PARTIAL DIFFERENTIAL EQUATIONS

TRANSFORMATIONS

Question 1

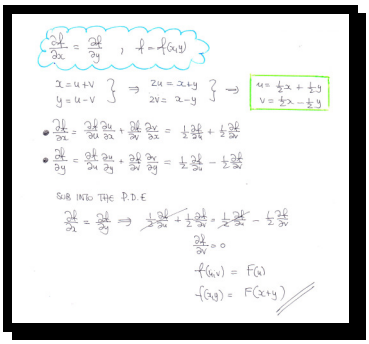
The smooth function $f = f(x, y)$ satisfies

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}.$$

Find the general solution of the above partial differential equation by using the transformation equations

$$x = u + v \quad \text{and} \quad y = u - v.$$

$$\boxed{f(x, y) = F(x + y)}$$



Question 2

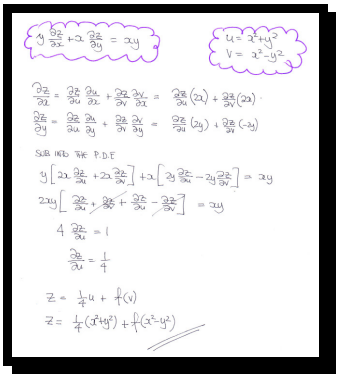
The smooth function $z = z(x, y)$ satisfies

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = xy\,.$$

Find the general solution of the above partial differential equation by using the transformation equations

$$u = x^2 + y^2 \quad \text{and} \quad v = x^2 - y^2\,.$$

$$\boxed{z(x, y) = \frac{1}{4}\left(x^2 + y^2\right) + f\left(x^2 - y^2\right)}$$



Question 3

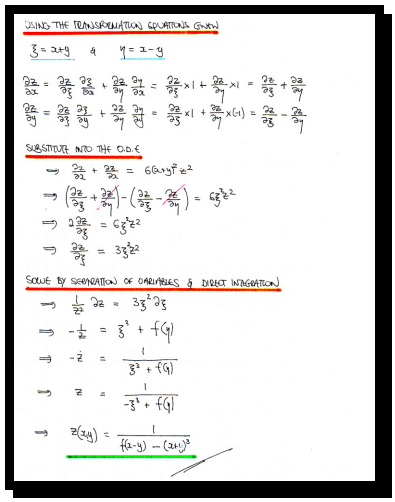
The smooth function $z = z(x, y)$ satisfies

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 6(x+y)^2 z^2 .$$

Find the general solution of the above partial differential equation by using the transformation equations

$$\xi = x + y \quad \text{and} \quad \eta = x - y .$$

$\square$, $\boxed{z(x, y) = -\dfrac{1}{(x+y)^3 - f(x-y)}}$



Question 4

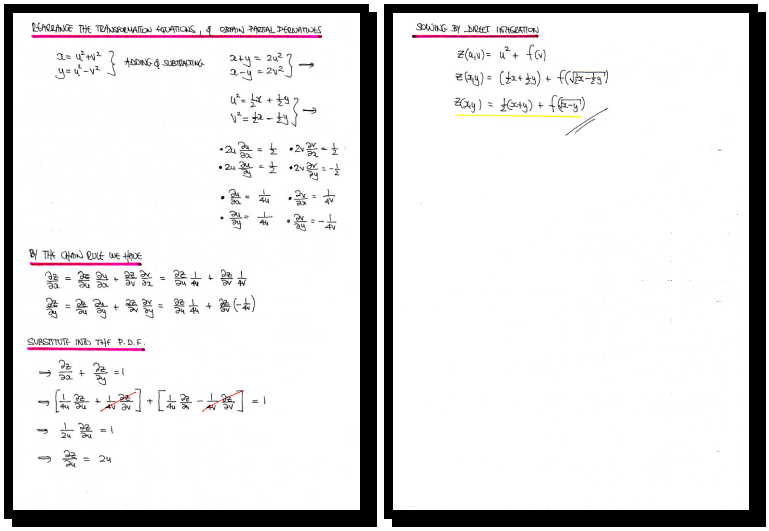
The smooth function $z = z(x, y)$ satisfies

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1 .$$

Find the general solution of the above partial differential equation by using the transformation equations

$$x = u^2 + v^2 \quad \text{and} \quad y = u^2 - v^2 .$$

, $\boxed{z(x, y) = \frac{1}{2}(x + y) + f\left(\sqrt{x - y}\right)}$



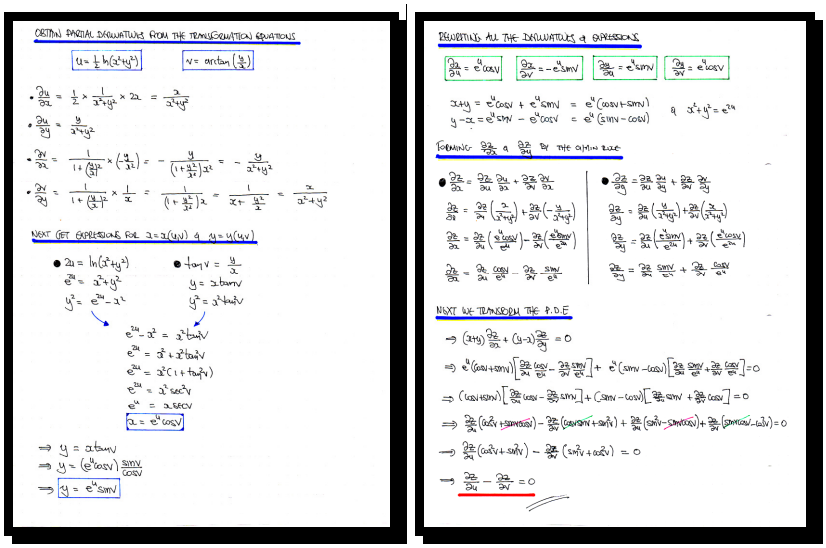
Question 5

$$(x+y)\frac{\partial z}{\partial x}+(y-x)\frac{\partial z}{\partial y}=0\,.$$

Transform the above partial differential equation using the equations

$$u=\frac{1}{2}\ln\left(x^2+y^2\right)\quad\text{and}\quad v=\arctan\left(\frac{y}{x}\right).$$

$$\square\,,\qquad \boxed{\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=0}$$



Question 6

The function z depends on x and y so that

$$z = f(u,v), \quad u = x - 2\sqrt{y} \quad \text{and} \quad v = x + 2\sqrt{y}.$$

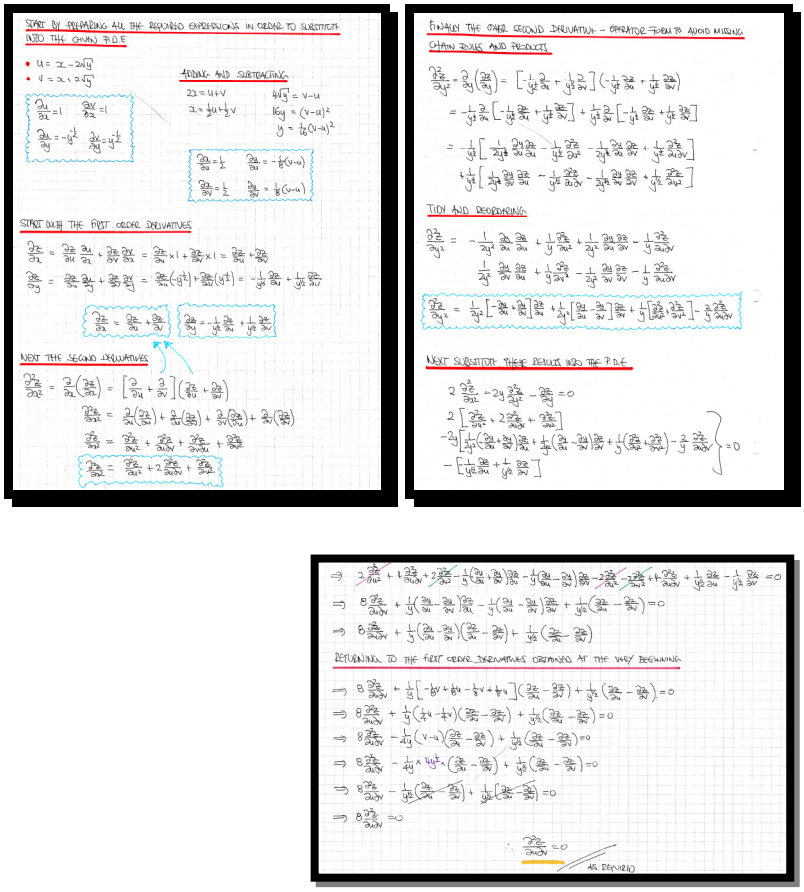
Show that the partial differential equation

$$2\frac{\partial^2 z}{\partial x^2} - 2y\frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial y} = 0,$$

can be simplified to

$$\frac{\partial^2 z}{\partial u \partial v} = 0.$$

, proof



Question 7

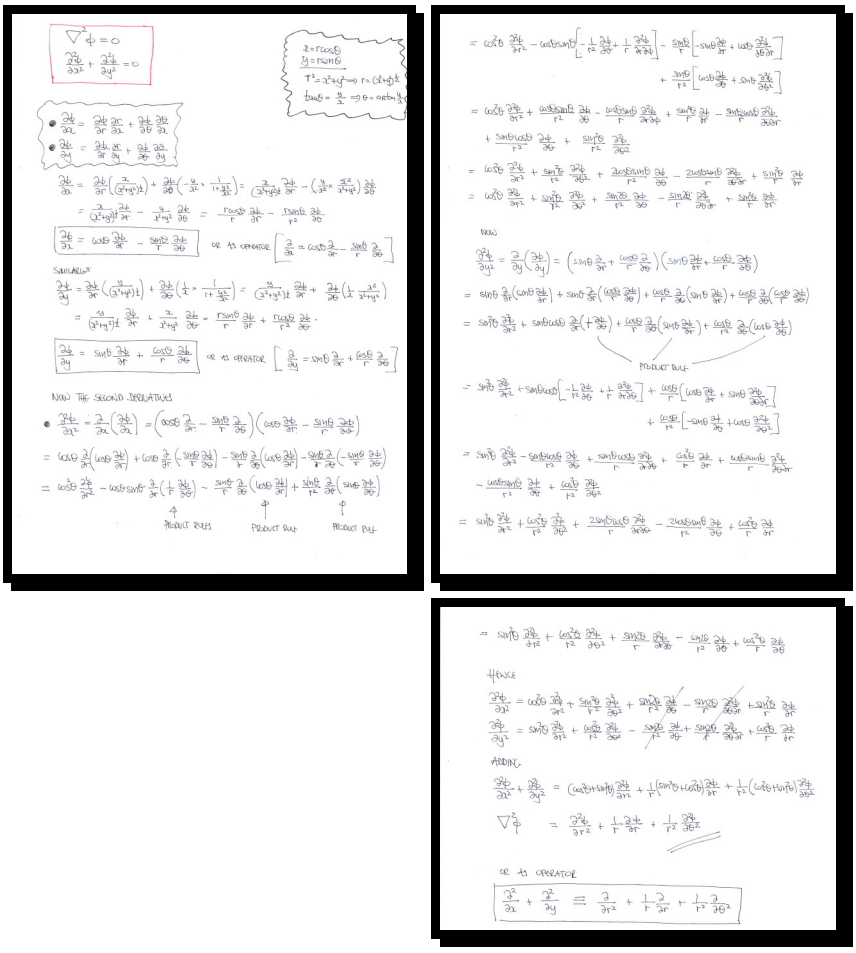
$$\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}=0\,.$$

The above partial differential equation is Laplace's equation in a two dimensional Cartesian system of coordinates.

Show clearly that Laplace's equation in the standard two dimensional Polar system of coordinates is given by

$$\frac{\partial^2 \varphi}{\partial r^2}+\frac{1}{r}\frac{\partial \varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}=0\,.$$

proof



FIRST ORDER P.D.E.s

$$\frac{\partial z}{\partial x} = F\left(x, y, z\right) \quad \text{or} \quad \frac{\partial z}{\partial y} = G\left(x, y, z\right) \quad \text{for} \quad z = z\left(x, y\right)$$

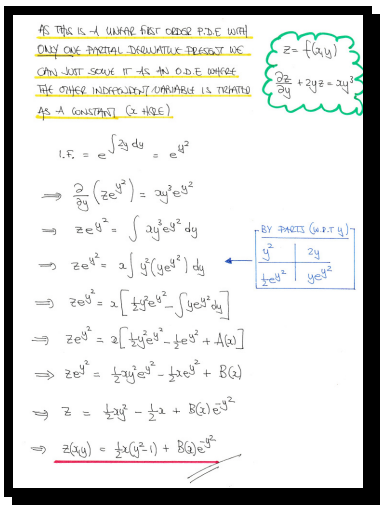
Question 1

It is given that $z = F(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial y} + 2yz = xy^3 .$$

Determine a general solution of the above partial differential equation.

$\square$, $\boxed{z = \frac{1}{2}x\left(y^2 - 1\right) + \mathrm{e}^{-y^2} f(x)}$



FIRST ORDER P.D.E.s

(by linear transformations)

$$A\frac{\partial z}{\partial x}+B\frac{\partial z}{\partial y}+Cz=G\left(x,y\right),\qquad z=z\left(x,y\right)$$

Question 1

It is given that $\psi = \psi(x, y)$ satisfies the partial differential equation

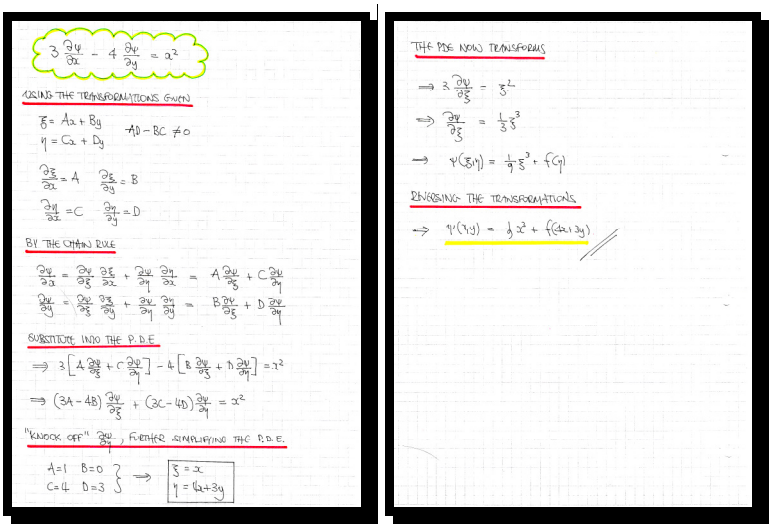
$$3\frac{\partial \psi}{\partial x} - 4\frac{\partial \psi}{\partial y} = x^2 .$$

Use the transformation equations

$$\xi = Ax + By \quad \text{and} \quad \eta = Cx + Dy , \quad AD - BC \neq 0$$

with suitable values of A, B, C and D, in order to determine a general solution of the above partial differential equation.

$\square$, $\boxed{\psi(x, y) = \frac{1}{9}x^3 + f(4x + 3y)}$



Question 2

It is given that $z = z(x, y)$ satisfies the partial differential equation

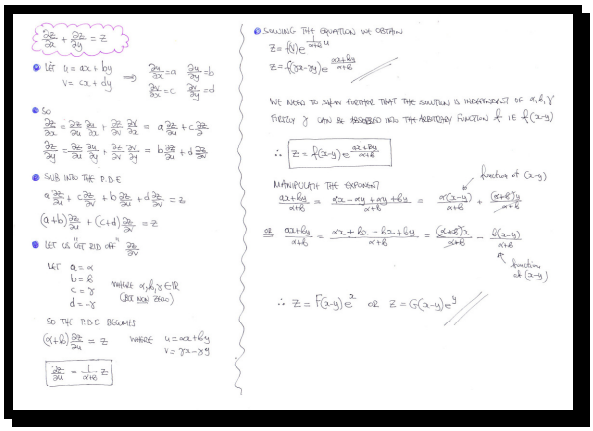
$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z \, .$$

Use the transformation equations

$$u = ax + by \quad \text{and} \quad v = cx + dy \, , \quad ad - bc \neq 0$$

in order to determine a general solution of the above partial differential equation, showing further that this general solution is independent of the choice of values of the constants of a, b, c and d.

$$\boxed{z = \mathrm{e}^{x} F(x - y) \quad \text{or} \quad z = \mathrm{e}^{y} G(x - y)}$$



Question 3

It is given that $z = z(x, y)$ satisfies the partial differential equation

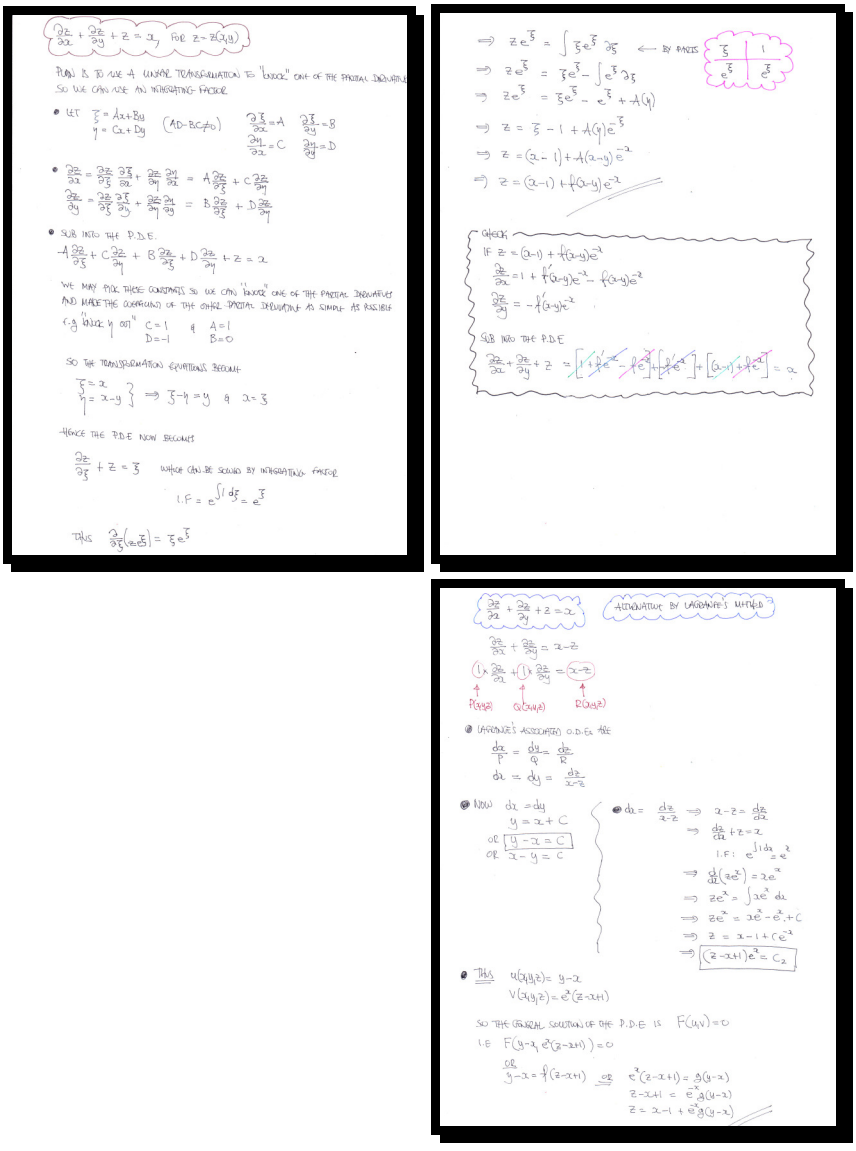
$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + z = x \, .$$

Use the transformation equations

$$\xi = Ax + By \quad \text{and} \quad \eta = Cx + Dy \, , \quad AD - BC \neq 0$$

with suitable values of A, B, C and D, in order to determine a general solution of the above partial differential equation.

$$\boxed{z = x - 1 + \mathrm{e}^{-x} f(x - y)}$$



Question 4

It is given that $\varphi = \varphi(x, y)$ satisfies the partial differential equation

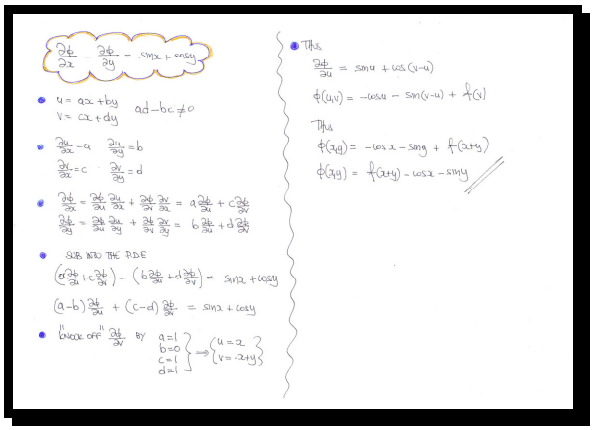
$$\frac{\partial \varphi}{\partial x} - \frac{\partial \varphi}{\partial y} = \sin x + \cos y \, .$$

Use the transformation equations

$$u = ax + by \quad \text{and} \quad v = cx + dy \, , \quad ad - bc \neq 0$$

with suitable values of a, b, c and d, in order to determine a general solution of the above partial differential equation.

$$\boxed{\varphi(x, y) = F(x + y) - \cos x - \sin y}$$



Question 5

It is given that $z = z(x, y)$ satisfies the partial differential equation

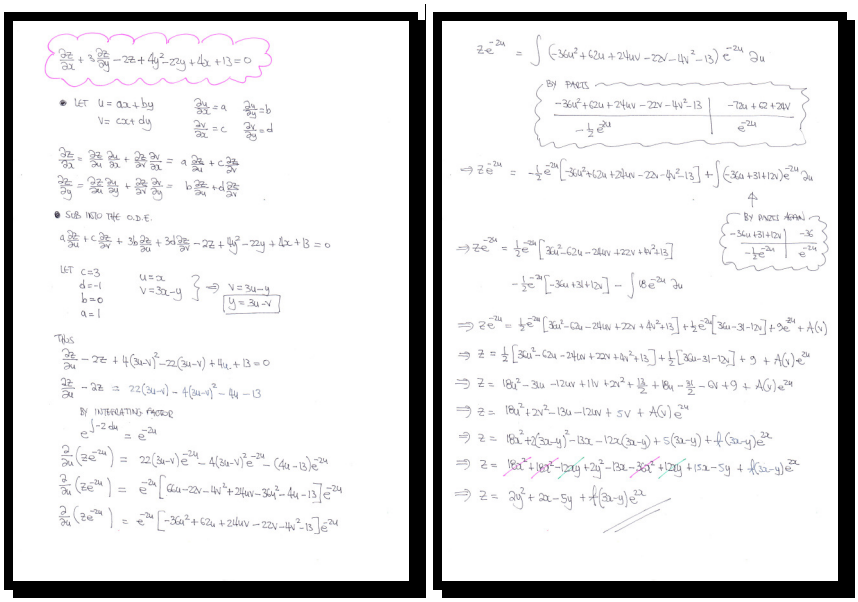
$$\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z + 4y^2 - 22y + 4x + 13 = 0 .$$

Use the transformation equations

$$u = ax + by \quad \text{and} \quad v = cx + dy , \quad ad - bc \neq 0$$

with suitable values of a, b, c and d, in order to determine a general solution of the above partial differential equation.

$$\boxed{z = 2y^2 + 2x - 5y + \mathrm{e}^{2x} f(3x - y)}$$



Question 6

It is given that $\varphi = \varphi(x, y)$ satisfies the partial differential equation

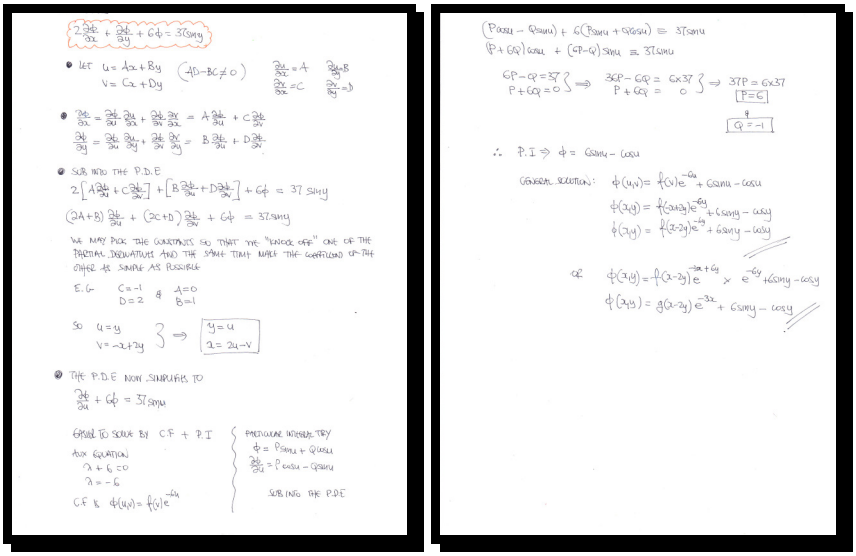
$$2\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + 6\varphi = 37\sin y\,.$$

Use the transformation equations

$$u = Ax + By \quad \text{and} \quad v = Cx + Dy\,, \qquad AD - BC \neq 0$$

with suitable values of A, B, C and D, in order to determine a general solution of the above partial differential equation.

$$\boxed{\varphi(x, y) = 6\sin y - \cos y + \mathrm{e}^{-3x} f(x - 2y) = 6\sin y - \cos y + \mathrm{e}^{-6y} g(x - 2y)}$$



Question 7

It is given that $\varphi = \varphi(x, y, z)$ satisfies the partial differential equation

$$\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + \frac{\partial \varphi}{\partial z} = \varphi .$$

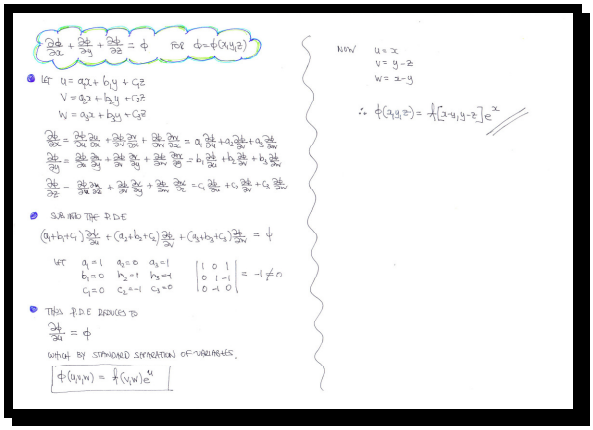
Use the transformation equations

$$u = a_1 x + b_1 y + c_1 z , \quad v = a_2 x + b_2 y + c_2 z \quad \text{and} \quad w = a_3 x + b_3 y + c_3 z ,$$

where $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0 ,$

in order to determine a general solution of the above partial differential equation.

$$\boxed{\varphi(x, y, z) = f\left[x - y, y - z\right] \mathrm{e}^{x}}$$



FIRST ORDER P.D.E.s

(by transformations)

$$A(x,y)\frac{\partial z}{\partial x}+B(x,y)\frac{\partial z}{\partial y}+C(x,y)z=G(x,y),$$

$$z=z(x,y)$$

Question 1

It is given that $\psi = f(x, y)$ satisfies the partial differential equation

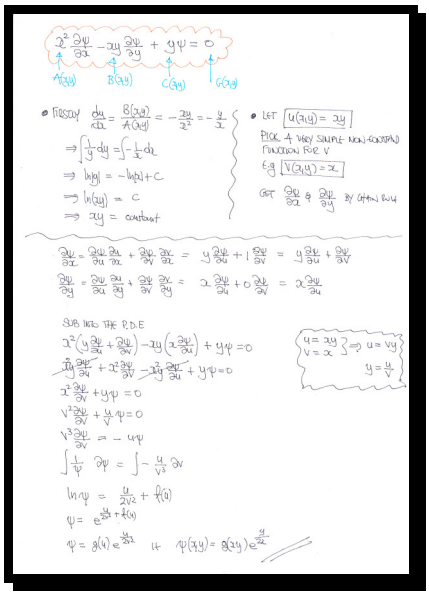
$$x^2 \frac{\partial \psi}{\partial x} - xy \frac{\partial \psi}{\partial y} + y\psi = 0 .$$

Use the transformation equations

$$u = u(x, y) \quad \text{and} \quad v = v(x, y),$$

for suitable functions u and v, in order to determine a general solution of the above partial differential equation.

$$\boxed{\psi(x, y) = e^{\frac{y}{2x}} g(xy)}$$



Question 2

It is given that $z = z(x, y)$ satisfies the partial differential equation

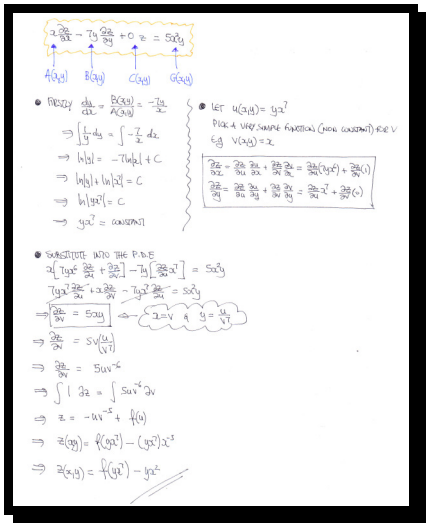
$$x\frac{\partial z}{\partial x} - 7y\frac{\partial z}{\partial y} = 5x^2 y\,.$$

Use the transformation equations

$$u = u(x, y) \quad \text{and} \quad v = v(x, y),$$

for suitable functions u and v, in order to determine a general solution of the above partial differential equation.

$$\boxed{z(x, y) = f\left(yx^7\right) - yx^2}$$



Question 3

It is given that $\varphi = \varphi(x, y)$ satisfies the partial differential equation

$$2\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + 6\varphi = 37\sin y\,.$$

Use the transformation equations

$$u = u(x, y) \quad \text{and} \quad v = v(x, y),$$

for suitable functions u and v, in order to determine a general solution of the above partial differential equation.

$$\boxed{\varphi(x, y) = 6\sin y - \cos y + \mathrm{e}^{-3x} f(x - 2y) = 6\sin y - \cos y + \mathrm{e}^{-6y} g(x - 2y)}$$



Question 4

It is given that $z = z(x, y)$ satisfies the partial differential equation

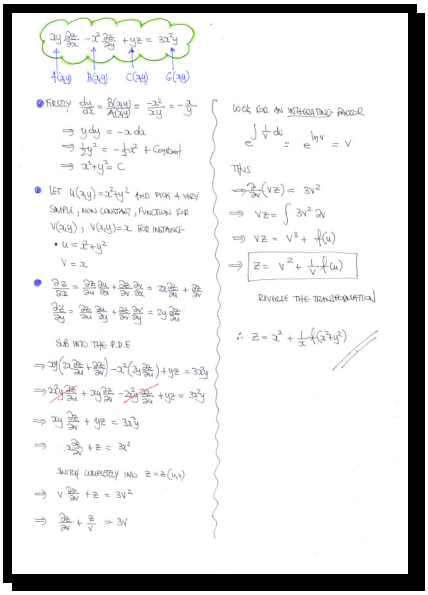
$$xy\frac{\partial z}{\partial x} - x^2\frac{\partial z}{\partial y} + yz = 3x^2 y\,.$$

Use the transformation equations

$$u = u(x, y) \quad \text{and} \quad v = v(x, y),$$

for suitable functions u and v, in order to determine a general solution of the above partial differential equation.

$$\boxed{z(x, y) = x^2 + \frac{1}{x} f\left(x^2 + y^2\right)}$$



FIRST ORDER P.D.E.s

(by Lagrange's method)

$$P(x,y,z)\frac{\partial z}{\partial x}+Q(x,y,z)\frac{\partial z}{\partial y}=R(x,y,z),\quad z=z(x,y)$$

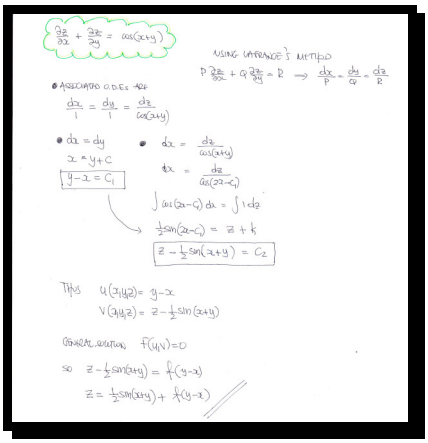
Question 1

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \cos(x + y).$$

Use Lagrange's method, to determine the general solution of the above partial differential equation.

$$\boxed{z = \frac{1}{2}\sin(x + y) + f(y - x)}$$



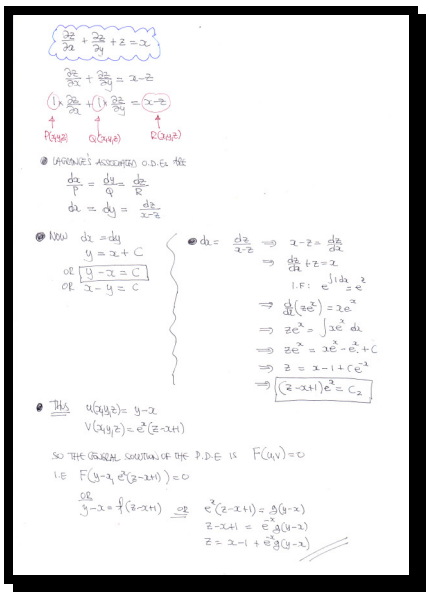
Question 2

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + z = x \, .$$

Use Lagrange's method to determine the general solution of the above partial differential equation.

$$\boxed{z = x - 1 + \mathrm{e}^{-x} f(x - y)}$$



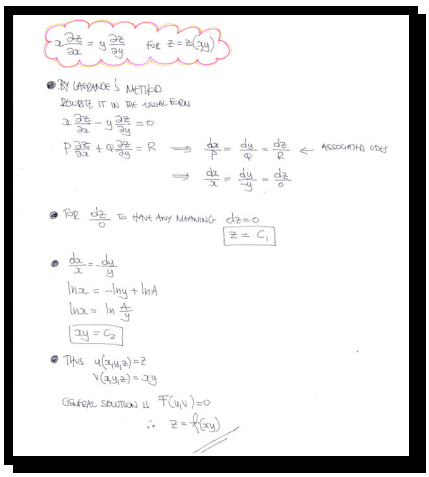
Question 3

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}.$$

Use Lagrange's method, to determine the general solution of the above partial differential equation.

$$\boxed{z = f(xy)}$$



Question 4

It is given that $z = z(x, y)$ satisfies the partial differential equation

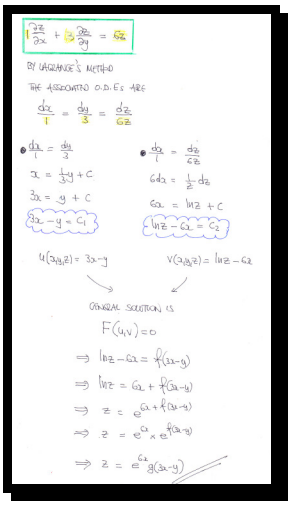
$$\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 6z .$$

Use Lagrange's method to show that the general solution of the above partial differential equation can be written as

$$z(x, y) = \mathrm{e}^{6x}\, g(3x - y),$$

where g is an arbitrary function of $3x - y$.

proof



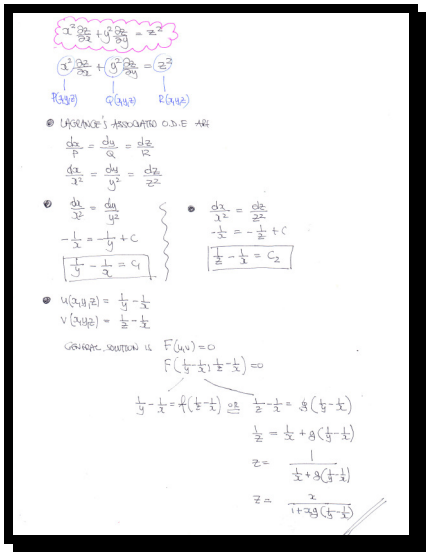
Question 5

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = z^2 .$$

Use Lagrange's method to determine the general solution of the above partial differential equation.

$$\boxed{z = \frac{x}{1 + x f\left(\frac{1}{y} - \frac{1}{x}\right)}}$$



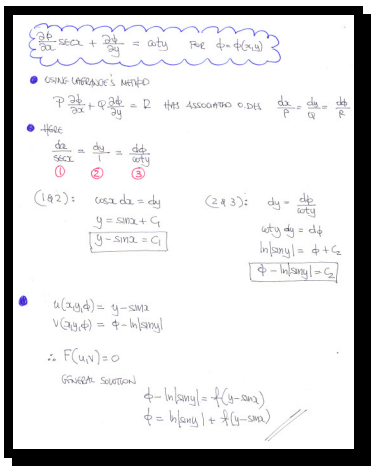
Question 6

It is given that $\varphi = \varphi(x, y)$ satisfies the partial differential equation

$$\frac{\partial \varphi}{\partial x}\sec x + \frac{\partial \varphi}{\partial y} = \cot y \, .$$

Use Lagrange's method, to determine the general solution of the above partial differential equation.

$$\boxed{\varphi(x, y) = \ln|\sin y| + f\left(y - \sin x\right)}$$



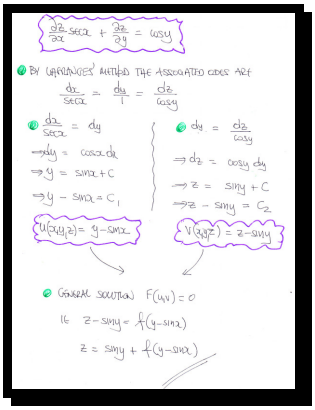
Question 7

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x}\sec x + \frac{\partial z}{\partial y} = \cos y \, .$$

Use Lagrange's method, to determine the general solution of the above partial differential equation.

$$\boxed{z(x, y) = \sin y + f(y - \sin x)}$$



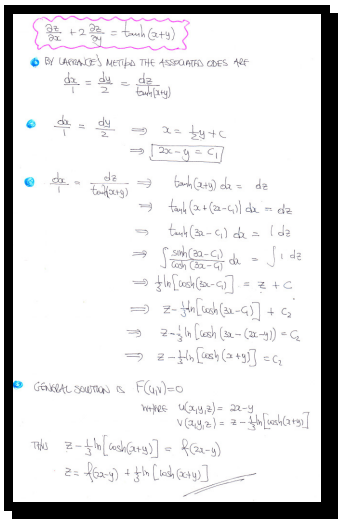
Question 8

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = \tanh(x + y).$$

Use Lagrange's method to determine the general solution of the above partial differential equation.

$$\boxed{z(x, y) = f(2x - y) + \frac{1}{3}\ln\left[\cosh(x + y)\right]}$$



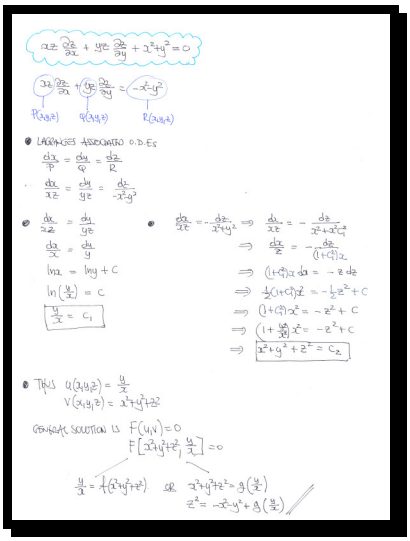
Question 9

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} + x^2 + y^2 = 0 .$$

Use Lagrange's method to determine the general solution of the above partial differential equation.

$$\boxed{z^2 = f\left(\frac{y}{x}\right) - x^2 - y^2}$$



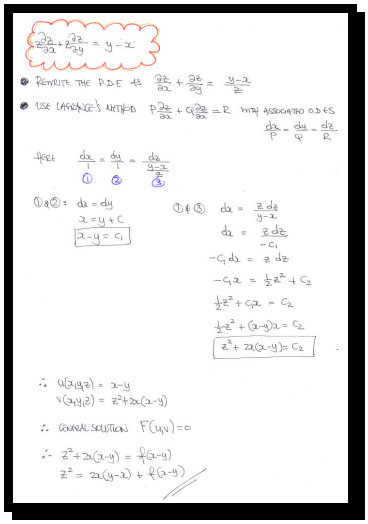
Question 10

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$z\frac{\partial z}{\partial x} + z\frac{\partial z}{\partial y} = y - x\,.$$

Use Lagrange's method, to determine the general solution of the above partial differential equation.

$$\boxed{z^2 = 2x(y-x) + f(x-y)}$$



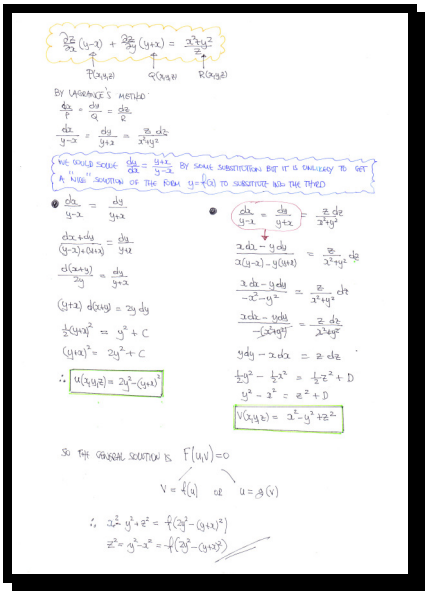
Question 11

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$(y-x)\frac{\partial z}{\partial x}+(y+x)\frac{\partial z}{\partial y}=\frac{x^2+y^2}{z}.$$

Use Lagrange's Multipliers method for derivatives, to find the general solution of the above partial differential equation.

$$\boxed{z^2 = y^2 - x^2 + f\left[2y^2 - (x+y)^2\right]}$$



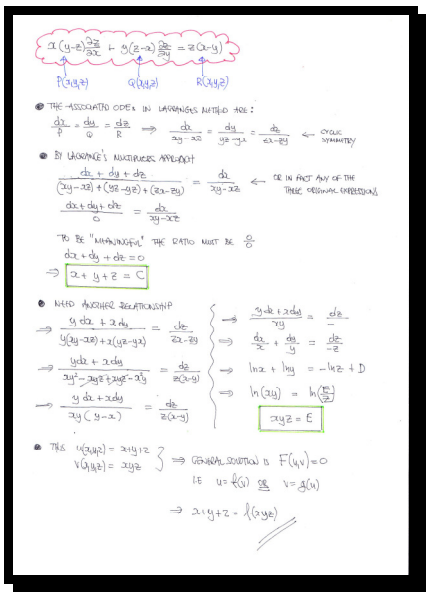
Question 12

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$x(y-z)\frac{\partial z}{\partial x} + y(z-x)\frac{\partial z}{\partial y} = z(x-y).$$

Use Lagrange's Multipliers method for derivatives, to find the general solution of the above partial differential equation.

$$\boxed{x+y+z = f(xyz) \quad \text{or} \quad xyz = g(x+y+z)}$$



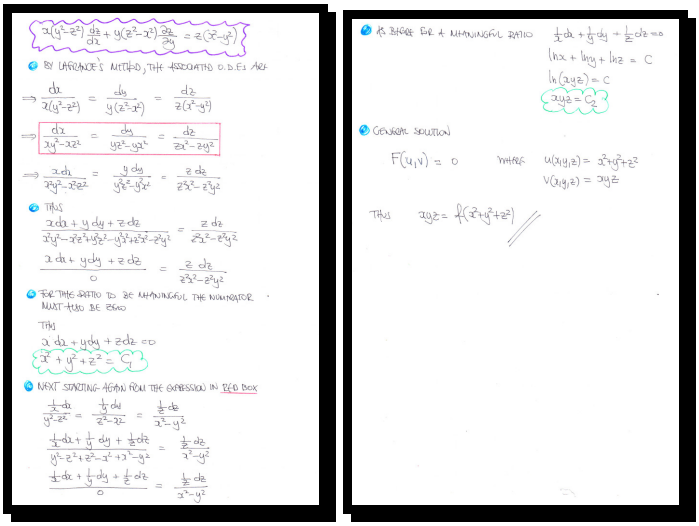
Question 13

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$x\left(y^2 - z^2\right)\frac{\partial z}{\partial x} + y\left(z^2 - x^2\right)\frac{\partial z}{\partial y} = z\left(x^2 - y^2\right).$$

Use Lagrange's Multipliers method for derivatives, to find the general solution of the above partial differential equation.

$$\boxed{xyz = f\left(x^2 + y^2 + z^2\right) \quad \text{or} \quad x^2 + y^2 + z^2 = g\left(xyz\right)}$$



Question 14

The surface S has Cartesian equation

$$z = f(x, y).$$

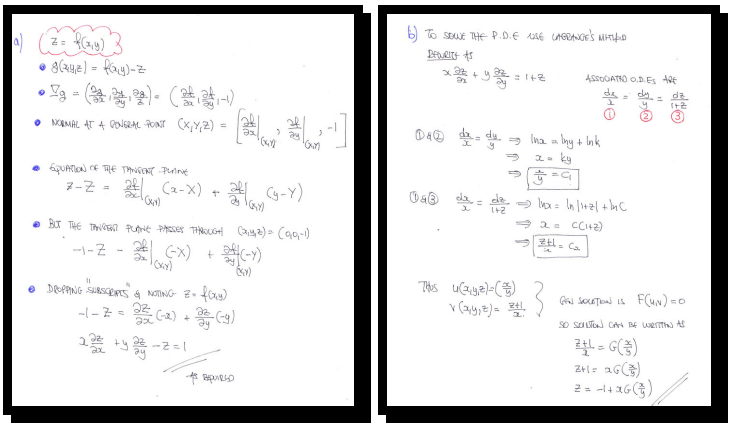
The tangent plane at any point on S passes through the point $(0,0,-1)$.

a) Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - z = 1.$$

b) Hence find the general expression for an equation for S.

$$\boxed{z = -1 + xG\left(\frac{x}{y}\right)}$$



FIRST ORDER P.D.E.s

(Boundary Value Problems)

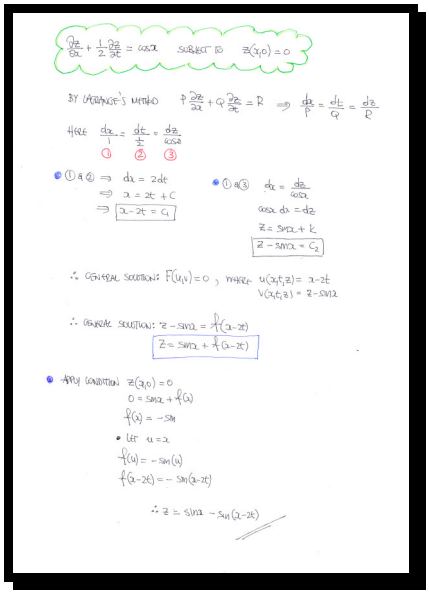
$$P(x,y,z)\frac{\partial z}{\partial x}+Q(x,y,z)\frac{\partial z}{\partial y}=R(x,y,z),\quad z=z(x,y)$$

Question 1

$$\frac{\partial z}{\partial x}+\frac{1}{2}\frac{\partial z}{\partial t}=\cos x \ .$$

Solve the above partial differential equation given that $z=z\left(x,t\right)$ and further satisfies the initial condition $z\left(x,0\right)=0$.

$$\boxed{z\left(x,y\right)=\sin x-\sin\left(x-2t\right)}$$



Question 2

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2z(x + y).$$

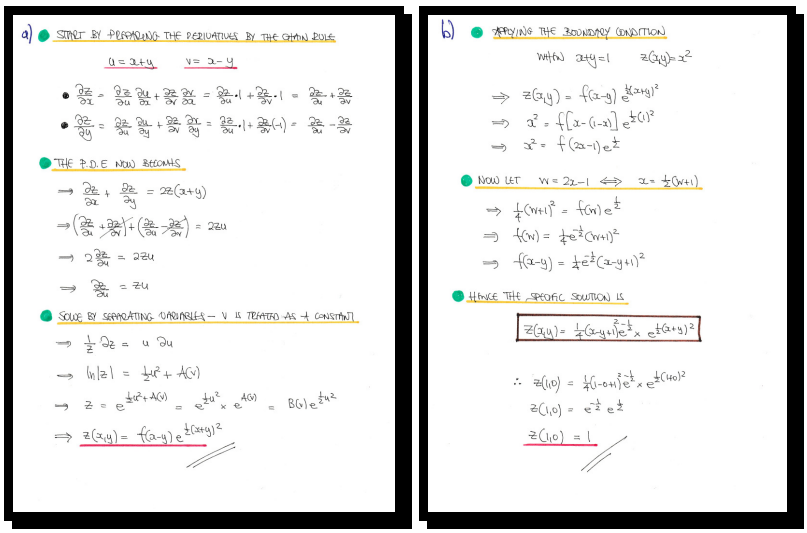
a) Use the transformation equations

$$u = x + y \quad \text{and} \quad v = x - y,$$

to find a general solution for the above partial differential equation.

b) Given further that when $z(x, y) = x^2$ at $x + y = 1$, find the value of $z(1,0)$.

$$\boxed{}, \quad \boxed{z(x, y) = g(x - y)e^{\frac{1}{2}(x+y)^2}}, \quad \boxed{z(1,0) = 1}$$



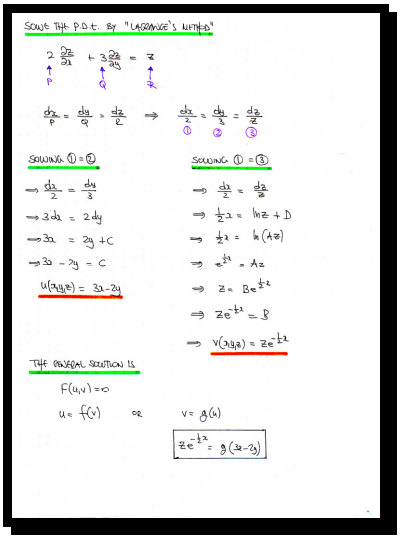
Question 3

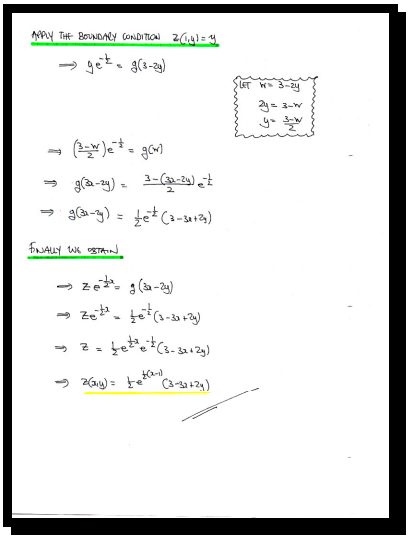
It is given that $z = z(x, y)$ satisfies the partial differential equation

$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = z \, .$$

Given further that $z = y$ at $x = 1$ for all y, find the solution of the above partial differential equation.

$\boxed{}$, $\boxed{z(x, y) = \frac{1}{2}(3 - 3x + 2y)\mathrm{e}^{\frac{1}{2}(x-1)}}$





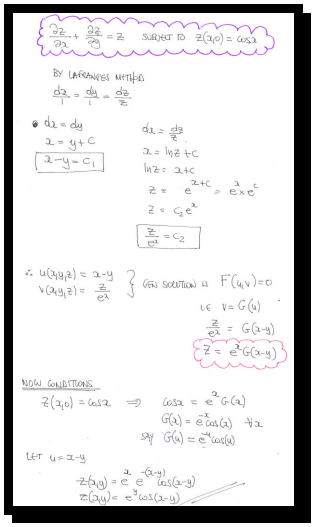
Question 4

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z \, .$$

Given further that $z(x,0) = \cos x$, find the solution of the above partial differential equation.

$$\boxed{z(x, y) = \mathrm{e}^{y} \cos(x - y)}$$



Question 5

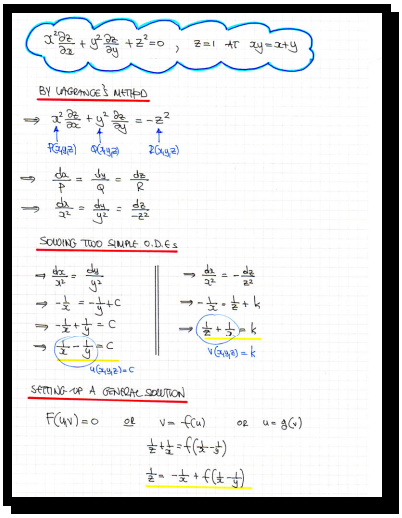
The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

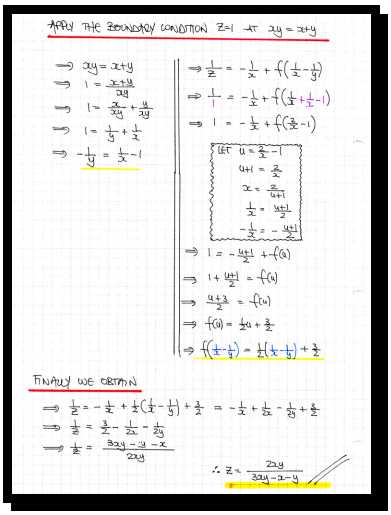
$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} + z^2 = 0 .$$

The plane with equation $z = 1$ meets S on the curve with equation $xy = x + y$.

Find a Cartesian equation of S, in the form $z = f(x, y)$.

$$\boxed{\phantom{z = \tfrac{1}{2}(3-3x+2y)e^{\frac{1}{2}(x-1)}}}, \quad \boxed{z = \tfrac{1}{2}(3-3x+2y)e^{\frac{1}{2}(x-1)}}$$





Question 6

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

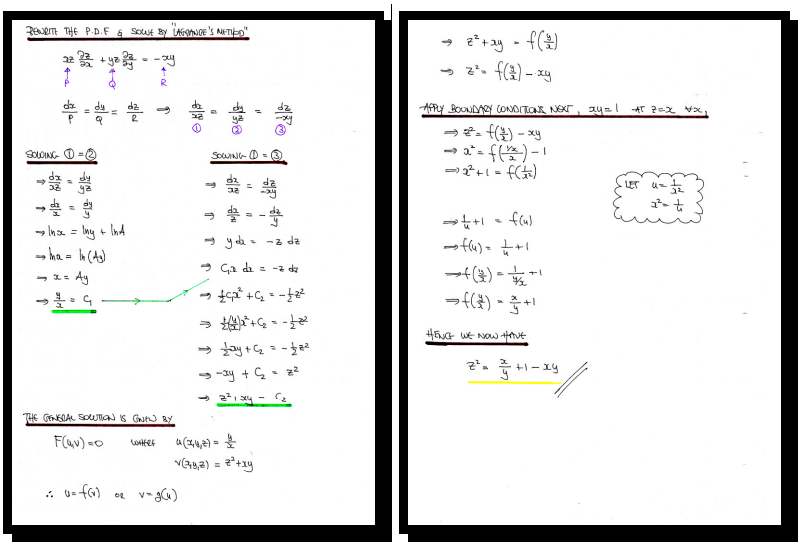
$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} + xy = 0.$$

S contains the curve with equation

$$xy = 1,\ z = x,\ \forall x\ .$$

Find a Cartesian equation of S, in the form $z = f(x, y)$.

$$\boxed{\phantom{z(x,y)=\frac{x}{y}-xy+1}}\ ,\quad \boxed{z(x,y)=\frac{x}{y}-xy+1}$$



Question 7

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

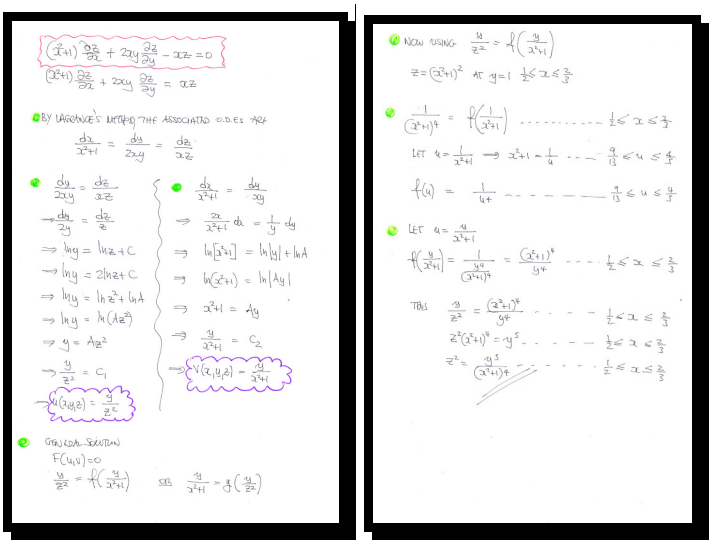
$$\left(x^2+1\right)\frac{\partial z}{\partial x}+2xy\frac{\partial z}{\partial y}-xy=0\,.$$

S contains the curve with equation

$$z(x,1)=\left(x^2+1\right)^2,\quad \frac{1}{2}\le x\le\frac{2}{3}\,.$$

Find a Cartesian equation of S, in the form $z^2 = f(x, y)$.

$$\boxed{z^2=\frac{y^5}{\left(x^2+1\right)^4}}$$



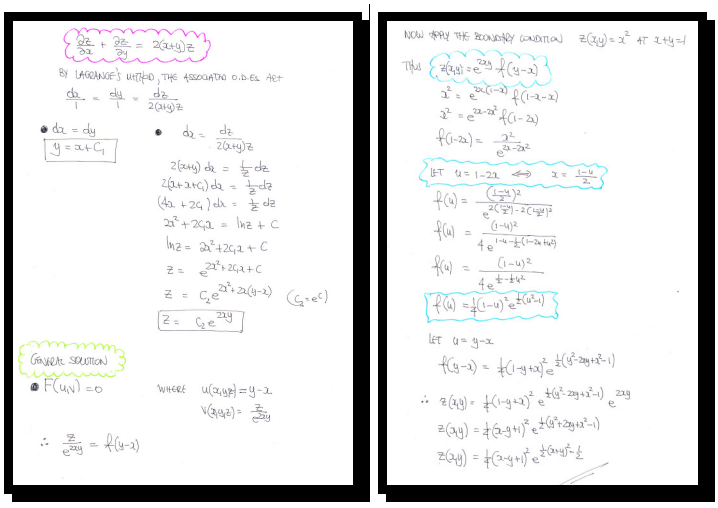
Question 8

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2z(x + y).$$

Given further that when $z(x, y) = x^2$ at $x + y = 1$, find the solution of the above partial differential equation.

$$\boxed{z(x, y) = \frac{1}{4}(x - y + 1)^2 \exp\left[\frac{1}{2}(x + y + 1)(x + y - 1)\right]}$$



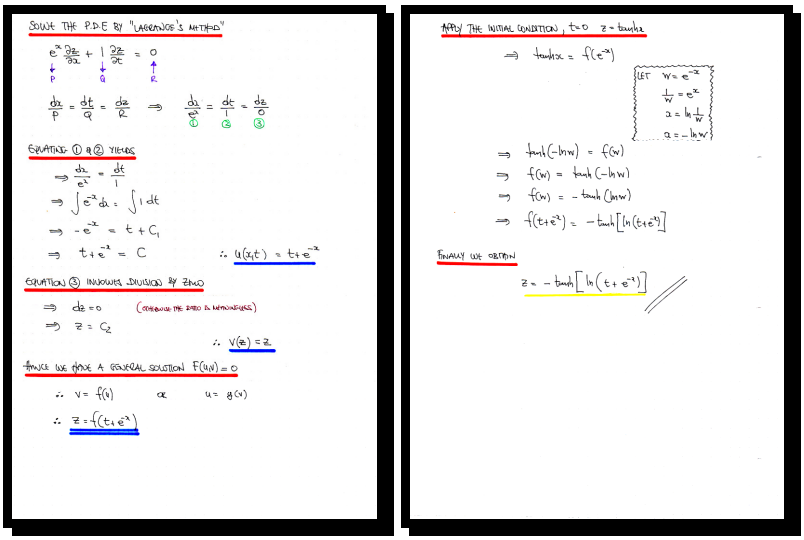
Question 9

It is given that $z = z(x,t)$ satisfies the partial differential equation

$$e^x \frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0, \quad z(x,0) = \tanh x .$$

Find the solution of the above partial differential equation, in the form $z = f(x,t)$.

$\square$, $\boxed{z(x,y) = -\tanh\left[\ln\left(t + e^{-x}\right)\right]}$



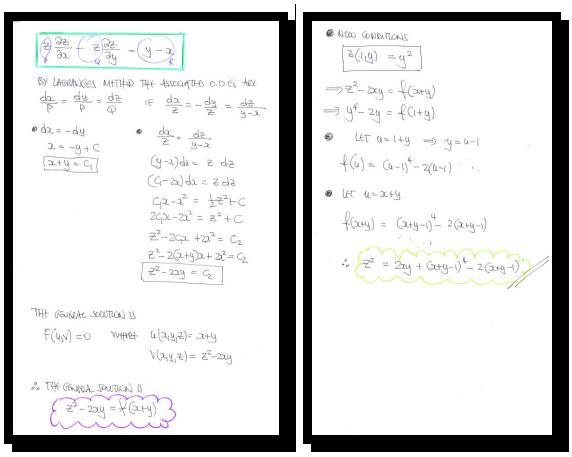
Question 10

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$z\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial t} = y - x, \qquad z(1, y) = y^2 .$$

Find the solution of the above partial differential equation, in the form $z^2 = f(x, y)$.

$$\boxed{z^2 = 2xy + (x + y - 1)^4 - 2(x + y - 1)}$$



Question 11

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

$$x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = x^2 y\,.$$

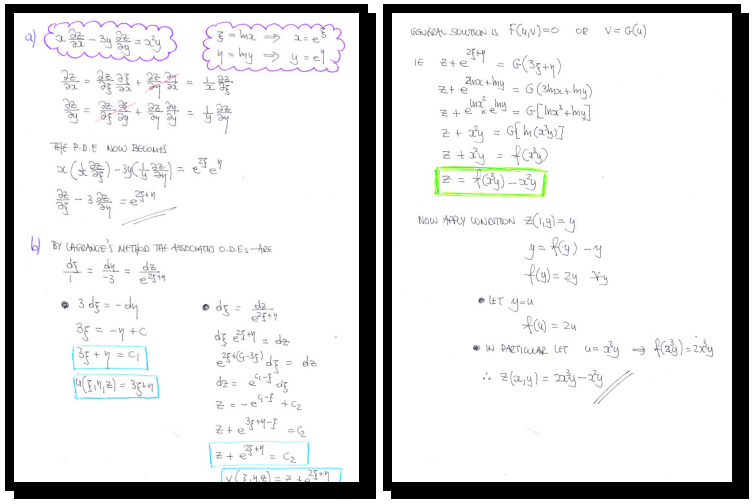
a) Use the transformation equations

$$\xi(x, y) = \ln x \quad \text{and} \quad \eta(x, y) = \ln y\,,$$

to transform the above partial differential equation into one with constant coefficients.

b) Given further that $z(1, y) = y$, find a Cartesian equation of S, giving the answer in the form $z = f(x, y)$.

$$\boxed{\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial y} = \mathrm{e}^{2\xi+\eta}}\,, \qquad \boxed{z(x, y) = 2x^3 y + x^2 y}$$



Question 12

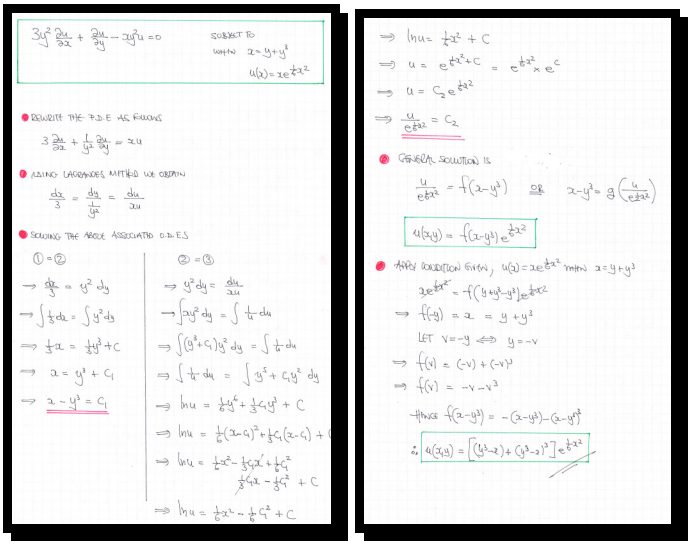
It is given that $u = u(x, y)$ satisfies the partial differential equation

$$3y^2 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - xy^2 u = 0 .$$

It is further given that when $x = y + y^3$, $u(x) = x e^{\frac{1}{6}x^2}$.

Find a simplified expression for $u = u(x, y)$, in the form $u(x, y) = f(x, y) e^{\frac{1}{6}x^2}$, where f is a function to be determined.

$$\boxed{u(x, y) = \left[\left(y^3 - x \right) + \left(y^3 - x \right)^3 \right] e^{\frac{1}{6}x^2}}$$



Question 13

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

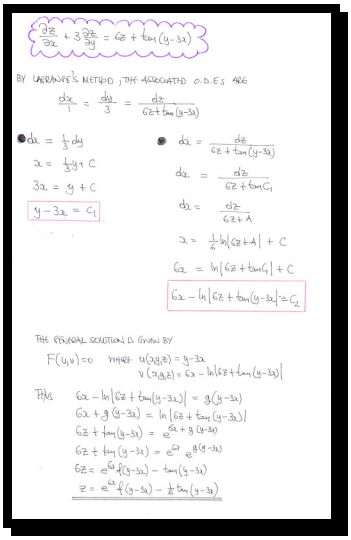
$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = z\,.$$

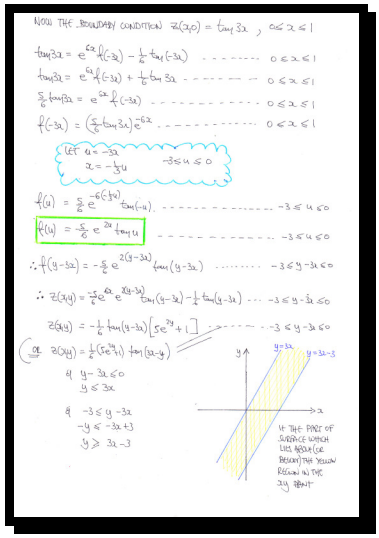
It is further given that

$$z(x,0) = \tan 3x\,,\ 0 \le x \le 1\,.$$

Find a Cartesian equation of S, in the form $z = f(x, y)$, further describing the relation of S to the x-y plane.

$$\boxed{z(x,y) = \frac{1}{6}\left(5\mathrm{e}^{x}+1\right)\tan(3x-y),\quad 3x-3 \le y \le 3x}$$





Question 14

The function f, with equation $z = f(x, y)$, satisfies the partial differential equation

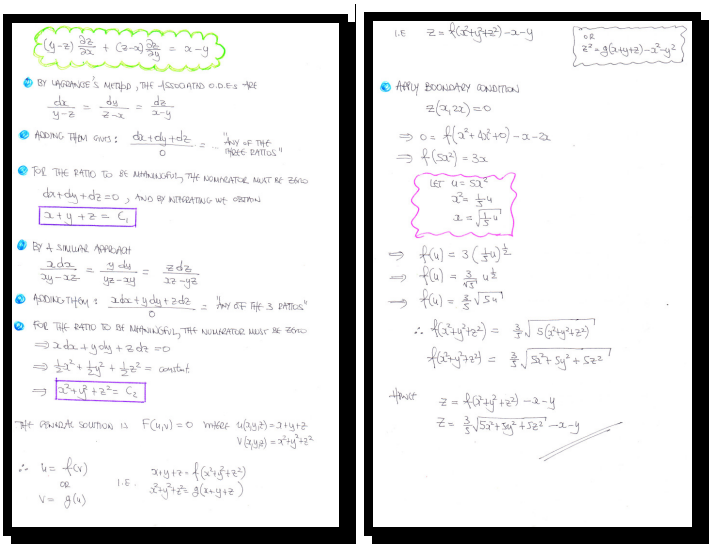
$$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y\,.$$

It is further given that

$$f(x, y)=0\,,\text{ when } y=2x\,.$$

Find a Cartesian equation of f, giving the answer in the form $z = f(x, y)$.

$$\boxed{z(x, y)=-x-y+\frac{3}{5}\sqrt{5x^2+5y^2+5z^2}}$$



Question 15

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

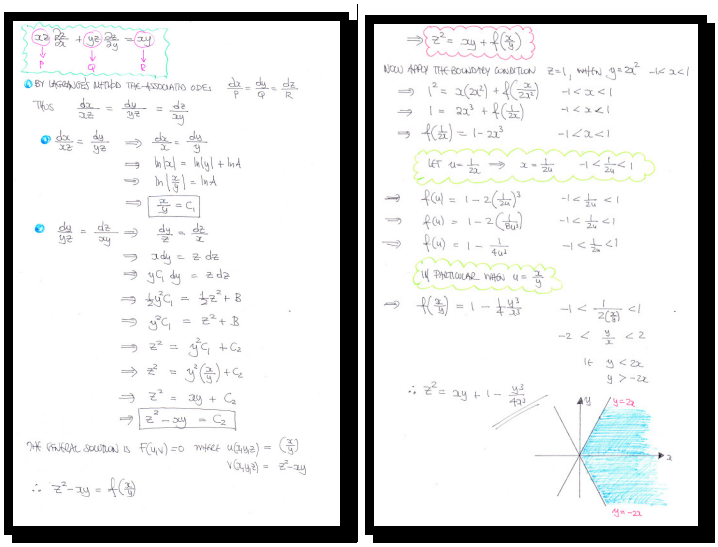
$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy\,.$$

The plane with equation $z = 1$ intersects S along the curve with equation

$$y = 2x^2\,,\ -1 < x < 1\,.$$

Determine a Cartesian equation of S, giving the answer in the form $z^2 = f(x, y)$, sketching the projection of S on the x-y plane.

$$\boxed{z(x, y) = xy + 1 - \frac{y^3}{4x^3}}$$



Question 16

The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

$$yz\frac{\partial z}{\partial x} - xz\frac{\partial z}{\partial y} = xy\,.$$

a) Find a general solution of the partial differential equation.

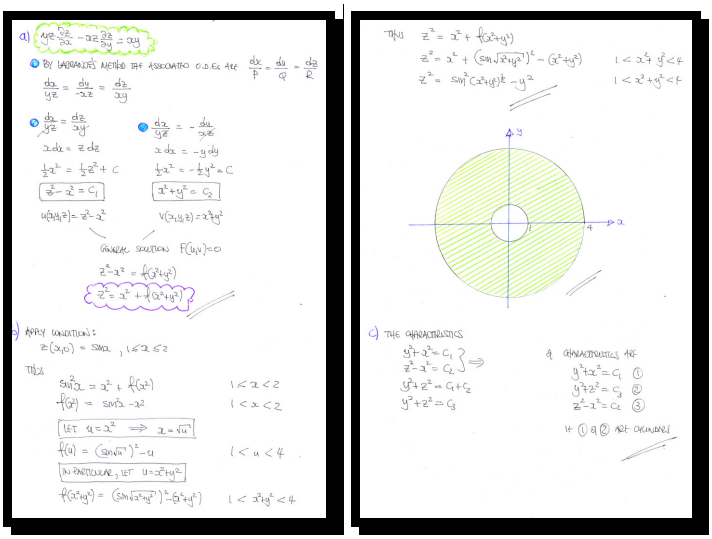
The plane with equation $y = 0$ intersects S along the curve with equation

$$z = \sin x\,,\ 1 < x < 2\,.$$

b) Find a Cartesian equation of S, giving the answer in the form $z^2 = f(x, y)$, sketching the projection of S on the x-y plane.

c) Show that the characteristic curves of the partial differential equation are the intersections of the families of two circular cylinders.

$$\boxed{z(x, y) = \frac{x}{y} - xy + 1}$$



Question 17

It is given that $z = z(x,t)$ satisfies the partial differential equation

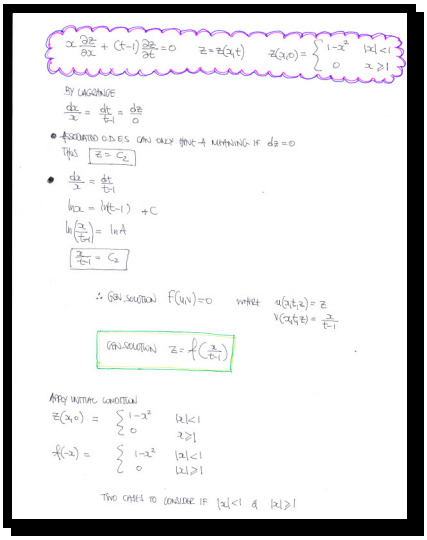
$$x\frac{\partial z}{\partial x} + (t-1)\frac{\partial z}{\partial t} = 0 .$$

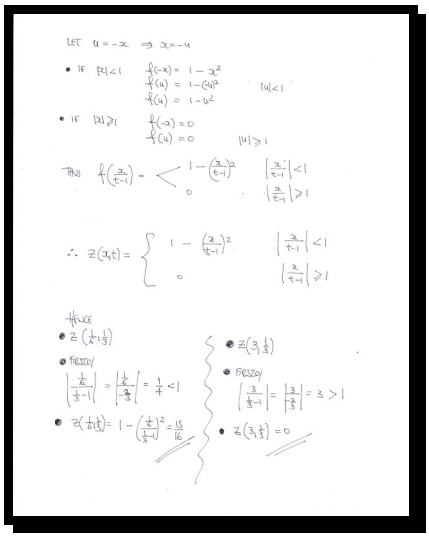
It is further given that

$$z(x,0) = \begin{cases} 1-x^2 & |x| < 1 \\ 0 & |x| \geq 1 \end{cases} .$$

Solve the above partial differential equation, and hence evaluate $z\left(\frac{1}{6},\frac{1}{3}\right)$ and $z\left(3,\frac{1}{3}\right)$.

$$\boxed{z\left(\frac{1}{6},\frac{1}{3}\right) = \frac{15}{16}}, \quad \boxed{z\left(3,\frac{1}{3}\right) = 0}$$





Question 18

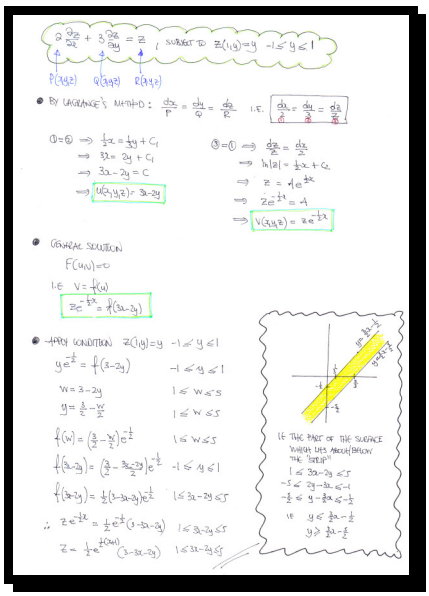
The surface S, with equation $z = z(x, y)$, satisfies the partial differential equation

$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = z\,.$$

It is further given that the plane with equation $x = 1$ meets S along the straight line with equation $z = y$, $-1 \le y \le 1$.

Find a Cartesian equation of S, in the form $z = f(x, y)$, further describing the relation of S to the x-y plane.

$$\boxed{z(x,y) = \tfrac{1}{2}(3 - 3x + 2y)\mathrm{e}^{\frac{1}{2}(x-1)}, \quad \tfrac{3}{2}x - \tfrac{5}{2} \le y \le \tfrac{3}{2}x - \tfrac{1}{2}}$$



Question 19

The surface S, with equation $z = z(x, y)$, is orthogonal to the sphere with Cartesian equation

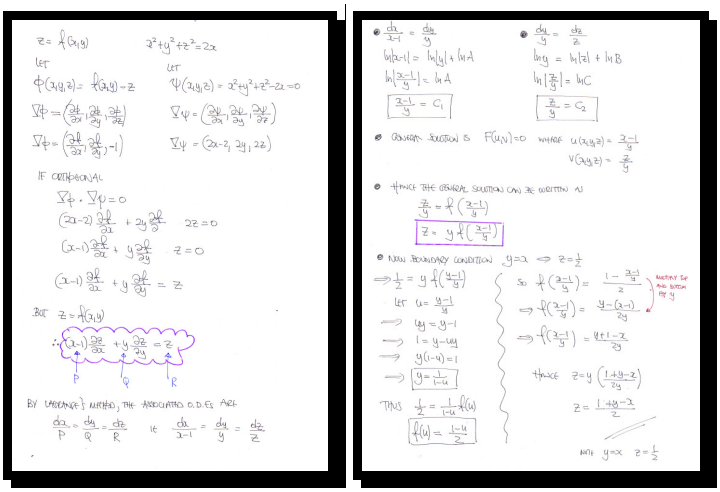
$$x^2 + y^2 + z^2 = 2x\,.$$

It is further given that S passes through the plane with equation $y = x$ at $z = \frac{1}{2}$.

$$z(x, 0) = \tan 3x\,,\ 0 \le x \le 1\,.$$

Find a Cartesian equation of S, in the form $z = f(x, y)$.

$$\boxed{z(x, y) = \frac{1}{2}(1 + y - x)}$$



SECOND ORDER P.D.E.s

$$P\frac{\partial^2 z}{\partial x^2}+Q\frac{\partial^2 z}{\partial x\,\partial y}+Q\frac{\partial^2 z}{\partial y^2}=0,\quad z=z\left(x,y\right)$$

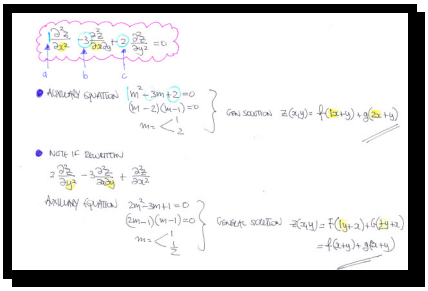
Question 1

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0 .$$

Find a general solution of the above partial differential equation.

$$\boxed{z = f(x+y) + g(2x+y)}$$



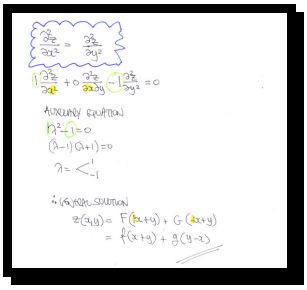
Question 2

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} .$$

Find a general solution of the above partial differential equation.

$$\boxed{z = f(y+x) + g(y-x)}$$



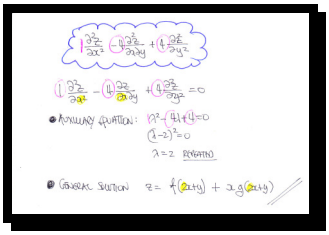
Question 3

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0 .$$

Find a general solution of the above partial differential equation.

$$\boxed{z = f(2x + y) + x\, g(2x + y)}$$



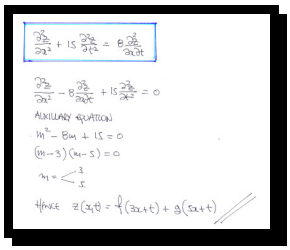
Question 4

It is given that $z = z(x,t)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 15\frac{\partial^2 z}{\partial t^2} = 8\frac{\partial^2 z}{\partial x \partial t} .$$

Find a general solution of the above partial differential equation.

$$\boxed{z(x,t) = f(3x + t) + g(5x + t)}$$



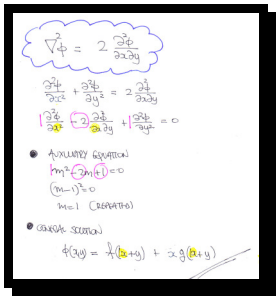
Question 5

It is given that $\varphi = \varphi(x, y)$ satisfies the partial differential equation

$$\nabla^2 \varphi = 2\frac{\partial^2 \varphi}{\partial x \partial y}.$$

Find a general solution of the above partial differential equation.

$$\boxed{z = f(x+y) + x\,g(x+y)}$$



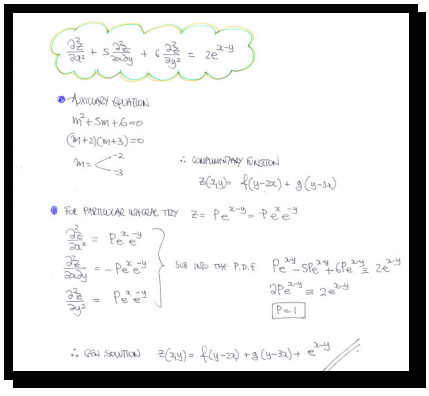
Question 6

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 2\mathrm{e}^{x-y}.$$

Find a general solution of the above partial differential equation.

$$\boxed{z(x, y) = f(y-2x) + g(y-3x) + \mathrm{e}^{x-y}}$$



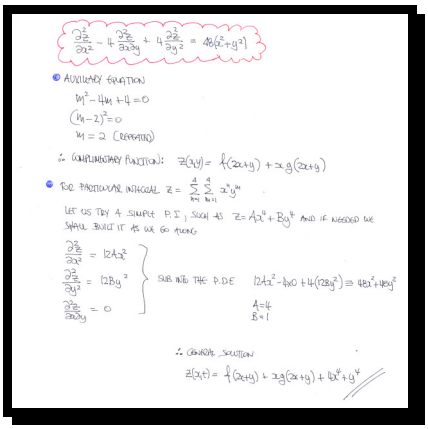
Question 7

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 48\left(x^2 + y^2\right).$$

Find a general solution of the above partial differential equation.

$$\boxed{z(x, y) = f(2x + y) + x\, g(2x + y) + 4x^4 + y^4}$$



Question 8

It is given that $z = z(x, y)$ satisfies the partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0 .$$

Find a general solution of the above partial differential equation.

$$\boxed{z(x, y) = f\left(\frac{y}{x}\right) + x f\left(\frac{y}{x}\right)}$$

