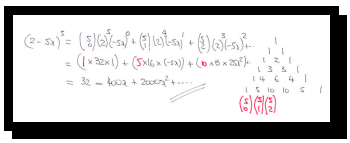
BINOMIAL EXPANSIONS

EXAM QUESTIONS

Question 1 (**)

Find, without any calculating aid, the first three terms in the expansion of $(2-5x)^5$, in ascending powers of x.

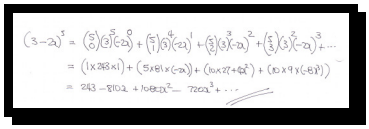
$\square$, $\boxed{32-400x+2000x^2-...}$



Question 2 (**)

Expand $(3-2x)^5$ in ascending powers of x, up and including the term in x^3.

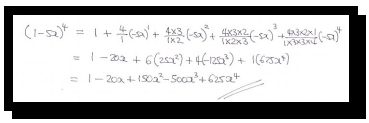
$\square$, $\boxed{243-810x+1080x^2-720x^3+...}$



Question 3 (**)

Find the binomial expansion of $(1-5x)^4$ in ascending powers of x.

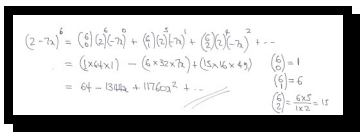
$\square$, $\boxed{1-20x+150x^2-500x^3+625x^4}$



Question 4 (**)

Find, without any calculating aid, the first three terms in the expansion of $(2-7x)^6$, in ascending powers of x.

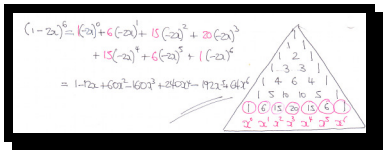
$$\boxed{64-1344x+11760x^2+...}$$



Question 5 (**)

Find the binomial expansion of $(1-2x)^6$ in ascending powers of x.

$$\boxed{1-12x+60x^2-160x^3+240x^4-192x^5+64x^6}$$

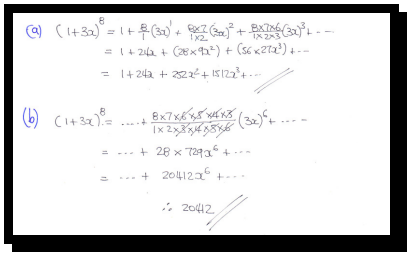


Question 6 (**)

a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1+3x)^8$.

b) Determine the coefficient of x^6 in the binomial expansion of $(1+3x)^8$.

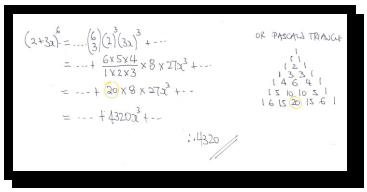
$$\boxed{}\text{, }\boxed{1+24x+252x^2+1512x^3+...}\text{, }\boxed{20412}$$



Question 7 (+)**

Find, without using a calculator, the coefficient of x^3 in the expansion of $(2+3x)^6$.

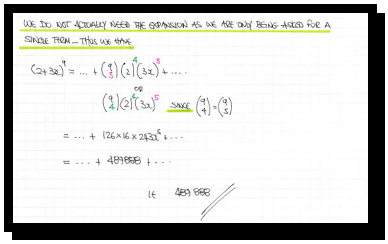
$\boxed{4320}$



Question 8 (+)**

Find the coefficient of x^5 in the binomial expansion of $(2+3x)^9$.

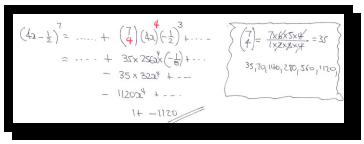
$\boxed{}$, $\boxed{489888}$



Question 9 (+)**

Find, without using a calculator, the coefficient of x^4 in the expansion of $\left(4x-\frac{1}{2}\right)^7$.

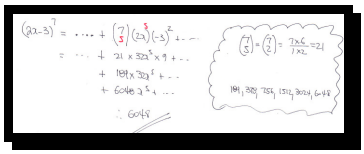
$\boxed{-1120}$



Question 10 (+)**

Find, without using a calculator, the coefficient of x^5 in the expansion of $(2x-3)^7$.

$\boxed{6048}$

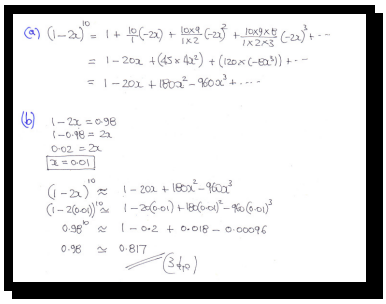


Question 11 (+)**

a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1-2x)^{10}$.

b) Use the answer of part (a) with a suitable value of x to find an approximate value for 0.98^{10}, giving the answer correct to three decimal places.

$\boxed{}$, $\boxed{1-20x+180x^2-960x^3+...}$, $\boxed{\approx 0.817}$



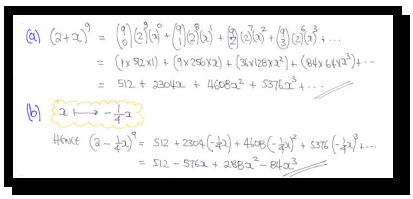
Question 12 (+)**

a) Find, in ascending powers of x, the first four terms in the binomial expansion of $(2+x)^9$.

b) By using the answer of part **(a)**, or otherwise, find the first four terms in the binomial expansion of $\left(2-\frac{1}{4}x\right)^9$.

$\square$, $\boxed{(2+x)^9 = 512+2304x+4608x^2+5376x^3+\ldots}$,

$\boxed{\left(2-\frac{1}{4}x\right)^9 = 512-576x+288x^2-84x^3+\ldots}$



Question 13 (+)**

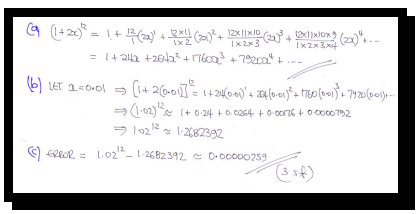
a) Find the first **five** terms, in ascending powers of x, in the binomial expansion of $(1+2x)^{12}$.

b) Use the answer of part **(a)** with a suitable value of x to find an approximate value for 1.02^{12}.

c) Determine the error in this approximation.

$\square$, $\boxed{1+24x+264x^2+1760x^3+7920x^4+\ldots}$, $\boxed{1.02^{12} \approx 1.2682392}$,

$\boxed{\text{error} \approx 0.00000259}$



Question 14 (+)**

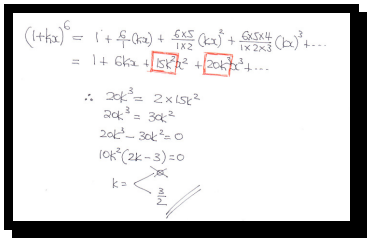
In the binomial expansion of

$$(1+kx)^6,$$

where k is constant, the coefficient of x^3 is twice as large as the coefficient of x^2.

Find the value of k.

$\square$, $\boxed{k=\frac{3}{2}}$



Question 15 (***)

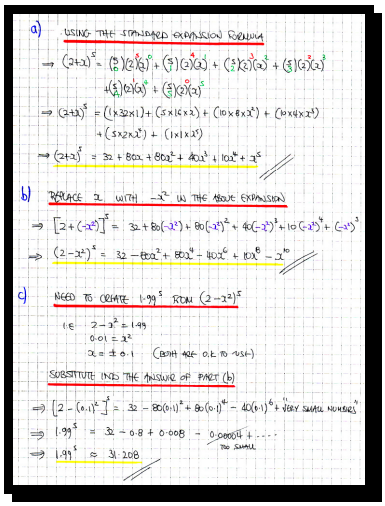
a) Find, in ascending powers of x, the binomial expansion of $(2+x)^5$.

b) By using the expression obtained in part **(a)**, or otherwise, find the binomial expansion of $\left(2-x^2\right)^5$.

c) Use the expression obtained in part **(b)** to estimate, correct to 3 decimal places, the value of 1.99^5.

$\square$, $\boxed{(2+x)^5=32+80x+80x^2+40x^3+10x^4+x^5}$,

$\boxed{\left(2-x^2\right)^5=32-80x^2+80x^4-40x^6+10x^8-x^{10}}$, $\boxed{1.99^5\approx 31.208}$



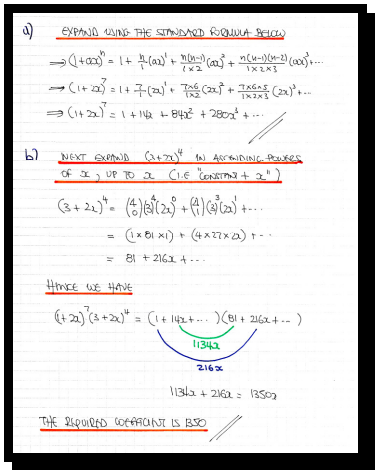
Question 16 (***)

a) Find the first four terms, in ascending powers of x, of the binomial expansion of $(1+2x)^7$.

b) Hence determine the coefficient of x in the expansion of

$$(1+2x)^7(3+2x)^4.$$

$\boxed{}$, $\boxed{1+14x+84x^2+280x^3+...}$, $\boxed{1350}$



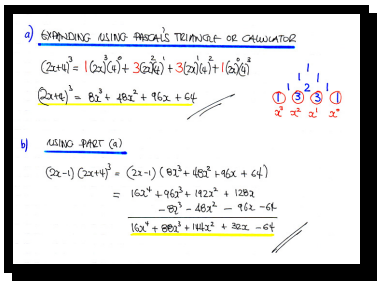
Question 17 **(***)**

a) Find the binomial expansion of $(2x+4)^3$, in descending powers of x.

b) Hence determine the expansion of

$$(2x-1)(2x+4)^3.$$

$\boxed{}$, $\boxed{8x^3+48x^2+96x+64}$, $\boxed{16x^4+88x^3+144x^2+32x-64}$



Question 18 (***)

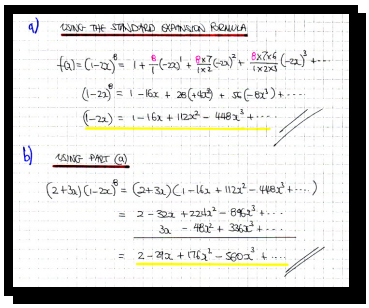
$$f(x)=(1-2x)^8$$

a) Find the first four terms in the expansion of $f(x)$, in ascending powers of x.

b) Hence determine, in ascending powers of x, the first four terms in the expansion of

$$(2+3x)(1-2x)^8 .$$

$\boxed{}$, $\boxed{1-16x+112x^2-448x^3+...}$, $\boxed{2-29x+176x^2-560x^3+...}$



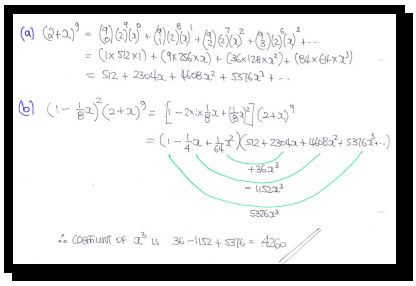
Question 19 (***)

a) Find, in ascending powers of x, the first four terms in the binomial expansion of $(2+x)^9$.

b) Hence find the coefficient of x^3 in the expansion of

$$\left(1-\frac{1}{8}x\right)^2(2+x)^9 \ .$$

$\square$, $\boxed{(2+x)^9=512+2304x+4608x^2+5376x^3+...}$, $\boxed{[x^3]=4260}$



Question 20 (***)

$$f(x)=(2+x)^4$$

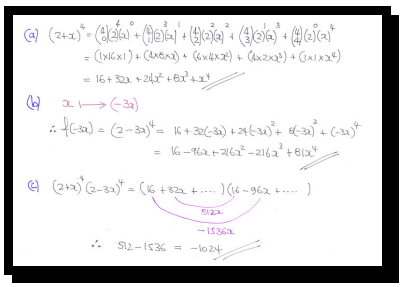
a) Find the expansion of $f(x)$, in ascending powers of x.

b) Deduce the expansion of $(2-3x)^4$, also in ascending powers of x.

c) Determine the coefficient of x in the expansion of

$$(2+x)^4(2-3x)^4.$$

$\boxed{f(x)=16+32x+24x^2+8x^3+x^4}$, $\boxed{16-96x+216x^2-216x^3+81x^4}$, $\boxed{-1024}$



Question 21 (***)

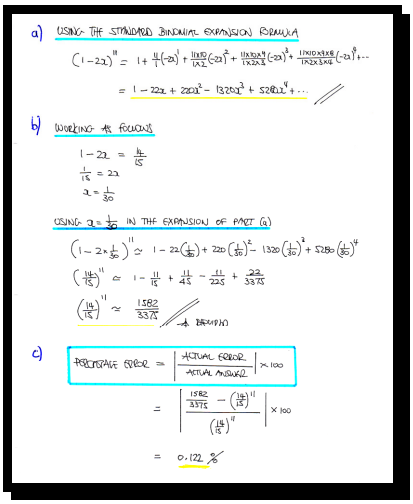
a) Find the first **five** terms, in ascending powers of x, in the binomial expansion of $(1-2x)^{11}$.

b) Use the answer of part **(a)** with a suitable value of x to show that

$$\left(\frac{14}{15}\right)^{11} \approx \frac{1582}{3375}.$$

c) Determine the percentage error in the approximation of part **(b)**.

$\boxed{}$, $\boxed{1-22x+220x^2-1320x^3+5280x^4+...}$, $\boxed{\%\text{ error} \approx 0.122\%}$


a) USING THE STANDARD BINOMIAL EXPANSION FORMULA

$(1-2x)^{11} = 1 + \frac{11}{1}(-2x)^1 + \frac{11\times10}{1\times2}(-2x)^2 + \frac{11\times10\times9}{1\times2\times3}(-2x)^3 + \frac{11\times10\times9\times8}{1\times2\times3\times4}(-2x)^4 + \ldots$

$= 1 - 22x + 220x^2 - 1320x^3 + 5280x^4 + \ldots$

b) WORKING AS FOLLOWS

$1-2x = \frac{14}{15}$

$\frac{1}{15} = 2x$

$x = \frac{1}{30}$

USING $x = \frac{1}{30}$ IN THE EXPANSION OF PART (a)

$\left(1-2\times\frac{1}{30}\right)^{11} \simeq 1 - 22\left(\frac{1}{30}\right) + 220\left(\frac{1}{30}\right)^2 - 1320\left(\frac{1}{30}\right)^3 + 5280\left(\frac{1}{30}\right)^4$

$\left(\frac{14}{15}\right)^{11} \simeq 1 - \frac{11}{15} + \frac{11}{45} - \frac{11}{225} + \frac{22}{3375}$

$\left(\frac{14}{15}\right)^{11} \simeq \frac{1582}{3375}$ AS REQUIRED

c) PERCENTAGE ERROR $= \left|\frac{\text{ACTUAL ERROR}}{\text{ACTUAL VALUE}}\right| \times 100$

$= \left|\frac{\frac{1582}{3375} - \left(\frac{14}{15}\right)^{11}}{\left(\frac{14}{15}\right)^{11}}\right| \times 100$

$= 0.122\,\%$

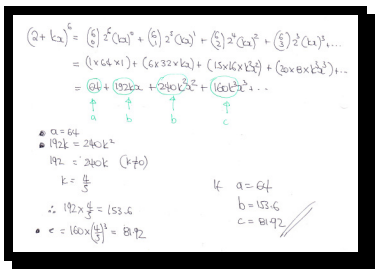
Question 22 (*+)**

It is given that if k is a non zero constant, then

$$(2+kx)^6 \equiv a+bx+bx^2+cx^3+\ldots$$

Determine the value of each of the constants a, b and c.

$\boxed{}$, $\boxed{a=64}$, $\boxed{b=153.6}$, $\boxed{c=81.92}$



Question 23 (*+)**

In the binomial expansion of

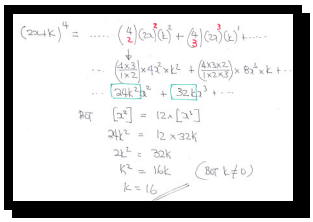
$$(2x+k)^4,$$

where k is a non zero constant,

the coefficient of x^2 is 12 times as large as the coefficient of x^3.

Find the value of k.

$\boxed{}$, $\boxed{k=16}$



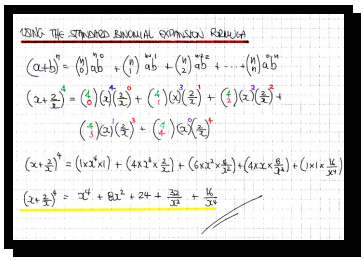
Question 24 (*+)**

Find the binomial expansion of

$$\left(x+\frac{2}{x}\right)^4, \ x\neq 0,$$

simplifying each term of the expansion.

$\square$, $\boxed{x^4+8x^2+24+\frac{32}{x^2}+\frac{16}{x^4}}$



Question 25 (*+)**

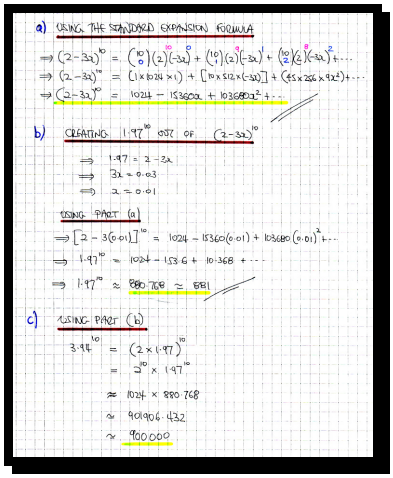
a) Determine, in ascending powers of x, the first three terms in the binomial expansion of $(2-3x)^{10}$.

b) Use the first three terms in the binomial expansion of $(2-3x)^{10}$, with a suitable value for x, to find an approximation for 1.97^{10}.

c) Use the answer of part **(b)** to estimate, correct to 2 significant figures, the value of 3.94^{10}.

$\boxed{}$, $\boxed{(2-3x)^{10}=1024-15360x+103680x^2+\ldots}$, $\boxed{1.97^{10}\approx 880.768\approx 881}$,

$\boxed{3.94^{10}\approx 900000}$

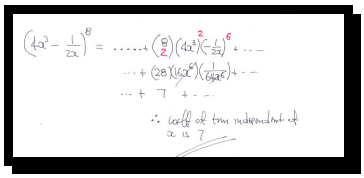


Question 26 (*+)**

Find the term which is independent of x, in the binomial expansion of

$$\left(4x^3-\frac{1}{2x}\right)^8,\ x\neq 0.$$

$\boxed{7}$

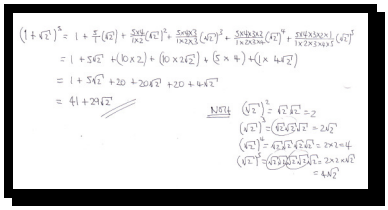


Question 27 (*+)**

$$\left(1+\sqrt{2}\right)^5\equiv a+b\sqrt{2}.$$

Determine the value of each of the constants a and b.

$\boxed{a=41,\ b=29}$



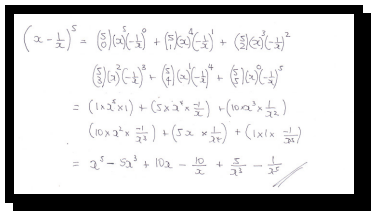
Question 28 (*+)**

Find the binomial expansion of

$$\left(x-\frac{1}{x}\right)^5,\ x\neq 0,$$

simplifying each term of the expansion.

$$\boxed{x^5-5x^3+10x-\frac{10}{x}+\frac{5}{x^3}-\frac{1}{x^5}}$$



Question 29 (*+)**

In the binomial expansion of

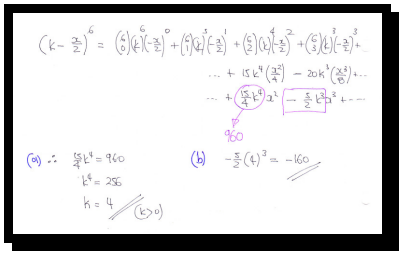
$$\left(k-\frac{x}{2}\right)^6,$$

where k is a positive constant, one of the terms is $960x^2$.

a) Find the value of k.

b) Determine the coefficient of x^3.

$$\boxed{k=4},\ \boxed{-160}$$



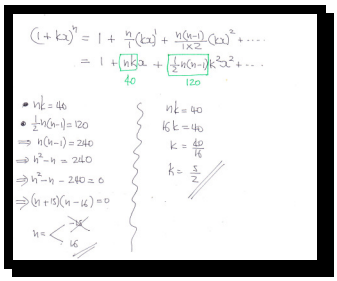
Question 30 (*+)**

Given that k is a non zero constant and n is a positive integer, then

$$(1+kx)^n \equiv 1+40x+120k^2x^2+\ldots$$

Find the value of k and the value of n.

$\square$, $\boxed{k=\frac{5}{2}}$, $\boxed{n=16}$



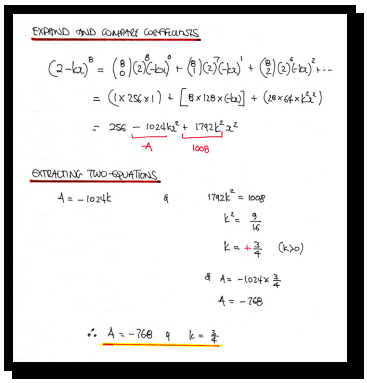
Question 31 (*+)**

Given that k and A are constants with $k>0$, then

$$(2-kx)^8 \equiv 256+Ax+1008x^2+\ldots$$

Find the value of k and the value of A.

$\square$, $\boxed{k=\frac{3}{4}}$, $\boxed{A=-768}$

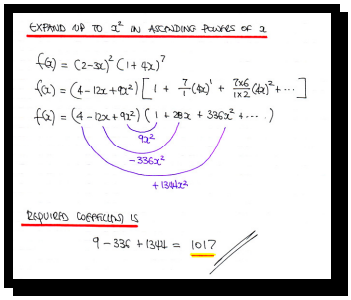


Question 32 **(***+)**

$$f(x)=(2-3x)^2(1+4x)^7.$$

Find the coefficient of x^2 in the polynomial expansion of $f(x)$.

$\boxed{}$, $\boxed{1017}$



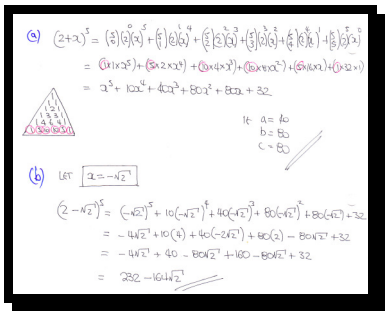
Question 33 **(***+)**

$$(2+x)^5=x^5+10x^4+ax^3+bx^2+cx+32.$$

a) Find the value of each of the constants a, b and c.

b) Hence, or otherwise, simplify $\left(2-\sqrt{2}\right)^5$, giving the final answer in the form $p+q\sqrt{2}$, where p and q are constants.

$\boxed{a=40,\ b=80,\ c=80}$, $\boxed{232-164\sqrt{2}}$



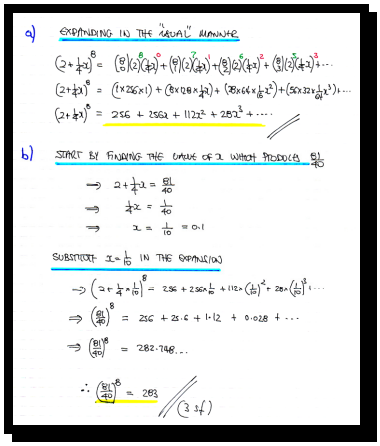
Question 34 (*+)**

$$f(x)=\left(2+\frac{1}{4}x\right)^8.$$

a) Find the first four terms in ascending powers of x in the expansion of $f(x)$.

b) Use the expansion found in part (a) to find an approximation, to 3 significant figures, for $\left(\frac{81}{40}\right)^8$.

$\square$, $\boxed{f(x)=256+256x+112x^2+28x^3+...}$, $\boxed{\left(\frac{81}{40}\right)^8\approx 283}$



Question 35 (*+)**

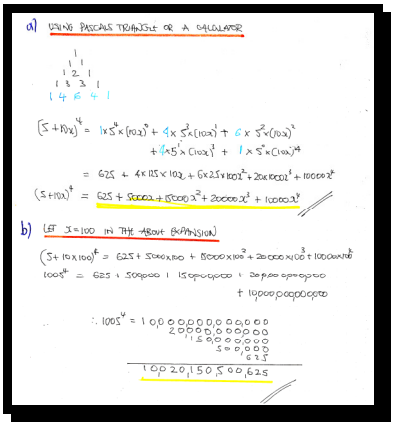
a) Find the binomial expansion of

$$(5+10x)^4 .$$

b) Hence, by using the answer of part (a) with a suitable value of x find the exact value of 1005^4.

You may not give the answer in standard index form.

$\boxed{}$, $\boxed{625+5000x+15000x^2+20000x^3+10000x^4}$, $\boxed{1,020,150,500,625}$



Question 36 (*+)**

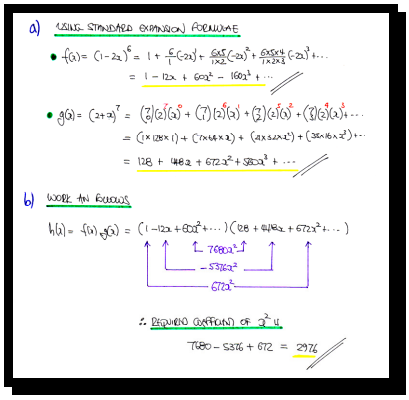
$$f(x)=(1-2x)^6, \qquad g(x)=(2+x)^7, \qquad h(x)=f(x)g(x).$$

a) Find the first four terms in ascending powers of x in the binomial expansion of $f(x)$ and in the binomial expansion of $g(x)$.

b) Hence determine the coefficient of x^2 in the binomial expansion of $h(x)$.

$\square$, $\boxed{f(x)=1-12x+60x^2-160x^3+...}$,

$\boxed{g(x)=128+448x+672x^2+560x^3+...}$, $\boxed{2976}$



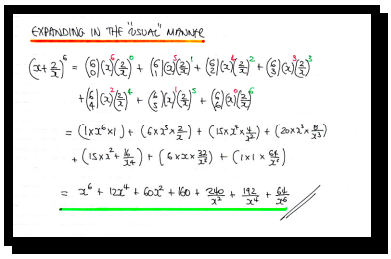
Question 37 **(***+)**

Find the binomial expansion of

$$\left(x+\frac{2}{x}\right)^6,\ x\neq 0,$$

simplifying each term of the expansion.

$\square$, $\boxed{x^6+12x^4+60x^2+160+\frac{240}{x^2}+\frac{192}{x^4}+\frac{64}{x^6}}$

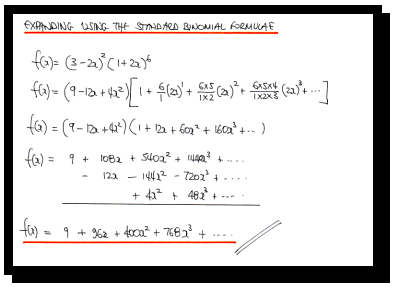


Question 38 **(***+)**

$$f(x)=(3-2x)^2(1+2x)^6.$$

Find the binomial expansion of $f(x)$ in ascending powers of x, up and including the term in x^3.

$\square$, $\boxed{9+96x+400x^2+768x^3+\ldots}$



Question 39 (*+)**

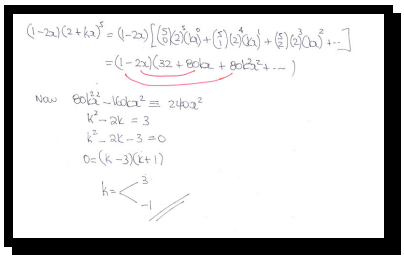
It is given that

$$(1-2x)(2+kx)^5 \equiv A+Bx+240x^2+\ldots,$$

where k, A and B are constants.

Determine the possible values of k

$\square$, $\boxed{k=-1,\ 3}$



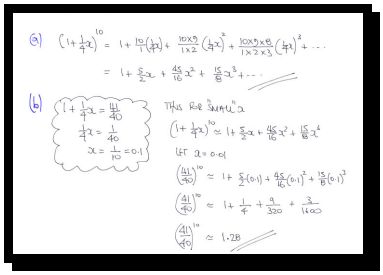
Question 40 (*+)**

a) Find the binomial expansion of $\left(1+\frac{1}{4}x\right)^{10}$ in ascending powers of x up and including the term in x^3, simplifying fully each coefficient.

b) Use the expansion of part **(a)** to show that

$$\left(\frac{41}{40}\right)^{10} \approx 1.28\,.$$

$\square$, $\boxed{1+\frac{5}{2}x+\frac{45}{16}x^2+\frac{15}{8}x^3+\ldots}$



Question 41 (*+)**

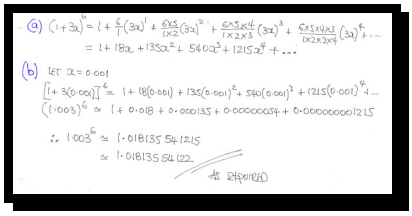
$$f(x)=(1+3x)^6 .$$

a) Find, in ascending powers of x, the binomial expansion of $f(x)$ up and including the term in x^4.

b) Use the expansion found in part **(a)** to show that

$$(1.003)^6 \approx 1.01813554122 .$$

$$\boxed{f(x)=1+18x+135x^2+540x^3+1215x^4+...}$$



Question 42 (*+)**

a) Find the binomial expansion of $(3+2x)^4$.

b) State the binomial expansion of $(3-2x)^4$.

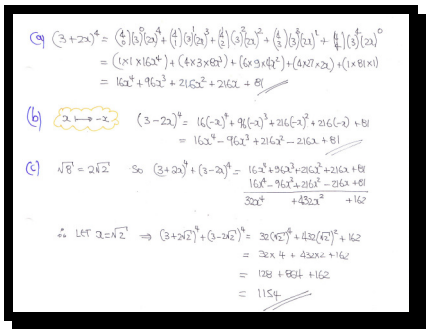
c) Use the answers of part (a) and (b) to find

$$\left(3+\sqrt{8}\right)^4+\left(3-\sqrt{8}\right)^4.$$

No credit will be given for any other type of evaluation.

$$\boxed{(3+2x)^4=16x^4+96x^3+216x^2+216x+81},$$

$$\boxed{(3-2x)^4=16x^4-96x^3+216x^2-216x+81},\ \boxed{\left(3+\sqrt{8}\right)^4+\left(3-\sqrt{8}\right)^4=1154}$$



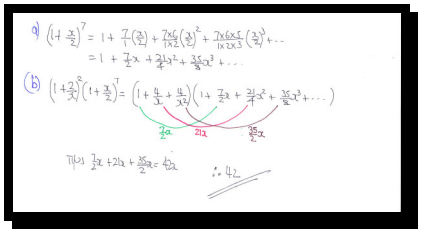
Question 43 (*+)**

a) Find the first four terms, in ascending powers of x, of the binomial expansion of $\left(1+\frac{x}{2}\right)^7$, giving each coefficient in exact simplified form.

b) Hence determine the coefficient of x in the expansion of

$$\left(1+\frac{2}{x}\right)^2\left(1+\frac{x}{2}\right)^7.$$

$\square$, $\boxed{1+\frac{7}{2}x+\frac{21}{4}x^2+\frac{35}{8}x^3+...}$, $\boxed{[x]=42}$



Question 44 **(***+)**

$$f(x)=(1+2x)^7$$

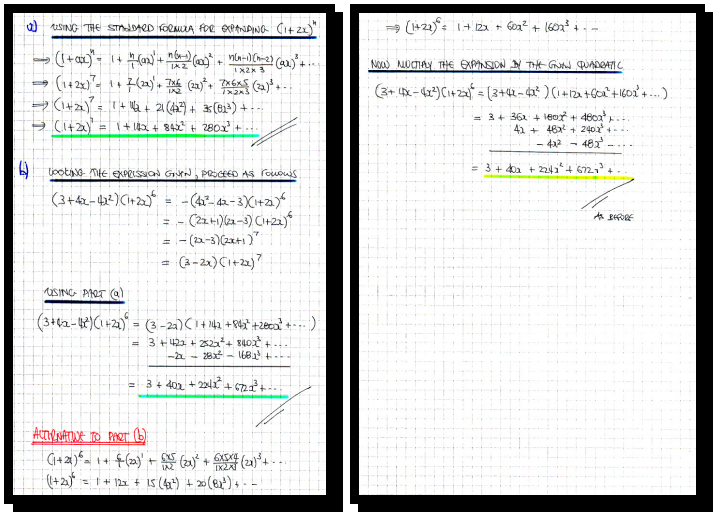
a) Determine the first four terms, in ascending powers of x, in the binomial expansion of $f(x)$.

b) Hence, or otherwise, find the first four terms in the expansion of

$$\left(3+4x-4x^2\right)(1+2x)^6,$$

giving the answer in ascending powers of x.

$\boxed{}$, $\boxed{1+14x+84x^2+280x^3+\ldots}$, $\boxed{3+40x+224x^2+672x^3+\ldots}$



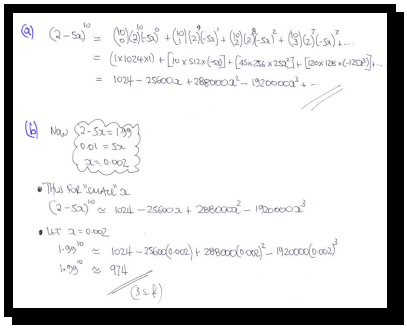
Question 45 (*+)**

a) Find the binomial expansion of $(2-5x)^{10}$ in ascending powers of x up and including the term in x^3, simplifying fully each coefficient.

b) Use the expansion of part (a) to show that

$$(1.99)^{10} \approx 974 .$$

$$\boxed{1024-25600x+288000x^2-1920000x^3+...}$$



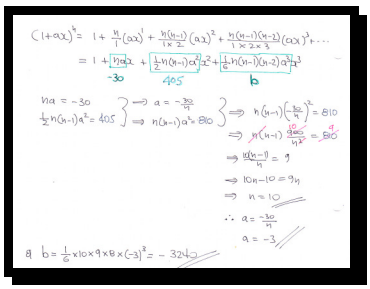
Question 46 (****)

$$(1+ax)^n = 1-30x+405x^2+bx^3+...,$$

where a and b are constants, and n is a positive integer.

Determine the value of n, a and b.

$$\boxed{\quad}, \ \boxed{n=10}, \ \boxed{a=-3}, \ \boxed{b=-3240}$$



Question 47 (****)

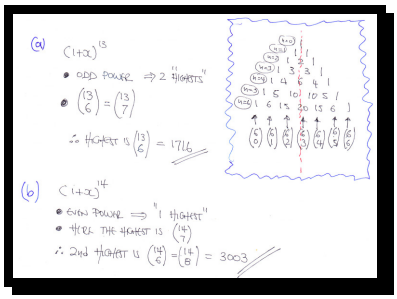
$$f(x)\equiv(1+x)^n,\ x\in\mathbb{R},\ n\in\mathbb{N}.$$

Determine showing a clear complete method the coefficient …

a) … of the **highest** power of x in the binomial expansion of $f(x)$, when $n=13$.

b) … of the **second highest** power of x in the binomial expansion of $f(x)$, when $n=14$.

$\boxed{}$, $\boxed{1716}$, $\boxed{3003}$

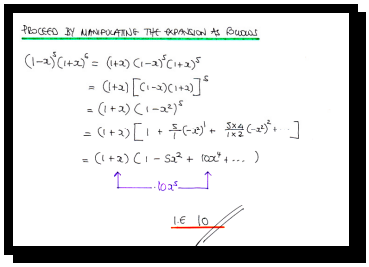


Question 48 (****)

Find the coefficient of x^5 in the binomial expansion of

$$(1-x)^5(1+x)^6.$$

$\boxed{}$, $\boxed{[x^5]=10}$



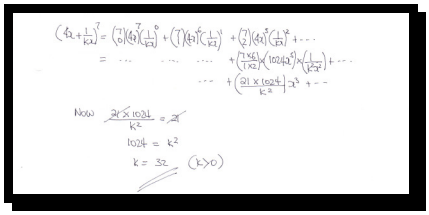
Question 49 (****)

$$f(x)=\left(4x+\frac{1}{kx}\right)^7,$$

where k is a positive constant.

Given the coefficient of x^3 in the binomial expansion of $f(x)$ is 21, determine the value of k.

$\boxed{}$, $\boxed{k=32}$



Question 50 (****)

$$f(x)=(1+2x)^5,\ x\in\mathbb{R}.$$

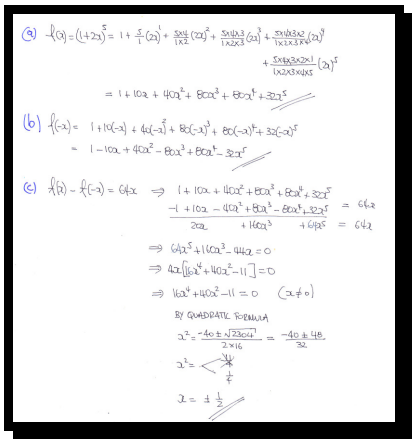
a) Find the binomial expansion of $f(x)$.

b) Hence state the binomial expansion of $f(-x)$.

c) Find the two non zero solutions of the equation

$$f(x)-f(-x)=64x.$$

$\square$, $\boxed{f(x)=1+10x+40x^2+80x^3+80x^4+32x^5}$,
$\boxed{f(-x)=1-10x+40x^2-80x^3+80x^4-32x^5}$ $\boxed{x=\pm\frac{1}{2}}$



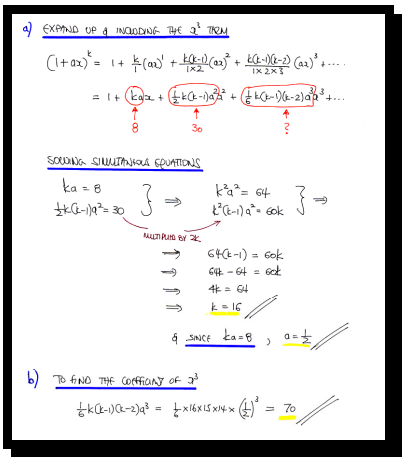
Question 51 (****)

In the binomial expansion of $(1+ax)^k$, where a and k are non zero constants, the coefficient of x is 8 and the coefficient of x^2 is 30.

a) Determine the value of a and the value of k.

b) Find the coefficient of x^3.

$\square$, $\boxed{a=\frac{1}{2}}$, $\boxed{k=16}$, $\boxed{70}$



Question 52 (****)

$$f(x)\equiv(k+x)^n,\ x\in\mathbb{R},$$

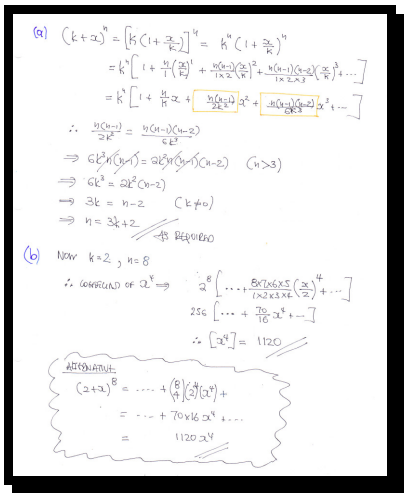
where k and n are constants such that $k\in\mathbb{R}$, $k\neq 0$, $n\in\mathbb{N}$, $n>3$.

a) Given the coefficients of x^2 and x^3 in the binomial expansion of $f(x)$ are equal, show clearly that

$$n=3k+2.$$

b) Given further that $k=2$, determine the coefficient of x^4 in the binomial expansion of $f(x)$.

$\square$, $\boxed{[x^4]=1120}$



Question 53 (****)

a) Find the binomial expansion of $(3+x)^4$, simplifying fully each coefficient.

b) Hence solve the equation

$$(3+x)^4+(3-x)^4=386.$$

$\boxed{81+108x+54x^2+12x^3+x^4}$, $\boxed{x=\pm\sqrt{2}}$



(a) $(3+x)^4=\binom{4}{0}(3)^4(x)^0+\binom{4}{1}(3)^3(x)^1+\binom{4}{2}(3)^2(x)^2+\binom{4}{3}(3)^1(x)^3+\binom{4}{4}(3)^0(x)^4$

$=(1\times81\times1)+(4\times27\times x)+(6\times9\times x^2)+(4\times3\times x^3)+(1\times1\times x^4)$

$=81+108x+54x^2+12x^3+x^4$

(b) $(3-x)^4=81+108(-x)+54(-x)^2+12(-x)^3+(-x)^4$

$=81-108x+54x^2-12x^3+x^4$

$\therefore (3+x)^4+(3-x)^4=81+108x+54x^2+12x^3+x^4$

$81-108x+54x^2-12x^3+x^4$

$=162\qquad +108x^2\qquad +2x^4$

HENCE $2x^4+108x^2+162=386$

$2x^4+108x^2-224=0$

$x^4+54x^2-112=0$

$y^2+54y-112=0\qquad (y=x^2)$

$(y+56)(y-2)=0$

$y=\begin{cases}-56\\2\end{cases}$

$x^2=2$

$x=\pm\sqrt{2}$

Question 54 (****)

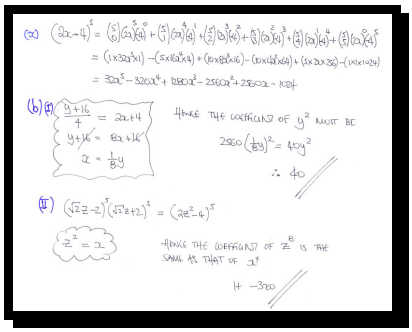
a) Find the binomial expansion of $(2x-4)^5$, simplifying fully each coefficient.

b) Hence find the coefficient of …

i. … y^2 in the binomial expansion of $\left(\frac{y+16}{4}\right)^5$.

ii. … z^8 in the binomial expansion of $\left(\sqrt{2}z-2\right)^5\left(\sqrt{2}z+2\right)^5$.

$\boxed{}$, $\boxed{32x^5-320x^4+1280x^3-2560x^2+2560x-1024}$, $\boxed{[y^2]=40}$, $\boxed{[z^8]=-320}$

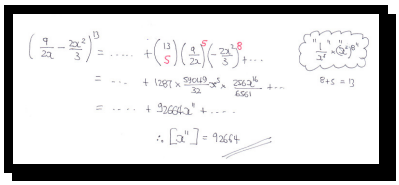


Question 55 (****)

Find the coefficient of x^{11} in the binomial expansion of

$$\left(\frac{9}{2x}-\frac{2x^2}{3}\right)^{13}.$$

$\boxed{}$, $\boxed{[x^{11}]=92664}$



Question 56 (****)

If $k>0$ and n is a positive integer, then

$$(1+kx)^n \equiv 1+\frac{7}{2}x+Bx^2+Bx^3+\ldots,$$

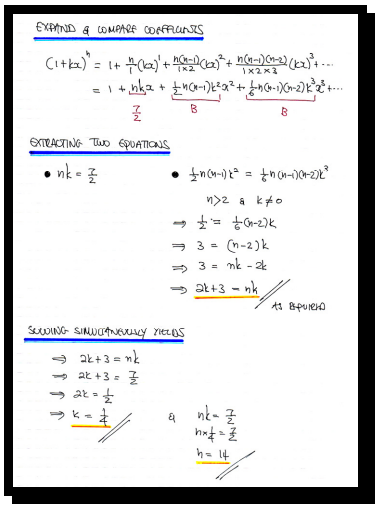
where B is a non zero constant.

By considering the coefficients of x^2 and x^3, show that

$$nk=2k+3,$$

and hence find the value of n and the value of k.

$\boxed{}$, $\boxed{n=14}$, $\boxed{k=\frac{1}{4}}$



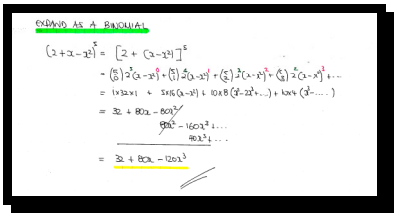
Question 57 **(****)**

Show that if x is numerically small

$$\left(2+x-x^2\right)^5 \approx A+Bx+Cx^3$$

where A, B and C are integers to be found.

$\boxed{}$, $\boxed{A=32}$, $\boxed{B=80}$, $\boxed{C=-120}$



Question 58 **(****)**

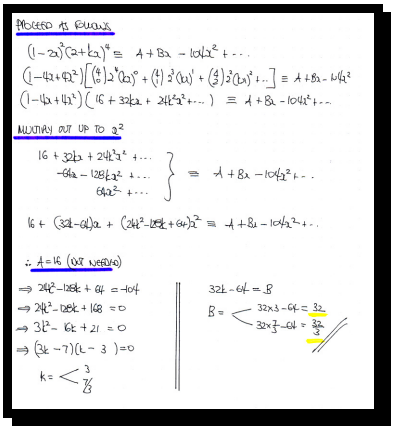
It is given that

$$(1-2x)^2(2+kx)^4 \equiv A+Bx-104x^2+\ldots,$$

where k, A and B are non zero constants.

Determine the possible values of B.

$\boxed{}$, $\boxed{B=32 \cup \frac{32}{3}}$



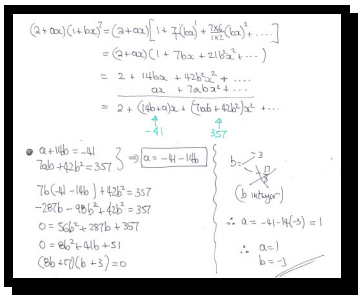
Question 59 (****)

$$(2+ax)(1+bx)^7 = 2-41x+357x^2+\ldots,$$

where a and b are integers.

Show that $b=-3$ and find the value of a.

$\boxed{}$, $\boxed{a=1}$



Question 60 (**+)**

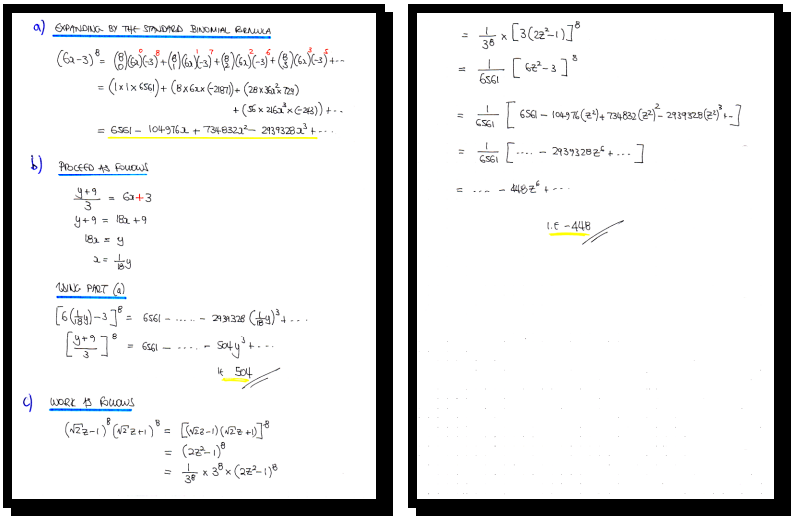
a) Find the first four terms, in ascending powers of x, of the binomial expansion of $(6x-3)^8$, simplifying fully each coefficient.

b) Hence find the coefficient of …

i. … y^3 in the binomial expansion of $\left(\frac{y+9}{3}\right)^8$.

ii. … z^6 in the binomial expansion of $\left(\sqrt{2}z-1\right)^8\left(\sqrt{2}z+1\right)^8$.

$\boxed{}$, $\boxed{6561-104976x+734832x^2-2939328x^3+\ \ldots}$, $\boxed{\left[y^3\right]=504}$, $\boxed{\left[z^6\right]=-448}$



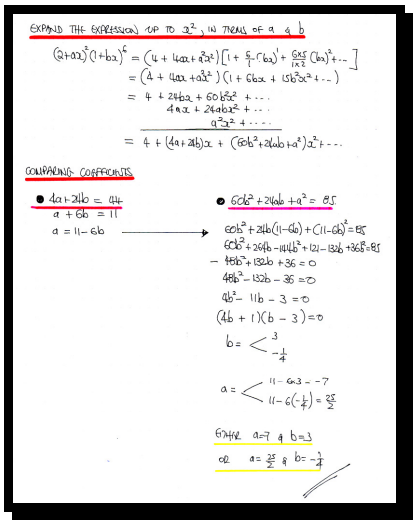
Question 61 (**+)**

$$(2+ax)^2(1+bx)^6 = 4+44x+85x^2+\ldots,$$

where a and b are integers.

Find the possible values of a and the possible values of b.

$\square$, $\boxed{[a,b]=[-7,3]\cup\left[\frac{25}{2},-\frac{1}{4}\right]}$



Question 62 (**+)**

a) Given that c is a non zero constant, determine the first four terms, in ascending powers of x, in the binomial expansion of $(1+cx)^6$.

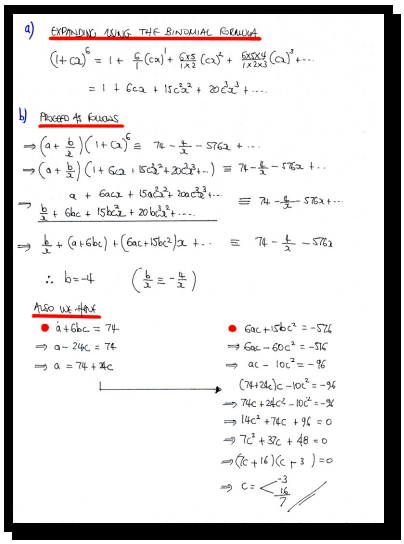
It is further given that

$$\left(a+\frac{b}{x}\right)(1+cx)^6 \equiv -\frac{4}{x}+74-576x+\ldots,$$

where a and b are non zero constants.

b) Show that one of the two possible values of c is -3, and find the other.

$\boxed{}$, $\boxed{1+6cx+15c^2x^2+20c^3x^3+\ldots}$, $\boxed{c=-\frac{16}{7}}$

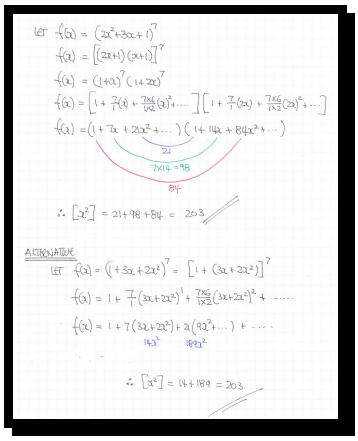


Question 63 (**+)**

Find the coefficient of x^2 in the binomial expansion of

$$\left(2x^2+3x+1\right)^7 .$$

$\square$, $\boxed{\left[x^2\right]=203}$



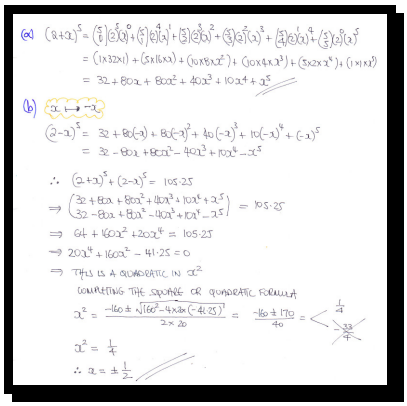
Question 64 **(****+)**

a) Find the binomial expansion of $(2+x)^5$.

b) Hence solve the equation

$$(2+x)^5+(2-x)^5=105.25\,.$$

$$\boxed{(2+x)^5=32+80x+80x^2+40x^3+10x^4+x^5}\,,\quad \boxed{x=\pm\frac{1}{2}}$$

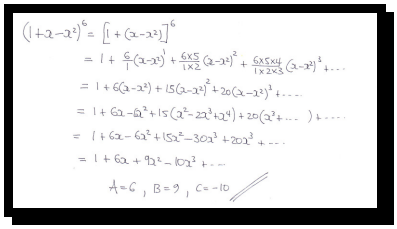


Question 65 **(****+)**

$$(1+x-x^2)^6=1+Ax+Bx^2+Cx^3+\ldots$$

Determine the value of each of the constants A, B and C.

$$\boxed{}\,,\quad \boxed{A=6}\,,\quad \boxed{B=-21}\,,\quad \boxed{C=-10}$$



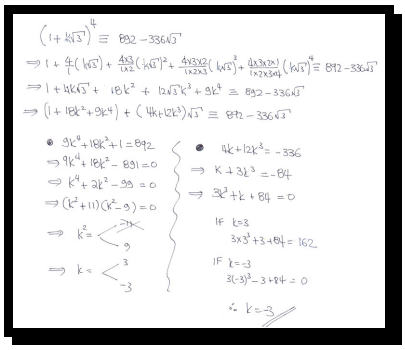
Question 66 (**+)**

It is given that if k is a constant then

$$\left(1+k\sqrt{3}\right)^4 \equiv 892-336\sqrt{3}\,.$$

Determine the value of k.

$$\boxed{k=-3}$$



Question 67 (**+)**

If A, k and n are constants, with $n \in \mathbb{N}$, then

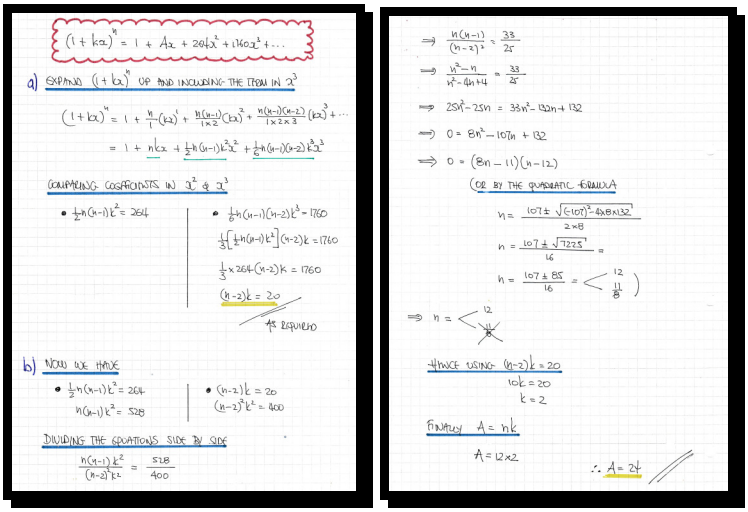
$$(1+kx)^n = 1 + Ax + 264x^2 + 1760x^3 + \ldots$$

a) Show that

$$(n-2)k = 20 .$$

b) Determine the value of A.

$\boxed{}$, $\boxed{A = 24}$



Question 68 (*****)

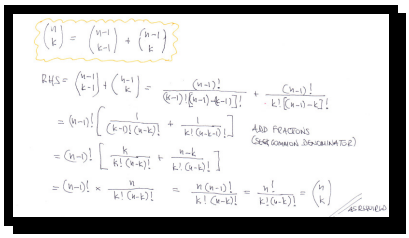
The binomial coefficients are given by

$$\binom{n}{k}=\frac{n!}{k!(n-k)!},\ k\in\mathbb{N},\ n\in\mathbb{N}\cup\{0\}.$$

Show directly from the above definition that

$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}.$$

, proof

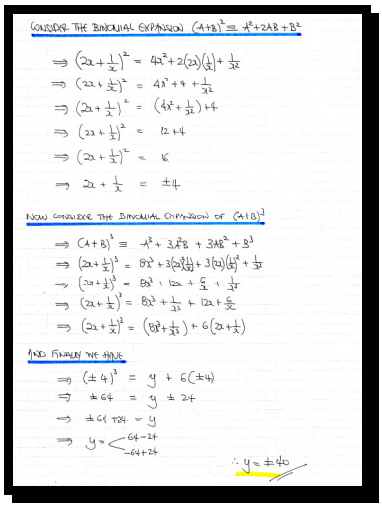


Question 69 (*****)

Find the value, or possible values, of y if

$$y = 8x^3 + \frac{1}{x^3} \qquad \text{and} \qquad 4x^2 + \frac{1}{x^2} = 12 .$$

$\boxed{}$, $\boxed{y = \pm 40}$



Question 70 **(*****)**

It is given that

$$f(x)=\sum_{r=0}^{n}\left[\binom{n}{r}x^r\left(1+x+x^2\right)^{n-r}\right],$$

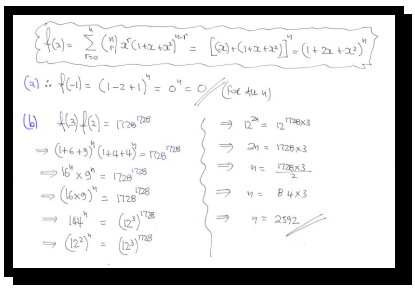
where n is a positive integer constant.

a) Evaluate $f(-1)$.

b) Find the value of n that satisfies the equation

$$f(3)f(2)=1728^{1728}.$$

$\square$, $\boxed{f(-1)=0}$, $\boxed{n=2592}$

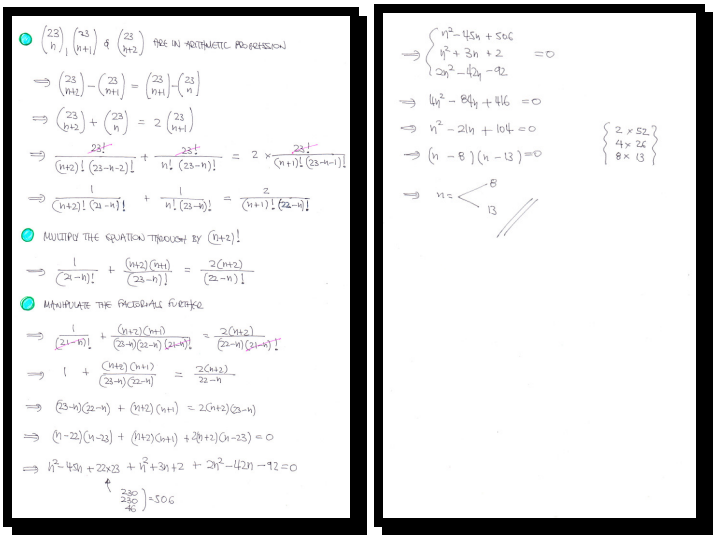


Question 71 (*****)

The coefficients of x^n, x^{n+1} and x^{n+2} in the binomial expansion of $(1+x)^{23}$ are in arithmetic progression.

Determine the possible values of n.

$\boxed{}$, $\boxed{n=8,\ 13}$



Question 72 **(*****)**

It is given that

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n-1}+\binom{n}{n}=k^n,$$

where n and k are positive integer constants.

a) By considering the binomial expansion of $(1+x)^n$, determine the value of k.

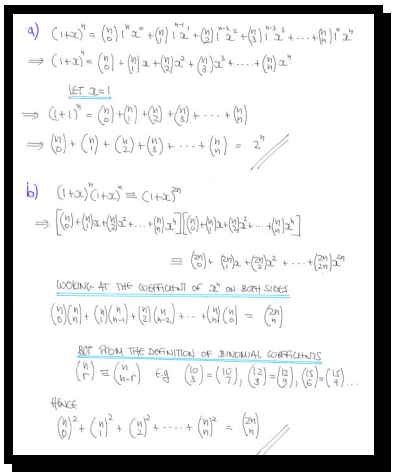
b) By considering the coefficient of x^n in

$$(1+x)^n(1+x)^n\equiv(1+x)^{2n},$$

simplify fully

$$\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\ldots+\binom{n}{n-1}^2+\binom{n}{n}^2.$$

$\boxed{}$, $\boxed{k=2}$



Question 73 **(*****)**

It is given that

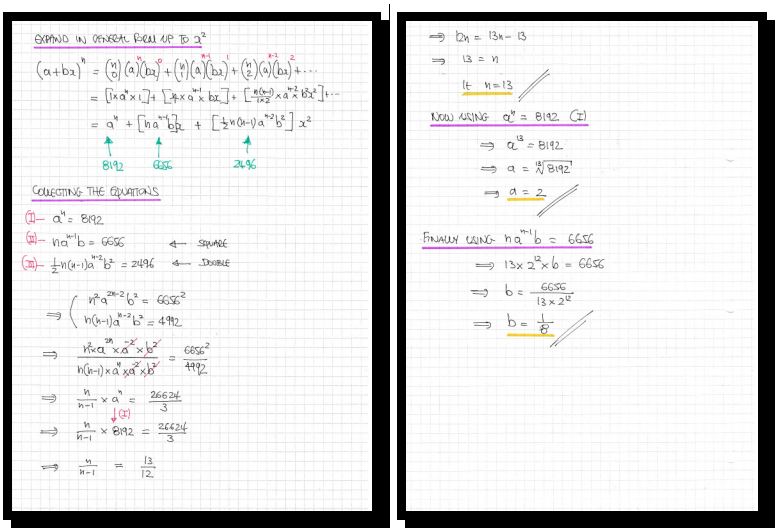
$$(a+bx)^n = 8192 + 6656x + 2496x^2 + \ldots,$$

where a, b and n are non zero constants.

Use algebra to determine the values of a, b and n.

No credit will be given to solutions by inspection and/or verification

$\square$, $\boxed{[a,b,n]=\left[2,\frac{1}{8},13\right]}$



Question 74 **(*****)**

It is given that

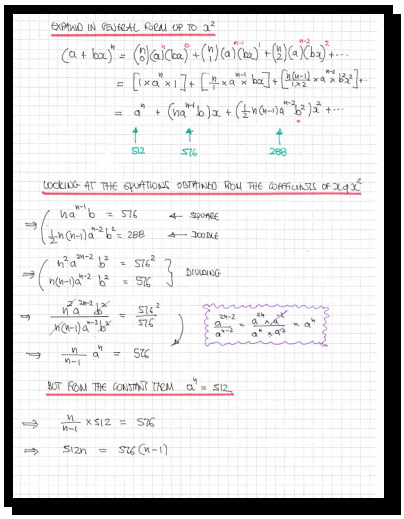
$$(a+bx)^n = 512+576x+288x^2+\ldots,$$

where a, b and n are non zero constants.

Use algebra to determine the values of a, b and n.

No credit will be given to solutions by inspection and/or verification

$\square$, $\boxed{[a,b,n]=\left[2,\frac{1}{4},9\right]}$

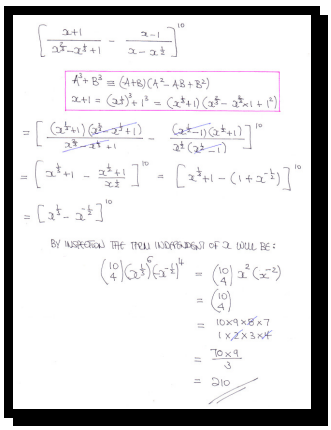


Question 75 (*****)

Find the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}.$$

, 210



Question 76 (*****)

The binomial coefficients are given by

$$\binom{n}{k}=\frac{n!}{k!(n-k)!},\ k\in\mathbb{N},\ n\in\mathbb{N}\cup\{0\}.$$

Show directly from the above definition that if $n\geq r\geq m$, then

$$\binom{n}{m}\binom{n-m}{r-m}=\binom{n}{r}\binom{r}{m}.$$

, proof

