3-DIMENSIONAL SKETCHING

SURFACES and CURVES

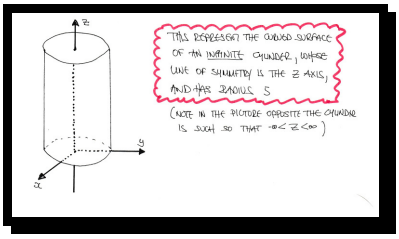
Question 1

A surface S is given by the Cartesian equation

$$x^2 + y^2 = 25 .$$

Draw a sketch of S, and describe it geometrically.

[ ], cylinder



Question 2

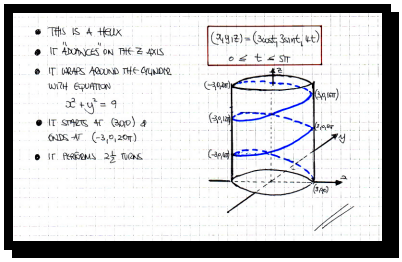
A curve C is defined parametrically

$$(x, y, z) = (3\cos t, 3\sin t, 4t), \quad 0 \le t \le 5\pi .$$

where t is a parameter.

Sketch the graph of C.

[ ], graph



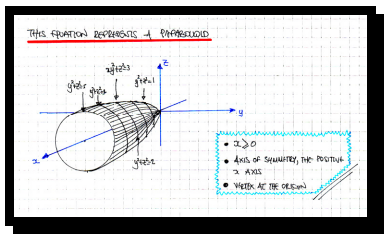
Question 3

The surface S has Cartesian equation

$$x = y^2 + z^2 .$$

Sketch the graph of S.

[ ], [graph]



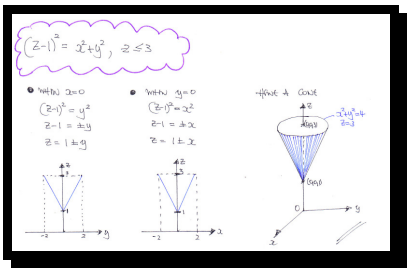
Question 4

The surface S has Cartesian equation

$$(z-1)^2 = x^2 + y^2 , \ 1 \le z \le 3 .$$

Sketch the graph of S.

[graph]



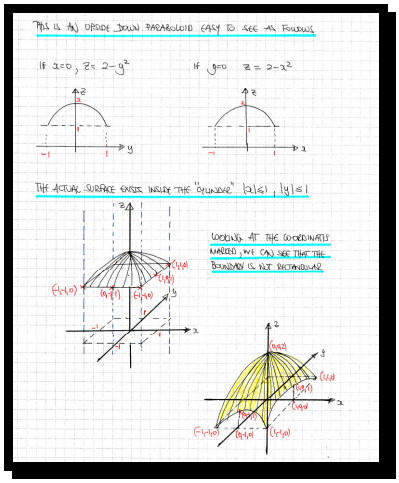
Question 5

The surface S has Cartesian equation

$$z = 2 - x^2 - y^2, \ |x| \leq 1, \ |y| \leq 1.$$

Sketch the graph of S.

[ ], graph



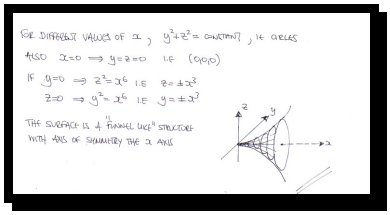
Question 6

A surface S has Cartesian equation

$$y^2 + z^2 = x^6, \ \ 0 \leq x \leq \sqrt[4]{\frac{5}{3}}.$$

Sketch the graph of S.

graph



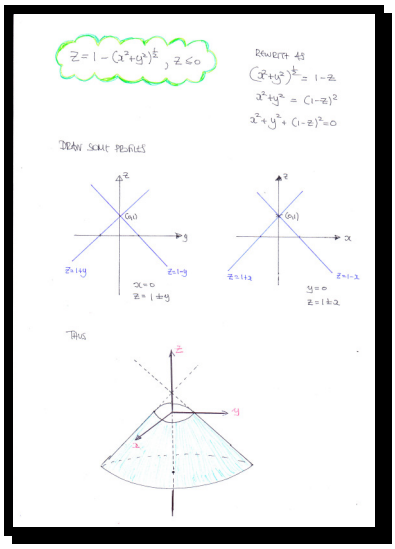
Question 7

A surface S is has Cartesian equation

$$z = 1 - \sqrt{x^2 + y^2}\ , \quad z \leq 0 .$$

Sketch the graph of S.

graph



Question 8

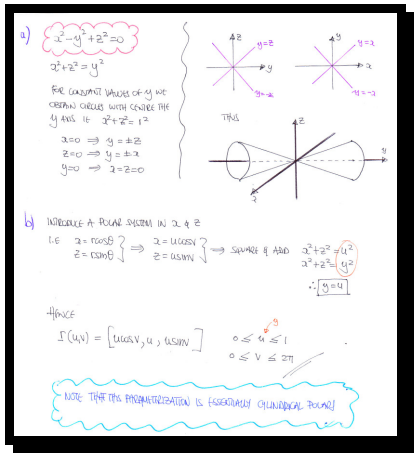
A surface S has Cartesian equation

$$x^2 - y^2 + z^2 = 0 .$$

a) Sketch the graph of S.

b) Find a parameterization for the equation of S, for $0 \le y \le 1$, in terms of the parameters u and v.

$$\boxed{\mathbf{r}(u,v) = \langle u\cos v, u, u\sin v \rangle \quad 0 \le u \le 1, \ 0 \le v \le 2\pi}$$



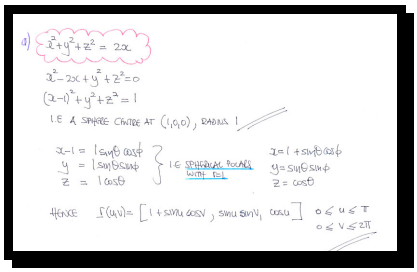
Question 9

A surface S has Cartesian equation

$$x^2 + y^2 + z^2 = 2x\,.$$

Describe fully the graph of S, and hence find a parameterization for its equation in terms of the parameters u and v.

$$\boxed{\mathbf{r}(u,v) = \langle 1+\sin u\cos v, \sin u\sin v, \cos u\rangle, \quad 0 \le u \le \pi,\ 0 \le v \le 2\pi}$$



Question 10

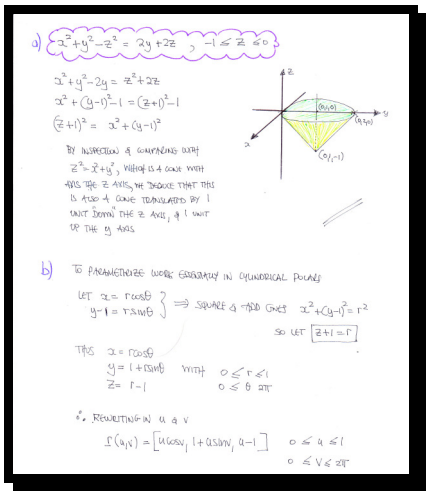
A surface S has Cartesian equation

$$x^2+y^2-z^2=2y+2z\,,\quad -1\le z\le 0\,.$$

a) Sketch the graph of S.

b) Find a parameterization for the equation of S, in terms of the parameters u and v.

$$\boxed{\mathbf{r}(u,v)=\langle u\cos v,1+u\sin v,u-1\rangle,\ \ 0\le u\le 1,\ 0\le v\le 2\pi}$$



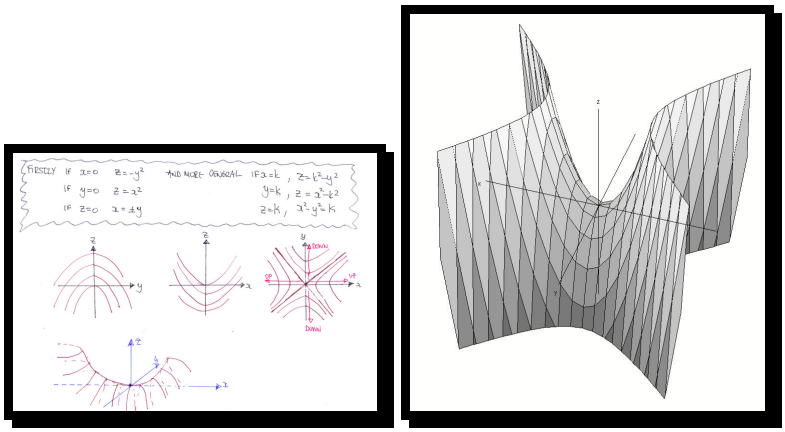
Question 11

A surface S has Cartesian equation

$$z = x^2 - y^2 .$$

Sketch contour profiles of S, parallel to the y-z plane, parallel to the x-z plane, and parallel to the x-y plane.

graph



Question 12

A surface S is given parametrically by

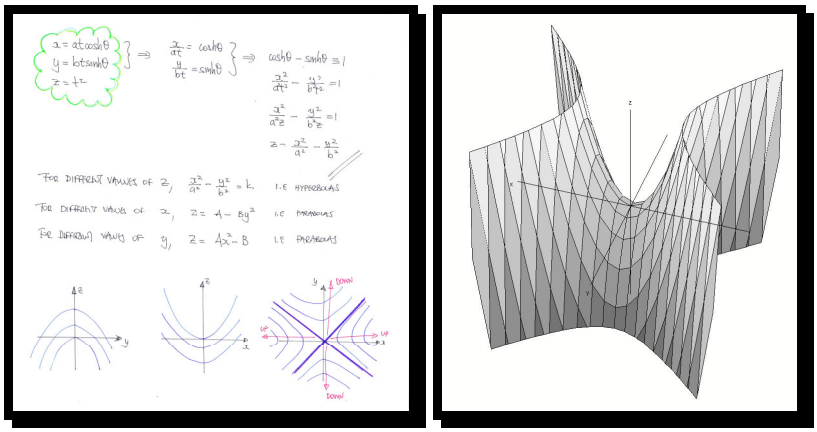
$$x = at\cosh\theta\,,\ x = bt\sinh\theta\,,\ z = t^2\,,$$

where t and θ are real parameters, and a and b are non zero constants .

a) Find a Cartesian equation for S .

b) Sketch profiles of S parallel to the y-z plane, parallel to the x-z plane, and parallel to the x-y plane,

$$\boxed{z = \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$



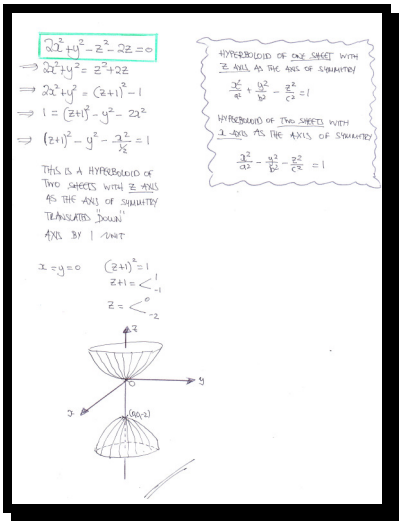
Question 13

A surface S is has Cartesian equation

$$2x^2 + y^2 - z^2 - 2z = 0 .$$

Sketch the graph of S.

graph



Question 14

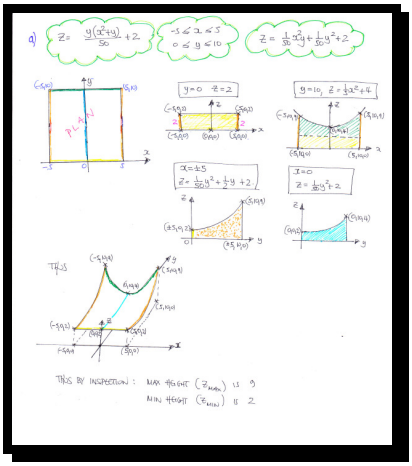
A building whose plan measures 10 m long by 10 m wide has vertical walls and a suspended fabric roof. The height, z m, of the roof above the ground is modelled in three dimensional Cartesian space by the equation

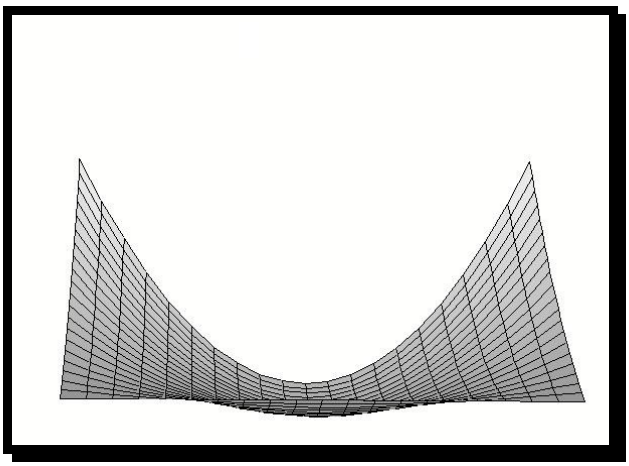
$$z = \frac{y\left(x^2 + y\right)}{50} + 2, \quad -5 \le x \le 5, \quad 0 \le y \le 10.$$

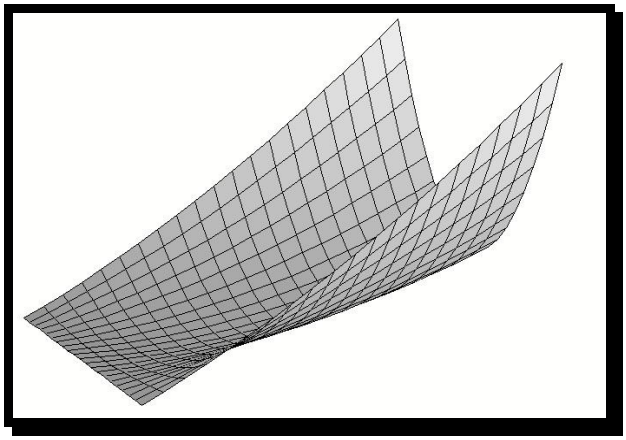
Sketch the graph of the surface which models the roof of the building.

Give a brief description of its shape including its key features with relevant coordinates such as the maximum height and minimum height of the roof.

graph







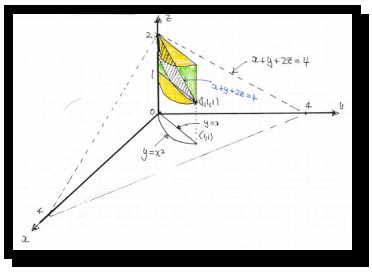
Question 15

A solid is bounded by the surfaces

$$y = x^2, \quad y = x, \quad z = 1 \quad \text{and} \quad z = 2.$$

Sketch in the same set of axes the solid and the plane with equation $x + y + 2z = 4$.

graph



Question 16

The curve C is the intersection of the surfaces with respective Cartesian equations

$$x^2+y^2+z^2=1, \quad z\geqslant 0 \qquad \text{and} \qquad x^2+y^2=x, \quad z\geqslant 0\,.$$

Find a suitable clockwise parameterisation for C, in the form

$$\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j}+t\mathbf{k}\,,$$

fully defining the functions f and g, and the range of the parameter t in a suitably labelled graph or diagram.

graph

