ALGEBRAIC FRACTIONS

Exam Questions

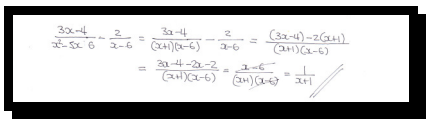
Question 1 (**)

Express

$$\frac{3x-4}{x^2-5x-6}-\frac{2}{x-6},$$

as a single fraction in its simplest form.

$\square$, $\boxed{\dfrac{1}{x+1}}$



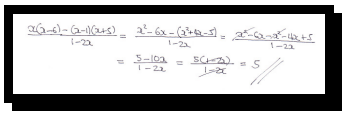
Question 2 (**)

Show that

$$\frac{x(x-6)-(x-1)(x+5)}{1-2x}=k,$$

where k is an integer to be found.

$\boxed{k=5}$

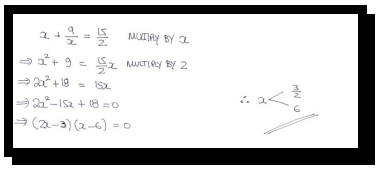


Question 3 (**)

Solve the equation

$$x+\frac{9}{x}=\frac{15}{2},\ x\neq 0.$$

$$\boxed{x=\frac{3}{2},6}$$

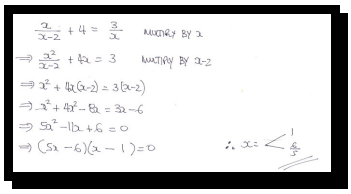


Question 4 (+)**

Solve the equation

$$\frac{x}{x-2}+4=\frac{3}{x},\ x\neq 0.$$

$$\boxed{x=1,\frac{6}{5}}$$



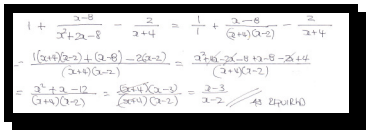
Question 5 (+)**

Show clearly that

$$1+\frac{x-8}{x^2+2x-8}-\frac{2}{x+4}\equiv\frac{x-p}{x-q},$$

stating the value of each of the integer constants, p and q.

$\boxed{}$, $\boxed{p=3}$, $\boxed{q=2}$



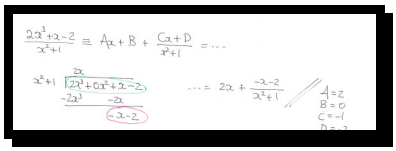
Question 6 (+)**

Given that

$$\frac{2x^3+x-2}{x^2+1}\equiv Ax+B+\frac{Cx+D}{x^2+1},$$

use polynomial division, or another appropriate method, to find the value of each of the constants A, B, C and D.

$\boxed{}$, $\boxed{(A,B,C,D)=(2,0,-1,-2)}$

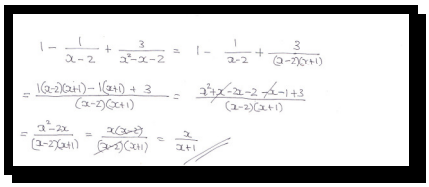


Question 7 (+)**

$$1-\frac{1}{x-2}+\frac{3}{x^2-x-2},\ x\neq 2,\ x\neq 1.$$

Write the above algebraic expression as a single simplified fraction

$\square$, $\boxed{\dfrac{x}{x+1}}$

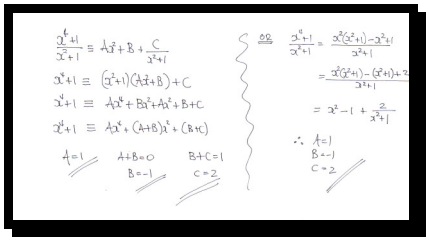


Question 8 (+)**

$$\frac{x^4+1}{x^2+1}\equiv Ax^2+B+\frac{C}{x^2+1}.$$

Find the value of each of the constants A, B and C.

$\boxed{A=1}$, $\boxed{B=-1}$, $\boxed{C=2}$



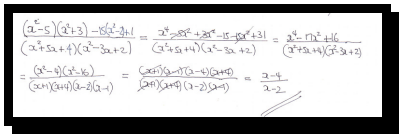
Question 9 (***)

Show that

$$\frac{\left(x^2-5\right)\left(x^2+3\right)-15\left(x^2-2\right)+1}{\left(x^2+5x+4\right)\left(x^2-3x+2\right)}\equiv\frac{x+A}{x+B},$$

stating the value of each of the constants A and B.

$\boxed{A=-4}$, $\boxed{B=-2}$

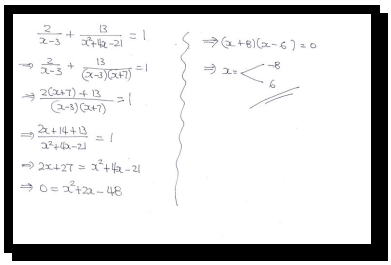


Question 10 (***)

Solve the equation

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}=1,\ x\neq 3,\ x\neq 7.$$

$\boxed{x=-8,6}$

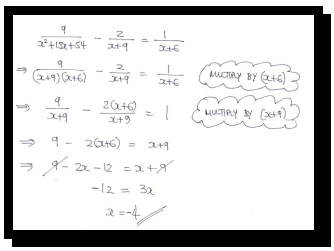


Question 11 (***)

Solve the equation

$$\frac{9}{x^2+15x+54}-\frac{2}{x+9}=\frac{1}{x+6},\ x\neq -6,\ x\neq -9.$$

$\boxed{x=-4}$



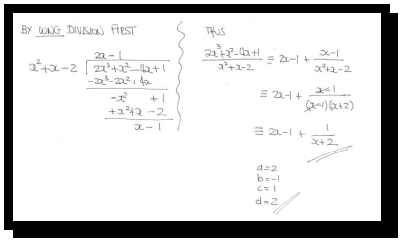
Question 12 (***)

Given that

$$\frac{2x^3+x^2-4x+1}{x^2+x-2}\equiv Ax+B+\frac{C}{x+D},$$

use polynomial division, or another appropriate method, to find the value of each of the constants A, B, C and D.

$\boxed{\ }$, $\boxed{(A,B,C,D)=(2,-1,1,2)}$

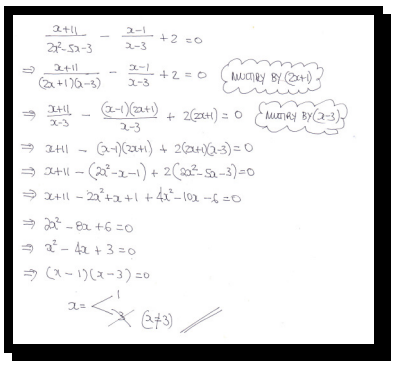


Question 13 (***)

Solve the equation

$$\frac{x+11}{2x^2-5x-3}-\frac{x-1}{x-3}+2=0\,,\ x\neq -\frac{1}{2}\,,\ x\neq 3\,.$$

$\boxed{x=1}$

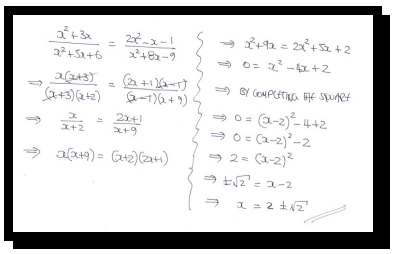


Question 14 (***)

Find, in exact surd form, the roots of the equation

$$\frac{x^2+3x}{x^2+5x+6}=\frac{2x^2-x-1}{x^2+8x-9}\,,\ x\neq -3\,,\ x\neq 1\,.$$

$\boxed{x=2\pm\sqrt{2}}$



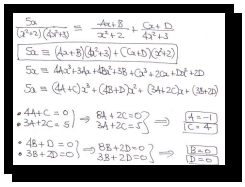
Question 15 (***)

$$\frac{5x}{\left(x^2+2\right)\left(4x^2+3\right)}\equiv\frac{Ax+B}{\left(x^2+2\right)}+\frac{Cx+D}{\left(4x^2+3\right)},$$

where A, B, C and D are constants.

Determine the value of each of the constants A, B, C and D.

$\boxed{A=-1}$, $\boxed{B=0}$, $\boxed{C=4}$, $\boxed{D=0}$



Question 16 (***)

Show clearly that the expression

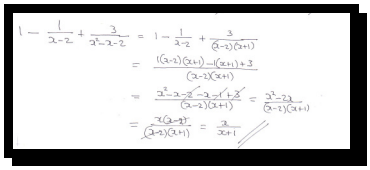
$$1-\frac{1}{x-2}+\frac{3}{x^2-x-2},$$

can be written in the form

$$\frac{x+a}{x+b},$$

where a and b are integer constants to be found.

$\boxed{}$, $\boxed{a=0}$, $\boxed{b=1}$



Question 17 (***)

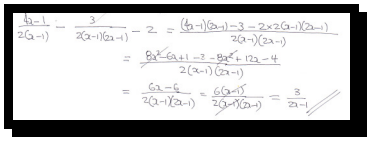
$$f(x)\equiv\frac{4x-1}{2(x-1)}-\frac{3}{2(x-1)(2x-1)}-2,\ x\in\mathbb{R},\ x\neq 1,\ x\neq\frac{1}{2}.$$

Show clearly that

$$f(x)\equiv\frac{k}{2x-1},\ x\in\mathbb{R},\ x\neq\frac{1}{2},$$

where k is an integer to be found.

, $\boxed{k=3}$



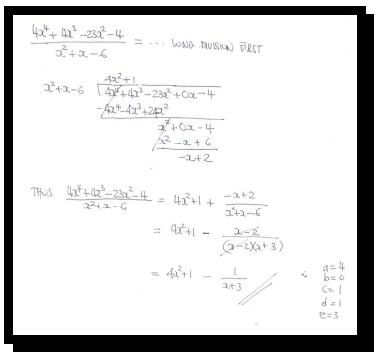
Question 18 (***)

Given that

$$\frac{4x^4+4x^3-23x^2-4}{x^2+x-6}\equiv Ax^2+Bx+C-\frac{C}{x+E},$$

find the value of each of the constants A, B, C, D and E.

, $\boxed{(A,B,C,D,E)=(4,0,1,1,3)}$



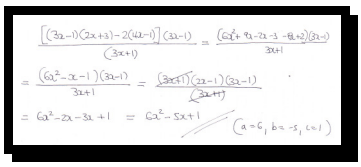
Question 19 (***)

Show clearly that

$$\frac{\big[(3x-1)(2x+3)-2(4x-1)\big](3x-1)}{3x+1}\equiv ax^2+bx+c\,,$$

where a, b and c are integers to be found.

$\boxed{}$, $\boxed{a=6}$, $\boxed{b=-5}$, $\boxed{c=1}$



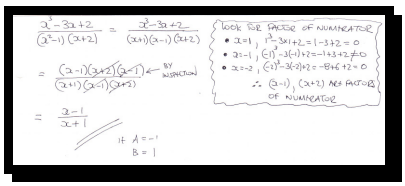
Question 20 (*+)**

Show clearly that

$$\frac{x^3-3x+2}{\big(x^2-1\big)(x+2)}\equiv\frac{x+A}{x+B},$$

stating the value of each of the constants A and B.

$\boxed{A=-1}$, $\boxed{B=1}$

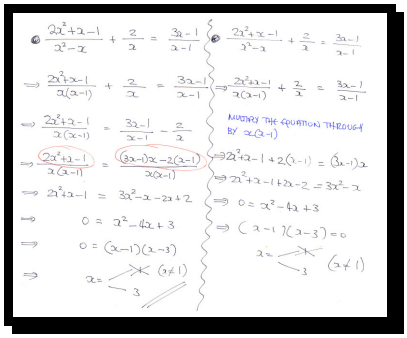


Question 21 (*+)**

Find the solution of the equation

$$\frac{2x^2+x-1}{x^2-x}+\frac{2}{x}=\frac{3x-1}{x-1}.$$

$$\boxed{x=3, x\neq 1}$$



Question 22 (*+)**

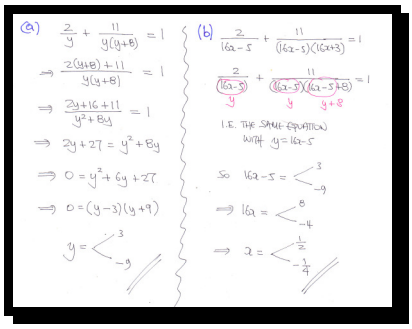
a) Find the solutions of the equation

$$\frac{2}{y}+\frac{11}{y(y+8)}=1,\ y\neq -8,\ y\neq 0.$$

b) Hence, or otherwise, solve the equation

$$\frac{2}{16x-5}+\frac{11}{(16x-5)(16x+3)}=1,\ x\neq -\frac{3}{16},\ x\neq \frac{5}{16}.$$

$\boxed{x=-9,3}$, $\boxed{x=-\frac{1}{4},\frac{1}{2}}$

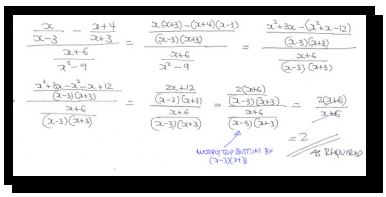


Question 23 (*+)**

Show clearly that

$$\frac{\dfrac{x}{x-3}-\dfrac{x+4}{x+3}}{\dfrac{x+6}{x^2-9}}\equiv 2.$$

$\boxed{\text{proof}}$



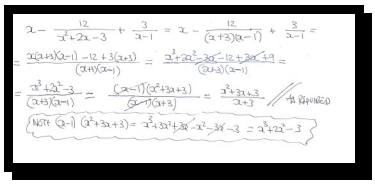
Question 24 (*+)**

$$f(x)\equiv x-\frac{12}{x^2+2x-3}+\frac{3}{x-1},\ x\in\mathbb{R},\ x\neq -3,\ x\neq 1\ .$$

Show clearly that

$$f(x)\equiv\frac{x^2+3x+3}{x+3},\ \ x\in\mathbb{R},\ x\neq -3,\ x\neq 1\ .$$

$\boxed{}$, $\boxed{\text{proof}}$



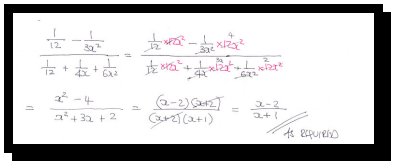
Question 25 (*+)**

Show clearly that

$$\frac{\dfrac{1}{12}-\dfrac{1}{3x^2}}{\dfrac{1}{12}+\dfrac{1}{4x}+\dfrac{1}{6x^2}}\equiv\frac{x+a}{x+b}\ .$$

where a and b are integers to be found

$\boxed{}$, $\boxed{\dfrac{x-2}{x+1}}$

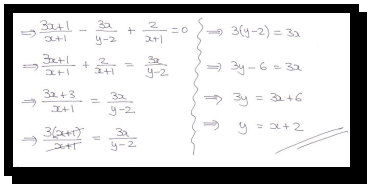


Question 26 (****)

$$\frac{3x+1}{x+1}-\frac{3x}{y-2}+\frac{2}{x+1}=0,\ x\neq -1,\ y\neq 2.$$

Show that the above expression represents a straight line and give its equation.

$$\boxed{y=x+2}$$



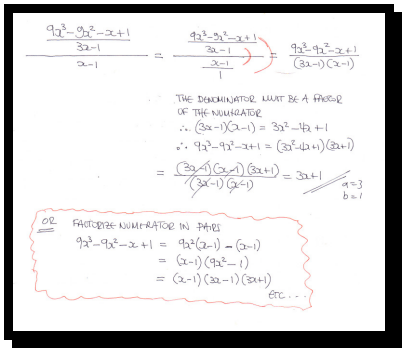
Question 27 (****)

Show clearly that

$$\frac{9x^3-9x^2-x+1}{\dfrac{3x-1}{x-1}}\equiv ax+b,$$

where a and b are integers to be found.

$$\boxed{a=1},\ \boxed{b=1}$$



Question 28 **(****)**

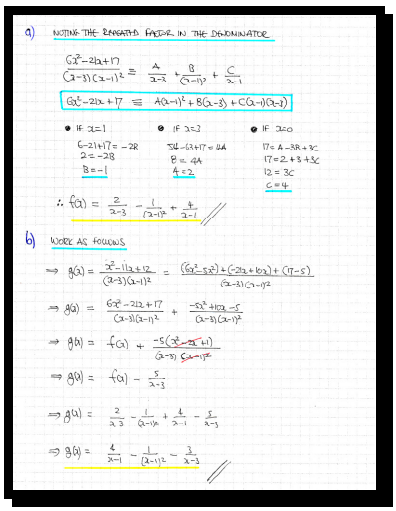
$$f(x)\equiv\frac{6x^2-21x+17}{(x-3)(x-1)^2},\quad x\in\mathbb{R},\quad x\neq 3,\quad x\neq 1.$$

a) Express $f(x)$ into partial fractions.

$$g(x)\equiv\frac{(x-12)(x+1)}{(x-3)(x-1)^2},\quad x\in\mathbb{R},\quad x\neq 3,\quad x\neq 1.$$

b) Use the result of part **(a)** to express $g(x)$ into partial fractions.
*No credit will be given in this part by repeating the method used in part **(a)***

$\square$, $f(x)\equiv\dfrac{4}{x-1}+\dfrac{2}{x-3}-\dfrac{1}{(x-1)^2}$ $f(x)\equiv\dfrac{4}{x-1}-\dfrac{3}{x-3}-\dfrac{1}{(x-1)^2}$



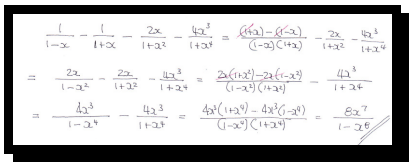
Question 29 **(****+)**

Simplify the following expression

$$\frac{1}{1-x}-\frac{1}{1+x}-\frac{2x}{1+x^2}-\frac{4x^3}{1+x^4},$$

giving the final answer as a single algebraic fraction in its simplest form.

$$\boxed{\frac{8x^7}{1-x^8}}$$

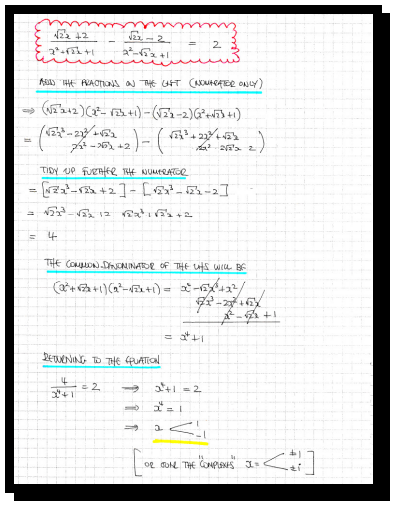


Question 30 **(****+)**

Solve the following equation.

$$\frac{2+\sqrt{2}\,x}{x^2+\sqrt{2}\,x+1}+\frac{2-\sqrt{2}\,x}{x^2-\sqrt{2}\,x+1}=2,\quad x\in\mathbb{R}.$$

$$\boxed{},\ \boxed{x=\pm1}$$

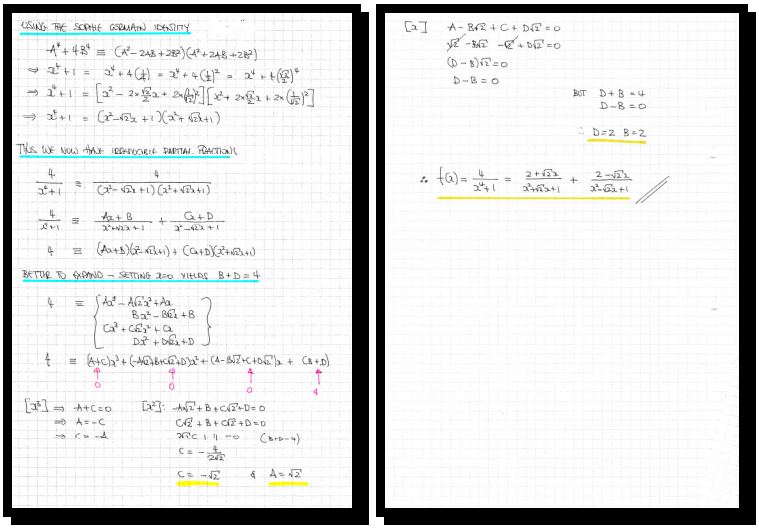


Question 31 **(*****)**

$$f(x)\equiv\frac{4}{x^4+1},\quad x\in\mathbb{R}.$$

Express $f(x)$ into partial fractions over the set of real numbers.

$$\boxed{},\quad \boxed{f(x)\equiv\frac{2+\sqrt{2}\,x}{x^2+\sqrt{2}\,x+1}+\frac{2-\sqrt{2}\,x}{x^2-\sqrt{2}\,x+1}=2}$$



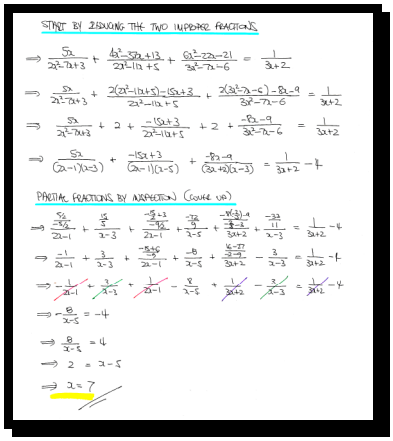
Question 32 (*****)

Solve the following rational equation, over the set of real numbers.

$$\frac{5x}{2x^2-7x+3}+\frac{4x^2-37x+13}{2x^2-11x+5}+\frac{6x^2-22x-21}{3x^2-7x-6}=\frac{1}{3x+2}.$$

You may ignore non finite solutions.

$\boxed{}$, $\boxed{x=7}$

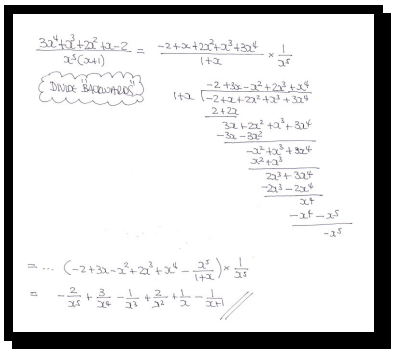


Question 33 (*****)

$$f(x)\equiv\frac{3x^4+x^3+2x^2+x-2}{(x+1)x^5},\ x\in\mathbb{R},\ x\neq 0,\ x\neq -1$$

Express $f(x)$ into partial fractions in their simplest form.

$\square$, $\boxed{f(x)\equiv -\frac{2}{x^5}+\frac{3}{x^4}-\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}-\frac{1}{x+1}}$



Question 34 **(*****)**

$$f(x)\equiv\frac{(2x+1)^2}{x(x+1)^4},\ x\in\mathbb{R},\ x\neq 0, x\neq -1$$

Express $f(x)$ into partial fractions in their simplest form.

$\square$, $\boxed{f(x)\equiv\frac{1}{x}-\frac{1}{x+1}-\frac{1}{(x+1)^2}+\frac{3}{(x+1)^3}-\frac{1}{(x+1)^4}}$

