PRODUCT OPERATOR

Question 1

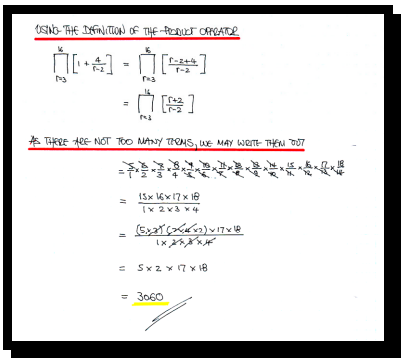
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

Find the value of

$$\prod_{r=3}^{16}\left[1+\frac{4}{r-2}\right].$$

$\square$, $\boxed{3060}$



Question 2

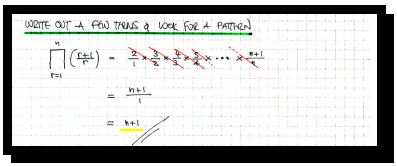
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

Simplify, showing a clear method

$$\prod_{r=1}^{n}\left[\frac{r+1}{r}\right].$$

$\boxed{}$, $\boxed{n+1}$



Question 3

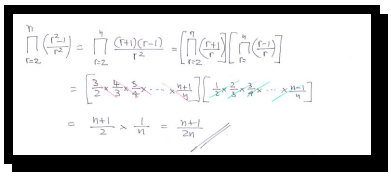
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Simplify, showing a clear method

$$\prod_{r=2}^{n}\left[\frac{r^2-1}{r^2}\right].$$

$$\boxed{\frac{n+1}{2n}}$$



Question 4

The product operator $\prod$, is defined as

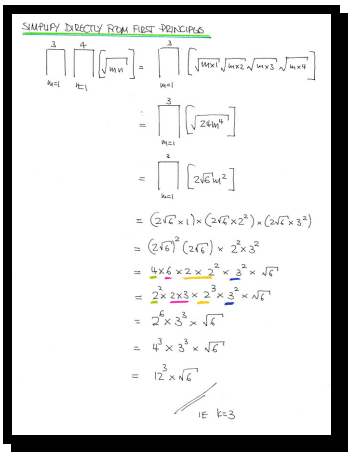
$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Use a clear method to show that

$$\prod_{m=1}^{3}\prod_{n=1}^{4}\left[\sqrt{mn}\right] = k^3\sqrt{6}\,,$$

where k is a positive integer to be found.

$\boxed{}$, $\boxed{}$, $\boxed{k=12}$



Question 5

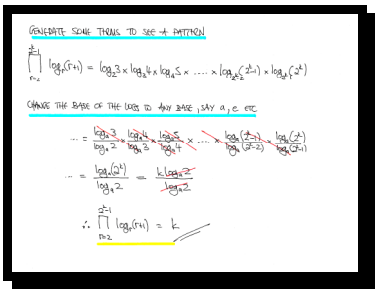
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Given that $k\in\mathbb{N}$, use a detailed method to find the value of

$$\prod_{r=2}^{2^k-1}\left[\log_r\left(r+1\right)\right].$$

$\square$, $\boxed{k}$



Question 6

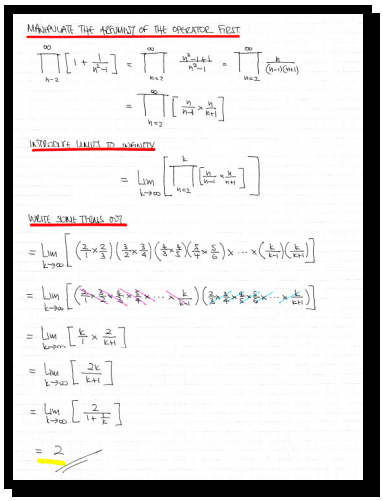
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

Evaluate, showing a clear method

$$\prod_{n=2}^{\infty}\left[1+\frac{1}{n^2-1}\right].$$

$\boxed{}$, $\boxed{2}$



Question 7

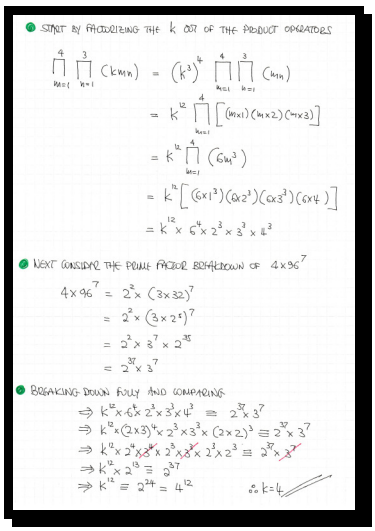
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Use a clear method to determine the value of k given that

$$\prod_{m=1}^{4}\prod_{n=1}^{3}[kmn] = 4 \times 96^7 \, .$$

$\boxed{}$, $\boxed{k=4}$



Question 8

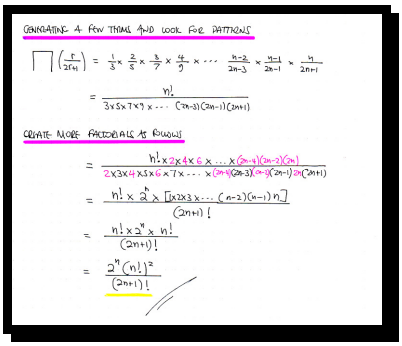
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Simplify, showing a clear method

$$\prod_{r=1}^{n}\left[\frac{r}{2r+1}\right].$$

$\square$, $\boxed{\dfrac{2^n (n!)^2}{(2n+1)!}}$



Question 9

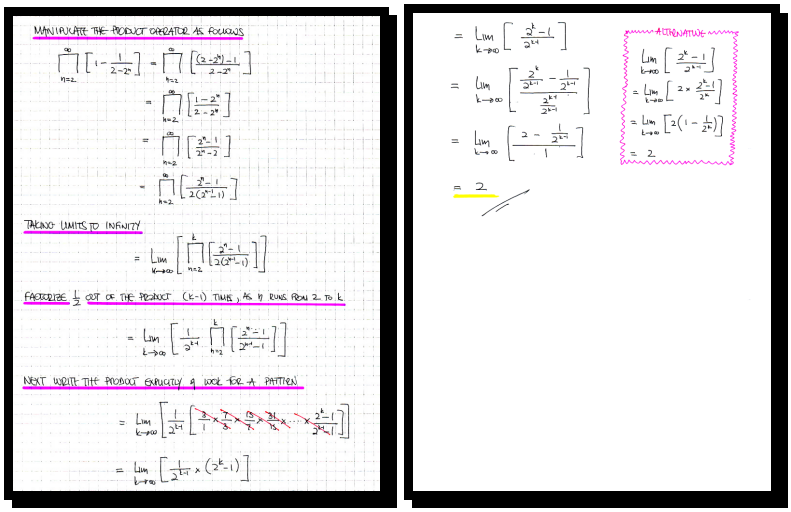
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Evaluate, showing a clear method

$$\prod_{n=2}^{\infty}\left[1-\frac{1}{2-2^n}\right].$$

$\square$, $\square$, $\boxed{2}$



Question 10

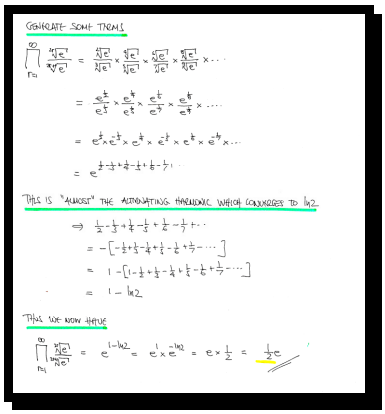
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Given that e is Euler's number, use a detailed method to find the exact value of

$$\prod_{r=1}^{\infty}\left[\frac{\sqrt[(2r)]{e}}{\sqrt[(2r+1)]{e}}\right].$$

$\square$, $\boxed{\frac{1}{2}e}$



Question 11

The product operator $\prod$, is defined as

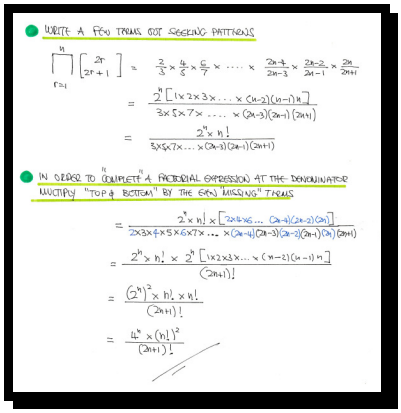
$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Simplify, showing a clear method, the following expression.

$$\prod_{r=1}^{n}\left[\frac{2r}{2r+1}\right].$$

Give the final answer as a single simplified fraction.

$\square$, $\boxed{\dfrac{4^n (n!)^2}{(2n+1)!}}$



Question 12

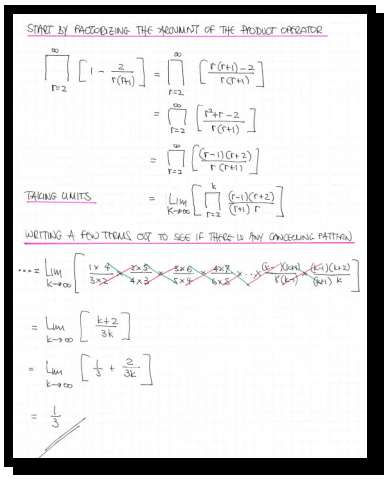
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Evaluate, showing a clear method

$$\prod_{r=2}^{\infty}\left[1-\frac{2}{r(r+1)}\right].$$

$\square$, $\boxed{\frac{1}{3}}$



Question 13

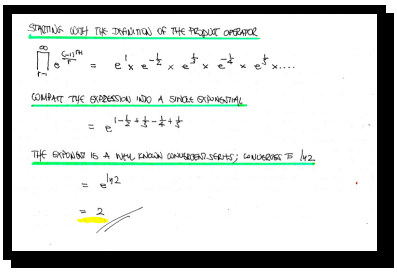
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Find the value of

$$\prod_{r=1}^{\infty}\left[\mathrm{e}^{\frac{(-1)^{r+1}}{r}}\right].$$

V, [ ], 2



Question 14

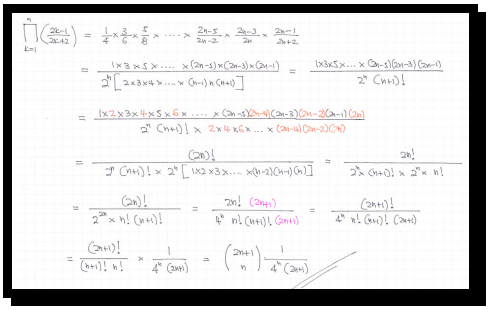
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

By showing a detailed method prove that

$$\prod_{k=1}^{n}\left[\frac{2k-1}{2k+2}\right] = \binom{2n+1}{n}\frac{1}{4^n(2n+1)} .$$

, proof



Question 15

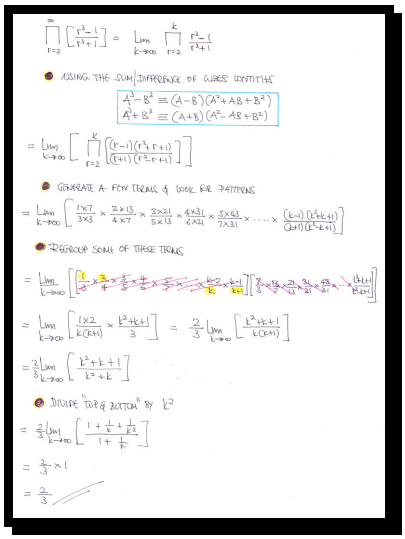
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Evaluate, showing a clear method

$$\prod_{r=2}^{\infty}\left[\frac{r^3-1}{r^3+1}\right].$$

$\square$, $\boxed{\frac{2}{3}}$



Question 16

The product operator $\prod$, is defined as

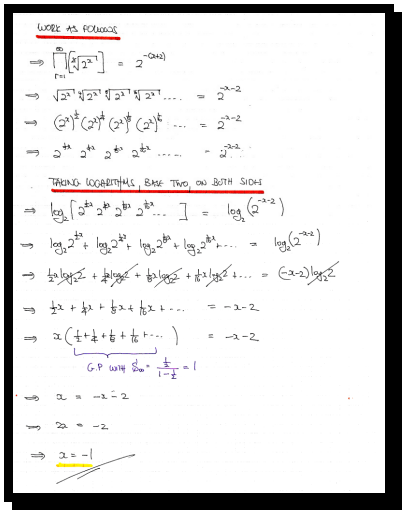
$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

Solve the equation

$$\prod_{r=1}^{\infty}\left[\sqrt[2r]{2^x}\right] = 2^{-(x+2)}.$$

You may assume that the left hand side of the equation converges.

$\boxed{\quad}$, $\boxed{x=-1}$



Question 17

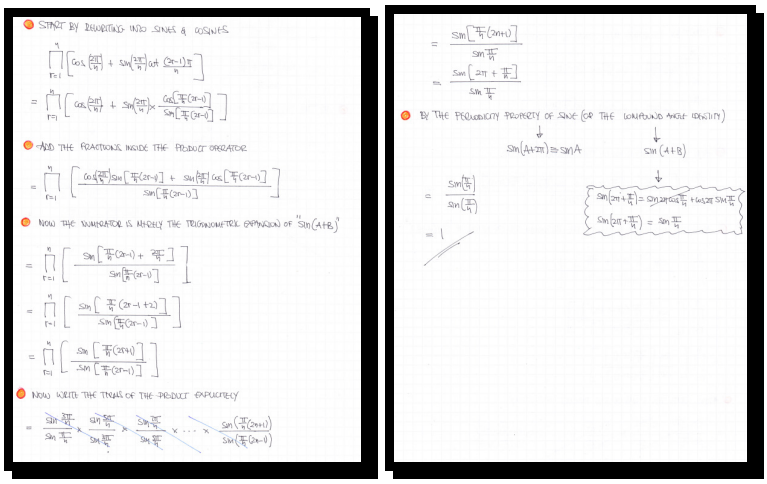
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

By showing a detailed method prove that if n is even

$$\prod_{r=1}^{n}\left[\cos\frac{2\pi}{n}+\cos\frac{2\pi}{n}\cot\frac{(2r-1)\pi}{n}\right]=1 .$$

, proof



Question 18

The product operator $\prod$, is defined as

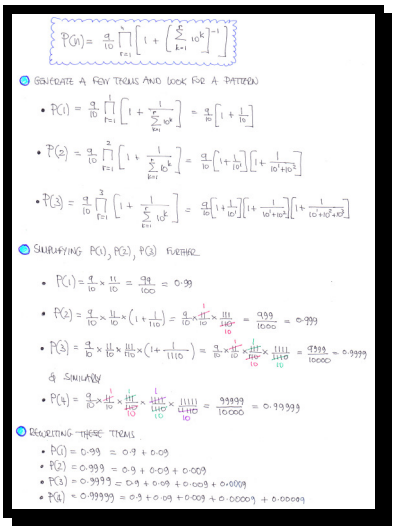
$$\prod_{i=1}^{k}[u_i] \;=\; u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

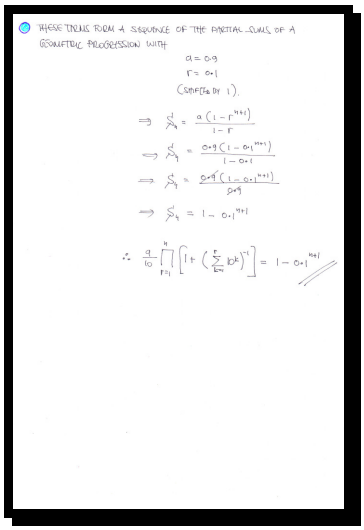
A sequence of numbers, $P(1)$, $P(2)$, $P(3)$ ... $P(n)$ is defined by the equation

$$P(n)=\frac{9}{10}\prod_{r=1}^{n}\left[1+\left[\sum_{k=1}^{r}10^k\right]^{-1}\right].$$

Express $P(n)$ in a simplified form not involving a sigma or product operators.

$\square$, $\boxed{P(n)=1-0.1^{n+1}}$





Question 19

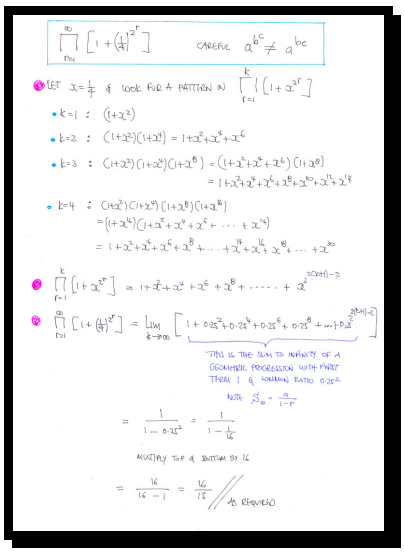
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

Show by a detailed method that

$$\prod_{r=1}^{\infty}\left[1+\left(\frac{1}{4}\right)^{2^r}\right]=\frac{16}{15}\,.$$

, proof



Question 20

The product operator $\prod$, is defined as

$$\prod_{r=1}^{k}[u_r] \;=\; u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

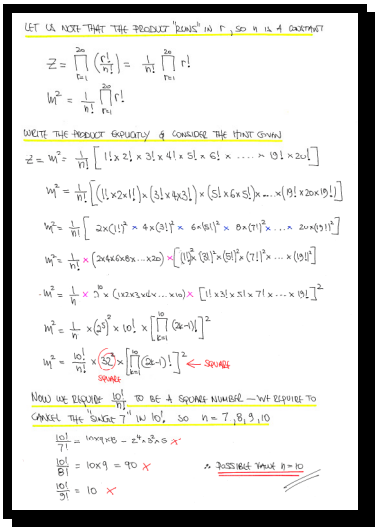
The integer Z is a **square number** and defined as

$$Z=\prod_{r=1}^{20}\left(\frac{r!}{n!}\right),\ \{n\in\mathbb{N}:1\leq n\leq 20\}.$$

By considering the terms inside the product operator in pairs, or otherwise, determine a possible value of n.

You must show a detailed method in this question.

$\boxed{}$, $\boxed{n=10}$



Question 21

$$I=\int_0^1 \left[\prod_{r=1}^{10}(x+r)\right]\left[\sum_{r=1}^{10}\left(\frac{1}{x+r}\right)\right]dx\,.$$

Show by a detailed method that

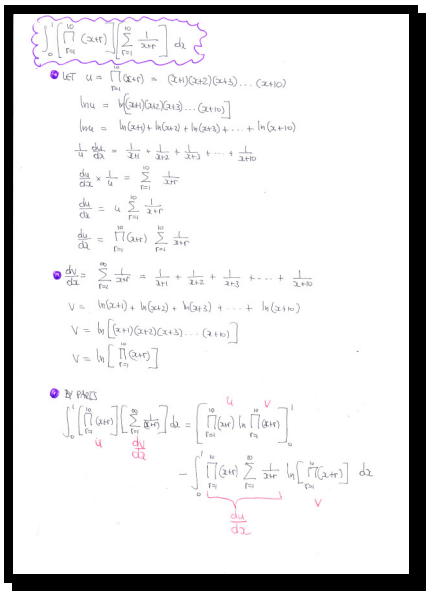
$$I=a\times b!,$$

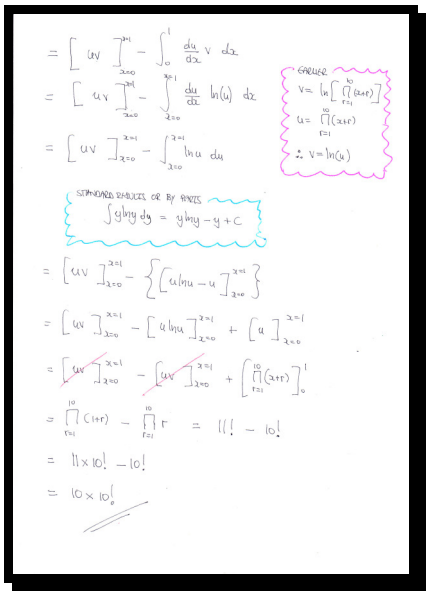
where a and b are positive integers to be found.

The product operator $\prod$, ***is defined as***

$$\prod_{i=1}^{k}[u_i] \;=\; u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

$\boxed{}$, proof





Question 22

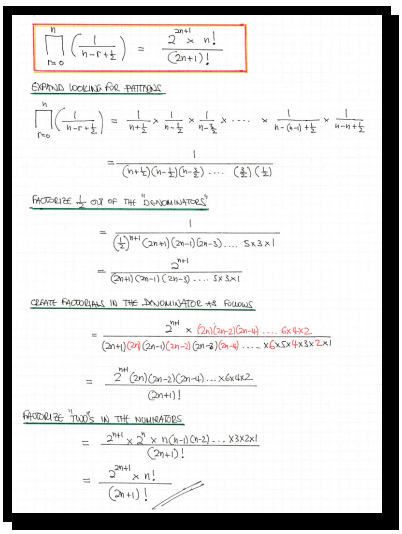
The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Show in detail that

$$\prod_{r=1}^{n}\left(\frac{1}{n-r+\frac{1}{2}}\right) = \frac{2^{2n+1}\times n!}{(2n+1)!}.$$

, proof



Question 23

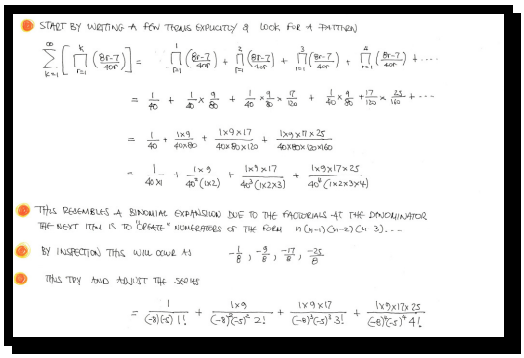
The product operator $\prod$, is defined as

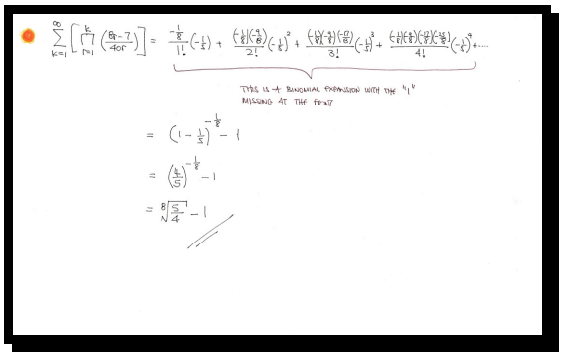
$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k \, .$$

Find the sum to infinity of the following expression

$$\sum_{k=1}^{\infty}\left[\prod_{r=1}^{k}\left(\frac{8r-7}{40r}\right)\right].$$

$\square$, $\sqrt[8]{\frac{5}{4}}-1$





Question 24

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}\left[u_i\right] = u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

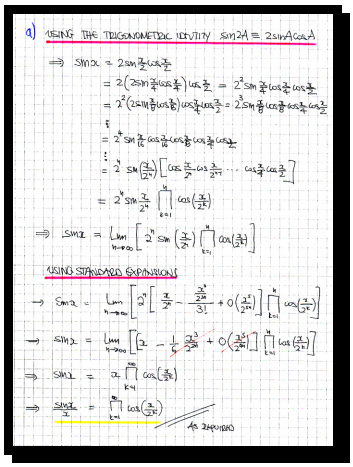
a) By considering the sine double angle identity show that

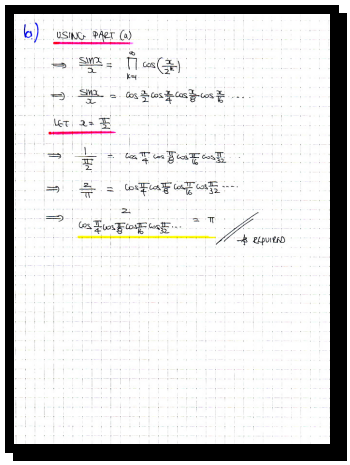
$$\frac{\sin x}{x} = \prod_{k=1}^{\infty}\left[\cos\left(\frac{x}{2^k}\right)\right].$$

b) Deduce that

$$\frac{2}{\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{32}\right)....} = \pi\,.$$

[ ], [proof]





Question 25

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

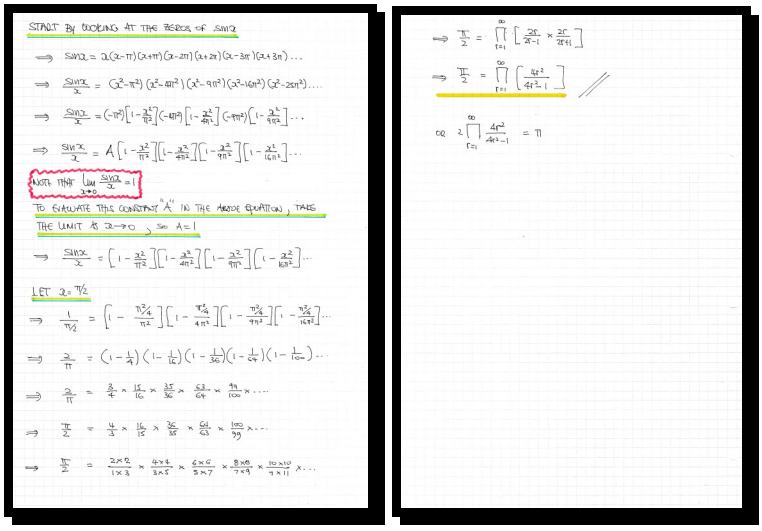
By writing $\sin x$ as an infinite factorized polynomial, where each of the factors represents a zero of $\sin x$, derive Wallis's formula

$$\frac{\pi}{2} = \prod_{r=1}^{\infty} \frac{4r^2}{4r^2 - 1}.$$

You may assume without proof that

$$\lim_{x \to 0}\left[\frac{\sin x}{x}\right] = 1.$$

[ ], [proof]



Question 26

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] \;=\; u_1\times u_2\times u_3\times u_4\times ...\times u_{k-1}\times u_k\,.$$

The function $f(n,k)$ is defined as

$$f(n,k)=\left[\prod_{m=1}^{n}\left[\frac{2n-2m+1}{n!}\right]\right]\left[\prod_{r=1}^{k}\left[\frac{n-r+1}{2n-2r+1}\right]\right]\left[\prod_{l=1}^{k}\left[\frac{1}{(2l)}\right]\right],\ n\geq 2k\,.$$

Show by a detailed method that

$$f(n,k)=\frac{(2n-2k)!}{2^n\ k!(n-k)!\ (n-2k)!}$$

, proof

