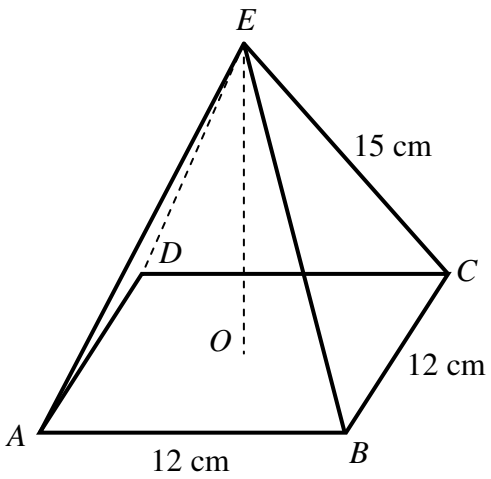
# GEOMETRIC MENSURATION IN 3 DIMENSIONS

**Question 1 (**)**




A square pyramid $ABCDE$ stands on level horizontal ground.

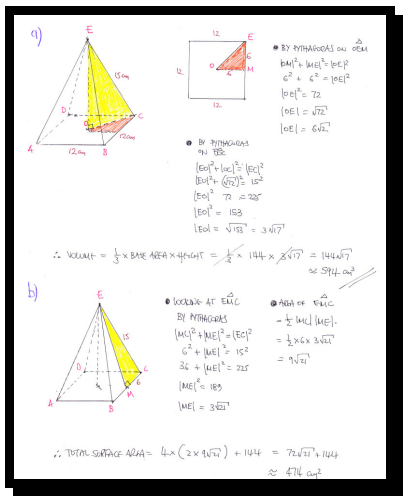
The vertex of the pyramid is at $E$. The points $A$, $B$, $C$ and $D$ are the corners of a square of side 12 cm, whose diagonals intersect at the point $O$.

Each of the sloping edges of the pyramid has length 15 cm.

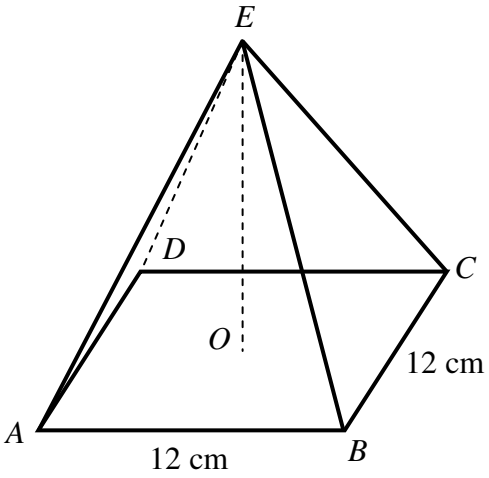
**a)** Calculate the volume of the pyramid.

**b)** Calculate the total surface area of the pyramid.

$$\boxed{V=144\sqrt{17}\approx 594\text{ cm}^3}\,,\qquad \boxed{A=144+72\sqrt{21}\approx 474\text{ cm}^2}$$



**Question 2 (***)**




A square pyramid $ABCDE$ stands on level horizontal ground. The points $A$, $B$, $C$ and $D$ are the corners of a square of side 12 cm, whose diagonals intersect at the point $O$. The vertex of the pyramid is at $E$. Each of the sloping edges of the pyramid makes an angle of $30°$ with the ground.

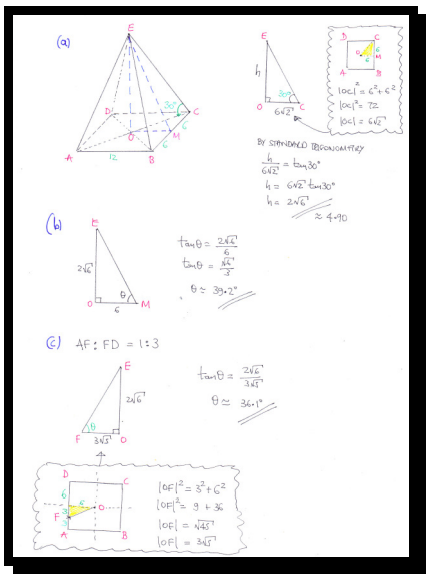
**a)** Determine the height of the pyramid, $OE$.

**b)** Calculate the angle the face $EBC$, makes with the ground.

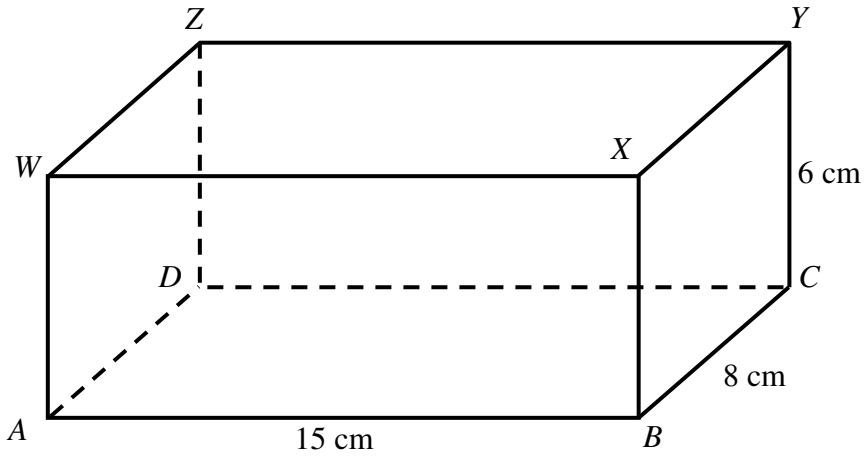
The point $F$ lies on $AD$ so that $AF:FD=1:3$.

**c)** Calculate the angle $EFO$.

$\boxed{|OE|=2\sqrt{6}\approx 4.90 \text{ cm}}$, $\boxed{\approx 39.2°}$, $\boxed{\measuredangle EFO\approx 36.1°}$



**Question 3 (***)**




The figure above shows a cuboid $ABCDWXYZ$, standing on level horizontal ground. The lengths of $AB$, $BC$ and $CY$ are 15 cm, 8 cm and 6 cm, respectively.

**a)** Find the length of $AY$.

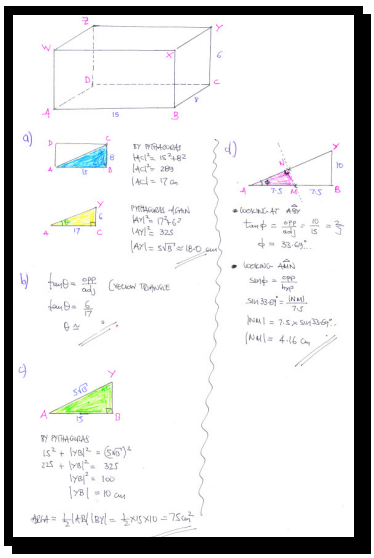
**b)** Calculate the angle $AY$ makes with the ground.

**c)** Determine the area of the triangle $ABY$.

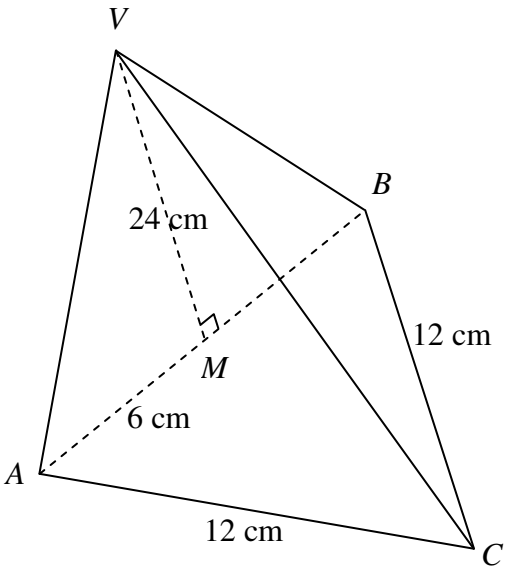
The point $M$ is the midpoint of $AB$ and the point $N$ lies on $AY$.

**d)** Calculate the length of $MN$, given further that $MN$ is perpendicular to $AY$.

$\boxed{|AY| = 5\sqrt{13} \approx 18.0 \text{ cm}}$, $\boxed{\approx 19.4^\circ}$, $\boxed{\text{area} = 75 \text{ cm}^2}$, $\boxed{\approx 4.16 \text{ cm}}$



**Question 4 (***)**




The figure above shows a pyramid $VABC$, standing on level horizontal ground. The base of the pyramid $ABC$ is an equilateral triangle of length 12 cm. The point $M$ is the midpoint of $AB$. The vertex of the pyramid $V$ lies vertically above $M$ so that the length of $VM$ is 24 cm.

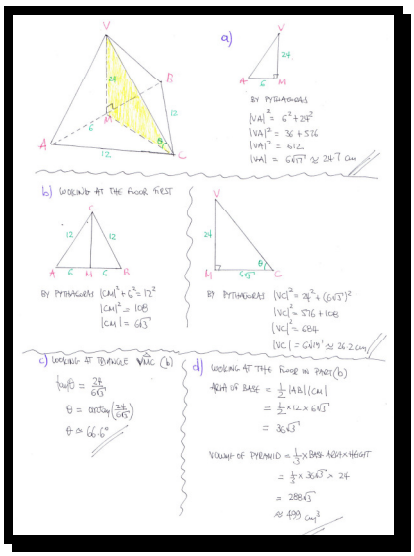
**a)** Find the length of $VA$.

**b)** Find the length of $VC$.

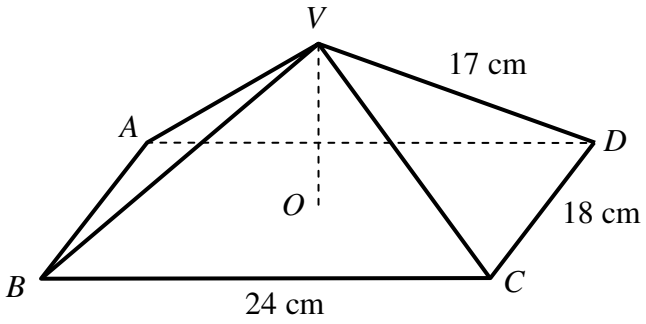
**c)** Calculate the angle $VC$ makes with the ground.

**d)** Determine the volume of the pyramid $VABC$.

$\boxed{|VA| = 6\sqrt{17} \approx 24.7 \text{ cm}}$, $\boxed{|VC| = 6\sqrt{19} \approx 26.2 \text{ cm}}$, $\boxed{\approx 66.6°}$, $\boxed{\text{volume} = 499 \text{ cm}^3}$



**Question 5 (***)**




A pyramid $VABCD$ stands on level horizontal ground. The points $A$, $B$, $C$ and $D$ are the corners of a rectangle, where $|AD|=|BC|=24$ cm and $|AB|=|CD|=18$ cm. The vertex of the pyramid is at $V$ and the diagonals intersect at the point $O$.

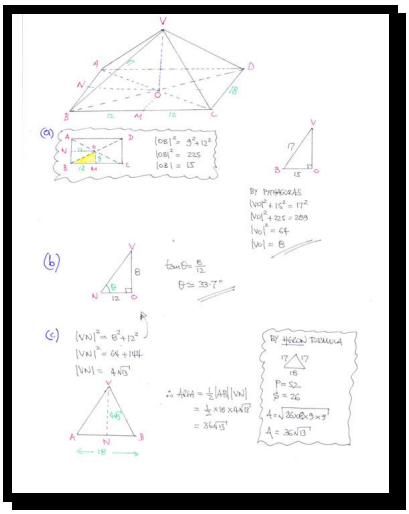
Each of the four sloping edges of the pyramid is $17$ cm.

**a)** Determine the height of the pyramid, $VO$.

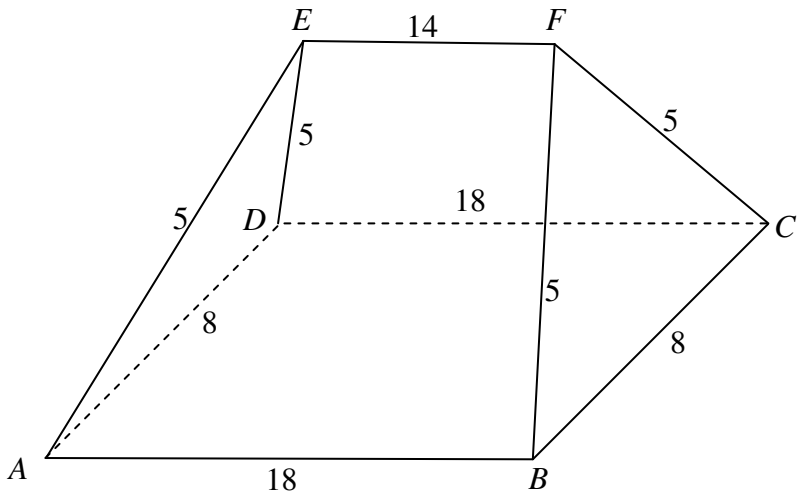
**b)** Calculate the angle the face $VAB$, makes with the base of the pyramid.

**c)** Calculate the exact area of the face $VAB$.

$\boxed{|VO|=8 \text{ cm}}$, $\boxed{\approx 33.7^\circ}$, $\boxed{\text{area}=36\sqrt{13} \text{ cm}^2}$



**Question 6 (***)**




The figure above shows a wooden pole structure $ABCDEF$, modelling a tent, standing on level horizontal ground. The base of the tent $ABCD$ is a rectangle. The pole $EF$ is horizontal.

The following measurements are given in metres.

$$|AB|=|CD|=18\,,\quad |BC|=|DA|=8\,,\quad |EF|=14\,,\quad |EA|=|ED|=|FB|=|FC|=5\,.$$

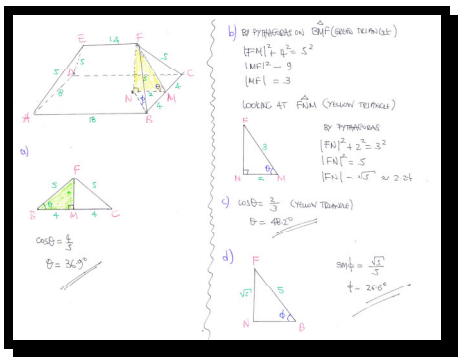
**a)** Calculate the angle $FBC$.

**b)** Find the height of the pole $EF$, from the ground.

**c)** Calculate the angle the face $BFC$ makes with the ground.

**d)** Determine the angle the pole $BF$ makes with the ground.

$$\boxed{\approx 36.9^\circ}\,,\quad \boxed{|FN|=\sqrt{5}\approx 2.24\text{ m}}\,,\quad \boxed{\approx 48.2^\circ}\,,\quad \boxed{\approx 26.6^\circ}$$



**Question 7 (****)**

A pyramid $PQRS$ has a triangular horizontal base $PQR$, where $|PQ|=|PR|=8$ m and $|RQ|=12$ m. The vertex of the pyramid $S$ lies directly above the level of $PQR$ so that $|SQ|=|SR|=10$ m and $|SP|=8$ m.

**a)** Show that the shortest distance of $S$ from the base $PQR$ is $\sqrt{57}$ m.

**b)** Calculate to the nearest degree the acute angle between …

**i.** … the plane $SQR$ and the plane $PQR$.

**ii.** … the edge $SQ$ and the plane $PQR$.

**c)** Determine as an exact surd the shortest distance of $P$ from the plane $SQR$.

$\boxed{71^\circ}$, $\boxed{49^\circ}$, $\boxed{d=\frac{1}{2}\sqrt{399}}$

