VECTOR INTEGRALS

$$\textbf{TYPE} \int_C \varphi \ \mathbf{dr}$$

Question 1

$$V(x,y,z)=60xyz^2 .$$

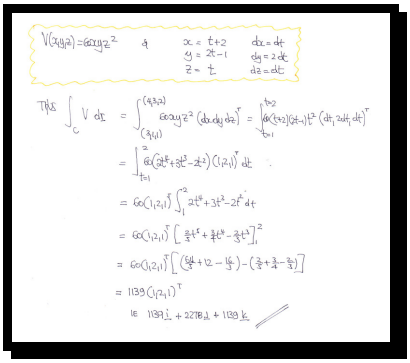
Evaluate the following integral along C, from $(3,1,1)$ to $(4,3,2)$,

$$\int_C V\ \mathbf{dr}, \qquad \mathbf{dr}=(dx,dy,dz)^{\mathrm{T}},$$

where C is the curve with parametric equations

$$x=t+2, \quad y=2t-1, \quad z=t .$$

$$\boxed{1139\mathbf{i}+2278\mathbf{j}+1139\mathbf{k}}$$



Question 2

$$\varphi(x,y,z) \equiv 3x+2y+z\,.$$

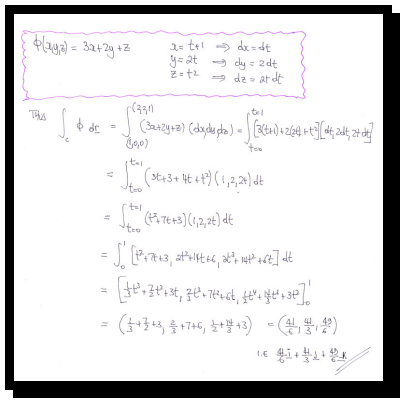
Evaluate the following integral along C, from $(1,0,0)$ to $(2,2,1)$,

$$\int_C \varphi\,\mathbf{dr}\,, \qquad \mathbf{dr}=(dx,dy,dz)^{\mathrm{T}}\,,$$

where C is the curve with parametric equations

$$x=t+1\,, \quad y=2t\,, \quad z=t^2\,.$$

$$\boxed{\frac{41}{6}\mathbf{i}+\frac{41}{3}\mathbf{j}+\frac{49}{6}\mathbf{k}}$$



Question 3

$$F(x,y,z)=xyz\,.$$

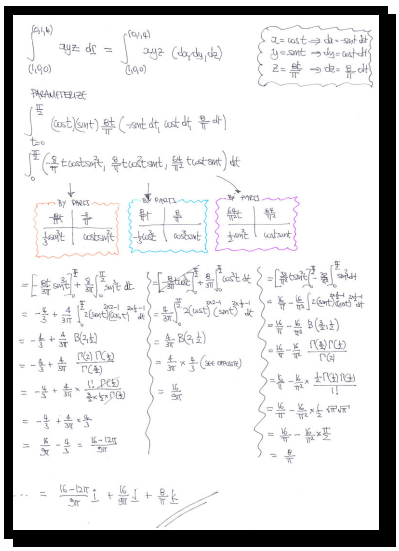
Evaluate the following integral along C, from $(1,0,0)$ to $(0,1,4)$,

$$\int_C F\ \mathbf{dr}\,, \qquad \mathbf{dr}=(dx,dy,dz)^{\mathrm{T}}\,,$$

where C is the curve with parametric equations

$$x=\cos t\,, \qquad y=\sin t\,, \qquad z=\frac{8t}{\pi}\,.$$

$$\boxed{\frac{16-12\pi}{9\pi}\mathbf{i}+\frac{16}{9\pi}\mathbf{j}+\frac{8}{\pi}\mathbf{k}}$$



$$\textbf{TYPE} \int_{V} \mathbf{F}\, dV$$

Question 1

$$\mathbf{F}(x,y,z) \equiv xy\mathbf{i} + z\mathbf{j} - x^2\mathbf{k} .$$

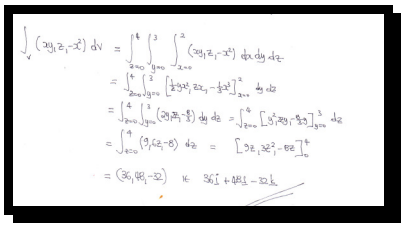
Evaluate the vector integral

$$\int_V \mathbf{F}\, dV ,$$

where V is the finite region in the first octant bounded by the planes with equations

$$x = 2 , \ y = 3 \text{ and } \ z = 4 .$$

$$\boxed{36\mathbf{i} + 48\mathbf{j} - 32\mathbf{k}}$$



Question 2

$$\mathbf{F}(x,y,z) \equiv z\mathbf{i} + \mathbf{j} + y\mathbf{k} \,.$$

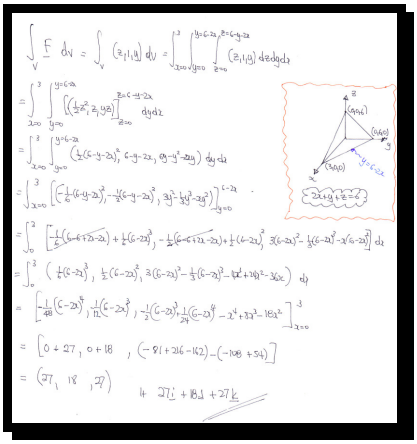
Evaluate the vector integral

$$\int_V \mathbf{F}\, dV \,,$$

where V is the finite region in the first octant bounded by the plane with equation

$$2x + y + z = 6 \,.$$

$$\boxed{27\mathbf{i} + 18\mathbf{j} + 27\mathbf{k}}$$



Question 3

$$\mathbf{F}(x,y,z)\equiv \mathbf{i}+2z\mathbf{j}+y\mathbf{k}\,.$$

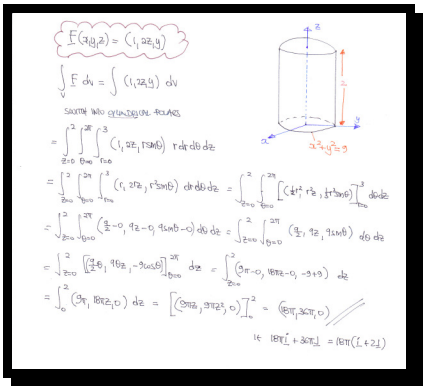
Evaluate the vector integral

$$\int_V \mathbf{F}\,dV\,,$$

where V is the finite region enclosed by the cylinder with equation

$$x^2+y^2=9\,,\ 0\le z\le 2\,.$$

$$\boxed{18\pi(\mathbf{i}+2\mathbf{j})}$$



Question 4

$$\mathbf{F}(x,y,z) \equiv \frac{1}{6\pi}\mathbf{i}+\frac{z}{18\pi}\mathbf{j}+\frac{y}{9\pi}\mathbf{k}\,.$$

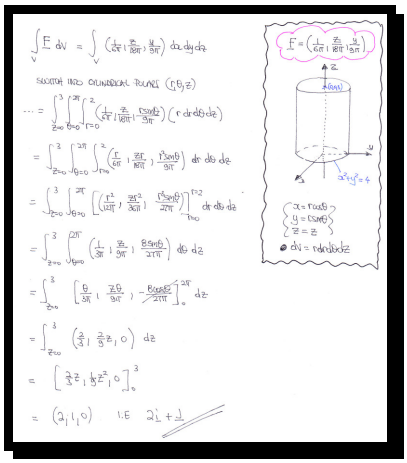
Evaluate the vector integral

$$\int_V \mathbf{F}\,dV\,,$$

where V is the finite region enclosed by the cylinder with equation

$$x^2+y^2=4\,,\ 0\le z\le 3\,.$$

$$\boxed{2\mathbf{i}+\mathbf{j}}$$



Question 5

$$\mathbf{F}(x,y,z) \equiv 3\mathbf{i} + -y\mathbf{j} + 6x\mathbf{k}\,.$$

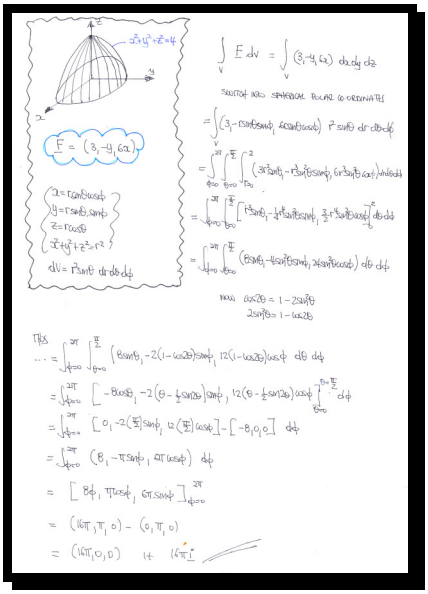
Evaluate the vector integral

$$\int_V \mathbf{F}\, dV\,,$$

where V is the finite region enclosed by the hemisphere with equation

$$x^2 + y^2 + z^2 = 4\,,\ z \geq 0\,.$$

$\boxed{16\pi\mathbf{i}}$



Question 6

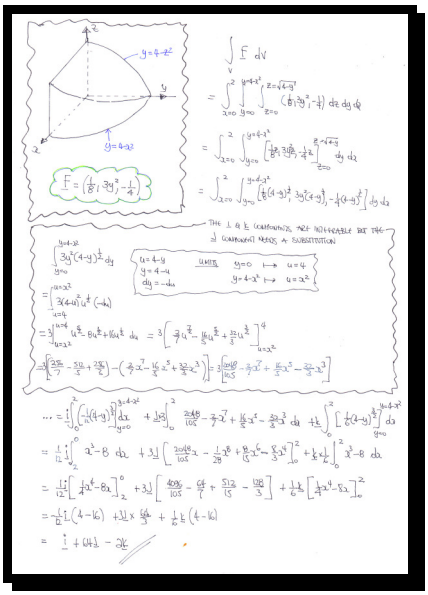
The finite region V in the first octant, is bounded by the surfaces with equations

$$y = 4 - x^2 \quad \text{and} \quad y = 4 - z^2 .$$

Given that $\mathbf{F} = \frac{1}{8}\mathbf{i} + 3y^2\mathbf{j} - \frac{1}{4}\mathbf{k}$ determine

$$\int_V \mathbf{F} \, dv .$$

$$\boxed{\mathbf{i} + 64\mathbf{j} - 2\mathbf{k}}$$



$$\textbf{TYPE} \int_{S} F \ \textbf{dS}$$

Question 1

$$F(x,y,z) \equiv x+y+z\,.$$

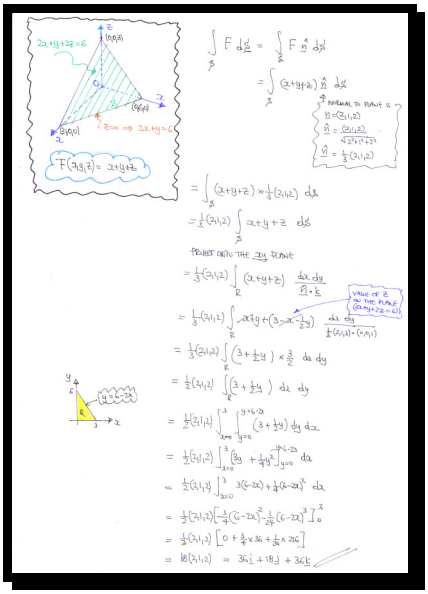
Evaluate the integral

$$\int_S F\ \mathbf{dS}\,,$$

where S is the plane surface with equation

$$2x+y+2z=6\,,\ x\geq 0\,,\ y\geq 0\,,\ z\geq 0\,.$$

$$\boxed{36\mathbf{i}+18\mathbf{j}+36\mathbf{k}}$$



Question 2

$$\varphi(x,y,z) \equiv \frac{3}{4}xyz\,.$$

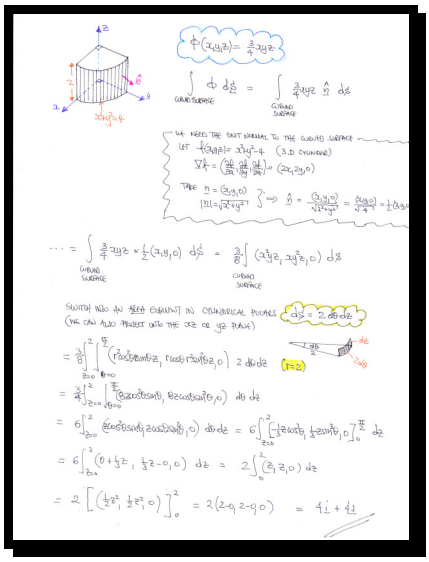
Evaluate the integral

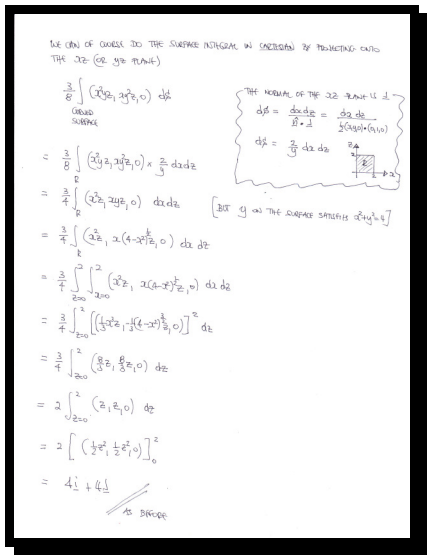
$$\int_S \varphi \, \mathbf{dS}\,,$$

where S is the curved surface of the cylinder with equation

$$x^2+y^2=4\,,\ x\geq 0\,,\ y\geq 0\,,\ 0\leq z\leq 2\,.$$

$\boxed{4\mathbf{i}+4\mathbf{j}}$





Question 3

$$\varphi(x,y,z) \equiv \frac{1}{2}xyz^2 .$$

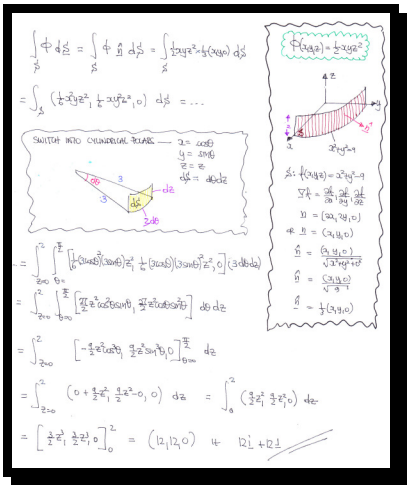
Evaluate the integral

$$\int_S \varphi \, \mathbf{dS} ,$$

where S is the curved surface of the cylinder with equation

$$x^2 + y^2 = 9 , \ x \geq 0 , \ y \geq 0 , \ 0 \leq z \leq 2 .$$

$\boxed{12\mathbf{i}+12\mathbf{j}}$



Question 4

$$\varphi(x,y,z) \equiv 2x+2y\,.$$

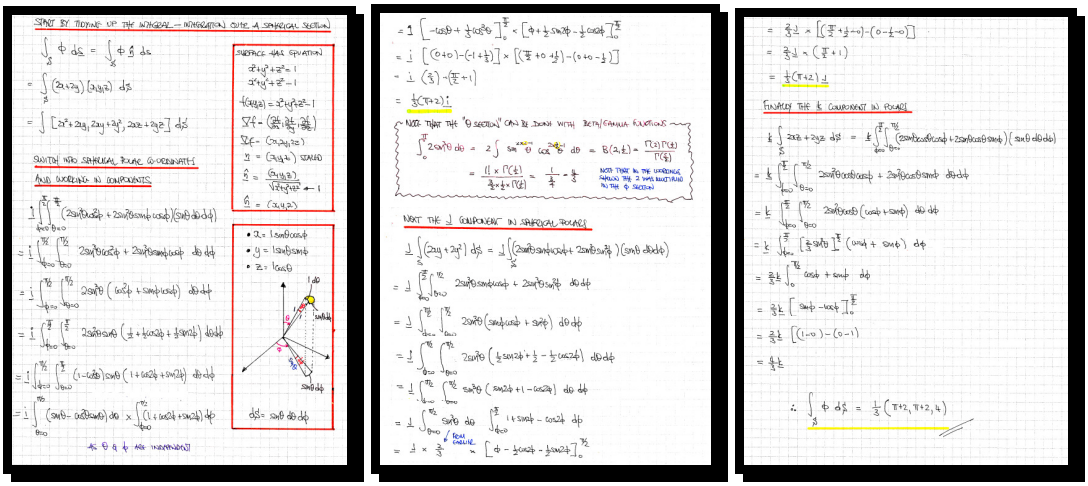
Evaluate the integral

$$\int_S \varphi \, \mathbf{dS}\,,$$

where S is the curved surface of the sphere with equation

$$x^2+y^2+z^2=1,\ x\geq 0,\ y\geq 0,\ z\geq 0\,.$$

$\square$, $\boxed{\frac{1}{3}\left[(\pi+2)\mathbf{i}+(\pi+2)\mathbf{j}+4\mathbf{k}\right]}$



$$\textbf{TYPE} \int_S \mathbf{F}\, dS$$

Question 1

The Cartesian equation of a surface S is

$$z = x^2 + y^2, \quad z \leq 1.$$

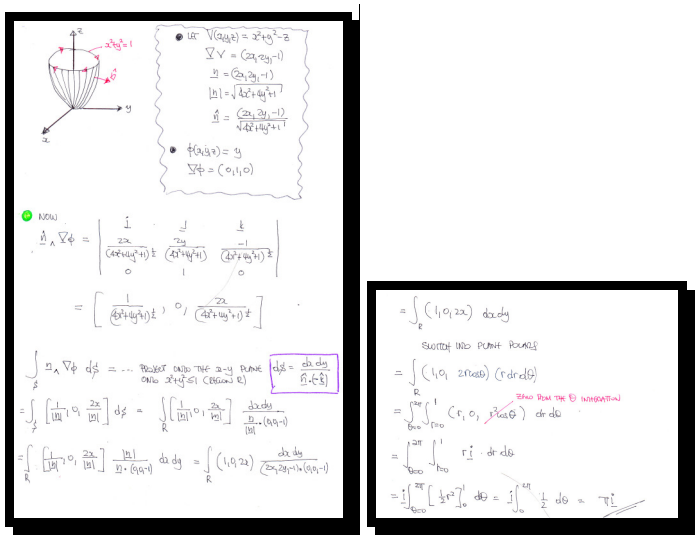
Evaluate the surface integral

$$\int_S \hat{\mathbf{n}} \wedge \nabla \varphi \, dS,$$

where $\hat{\mathbf{n}}$ is an outward normal unit vector field to S, and φ is the function with Cartesian equation

$$\varphi(x, y, z) = y.$$

$\boxed{\pi \mathbf{i}}$



Question 2

The Cartesian equation of a surface S is

$$z = 1 - x^2 - y^2, \quad z \geq 0 .$$

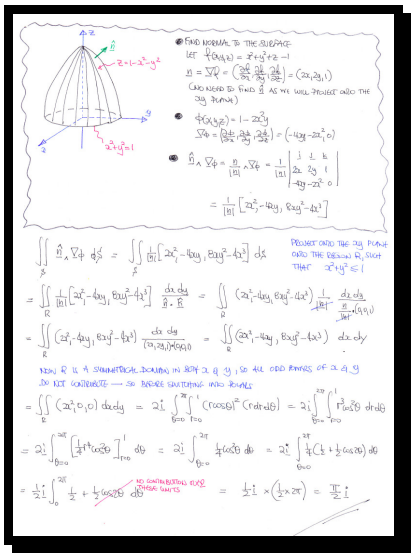
Evaluate the surface integral

$$\int_S \hat{\mathbf{n}} \wedge \nabla \varphi \, dS ,$$

where $\hat{\mathbf{n}}$ is an outward unit normal vector field to S, and φ is the function with Cartesian equation

$$\varphi(x, y, z) = 1 - 2x^2 y .$$

$$\frac{\pi}{2}\mathbf{i}$$



$$\textbf{TYPE} \int_S \mathbf{F} \cdot \mathbf{dS}$$

Question 1

Evaluate the surface integral

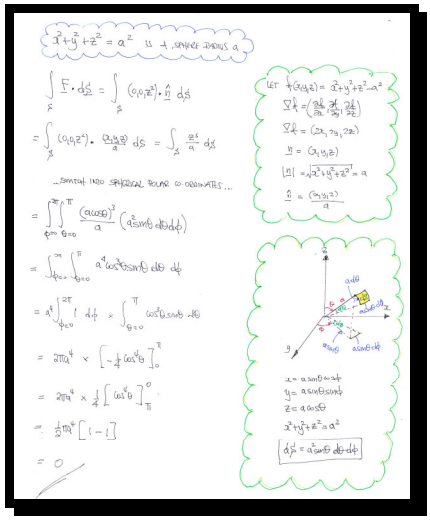
$$\int_S \mathbf{F}\cdot\mathbf{dS},$$

where S is the surface with equation

$$x^2+y^2+z^2=a^2,\ a>0,$$

and $\mathbf{F}=z^2\mathbf{k}$.

$\boxed{0}$



Question 2

$$\mathbf{F}(x,y,z) \equiv x^2\mathbf{i} - 2y\mathbf{j} - 2z\mathbf{k}\,.$$

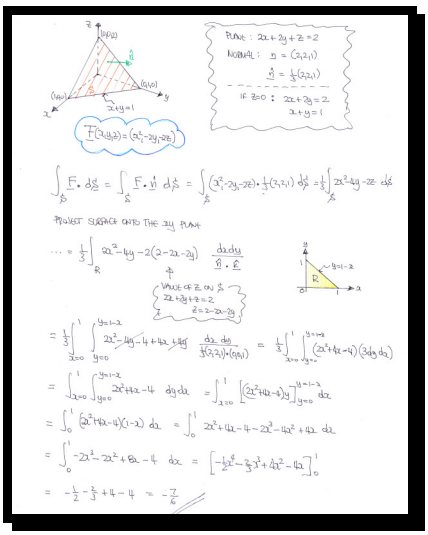
Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the plane surface with equation

$$2x+2y+z=2\,,\ x\geq 0\,,\ y\geq 0\,,\ z\geq 0\,.$$

$$\boxed{-\frac{7}{6}}$$



Question 3

$$\mathbf{F}(x,y,z) \equiv 4y\mathbf{i}+\mathbf{j}+2\mathbf{k}\,.$$

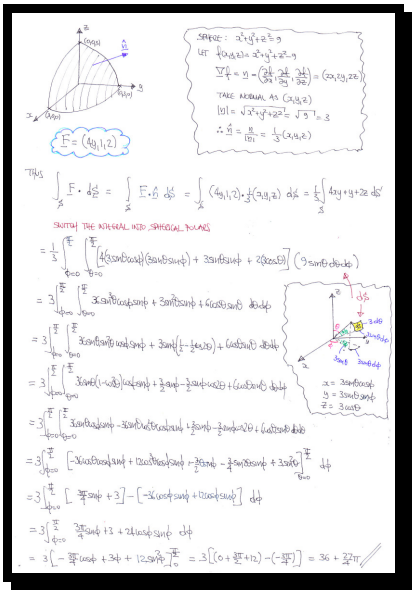
Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with equation

$$x^2+y^2+z^2=9\,,\ x\geq 0\,,\ y\geq 0\,,\ z\geq 0\,.$$

$$\boxed{36+\frac{9}{4}\pi}$$



Question 4

Evaluate the surface integral

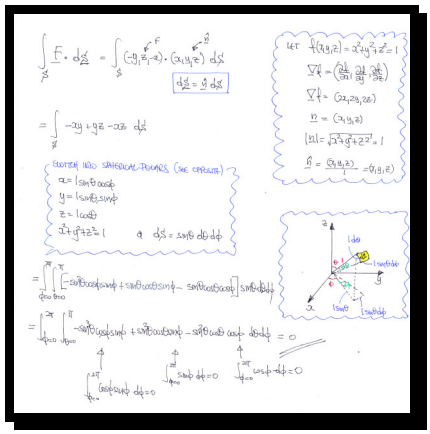
$$\int_S \mathbf{F}\cdot\mathbf{dS},$$

where S is the surface with equation

$$x^2+y^2+z^2=1,$$

and $\mathbf{F}=-y\mathbf{i}+z\mathbf{j}-x\mathbf{k}$.

$\boxed{0}$



Question 5

$$\mathbf{F}(x,y,z) \equiv \mathbf{i} + \frac{1}{2}y\mathbf{j} + z^2\mathbf{k}\,.$$

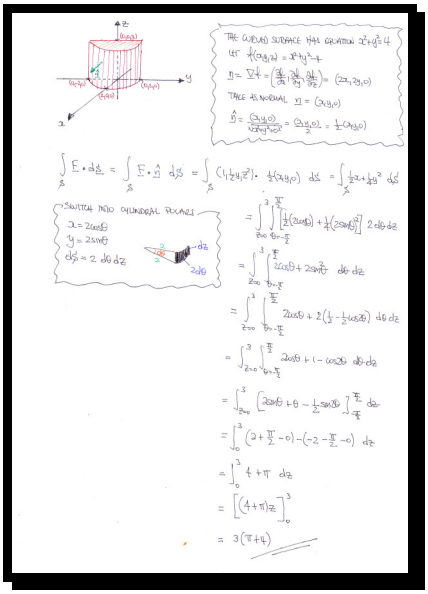
Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the curved cylindrical surface with equation

$$x^2 + y^2 = 4\,,\ x \geq 0\,,\ 0 \leq z \leq 3\,.$$

$$\boxed{3\pi + 12}$$



Question 6

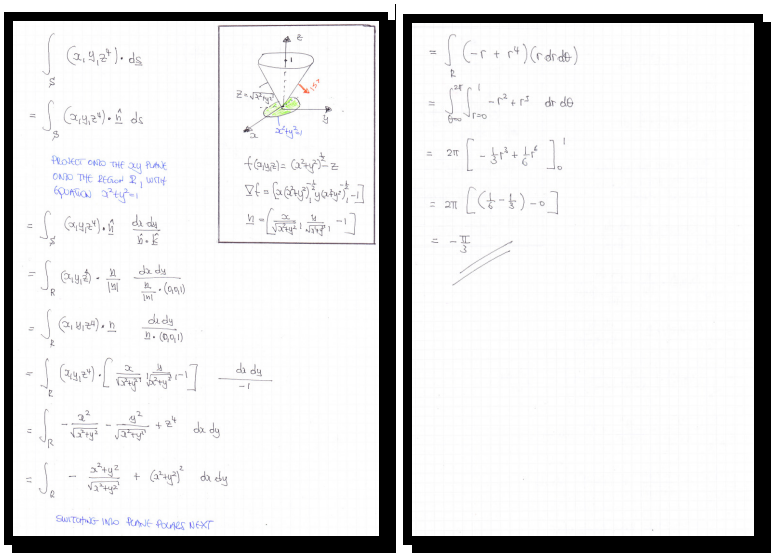
$$\mathbf{F}(x,y,z) \equiv x\,\mathbf{i} + y\,\mathbf{j} + z^4\,\mathbf{k}\,.$$

Calculate the flux of $\mathbf{F}$ through the open surface with equation

$$z = \sqrt{x^2 + y^2}\ ,\ z \le 1,$$

in the direction of z decreasing.

$$\boxed{-\frac{1}{3}\pi}$$



Question 7

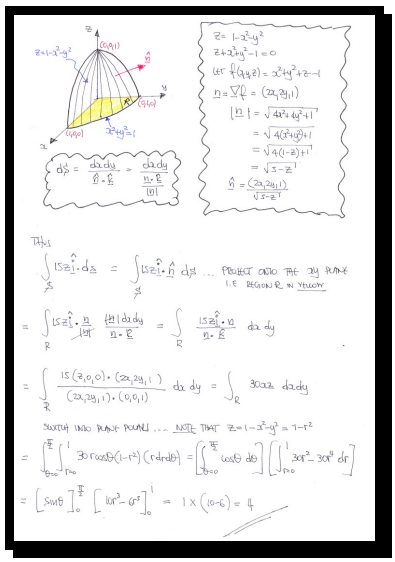
The surface S has Cartesian equation

$$z = 1 - x^2 - y^2, \ x \geq 0, \ y \geq 0, \ z \geq 0.$$

Evaluate the surface integral

$$\int_S 15z\mathbf{i} \cdot \mathbf{dS}.$$

4



Question 8

$$\mathbf{F}(x,y,z)\equiv -y\mathbf{i}+x\mathbf{j}+3z\mathbf{k}\,.$$

Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

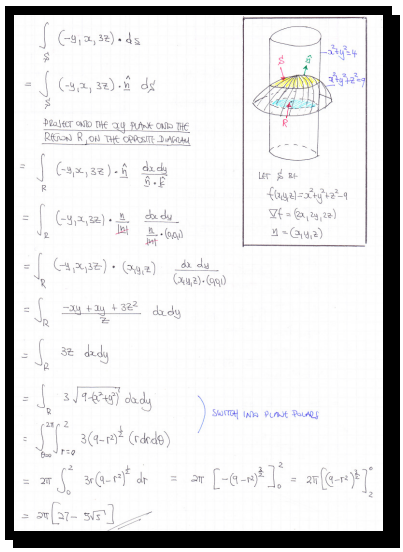
where S is the surface of the hemisphere with equation

$$x^2+y^2+z^2=9\,,\ z\geq 0\,,$$

contained inside the cylinder with equation

$$x^2+y^2=4\,,\ z\geq 0\,,$$

$$\boxed{2\pi\left[27-5\sqrt{5}\right]}$$



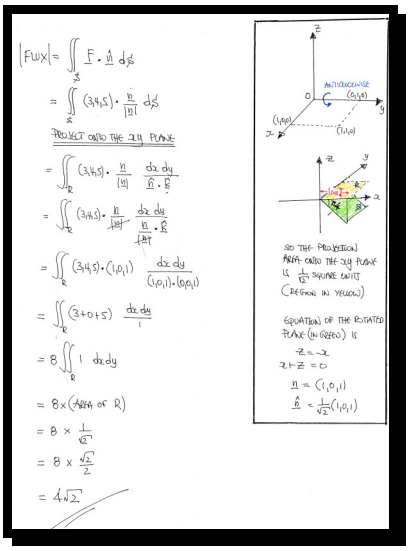
Question 9

Space is filled uniformly by the constant vector field $3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$.

A square lamina whose vertices are at $(0,0,0)$, $(1,0,0)$, $(1,1,0)$ and $(0,1,0)$ is rotated by $\frac{1}{4}\pi$, anticlockwise, about the y axis.

determine the magnitude of the flux of the field through the rotated lamina.

$$\boxed{4\sqrt{2}}$$



Question 10

The surface S has Cartesian equation

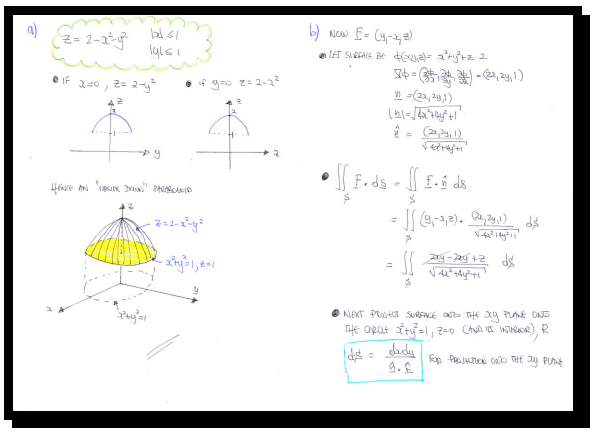
$$z = 2 - x^2 - y^2, \ x^2 + y^2 \leq 1.$$

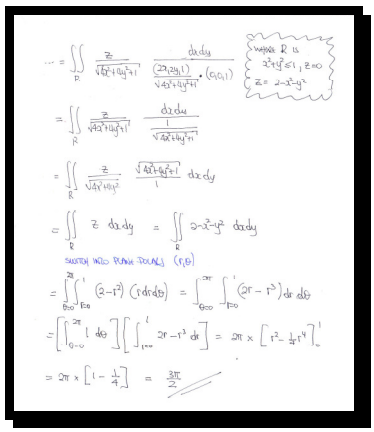
a) Sketch the graph of S.

b) Given that $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$, evaluate the integral

$$\int_S \mathbf{F} \cdot \mathbf{dS}.$$

$$\boxed{\frac{3\pi}{2}}$$





Question 11

The surface S has Cartesian equation

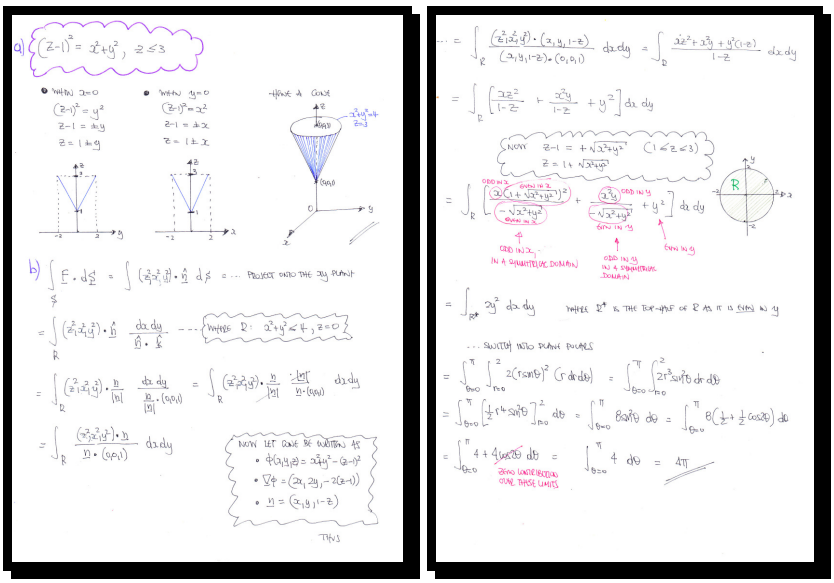
$$(z-1)^2 = x^2 + y^2,\ 1 \le z \le 3.$$

a) Sketch the graph of S.

b) Given that $\mathbf{F} = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$, evaluate the integral

$$\int_S \mathbf{F} \cdot \mathbf{dS}.$$

$\boxed{4\pi}$



Question 12

$$\mathbf{F}(x,y,z) \equiv 3x\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}\,.$$

Evaluate the surface integral

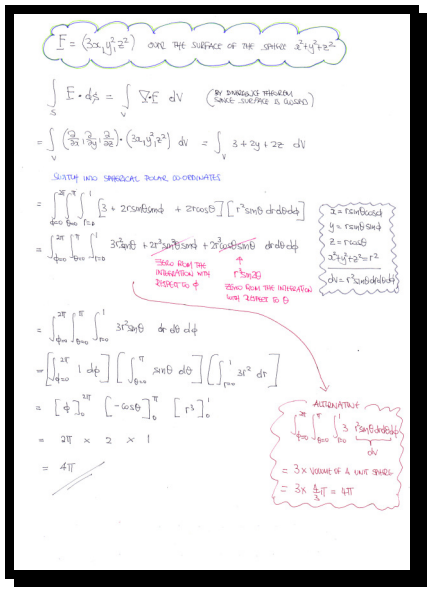
$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

$$x^2 + y^2 + z^2 = 1\,.$$

$\boxed{4\pi}$





Question 13

$$\mathbf{F}(x,y,z)\equiv(x+y)\mathbf{i}+(x-y)\mathbf{j}+(x+z)\mathbf{k}\,.$$

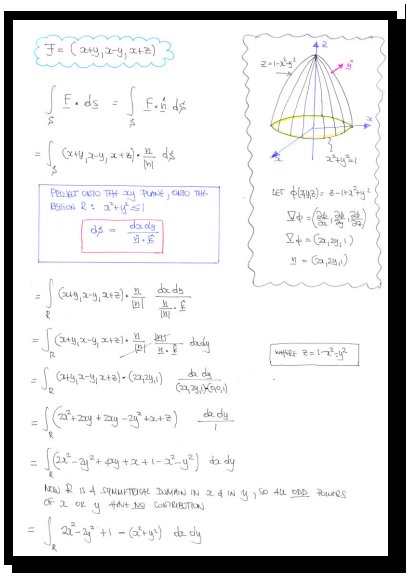
Evaluate the surface integral

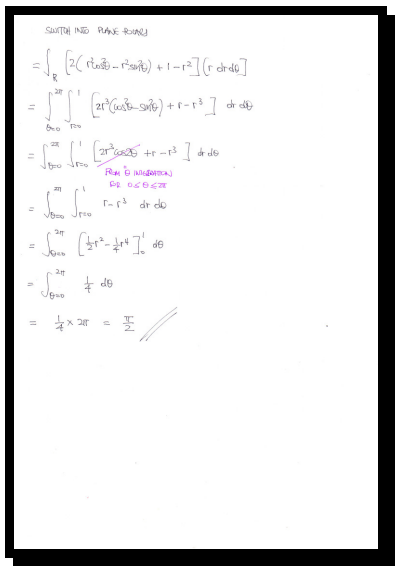
$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

$$z=1-x^2-y^2\,,\ z\geq 0\,.$$

$$\boxed{\frac{\pi}{2}}$$





Question 14

$$\mathbf{F}(x,y,z)\equiv(x+z+xy)\mathbf{i}+\left(z^2-2xz-y\right)\mathbf{j}+\mathbf{k}\,.$$

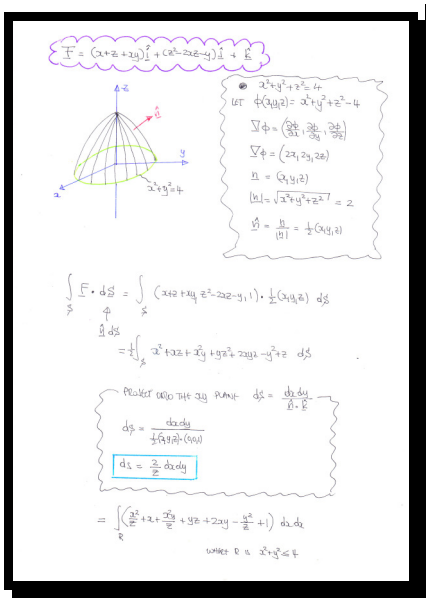
Evaluate the surface integral

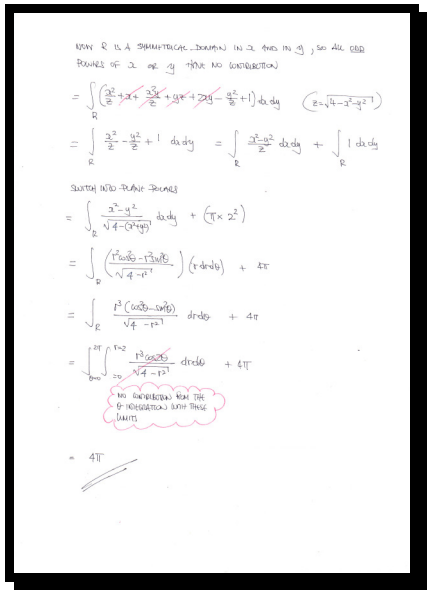
$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

$$x^2+y^2+z^2=4\,,\ z\geq 0\,.$$

$\boxed{4\pi}$





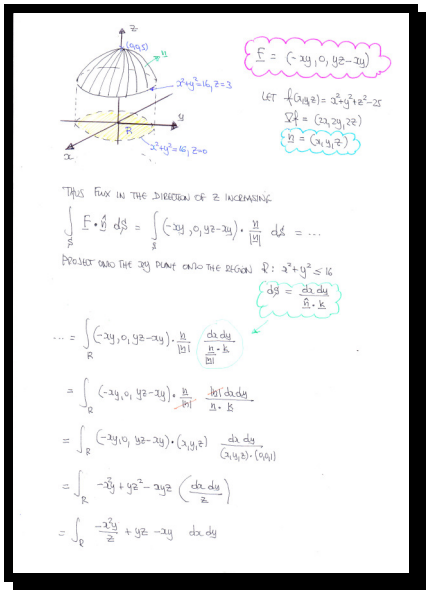
Question 15

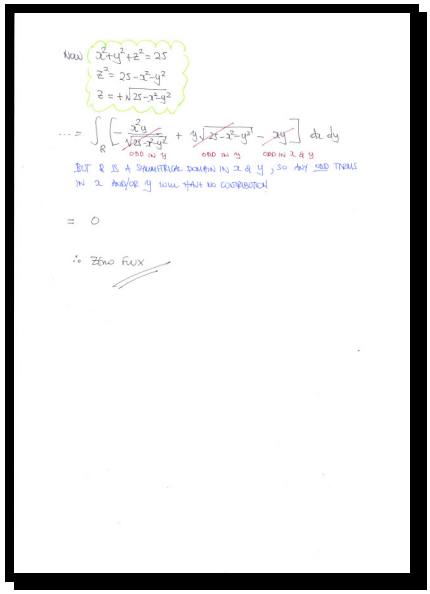
$$\mathbf{F}(x,y,z)\equiv -xy\,\mathbf{i}+(yz-xy)\mathbf{k}\,.$$

Show that there is zero net flux of $\mathbf{F}$ through the surface with Cartesian equation

$$x^2+y^2+z^2=25\,,\ z\geq 3\,.$$

proof





Question 16

$$\mathbf{F}(x,y,z)\equiv x\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k}\,.$$

a) Given that S is the surface with Cartesian equation

$$x^2+y^2+z^2=1,\ z\geq 0,$$

show that

$$\int_S \mathbf{F}\cdot\mathbf{dS}=4\int_R\left[\frac{x^2}{\sqrt{1-x^2-y^2}}+1-x^2-y^2\right]dx\,dy\,,$$

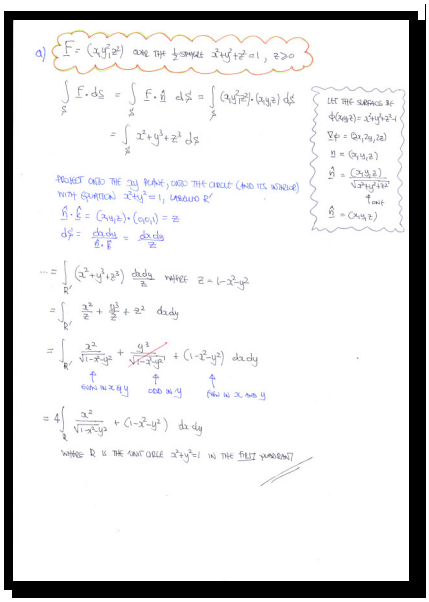
where R is the region in the first quadrant with Cartesian equation

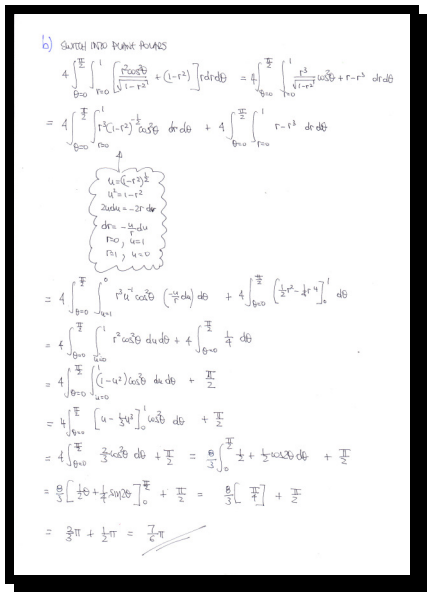
$$x^2+y^2\leq 1,\ x\geq 0,\ y\geq 0.$$

b) Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,.$$

$$\boxed{\frac{7}{6}\pi}$$





Question 17

$$\mathbf{F} = x^2 y^3 \mathbf{i} + z\mathbf{j} + x\mathbf{k} \, .$$

Show by **direct** evaluation that

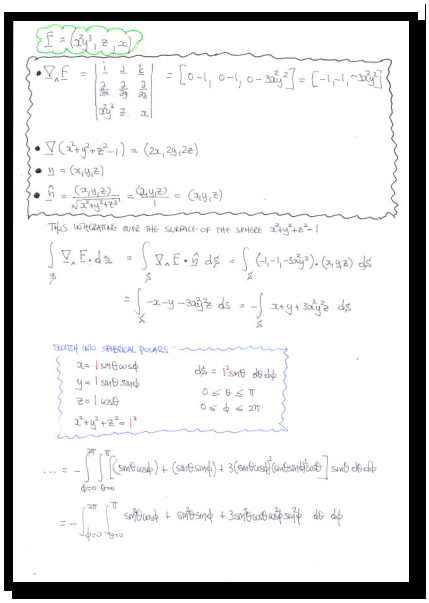
$$\int_S \nabla \wedge \mathbf{F} \cdot \hat{\mathbf{n}} \, dS = 0 \, ,$$

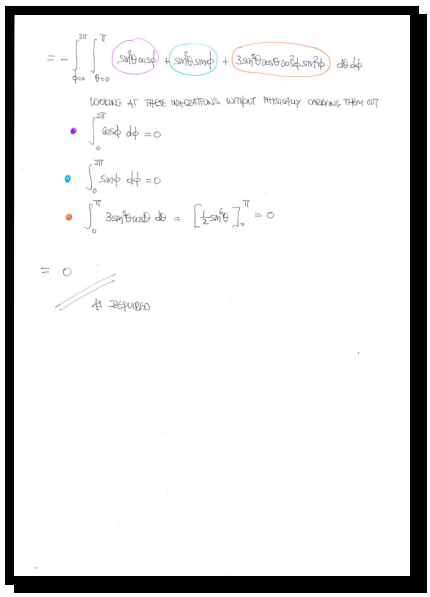
where S is the sphere with equation

$$x^2 + y^2 + z^2 = 1 \, ,$$

and $\hat{\mathbf{n}}$ is an outward unit normal to S.

proof





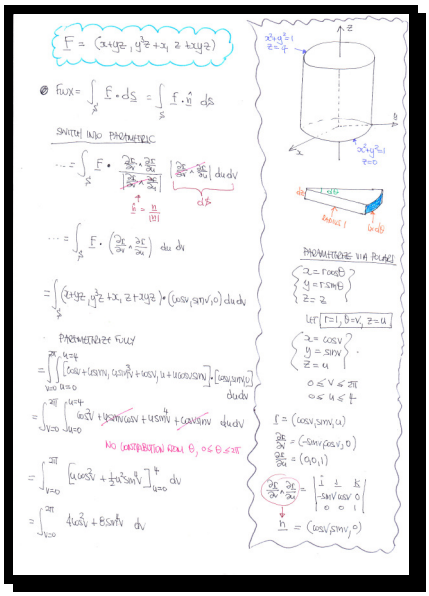
Question 18

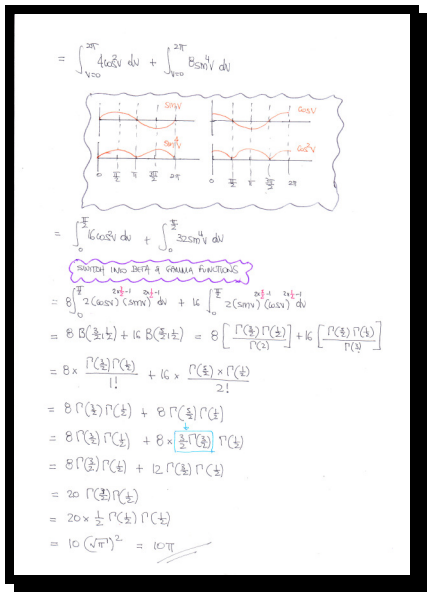
$$\mathbf{F}(x,y,z)\equiv(x+yz)\mathbf{i}+\left(y^3z+x\right)\mathbf{j}+(z+xyz)\mathbf{k}\,.$$

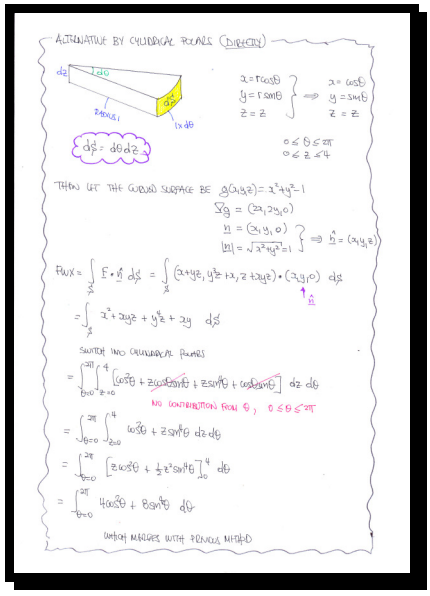
Calculate the magnitude of the flux of $\mathbf{F}$ through the open cylindrical surface with equation

$$x^2+y^2=1,\ 0\le z\le 4\,.$$

$\boxed{10\pi}$







Question 19

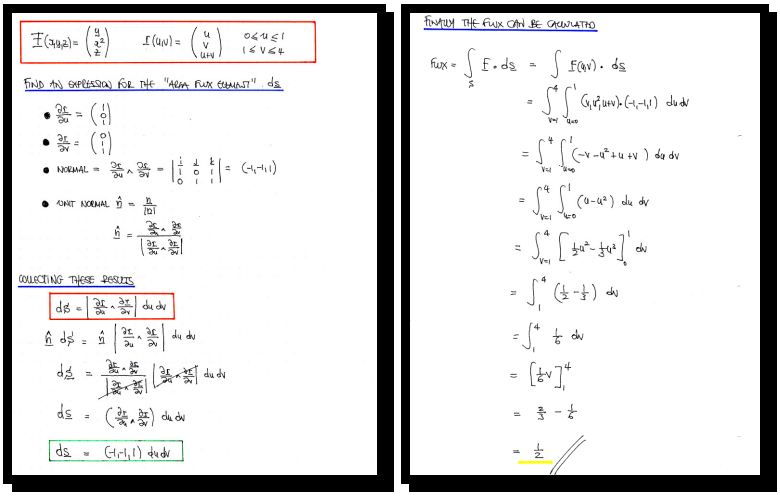
$$\mathbf{F}(x,y,z) \equiv y\,\mathbf{i} + x^2\,\mathbf{j} + z\,\mathbf{k}\,.$$

Find the magnitude of the flux through the surface with parametric equations

$$\mathbf{r}(u,v) = u\,\mathbf{i} + v\,\mathbf{j} + (u+v)\,\mathbf{k}\,, \quad 0 \le u \le 1,\ 1 \le v \le 4\ .$$

All integrations must be carried out in parametric.

$\square$, $\boxed{\frac{1}{2}}$



Question 20

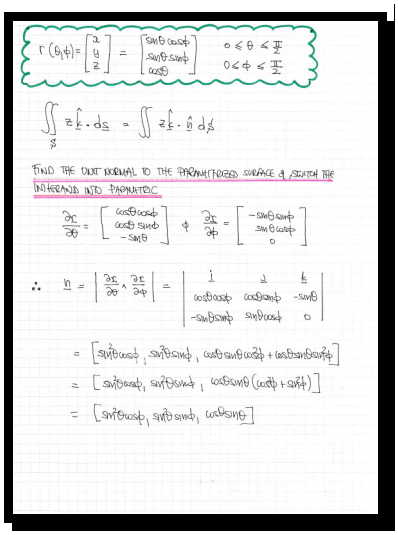
Evaluate the surface integral

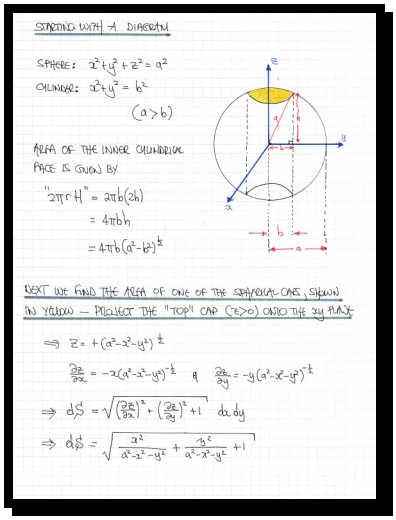
$$\int_S z\,\mathbf{k}\cdot\mathbf{dS}\,,$$

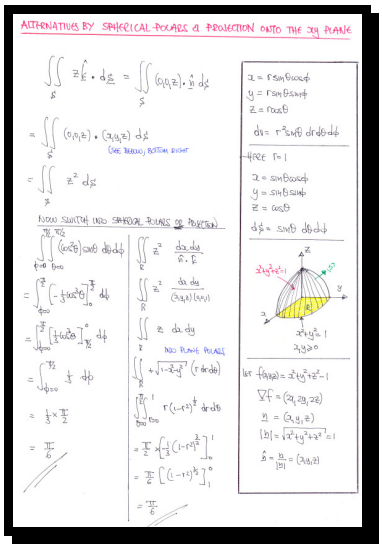
where S is the surface represented parametrically by

$$\mathbf{r}(\theta,\varphi)=\begin{bmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{bmatrix},\ 0\le\theta\le\frac{1}{2}\pi\,,\ 0\le\varphi\le\frac{1}{2}\pi\,.$$

$\square$, $\boxed{\frac{1}{6}\pi}$







Question 21

Evaluate the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS},$$

where S is the surface represented parametrically by

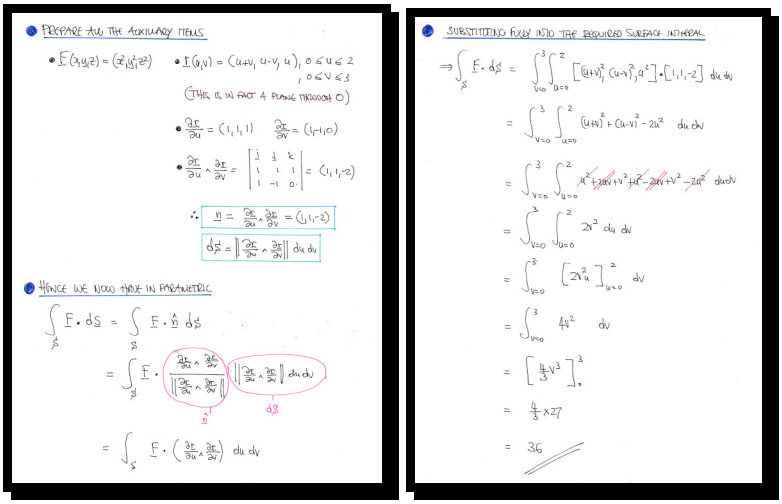
$$\mathbf{r}(u,v)=\begin{bmatrix} u+v \\ u-v \\ u \end{bmatrix}, \quad 0\le u\le 2, \quad 0\le v\le 3,$$

and $\mathbf{F}$ is the vector field

$$x^2\mathbf{i}+y^2\mathbf{j}+z^2\mathbf{k}.$$

All integrations must be carried out in parametric.

, 36



Question 22

$$\mathbf{F}(x,y,z)\equiv x\mathbf{i}+y\mathbf{j}+2z\mathbf{k}\,.$$

Find the magnitude of the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

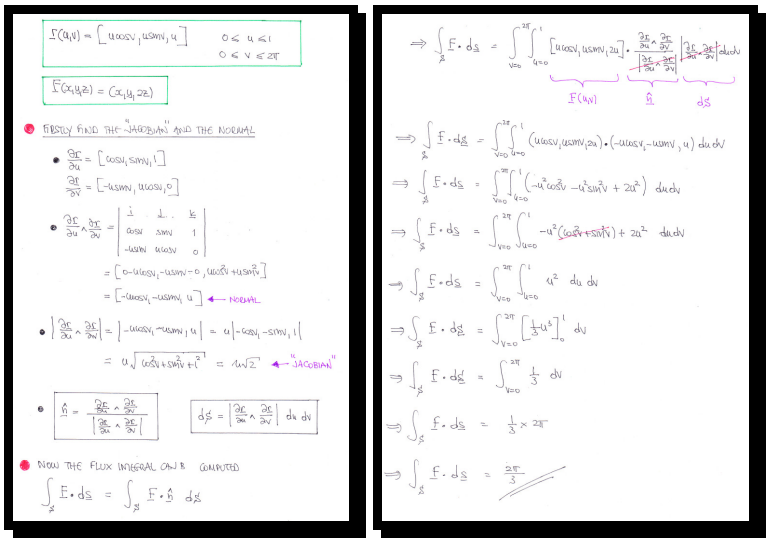
where S is the surface with parametric equations

$$\mathbf{r}(u,v)=(u\cos v)\mathbf{i}+(u\sin v)\mathbf{j}+u\mathbf{k}\,,$$

such that $0\le u\le 1$, $0\le v\le 2\pi$.

All integrations must be carried out in parametric.

$$\boxed{\frac{2}{3}\pi}$$



Question 23

$$\mathbf{F}(x,y,z)\equiv x\mathbf{i}+y\mathbf{j}+z\mathbf{k}\,.$$

Find the magnitude of the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

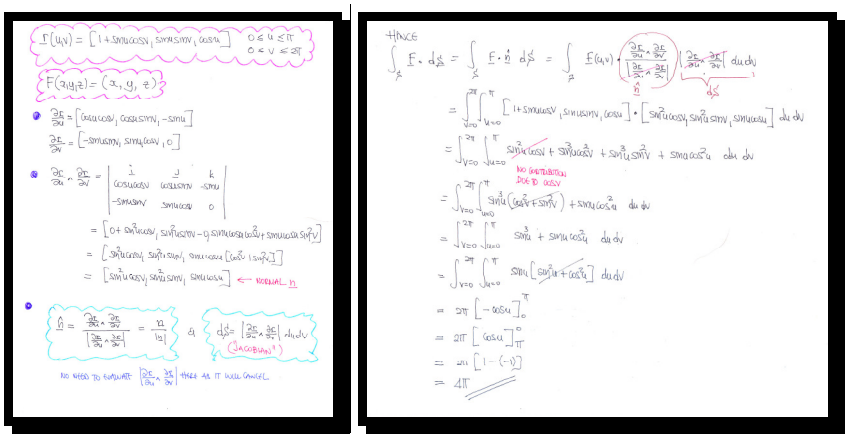
where S is the surface with parametric equations

$$\mathbf{r}(u,v)=(1+\sin u\cos v)\mathbf{i}+(\sin u\sin v)\mathbf{j}+(\cos u)\mathbf{k}\,,$$

such that $0\le u\le\pi$, $0\le v\le 2\pi$.

All integrations must be carried out in parametric.

$\boxed{4\pi}$



Question 24

$$\mathbf{F}(x,y,z)\equiv x\mathbf{i}+y\mathbf{j}+z\mathbf{k}\,.$$

Find the magnitude of the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

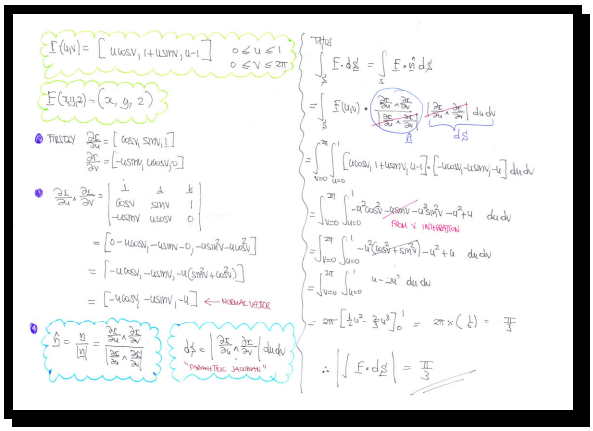
where S is the surface with parametric equations

$$\mathbf{r}(u,v)=(u\cos v)\mathbf{i}+(1+u\sin v)\mathbf{j}+(u-1)\mathbf{k}\,,$$

such that $0\le u\le 1$, $0\le v\le 2\pi$.

All integrations must be carried out in parametric.

$$\boxed{\frac{1}{3}\pi}$$



Question 25

$$\mathbf{F}(x,y,z)\equiv x\mathbf{i}+y\mathbf{j}+2z\mathbf{k}\,.$$

Find the magnitude of the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

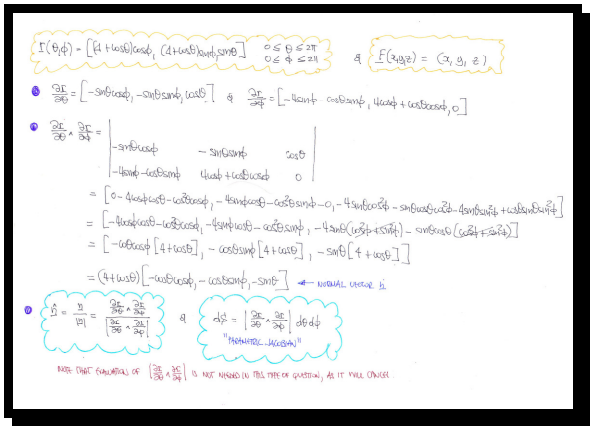
where S is the surface with parametric equations

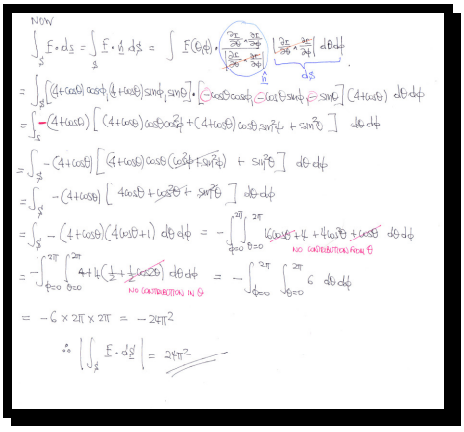
$$\mathbf{r}(\theta,\varphi)=\big[(4+\cos\theta)\cos\varphi\big]\mathbf{i}+\big[(4+\cos\theta)\sin\varphi\big]\mathbf{j}+(\sin\theta)\mathbf{k}\,,$$

such that $0\le\theta\le 2\pi$, $0\le\varphi\le 2\pi$.

All integrations must be carried out in parametric.

$24\pi^2$





Question 26

It is given that the vector field $\mathbf{F}$ satisfies

$$\mathbf{F} = 2y\mathbf{i} - 2x\mathbf{j} + \mathbf{k}\,.$$

Find the magnitude of the surface integral

$$\int_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

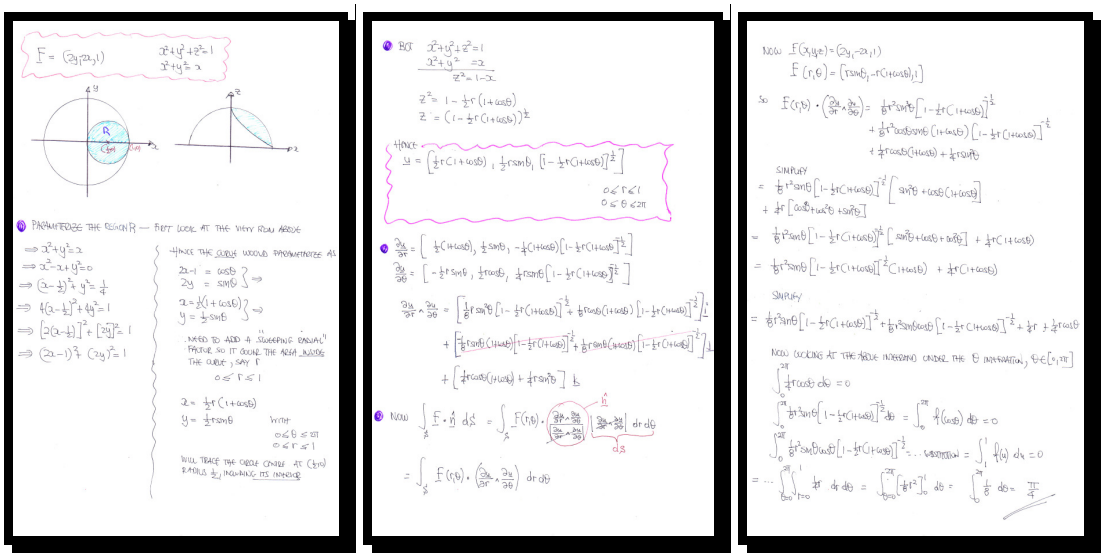
$$x^2 + y^2 + z^2 = 1, \quad z \geq 0\ ,$$

cut off by the cylinder with cartesian equation

$$x^2 + y^2 = x\,.$$

You **must** find a suitable parameterization for S, and carry out the **integration in parametric**, without using any integral theorems.

$$\frac{\pi}{4}$$



TYPE $\oint_S$ F·dS

Question 1

$$\mathbf{F}(x,y,z) \equiv xy\mathbf{i} + y\mathbf{j} + 4\mathbf{k}\,.$$

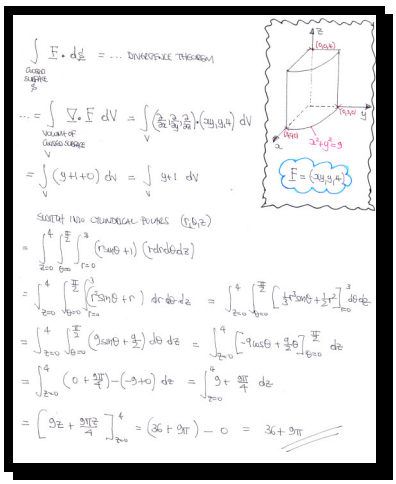
Evaluate the integral

$$\oiint_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the **closed** surface enclosing the finite region V, defined by

$$x^2 + y^2 \le 9\,, \quad x \ge 0\,, \quad y \ge 0\,, \quad 0 \le z \le 4\,.$$

$$\boxed{9\pi + 36}$$



Question 2

$$\mathbf{F}(x,y,z)\equiv\left(x+y^2\right)\mathbf{i}+\left(2y+xz\right)\mathbf{j}+\left(3z+xyz\right)\mathbf{k}\,.$$

Evaluate the integral

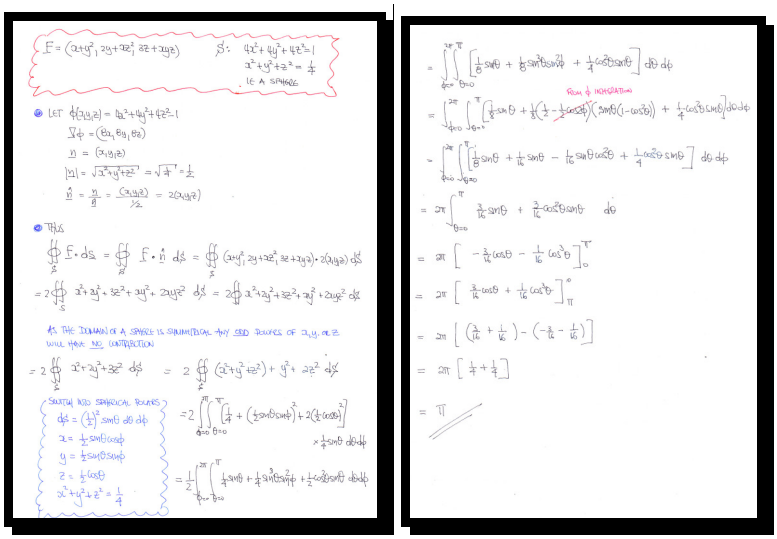
$$\iint_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

$$4x^2+4y^2+4z^2=1\,.$$

You may not use the Divergence Theorem in this question.

$\boxed{\pi}$



Question 3

It is given that

$$\mathbf{F}(x,y,z)\equiv \mathbf{k}\wedge\mathbf{r}\,,\quad \text{where}\quad \mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}\,.$$

Show by direct integration that

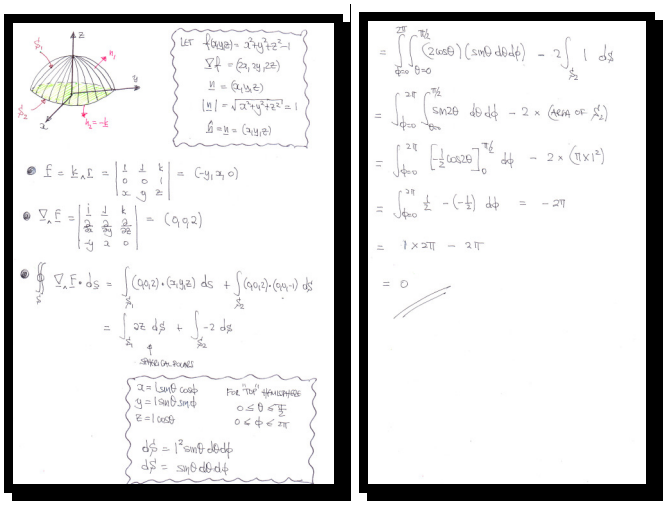
$$\oiint_S \nabla\wedge\mathbf{F}\cdot\mathbf{dS}=0\,,$$

where S is the **closed** surface enclosing the finite region V, defined by

$$x^2+y^2+z^2\leq 1\,,\quad z\geq 0\,,\quad \text{and}\quad x^2+y^2\leq 1\,.$$

You may not use any Integral Theorems in this question.

proof



Question 4

$$\mathbf{F}(x,y,z)\equiv(4yz)\mathbf{i}+\left(2y^2\right)\mathbf{j}+\left(5xyz+6z^2+3z\right)\mathbf{k}\,.$$

Evaluate the integral

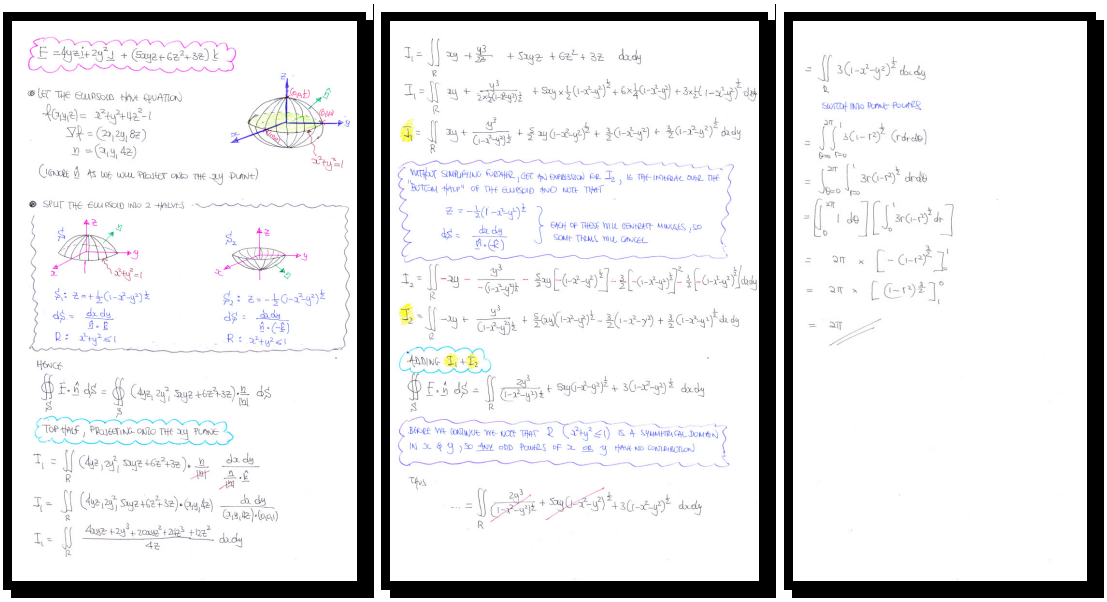
$$\iint_S \mathbf{F}\cdot\mathbf{dS}\,,$$

where S is the surface with Cartesian equation

$$x^2+y^2+4z^2=1\,.$$

You may not use the Divergence Theorem in this question.

$$\boxed{2\pi}$$



Question 5

The surface Ω is the sphere with Cartesian equation

$$(x-1)^2+(y-1)^2+(z-1)^2=1$$

Evaluate the surface integral

$$\iint_{\Omega}\left[(x+y)\mathbf{i}+\left(x^2+xy\right)\mathbf{j}+z^2\mathbf{k}\right]\cdot\mathbf{dS},$$

where $\mathbf{dS}$ is a unit surface element on Ω.

You may not use the Divergence Theorem in this question.

$$\frac{16}{3}\pi$$

