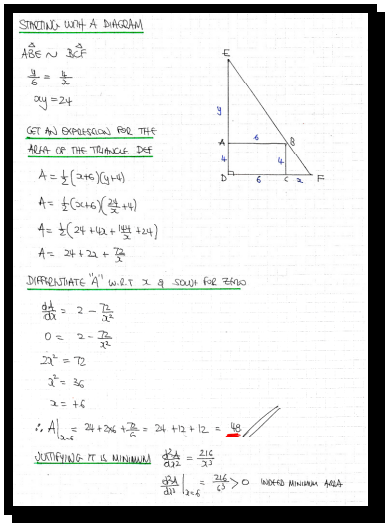
Question 302 (*****)

A rectangle $ABCD$ is such so that $|DC| = 6$ and $|DA| = 4$.

The side DA is extended to the point E and the side DC is extended to the point F so that EBF is a straight line.

Determine, with full justification, the minimum area of the triangle EDF.

$\boxed{}$, $\boxed{A_{\min} = 48}$



Question 303 (*****)

A curve C and a straight line L have respective equations

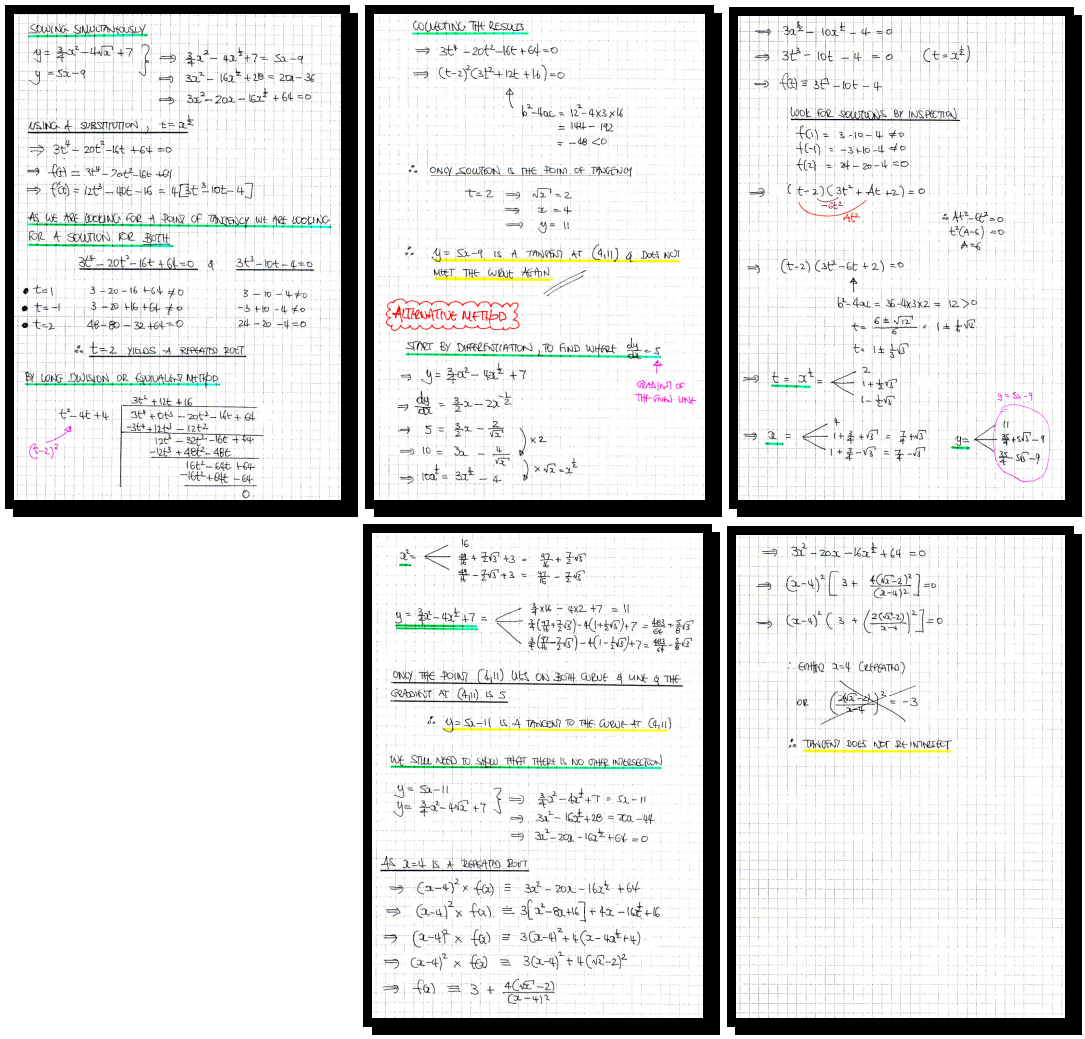
$$C:\ y=\frac{3}{4}x^2-4\sqrt{x}+7\ ,\ x\geq 0$$

$$L:\ y=5x-9$$

a) Show that L is a tangent to C at some point P, further determining the coordinates of P.

b) Show further that L does not meet C again.

$\boxed{}$, $\boxed{P(4,11)}$

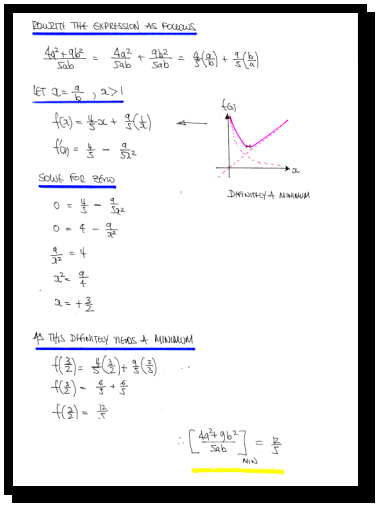


Question 304 (*****)

If a and b such that $a > b > 0$, find with justification the minimum value of

$$\frac{4a^2+9b^2}{5ab}.$$

$\square$, $\boxed{\frac{12}{5}}$

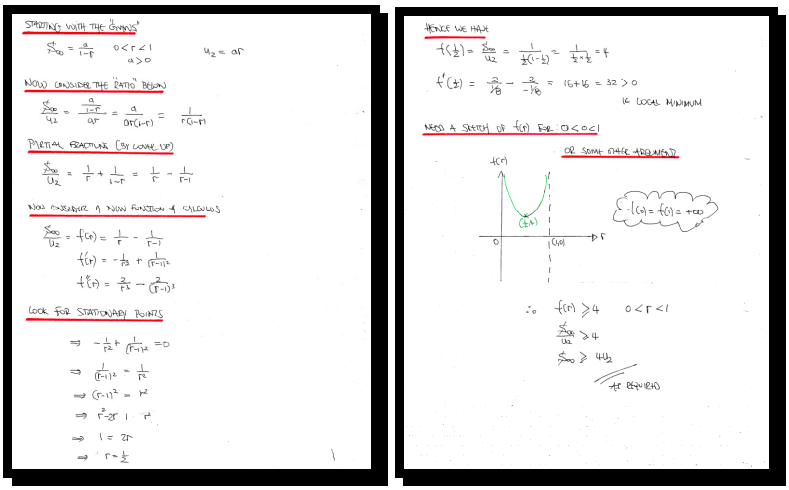


Question 305 (*****)

A convergent geometric progression has positive first term and positive common ratio.

Show that the sum to infinity of the progression is at least four times as large as its second term.

V, [ ], proof



Created by T. Madas