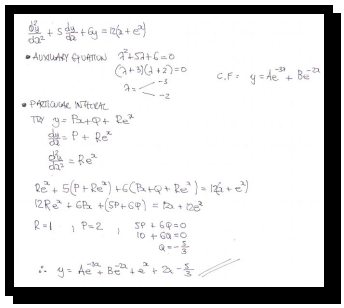
DIFFERENTIAL EQUATIONS

Question 1 (**)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=12\left(x+\mathrm{e}^x\right).$$

$$\boxed{y=A\mathrm{e}^{-3x}+B\mathrm{e}^{-2x}+\mathrm{e}^x+2x-\frac{5}{3}}$$



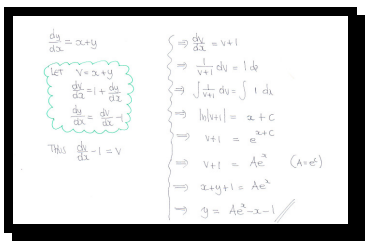
Question 2 (**)

By using a suitable substitution find a general solution of the differential equation

$$\frac{dy}{dx}=x+y,$$

giving the answer in the form $y=f\left(x\right)$.

$$\boxed{y=A\mathrm{e}^x-x-1}$$



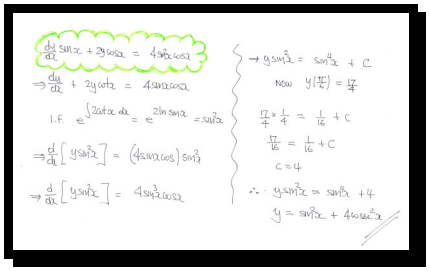
Question 3 (**)

Solve the differential equation

$$\frac{dy}{dx}\sin x+2y\cos x=4\sin^2 x\cos x\,,\quad y\left(\tfrac{1}{6}\pi\right)=\frac{17}{4}\,.$$

Give the answer in the form $y=f\left(x\right)$.

$$\boxed{y=\sin^2 x+4\operatorname{cosec}^2 x}$$

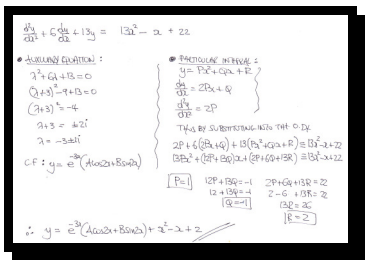


Question 4 (**)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=13x^2-x+22\,.$$

$$\boxed{y=\mathrm{e}^{-3x}\left(A\cos 2x+B\sin 2x\right)+x^2-x+2}$$

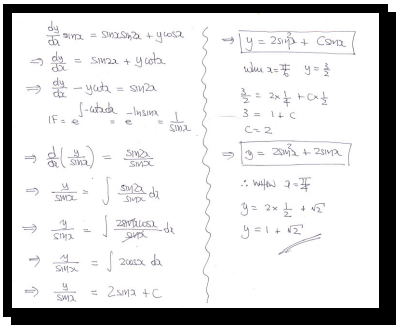


Question 5 (**)

$$\frac{dy}{dx}\sin x = \sin x \sin 2x + y\cos x\,.$$

Given that $y = \frac{3}{2}$ at $x = \frac{\pi}{6}$, find the exact value of y at $x = \frac{\pi}{4}$.

$\boxed{1+\sqrt{2}}$



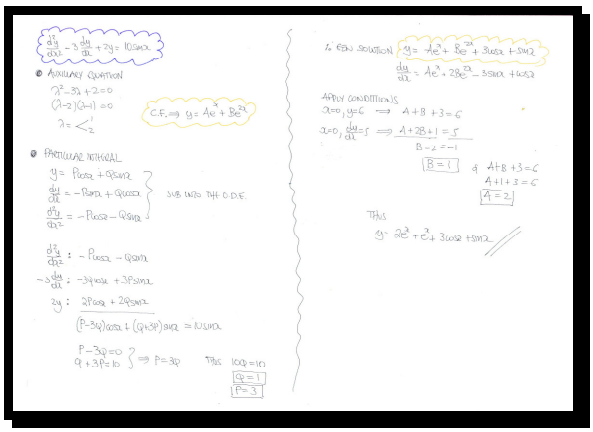
Question 6 (**)

Find a solution of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=10\sin x,$$

subject to the boundary conditions $y=6$ and $\frac{dy}{dx}=5$ at $x=0$.

$$\boxed{y=2\mathrm{e}^{x}+\mathrm{e}^{2x}+3\cos x+\sin x}$$



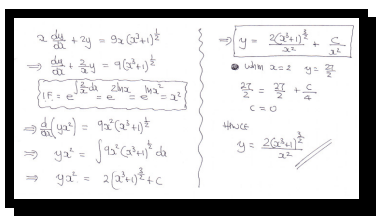
Question 7 (**)

$$x\frac{dy}{dx}+2y=9x\left(x^3+1\right)^{\frac{1}{2}}\text{ , with }y=\frac{27}{2}\text{ at }x=2.$$

Show that the solution of the above differential equation is

$$y=\frac{2}{x^2}\left(x^3+1\right)^{\frac{3}{2}}.$$

proof



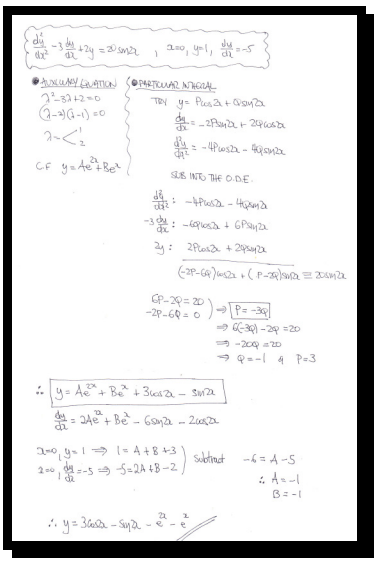
Question 8 (**)

Find a solution of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=20\sin 2x\,,$$

subject to the boundary conditions $y=1$ and $\dfrac{dy}{dx}=-5$ at $x=0$.

$$\boxed{y=3\cos 2x-\sin 2x-\mathrm{e}^{2x}-\mathrm{e}^{x}}$$



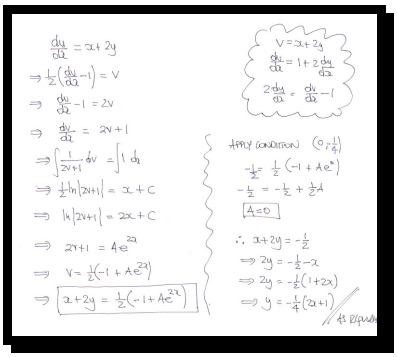
Question 9 (**)

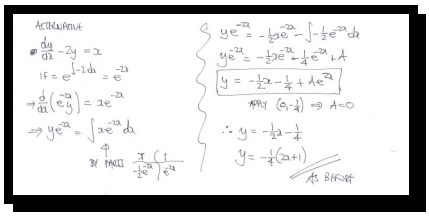
$$\frac{dy}{dx}=x+2y\text{, with }y=-\frac{1}{4}\text{ at }x=0\,.$$

By using a suitable substitution, show that the solution of the differential equation is

$$y=-\frac{1}{4}\left(2x+1\right).$$

proof



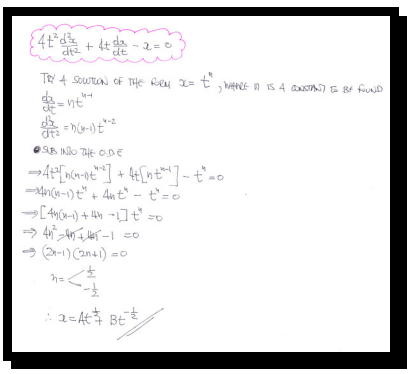


Question 10 (**)

Find the general solution of the following differential equation.

$$4t^2\frac{d^2x}{dt^2}+4t\frac{dx}{dt}-x=0\,.$$

$$\boxed{x=At^{\frac{1}{2}}+Bt^{-\frac{1}{2}}}$$

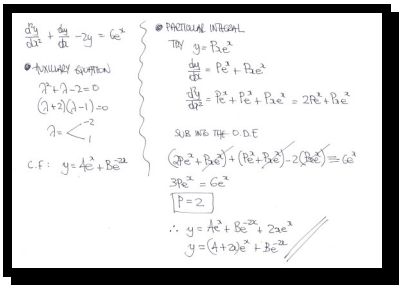


Question 11 (**)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=6\mathrm{e}^x.$$

$$\boxed{y=(A+2x)\mathrm{e}^x+B\mathrm{e}^{-2x}}$$



Question 12 (**)

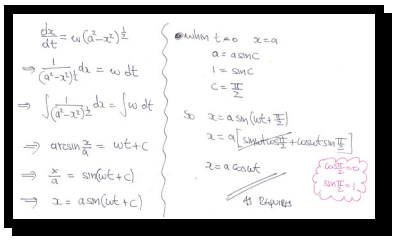
Show that if $y=a$ at $t=0$, the solution of the differential equation

$$\frac{dy}{dt}=\omega\left(a^2-y^2\right)^{\frac{1}{2}},$$

where a and ω are positive constants, can be written as

$$y=a\cos\omega t.$$

proof

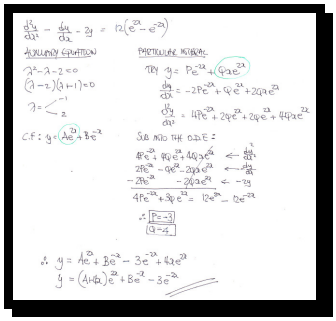


Question 13 (**)

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=12\left(\mathrm{e}^{2x}-\mathrm{e}^{-2x}\right).$$

$$\boxed{y=\left(A+4x\right)\mathrm{e}^{2x}+B\mathrm{e}^{-x}-3\mathrm{e}^{-2x}}$$



Question 14 (**)

20 grams of salt are dissolved into a beaker containing 1 litre of a certain chemical.

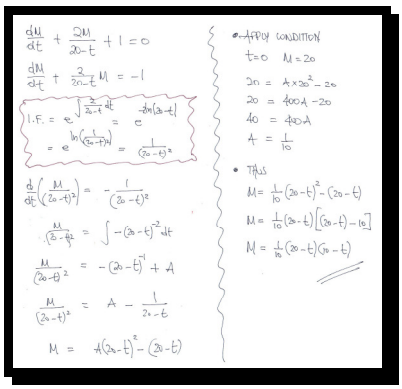
The mass of salt, M grams, which remains undissolved t seconds later, is modelled by the differential equation

$$\frac{dM}{dt}+\frac{2M}{20-t}+1=0\,,\ t\geq 0\,.$$

Show clearly that

$$M=\frac{1}{10}(10-t)(20-t)\,.$$

proof



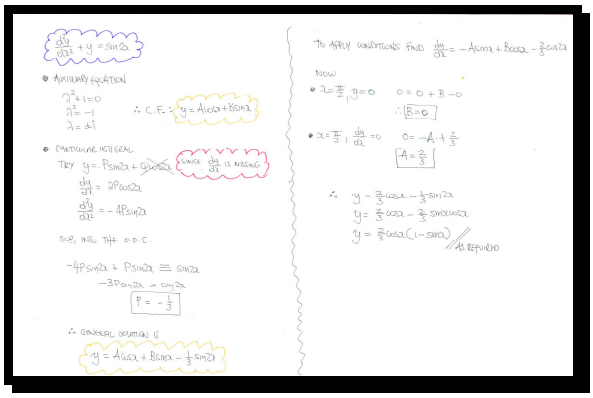
Question 15 (**)

$$\frac{d^2y}{dx^2}+y=\sin 2x,\ \text{with}\ y=0\ ,\ \frac{dy}{dx}=0\ \text{at}\ x=\frac{\pi}{2}.$$

Show that a solution of the above differential equation is

$$y=\frac{2}{3}\cos x\left(1-\sin x\right).$$

proof



Question 16 (+)**

Show that a general solution of the differential equation

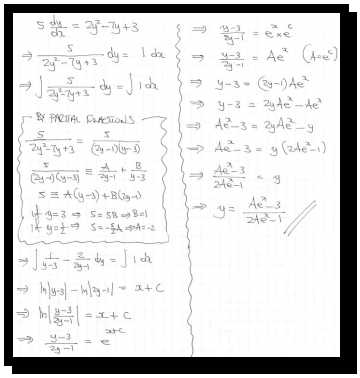
$$5\frac{dy}{dx}=2y^2-7y+3$$

is given by

$$y=\frac{A\mathrm{e}^x-3}{2A\mathrm{e}^x-1},$$

where A is an arbitrary constant.

proof



Question 17 (+)**

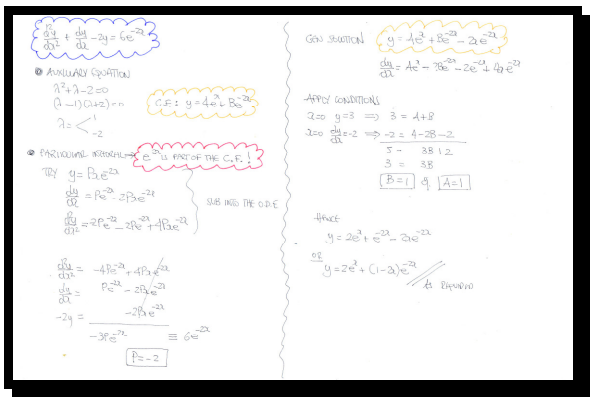
$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=6\mathrm{e}^{-2x},$$

with $y=3$ and $\frac{dy}{dx}=-2$ at $x=0$.

Show that the solution of the above differential equation is

$$y=2\mathrm{e}^{x}+(1-2x)\mathrm{e}^{-2x}.$$

proof

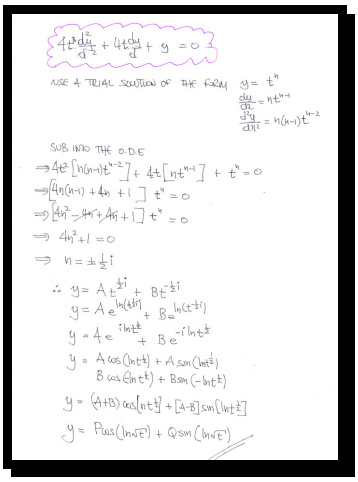


Question 18 (+)**

Find the general solution of the following differential equation.

$$4t^2\frac{d^2y}{dt^2}+4t\frac{dy}{dt}+y=0\,.$$

$$\boxed{y=P\cos\left[\ln\sqrt{t}\right]+P\sin\left[\ln\sqrt{t}\right]}$$

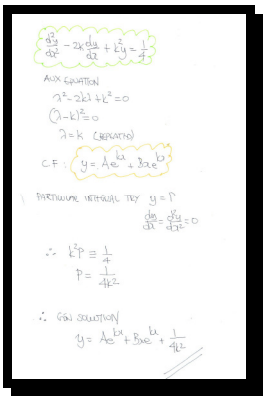


Question 19 (+)**

Find a general solution of the differential equation

$$\frac{d^2y}{dx^2}-2k\frac{dy}{dx}+k^2y=\frac{1}{4},\ k>0\,.$$

$$\boxed{y=A\mathrm{e}^{kx}+Bx\mathrm{e}^{kx}+\frac{1}{4k^2}}$$



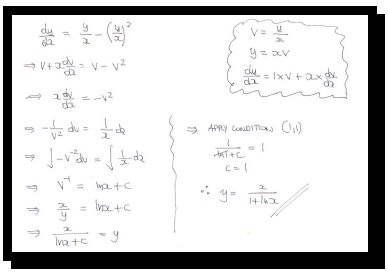
Question 20 (+)**

Solve the differential equation

$$\frac{dy}{dx}=\frac{y}{x}-\left(\frac{y}{x}\right)^2,\ x>0,$$

subject to the condition $y=1$ at $x=1$.

$$\boxed{y=\frac{x}{1+\ln x}}$$



Question 21 (+)**

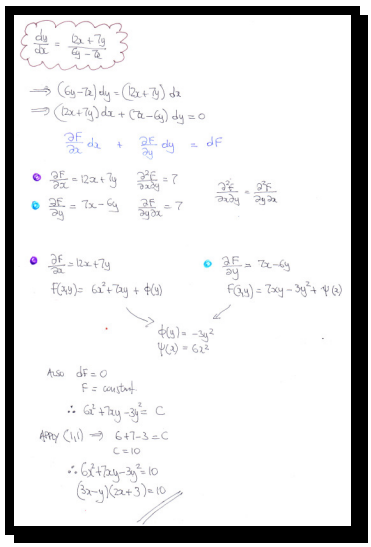
$$\frac{dy}{dx}=\frac{12x+7y}{6y-7x},\ y(1)=1.$$

Use a method involving partial differentiation to show that the solution of the above differential equation can be written as

$$(ax+by)(cx+dy)=10,$$

where a, b, c and d are integers to be found.

$$\boxed{(3x-y)(2x+3y)=k}$$



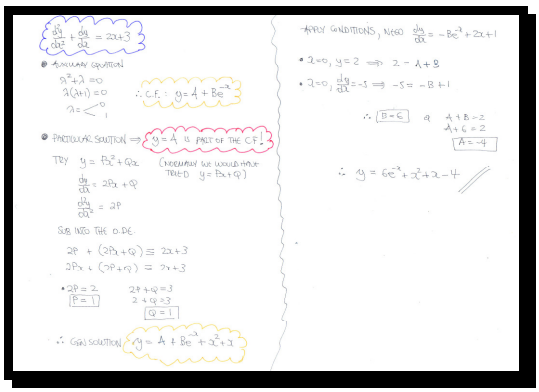
Question 22 (+)**

Find the solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=2x+3,$$

subject to the conditions $y=2$, $\frac{dy}{dx}=-5$ at $x=0$.

$$\boxed{y=x^2+x-4+6\mathrm{e}^{-x}}$$

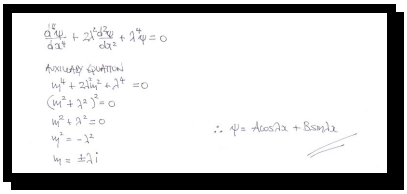


Question 23 (+)**

Find the general solution of the following differential equation.

$$\frac{d^4\psi}{dx^4}+2\lambda\frac{d^2\psi}{dx^2}+\lambda^4\psi=0.$$

$$\boxed{\psi=A\cos\lambda x+B\sin\lambda x}$$



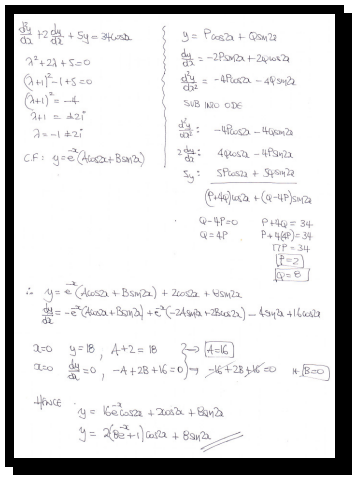
Question 24 (+)**

Find a solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=34\cos 2x,$$

subject to the boundary conditions $y=18$ and $\frac{dy}{dx}=0$ at $x=0$.

$$\boxed{y=2\left(8\mathrm{e}^{-x}+1\right)\cos 2x+8\sin 2x}$$



Question 25 (+)**

$$\frac{dy}{dx}=\frac{(4x+y)(x+y)}{x^2},\ x>0.$$

a) Use a suitable substitution to show that the above differential equation can be transformed to

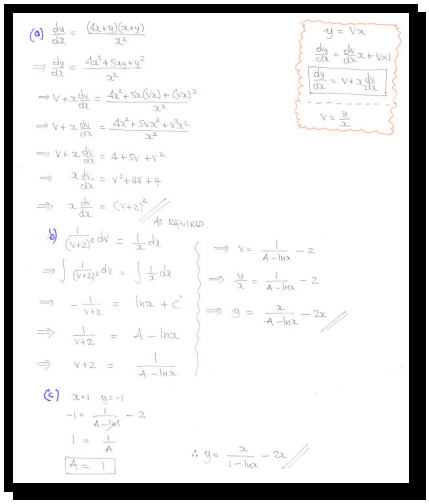
$$x\frac{dv}{dx}=(v+2)^2.$$

b) Hence find the general solution of the original differential equation, giving the answer in the form $y=f(x)$.

c) Use the boundary condition $y=-1$ at $x=1$, to show that a specific solution of the original differential equation is

$$y=\frac{x}{1-\ln x}-2x.$$

$$\boxed{y=\frac{x}{A-\ln x}-2x}$$



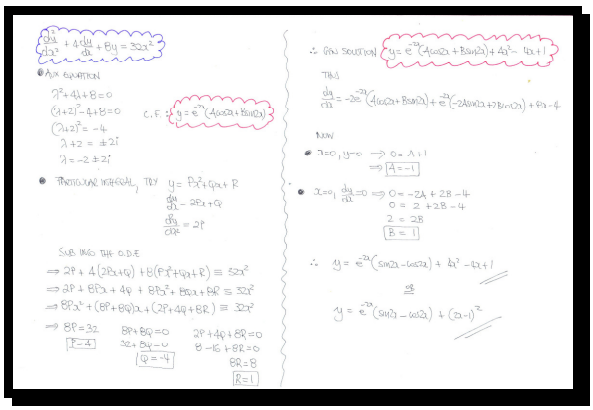
Question 26 (+)**

The curve C has a local minimum at the origin and satisfies the differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+8y=32x^2\,.$$

Find an equation for C.

$$\boxed{y=\mathrm{e}^{x}\left(\sin 2x+\cos 2x\right)+\left(2x-1\right)^2}$$



Question 27 (+)**

Show that a general solution of the differential equation

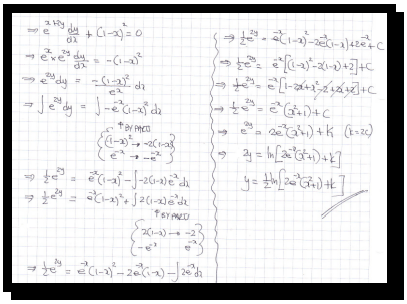
$$\mathrm{e}^{x+2y}\frac{dy}{dx}+(1-x)^2=0$$

is given by

$$y=\frac{1}{2}\ln\left[2\mathrm{e}^{-x}\left(x^2+1\right)+K\right],$$

where K is an arbitrary constant.

proof



Question 28 (+)**

$$\frac{d^2x}{dt^2}+9x+12\sin 3t=0,\ t\geq 0,$$

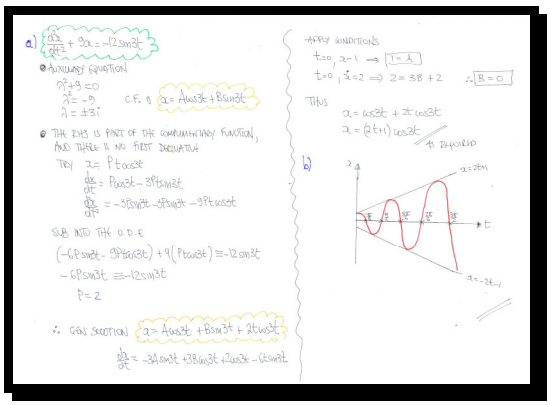
with $x=1$, $\frac{dx}{dt}=2$ at $t=0$.

a) Show that a solution of the differential equation is

$$x=(2t+1)\cos 3t.$$

b) Sketch the graph of x.

proof



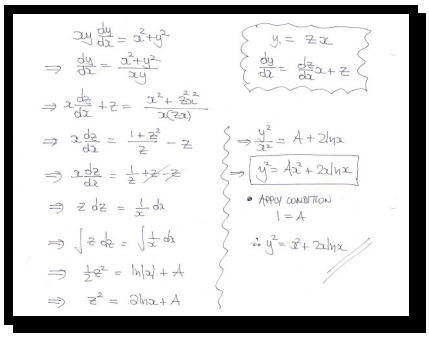
Question 29 (+)**

By using a suitable substitution, solve the differential equation

$$xy\frac{dy}{dx}=x^2+y^2,\ x>0,$$

subject to the boundary condition $y=1$ at $x=1$.

$$\boxed{y=x^2\left(1+2\ln x\right)}$$



Question 30 (+)**

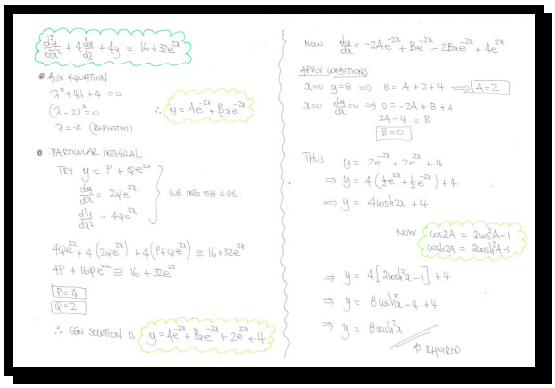
$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=16+32\mathrm{e}^{2x},$$

with $y=8$ and $\frac{dy}{dx}=0$ at $x=0$.

Show that the solution of the above differential equation is

$$y=8\cosh^2 x.$$

proof



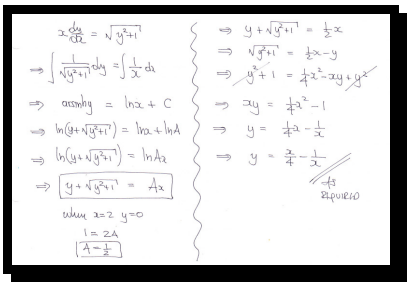
Question 31 (+)**

$$x\frac{dy}{dx}=\sqrt{y^2+1}\ ,\ x>0\text{, with }y=0\text{ at }x=2\text{.}$$

Show that the solution of the above differential equation is

$$y=\frac{x}{4}-\frac{1}{x}\text{.}$$

proof



Question 32 (+)**

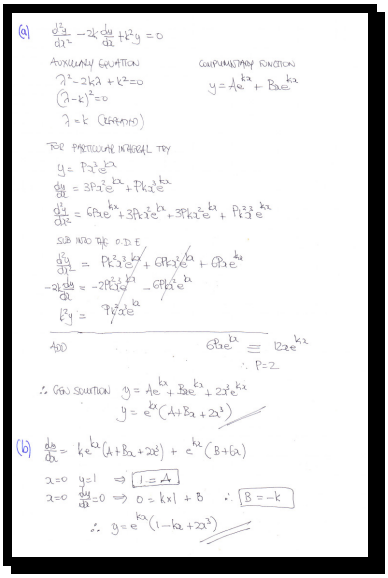
$$\frac{d^2y}{dx^2}-2k\frac{dy}{dx}+k^2y=12x\mathrm{e}^{kx},\ k>0$$

a) Find a general solution of the differential equation given that $y=Px^3\mathrm{e}^{kx}$, where P is a constant, is part of the solution.

b) Given further that $y=1,\ \frac{dy}{dx}=0$ at $x=0$ show that

$$y=\mathrm{e}^{kx}\left(2x^3-kx+1\right).$$

$$\boxed{y=\mathrm{e}^{kx}\left(2x^3+Ax+B\right)}$$



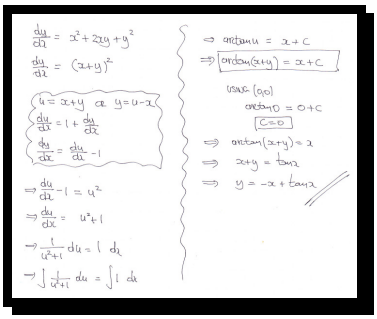
Question 33 (+)**

By using a suitable substitution, or otherwise, solve the differential equation

$$\frac{dy}{dx} = x^2 + 2xy + y^2,$$

subject to the condition $y(0) = 0$.

$$\boxed{y = -x + \tan x}$$



Question 34 (+)**

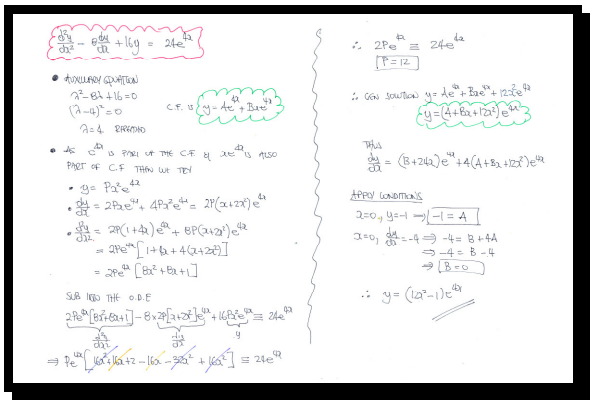
Show that the solution of the differential equation

$$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=24\mathrm{e}^{4x},$$

subject to the boundary conditions $y=-1$, $\frac{dy}{dx}=-4$ at $x=0$, can be written as

$$y=\left(12x^2-1\right)\mathrm{e}^{4x}.$$

proof

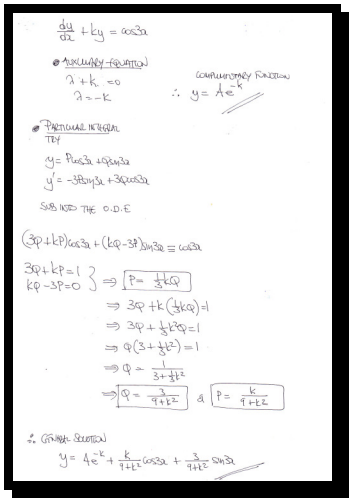


Question 35 (+)**

$$\frac{dy}{dx}+ky=\cos 3x\,,\ k \text{ is a non zero constant.}$$

By finding a complimentary function and a particular integral, or otherwise, find the general of the above differential equation.

$$\boxed{y=A\mathrm{e}^{-x}+\frac{k}{9+k^2}\cos 3x+\frac{3}{9+k^2}\sin 3x}$$



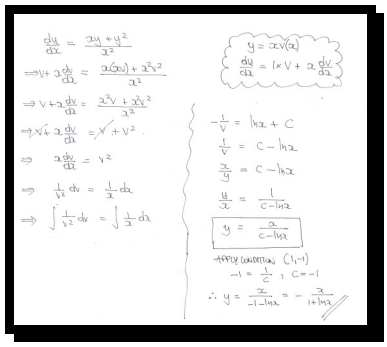
Question 36 (+)**

By using a suitable substitution, solve the differential equation

$$\frac{dy}{dx}=\frac{xy+y^2}{x^2},\ x>0,$$

subject to the condition $y=-1$ at $x=1$.

$$\boxed{y=-\frac{x}{1+\ln x}}$$



Question 37 (+)**

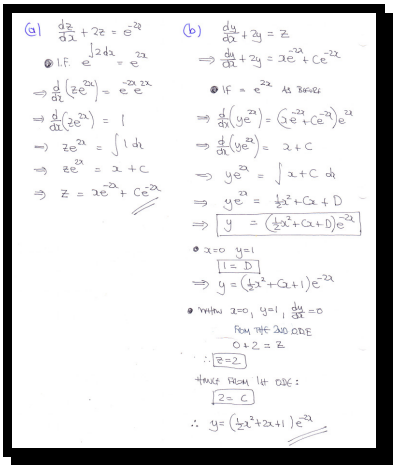
Given that $z=f(x)$ and $y=g(x)$ satisfy the following differential equations

$$\frac{dz}{dx}+2z=\mathrm{e}^{-2x} \quad \text{and} \quad \frac{dy}{dx}+2y=z\,,$$

a) Find z in the form $z=f(x)$

b) Express y in the form $y=g(x)$, given further that at $x=0$, $y=1$, $\dfrac{dy}{dx}=0$

$$\boxed{z=(x+C)\mathrm{e}^{-2x}}\,,\quad \boxed{y=\left(\frac{1}{2}x^2+2x+1\right)\mathrm{e}^{-2x}}$$



Question 38 (+)**

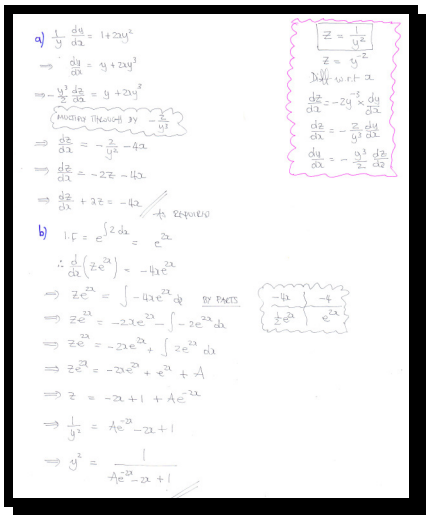
$$\frac{1}{y}\frac{dy}{dx}=1+2xy^2\,,\ \ y>0\,.$$

a) Show that the substitution $z=\dfrac{1}{y^2}$ transforms the above differential equation into the new differential equation

$$\frac{dz}{dx}+2z=-4x\,.$$

b) Hence find the general solution of the original differential equation, giving the answer in the form $y^2=f\left(x\right)$.

$$\boxed{y^2=\frac{1}{A\mathrm{e}^{x}-2x+1}}$$



Question 39 (***)

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=4\mathrm{e}^{3x}.$$

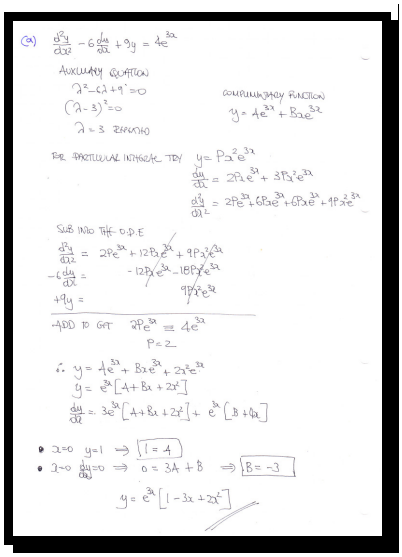
a) Find a solution of the differential equation given that $y=1$, $\dfrac{dy}{dx}=0$ at $x=0$.

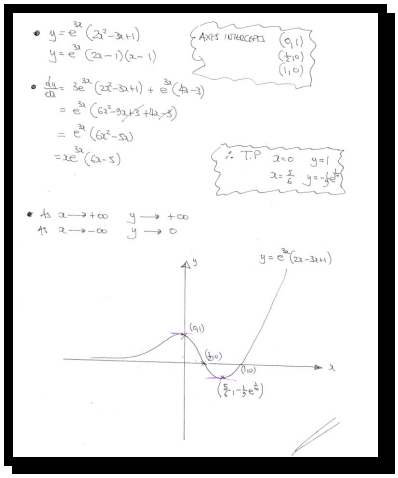
b) Sketch the graph of y.

The sketch must include …

- the coordinates of any points where the graph meets the coordinate axes.
- the coordinates of any stationary points of the curve.
- clear indications of how the graph looks for large positive or negative values of x.

$$\boxed{y=\mathrm{e}^{3x}\left(2x^2-3x+1\right)}$$





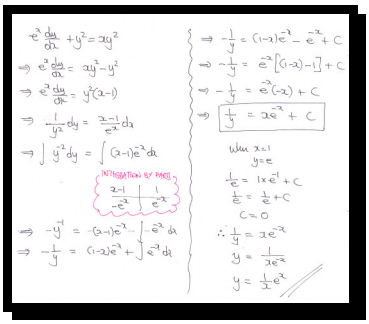
Question 40 (***)

$$\mathrm{e}^x \frac{dy}{dx} + y^2 = xy^2 , \ x > 0 , \ y > 0$$

Show that the solution of the above differential equation subject to $y = \mathrm{e}$ at $x = 1$, is

$$y = \frac{1}{x}\mathrm{e}^x .$$

proof



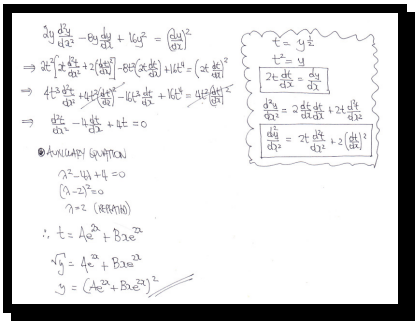
Question 41 (***)

$$2y\frac{d^2y}{dx^2}-8y\frac{dy}{dx}+16y^2=\left(\frac{dy}{dx}\right)^2,\ y\neq 0,$$

Find the general solution of the above differential equation by using the transformation equation $t=\sqrt{y}$.

Give the answer in the form $y=f\left(x\right)$.

$$\boxed{y=\left(Ae^{2x}+Bxe^{2x}\right)^2}$$



Question 42 (***)

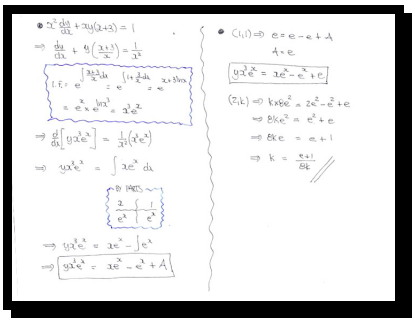
A curve C, with equation $y=f(x)$, passes through the points with coordinates $(1,1)$ and $(2,k)$, where k is a constant.

Given further that the equation of C satisfies the differential equation

$$x^2\frac{dy}{dx}+xy(x+3)=1,$$

determine the exact value of k.

$$\boxed{k=\frac{\mathrm{e}+1}{8e}}$$



Question 43 (***)

A curve C, with equation $y=f(x)$, meets the y axis the point $(0,1)$.

It is further given that the equation of C satisfies the differential equation

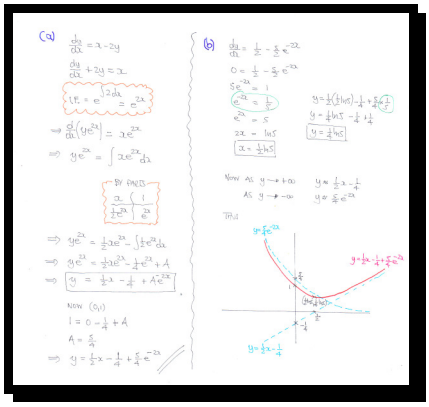
$$\frac{dy}{dx}=x-2y\,.$$

a) Determine an equation of C.

b) Sketch the graph of C.

The graph must include in exact simplified form the coordinates of the stationary point of the curve and the equation of its asymptote.

$$\boxed{y=\frac{1}{2}x-\frac{1}{4}+\frac{5}{4}\mathrm{e}^{-2x}}$$



Question 44 (***)

A curve $y=f\left(x\right)$ satisfies the differential equation

$$y=1-\frac{dy}{dx}\frac{x+1}{\left(x-1\right)\left(x+2\right)},\ y>1, x>-1$$

a) Solve the differential equation to show that

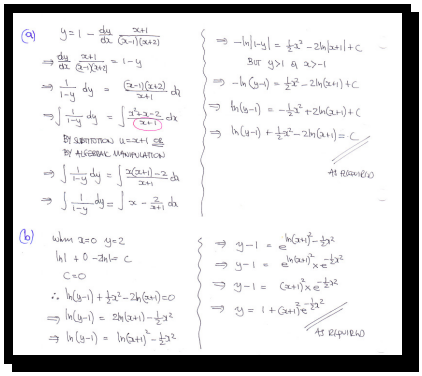
$$\ln\left(y-5\right)+\frac{1}{2}x^2+4x-2\ln\left(x+1\right)=C\,.$$

When $x=0$, $y=2$.

b) Show further that

$$y=1+\left(x+1\right)^2\mathrm{e}^{-\frac{1}{2}x^2}\,.$$

proof



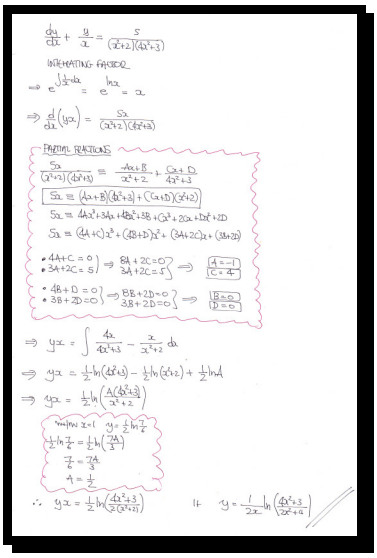
Question 45 (***)

$$\frac{dy}{dx}+\frac{y}{x}=\frac{5}{\left(x^2+2\right)\left(4x^2+3\right)},\ x>0\,.$$

Given that $y=\frac{1}{2}\ln\frac{7}{6}$ at $x=1$, show that the solution of the above differential equation can be written as

$$y=\frac{1}{2x}\ln\left(\frac{4x^2+3}{2x^2+4}\right).$$

proof



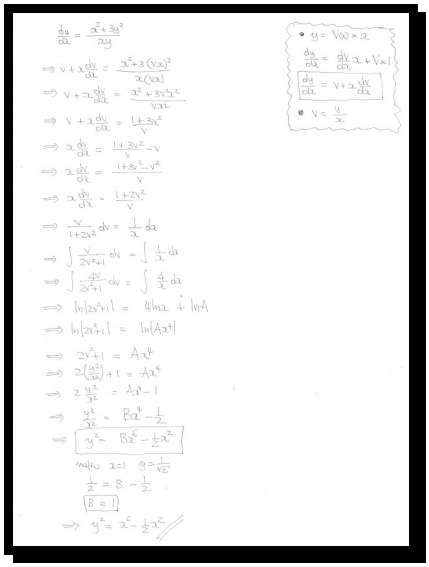
Question 46 (***)

$$\frac{dy}{dx}=\frac{x^2+3y^2}{xy},\ x>0,\ y>0.$$

Given the boundary condition $y(1)=\frac{1}{\sqrt{2}}$, show that

$$y^2=x^6-\frac{1}{2}x^2.$$

proof



Question 47 (***)

The differential equation

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=3x\,,\ x\neq 0\,,$$

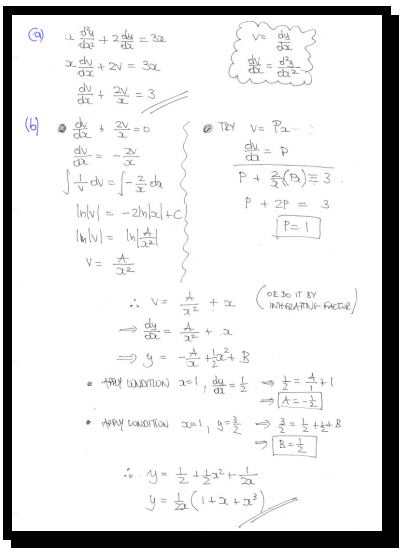
is to be solved subject to the boundary conditions $y=\frac{3}{2}$, $\frac{dy}{dx}=\frac{1}{2}$ at $x=1$.

a) Show that the substitution $v=\frac{dy}{dx}$, transforms the above differential equation into

$$\frac{dv}{dx}+\frac{2v}{x}=3\,.$$

b) Hence find the solution of the original differential equation, giving the answer in the form $y=f(x)$.

$$\boxed{y=\frac{1}{2}\left(x^2+\frac{1}{x}+1\right)}$$



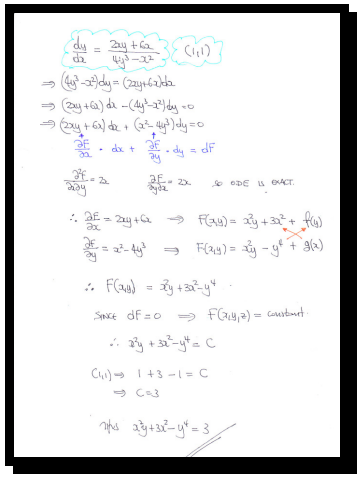
Question 48 (***)

Solve the differential equation

$$\frac{dy}{dx}=\frac{2xy+6x}{4y^3-x^2},$$

subject to the boundary condition $y=1$ at $x=1$.

$$\boxed{x^2y+3x^2-y^4=3}$$



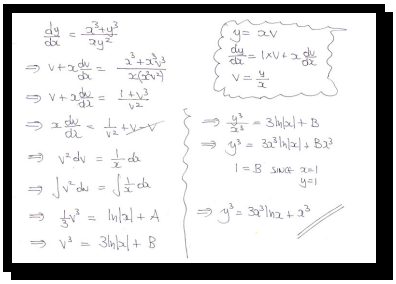
Question 49 (***)

By using a suitable substitution, solve the differential equation

$$\frac{dy}{dx}=\frac{x^3+y^3}{xy^2},$$

subject to the condition $y=1$ at $x=1$.

$$\boxed{y^3=x^3\left(3\ln x+1\right)}$$



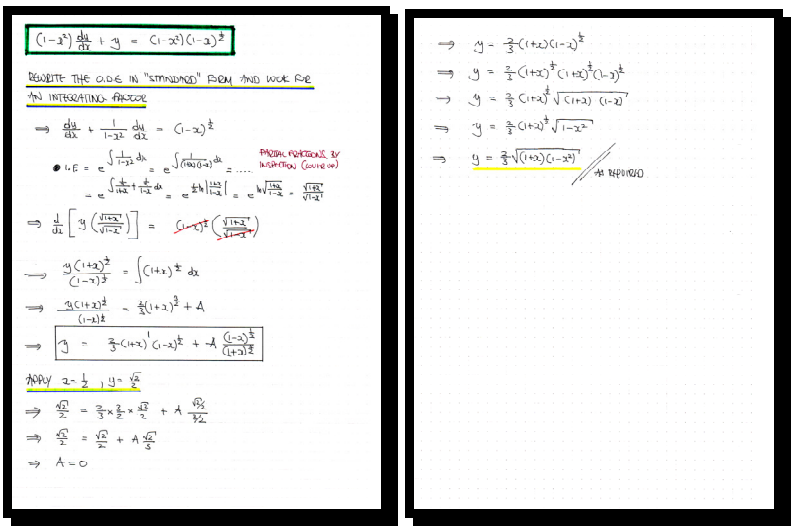
Question 50 (***)

$$\left(1-x^2\right)\frac{dy}{dx}+y=\left(1-x^2\right)(1-x)^{\frac{1}{2}}\ ,\ -1<x<1\,.$$

Given that $y=\dfrac{\sqrt{2}}{2}$ at $x=\dfrac{1}{2}$, show that the solution of the above differential equation can be written as

$$y=\frac{2}{3}\sqrt{\left(1-x^2\right)(1+x)}\,.$$

$\boxed{}$, $\boxed{\text{proof}}$



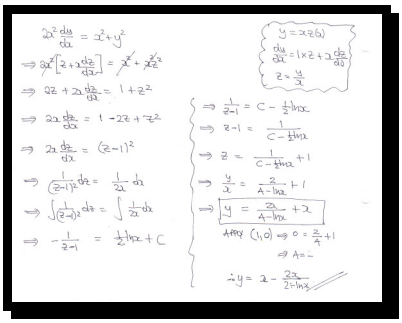
Question 51 (***)

By using a suitable substitution, solve the differential equation

$$2x^2\frac{dy}{dx}=x^2+y^2,\ x>0,$$

subject to the condition $y(1)=0$.

$$\boxed{y=x-\frac{2x}{2+\ln x}}$$



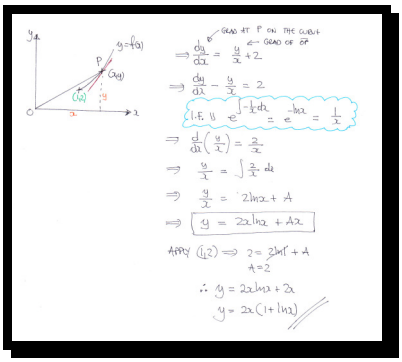
Question 52 (***)

The general point P lies on the curve with equation $y = f(x)$.

The gradient of the curve at P is 2 more than the gradient of the straight line segment OP.

Given further that the curve passes through $Q(1,2)$, express y in terms of x.

$$\boxed{y = 2x(1+\ln x)}$$



Question 53 (***)

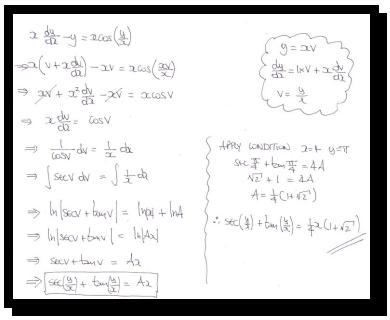
By using a suitable substitution, solve the differential equation

$$x\frac{dy}{dx}-y=x\cos\left(\frac{y}{x}\right),\ x\neq 0,$$

subject to the condition $y(4)=\pi$.

The final answer may not involve natural logarithms.

$$\boxed{\sec\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)=\frac{1}{4}x\left(1+\sqrt{2}\right)}$$



Question 54 **(***)**

The curve C has equation $y=f(x)$ and satisfies the differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-2y\left(2x^2-1\right)=3x^3\,\mathrm{e}^x\,,\quad x\neq 0$$

is to be solved subject to the boundary conditions $y=\frac{3}{2}$, $\frac{dy}{dx}=\frac{1}{2}$ at $x=1$.

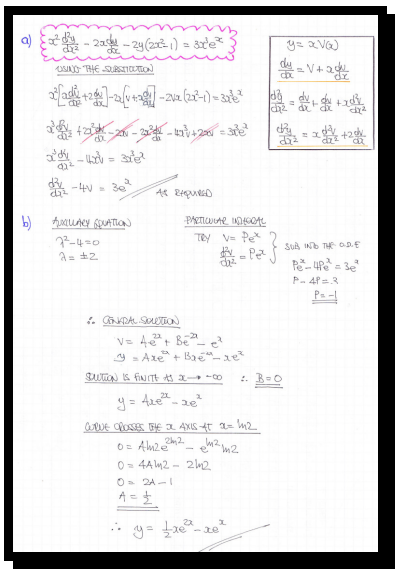
a) Show that the substitution $y=xv$, where v is a function of x transforms the above differential equation into

$$\frac{d^2v}{dx^2}-4v=3\mathrm{e}^x\,.$$

It is further given that C meets the x axis at $x=\ln 2$ and has a finite value for y as x gets infinitely negatively large.

b) Express the equation of C in the form $y=f(x)$.

$$\boxed{y=\frac{1}{2}x\mathrm{e}^{2x}-x\mathrm{e}^x}$$

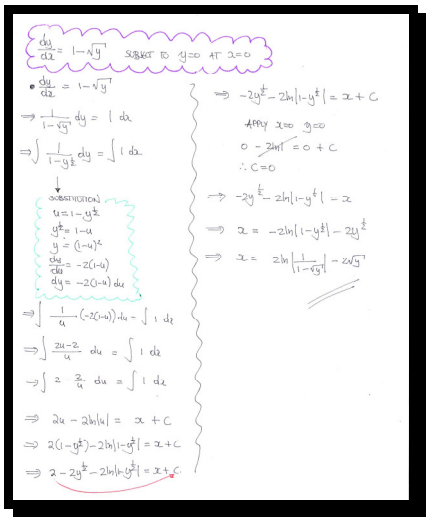


Question 55 (***)

$$\frac{dy}{dx}=1-\sqrt{y}\ ,\ \ y\geq 0\ ,\ \ y\neq 1\ .$$

Find the solution of the above differential equation subject to the condition $y=0$ at $x=0$, giving the answer in the form $x=f\left(y\right)$.

$$\boxed{x=2\ln\left|\frac{1}{1-\sqrt{y}}\right|-2\sqrt{y}}$$

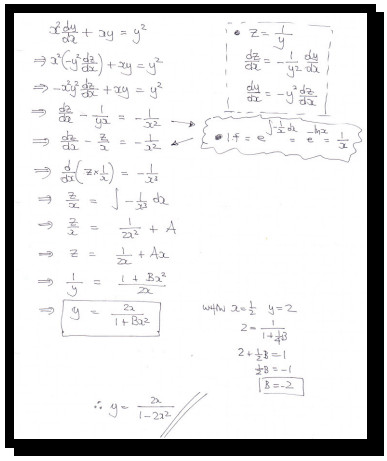


Question 56 (***)

Solve the differential equation

$$x^2\frac{dy}{dx}+xy=y^2,\ y\left(\frac{1}{2}\right)=2.$$

$$\boxed{y=\frac{2x}{1-2x^2}}$$

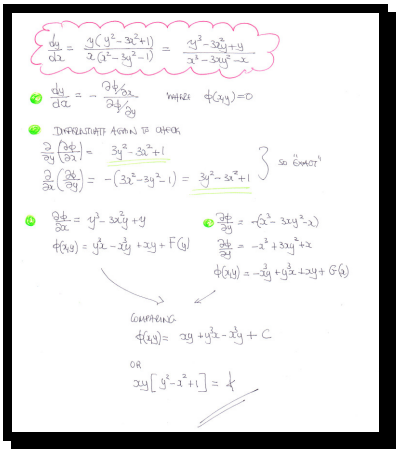


Question 57 (***)

Find a general solution of the following differential equation

$$\frac{dy}{dx}=\frac{y\left(y^2-3x^2+1\right)}{x\left(x^2-3y^2-1\right)}.$$

$$\boxed{xy\left(x^2-y^2-1\right)=\text{constant}}$$



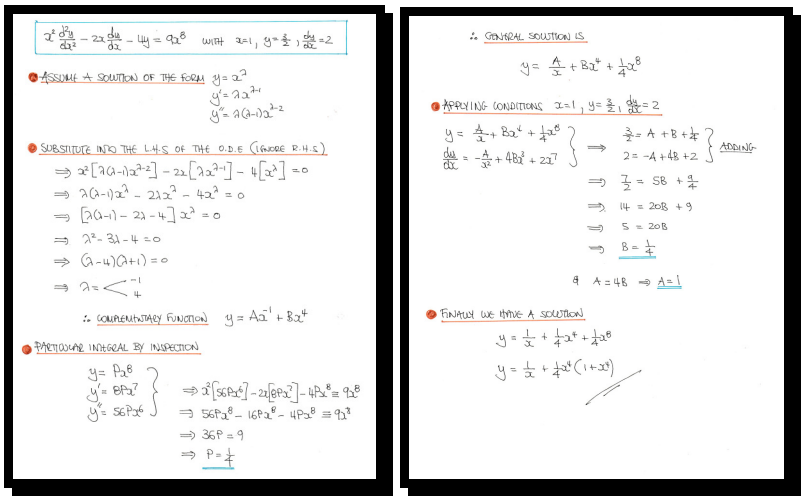
Question 58 (***)

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=9x^8.$$

Determine the solution of the above differential equation subject to the boundary conditions

$$y=\frac{3}{2},\ \frac{dy}{dx}=2 \text{ at } x=1.$$

$$\boxed{},\ \boxed{y=\frac{1}{4}x^4\left(x^4+1\right)+\frac{1}{x}}$$



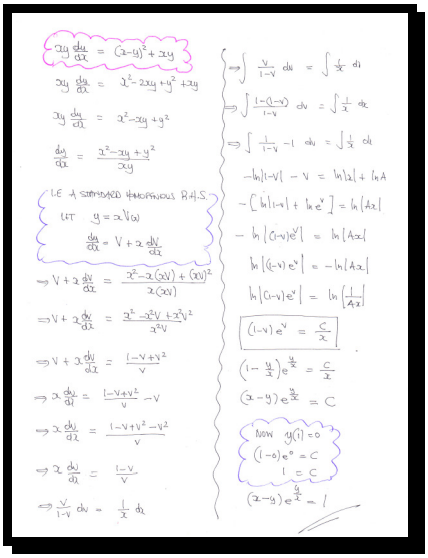
Question 59 (***)

$$xy\frac{dy}{dx}=(x-y)^2+xy\,,\quad y(1)=0\,.$$

Show that the solution of the above differential equation is

$$(x-y)\mathrm{e}^{\frac{y}{x}}=1\,.$$

proof



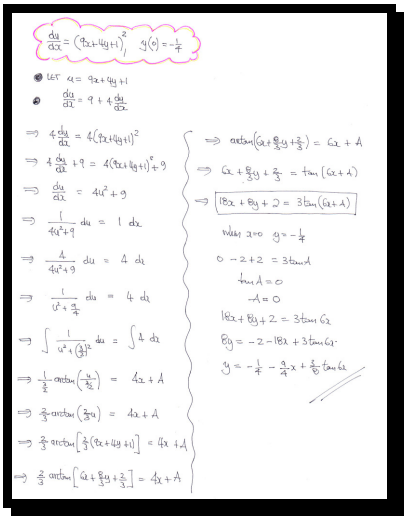
Question 60 (***)

Solve the differential equation

$$\frac{dy}{dx}=(9x+4y+1)^2,\quad y(0)=-\frac{1}{4}.$$

Give the answer in the form $y=f(x)$.

$$\boxed{y=-\frac{1}{4}-\frac{9}{4}x+\frac{3}{8}\tan 6x}$$

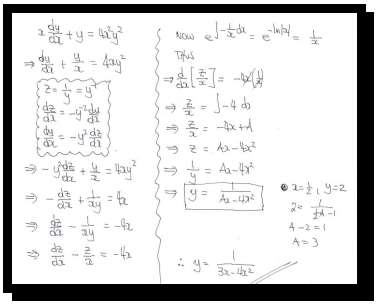


Question 61 (***)

Solve the differential equation

$$x\frac{dy}{dx}+y=4x^2y^2\,,\quad y\left(\frac{1}{2}\right)=2\,.$$

$$\boxed{y=\frac{1}{3x-4x^2}}$$



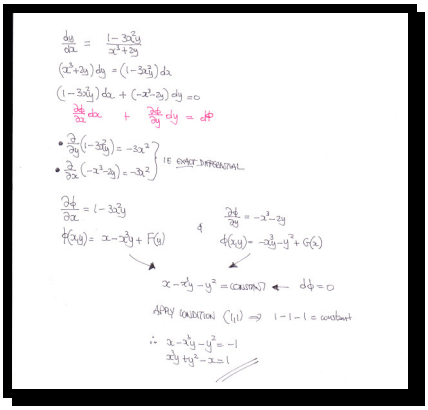
Question 62 (***)

Find the solution of the following differential equation

$$\frac{dy}{dx}=\frac{1-3x^2y}{x^3+2y}\,,$$

subject to the boundary condition $y=1$ at $x=1$.

$$\boxed{x^3y+y^2-x=1}$$



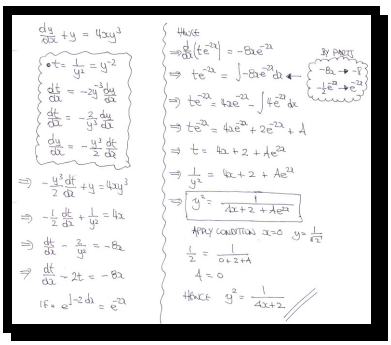
Question 63 (***)

Solve the differential equation

$$\frac{dy}{dx}+y=4xy^3,\ y(0)=\frac{1}{\sqrt{2}}.$$

Give the answer in the form $y^2=f(x)$.

$$\boxed{y^2=\frac{1}{4x+2}}$$



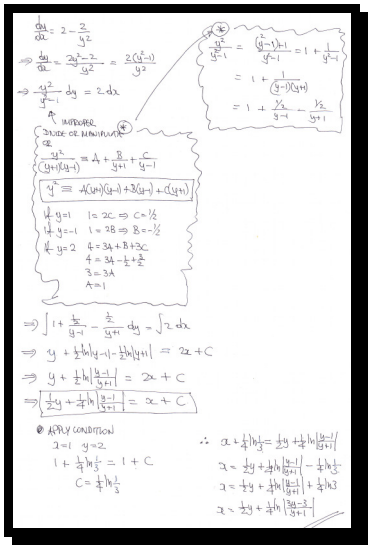
Question 64 (***)

Solve the differential equation

$$\frac{dy}{dx}=2-\frac{2}{y^2},$$

subject to the condition $y=2$ at $x=1$, giving the answer in the form $x=f\left(y\right)$.

$$\boxed{x=\frac{1}{2}y+\frac{1}{4}\ln\left|\frac{3y-3}{y+1}\right|}$$



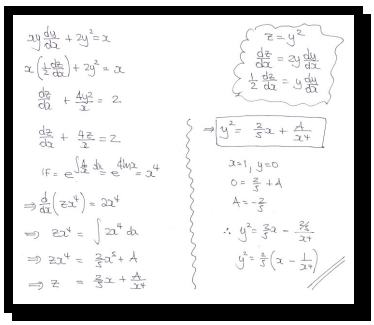
Question 65 (***)

By using a suitable substitution, solve the differential equation

$$xy\frac{dy}{dx}+2y^2=x,\ y(1)=0.$$

Give the answer in the form $y^2=f(x)$.

$$\boxed{y^2=\frac{2}{5}\left(x-\frac{1}{x^4}\right)}$$

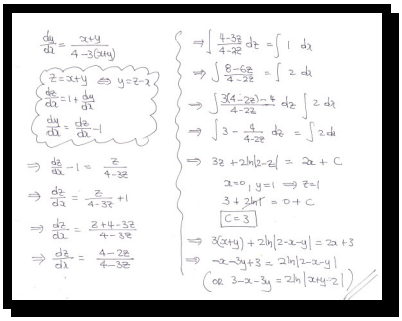


Question 67 (***)

Use a suitable substitution to solve the differential equation

$$\frac{dy}{dx}=\frac{x+y}{4-3(x+y)},\ y(0)=1.$$

$$\boxed{2\ln|x+y-2|=3-x-3y}$$



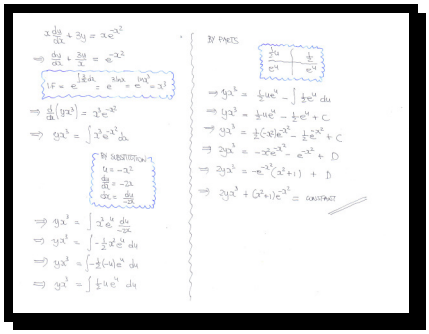
Question 68 (***)

$$x\frac{dy}{dx}+3y=xe^{-x^2},\ x>0.$$

Show clearly that the general solution of the above differential equation can be written in the form

$$2yx^3+\left(x^2+1\right)e^{-x^2}=\text{constant}.$$

$$\boxed{\text{proof}}$$



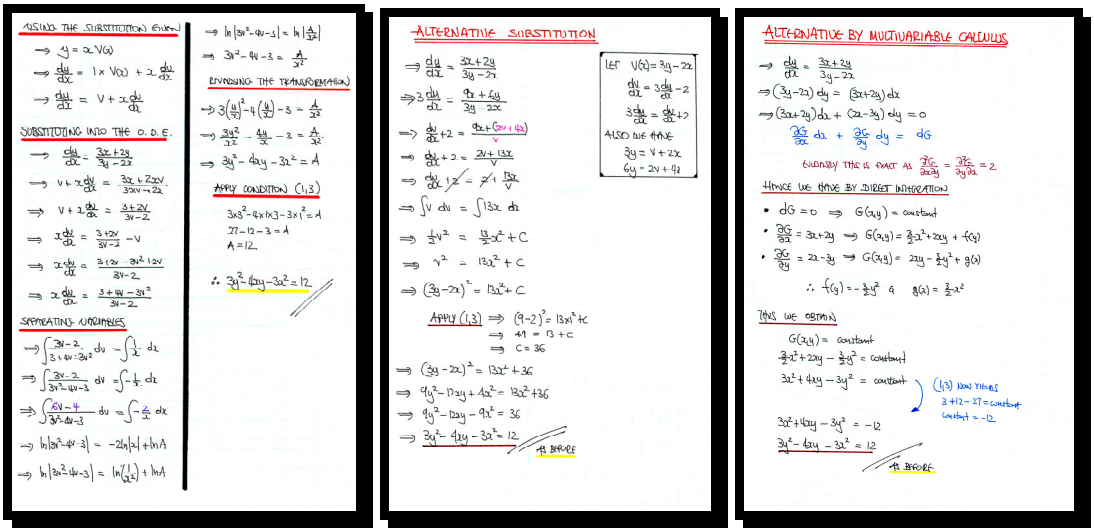
Question 69 (***)

Solve the following differential equation

$$\frac{dy}{dx}=\frac{3x+2y}{3y-2x},\ y(1)=3.$$

Give the final answer in the form $F(x,y)=12$

$\square$, $\boxed{3y^2-4xy-3x^2=12}$

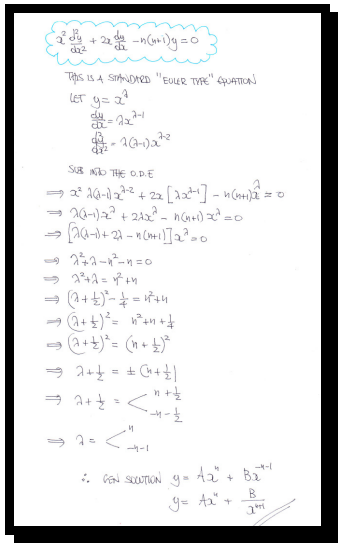


Question 70 (***)

Find the general solution of the following differential equation

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-n(n+1)y=0.$$

$$\boxed{y=Ax^n+\frac{B}{x^{n+1}}}$$

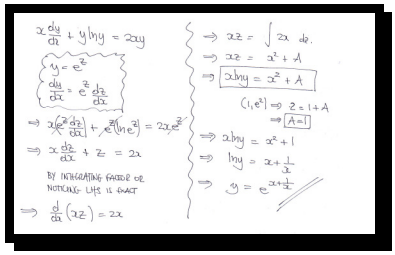


Question 71 (***)

Use the substitution $y=\mathrm{e}^z$ to solve the differential equation

$$x\frac{dy}{dx}+y\ln y=2xy,\ \ y(1)=\mathrm{e}^2.$$

$$\boxed{y=\mathrm{e}^{x+\frac{1}{x}}}$$



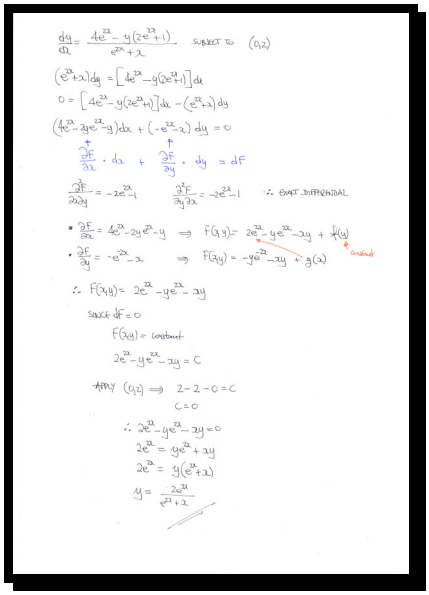
Question 72 (***)

Solve the differential equation

$$\frac{dy}{dx}=\frac{4\mathrm{e}^{2x}-y\left(2\mathrm{e}^{2x}+1\right)}{\mathrm{e}^{2x}+x},$$

subject to the boundary condition $y=2$ at $x=0$.

$$\boxed{y=\frac{2\mathrm{e}^{2x}}{\mathrm{e}^{2x}+x}}$$



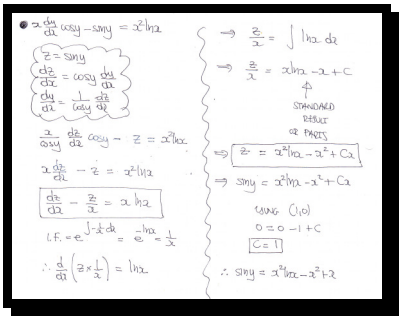
Question 73 (***)

Use the substitution $z = \sin y$ to solve the differential equation

$$x\frac{dy}{dx}\cos y - \sin y = x^2 \ln x\,, \quad y(1) = 0$$

subject to the condition $y = 0$ at $x = 1$.

$$\boxed{\sin y = x^2 \ln x - x^2 + x}$$



Question 74 (*+)**

The differential equation

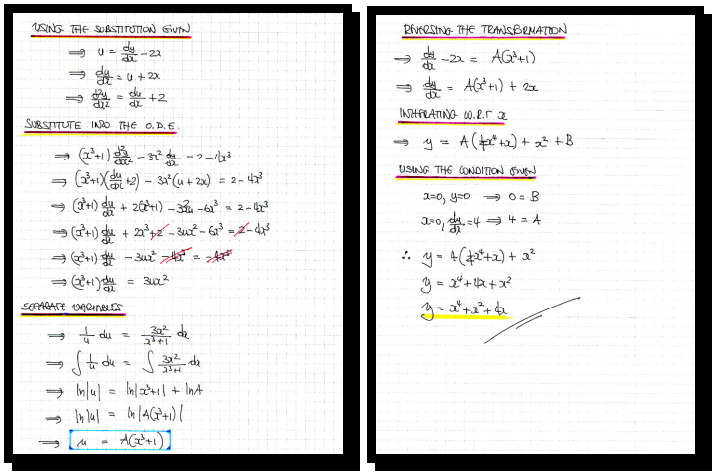
$$\left(x^3+1\right)\frac{d^2y}{dx^2}-3x^2\frac{dy}{dx}=2-4x^3,$$

is to be solved subject to the boundary conditions $y=0$, $\frac{dy}{dx}=4$ at $x=0$.

Use the substitution $u=\frac{dy}{dx}-2x$, where u is a function of x, to show that the solution of the above differential equation is

$$y=x^4+x^2+4x.$$

[ ], proof



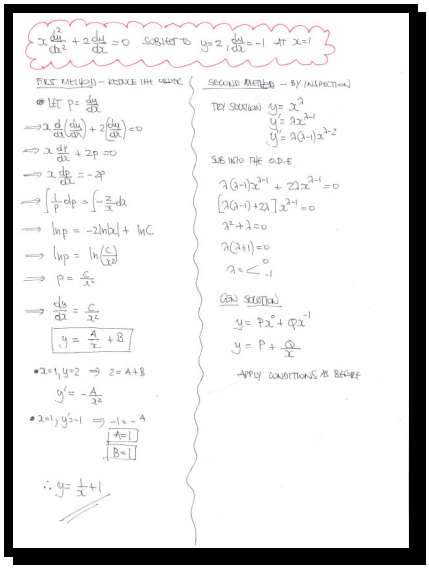
Question 75 (*+)**

Solve the differential equation

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0,$$

subject to the boundary conditions $y=2$, $\frac{dy}{dx}=-1$ at $x=1$.

$$\boxed{y=\frac{2\mathrm{e}^{2x}}{\mathrm{e}^{2x}+x}}$$



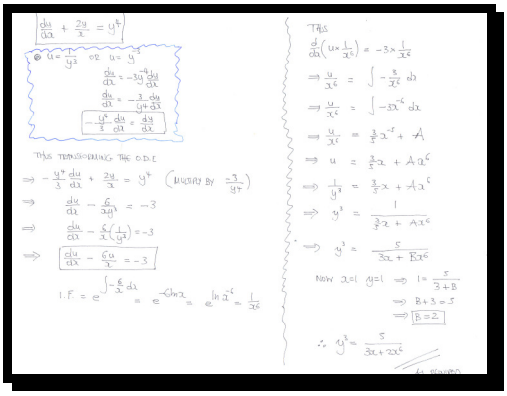
Question 76 (*+)**

$$\frac{dy}{dx}+\frac{2y}{x}=y^4,\ x>0,\ y>0.$$

Given that $y(1)=1$, show that

$$y^3=\frac{5}{3x+2x^6}.$$

proof



Question 77 (*+)**

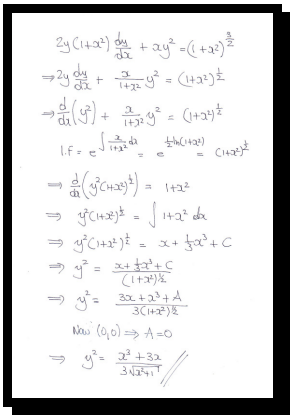
A curve with equation $y=f\left(x\right)$ passes through the origin and satisfies the differential equation

$$2y\left(1+x^2\right)\frac{dy}{dx}+xy^2=\left(1+x^2\right)^{\frac{3}{2}}.$$

By finding a suitable integrating factor, or otherwise, show that

$$y^2=\frac{x^3+3x}{3\sqrt{x^2+1}}.$$

proof



Question 78 (*+)**

Given that if $x=\mathrm{e}^t$ and $y=f\left(x\right)$, show clearly that …

a) … $x\dfrac{dy}{dx}=\dfrac{dy}{dt}$.

b) … $x^2\dfrac{d^2y}{dx^2}=\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}$.

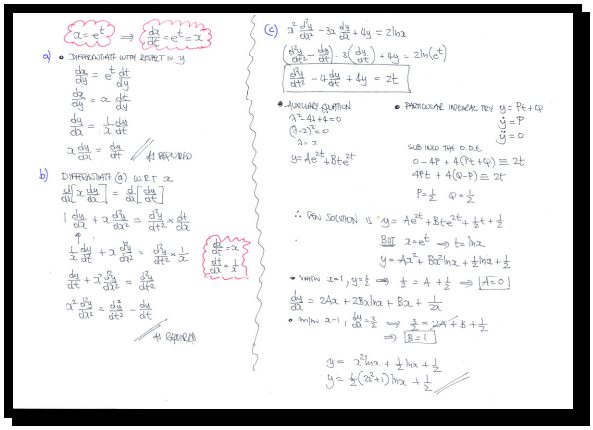
The following differential equation is to be solved

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2\ln x$$

subject to the boundary conditions $y=\dfrac{1}{2}, \dfrac{dy}{dx}=\dfrac{3}{2}$ at $x=1$.

c) Use the substitution $x=\mathrm{e}^t$ to solve the above differential equation.

$$\boxed{y=\frac{1}{2}+\frac{1}{2}\left(2x^2+1\right)\ln x}$$



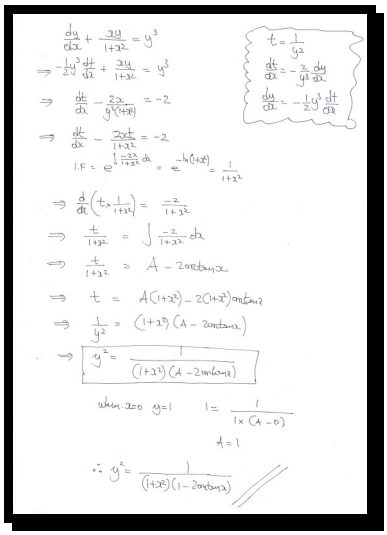
Question 79 (*+)**

Solve the differential equation

$$\frac{dy}{dx}+\frac{xy}{1+x^2}=y^3,\ y(0)=1.$$

Give the answer in the form $y^2=f(x)$.

$$\boxed{y^2=\frac{1}{\left(1+x^2\right)\left(1-2\arctan x\right)}}$$



Question 80 (*+)**

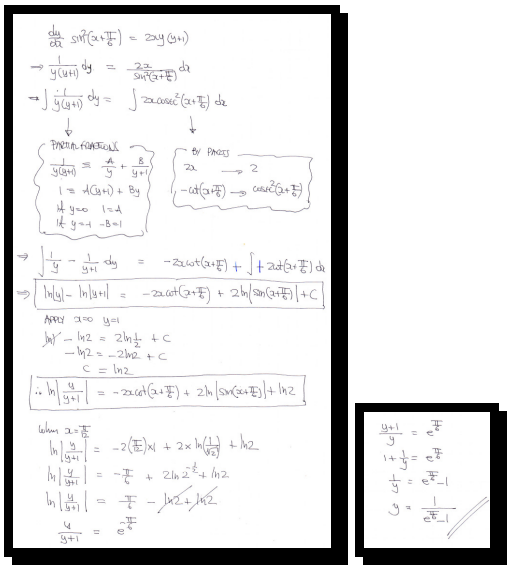
The function $y=f\left(x\right)$ satisfies the differential equation

$$\frac{dy}{dx}\sin^2\left(x+\frac{\pi}{6}\right)=2xy\left(y+1\right),$$

subject to the condition $y=1$ at $x=0$.

Find the exact value of y when $x=\dfrac{\pi}{12}$.

$$\boxed{y=\frac{1}{e^{\frac{\pi}{6}}-1}}$$

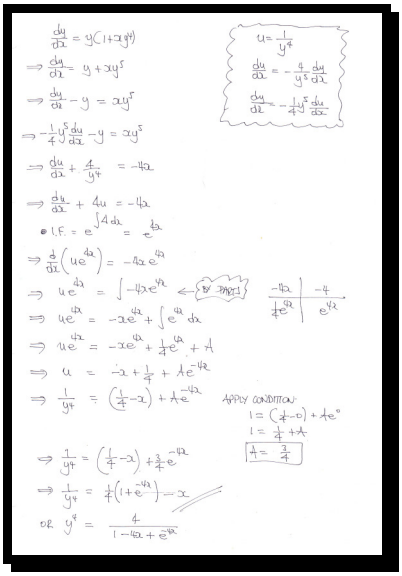


Question 81 (*+)**

Solve the differential equation

$$\frac{dy}{dx}=y\left(1+xy^4\right),\ y\left(0\right)=1.$$

$$\boxed{\frac{1}{y^4}=\frac{1}{4}\left(1+3\mathrm{e}^{-4x}\right)-x}$$

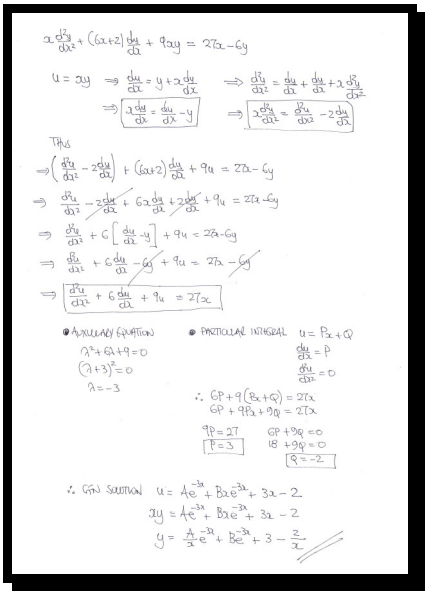


Question 82 (*+)**

$$x\frac{d^2y}{dx^2}+(6x+2)\frac{dy}{dx}+9xy=27x-6y\,.$$

Use the substitution $u=xy$, where u is a function of x, to find a general solution of the above differential equation.

$$\boxed{y=\frac{A}{x}\mathrm{e}^{-3x}+B\mathrm{e}^{-3x}+3-\frac{2}{x}}$$



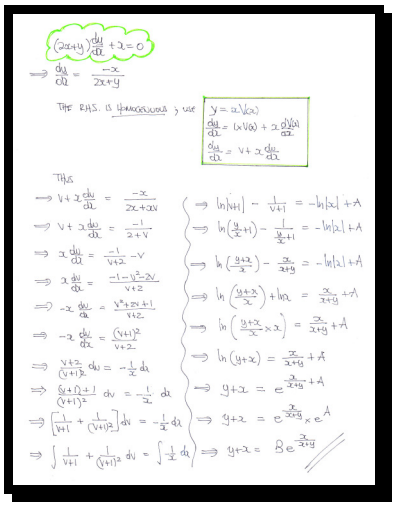
Question 83 (*+)**

Find a general solution for the following differential equation

$$(2x+y)\frac{dy}{dx}+x=0\,.$$

The final answer must not contain natural logarithms.

$$\boxed{y+x=A\,\mathrm{e}^{\frac{x}{x+y}}}$$



Question 84 (*+)**

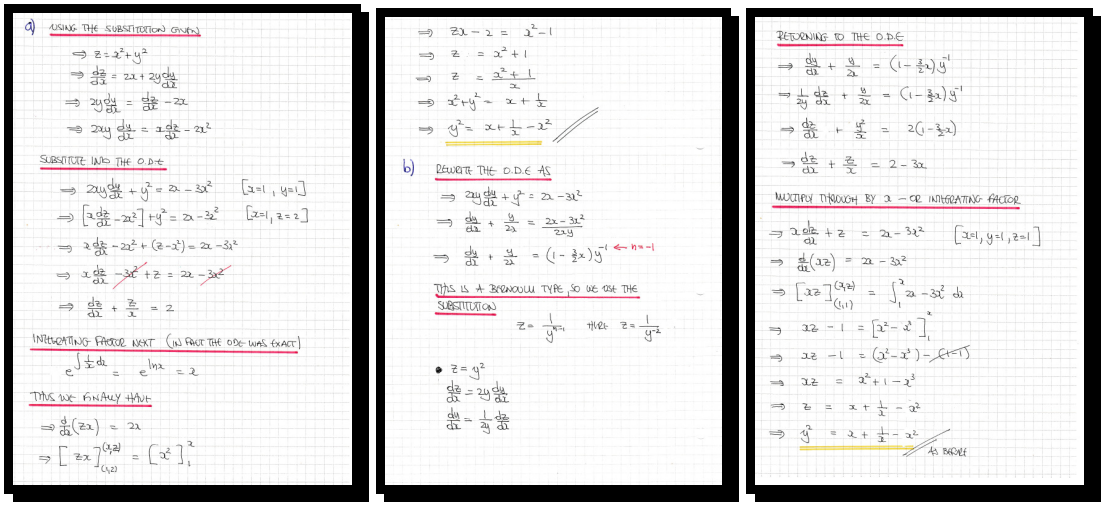
a) By using the substitution $z = x^2 + y^2$, solve the following differential equation

$$2xy\frac{dy}{dx} + y^2 = 2x - 3x^2,$$

subject to the condition $y = 1$ at $x = 1$.

b) Verify the answer to part **(a)** by using the substitution $z = y^2$ to solve the same differential equation and subject to the same condition.

$$\boxed{}, \quad \boxed{y^2 = x - x^2 + \frac{1}{x}}$$



Question 85 (*+)**

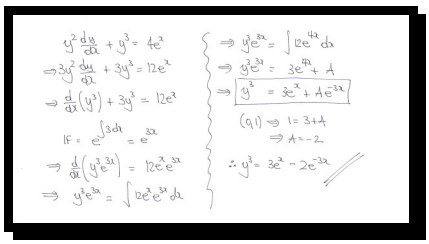
A curve with equation $y = f(x)$ passes through the point with coordinates $(0,1)$ and satisfies the differential equation

$$y^2 \frac{dy}{dx} + y^3 = 4\mathrm{e}^x .$$

By finding a suitable integrating factor, or otherwise, show that

$$y^3 = 3\mathrm{e}^x - 2\mathrm{e}^{-3x} .$$

proof



Question 86 **(***+)**

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}\tan x-y\sec^4 x=0.$$

The above differential equation is to be solved by a substitution.

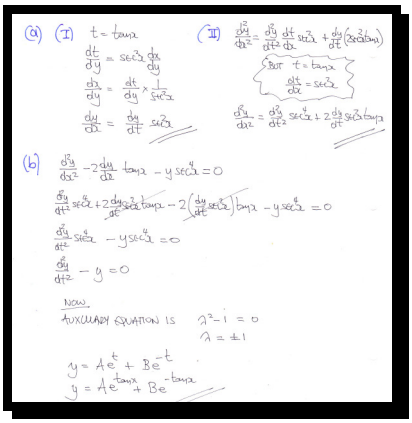
a) If $t=\tan x$ show that …

i. … $\dfrac{dy}{dx}=\dfrac{dy}{dt}\sec^2 x$

ii. … $\dfrac{d^2y}{dx^2}=\dfrac{d^2y}{dt^2}\sec^4 x+2\dfrac{dy}{dt}\sec^2 x\tan x$

b) Use the results obtained in part **(a)** to find a general solution of the differential equation in the form $y=f\left(x\right)$.

$$\boxed{y=A\mathrm{e}^{\tan x}+B\mathrm{e}^{-\tan x}}$$



Question 87 (*+)**

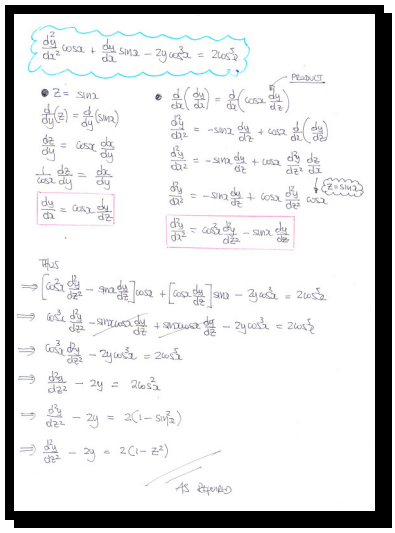
Show clearly that the substitution $z=\sin x$, transforms the differential equation

$$\frac{d^2y}{dx^2}\cos x+\frac{dy}{dx}\sin x-2y\cos^3 x=2\cos^5 x,$$

into the differential equation

$$\frac{d^2y}{dz^2}-2y=2\left(1-z^2\right)$$

proof

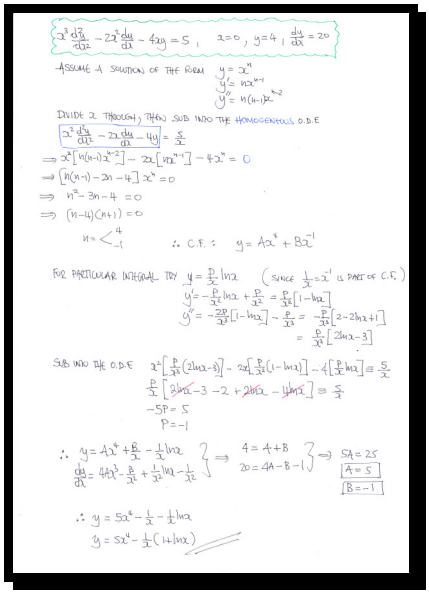


Question 88 (*+)**

$$x^3\frac{d^2y}{dx^2}-2x^2\frac{dy}{dx}-4xy=5\,.$$

Find the solution of the above differential equation subject to the boundary conditions $y=4$, $\dfrac{dy}{dx}=20$ at $x=0$.

$$\boxed{y=5x^4-\frac{1}{x}(1+\ln x)}$$

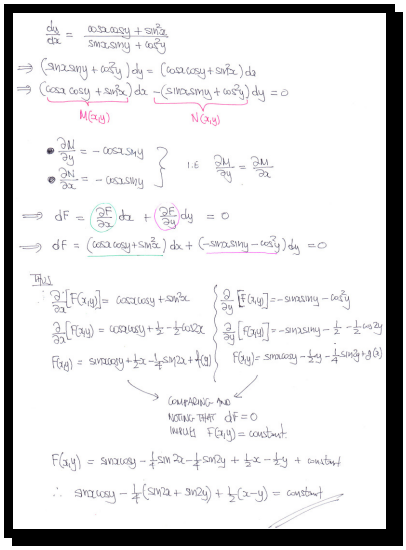


Question 89 (*+)**

Find a general solution of the following differential equation

$$\frac{dy}{dx}=\frac{\cos x\cos y+\sin^2 x}{\sin x\sin y+\cos^2 y}.$$

$$\boxed{\sin x\cos y-\frac{1}{4}\left(\sin 2x+\sin 2y\right)+\frac{1}{2}\left(x-y\right)=\text{constant}}$$



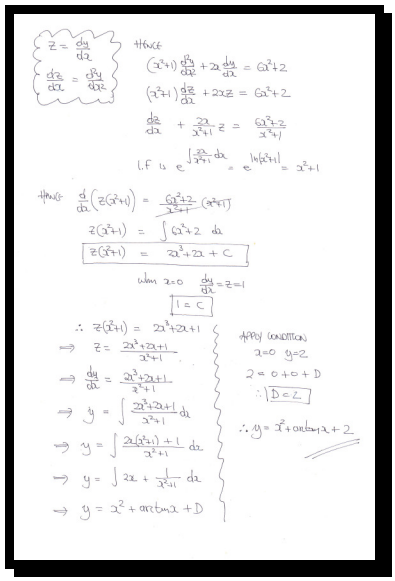
Question 90 (*+)**

By using the substitution $z=\dfrac{dy}{dx}$, or otherwise, solve the differential equation

$$\left(x^2+1\right)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=6x^2+2\,,$$

subject to the conditions $x=0$, $y=2$, $\dfrac{dy}{dx}=1$

$$\boxed{y=x^2+2+\arctan x}$$



Question 91 (****)

A curve C passes through the point $(1,1)$ and satisfies the differential equation

$$\frac{dy}{dx}-\frac{y}{x}=\frac{x^3}{4y^3},\ x>0,\ y>0,$$

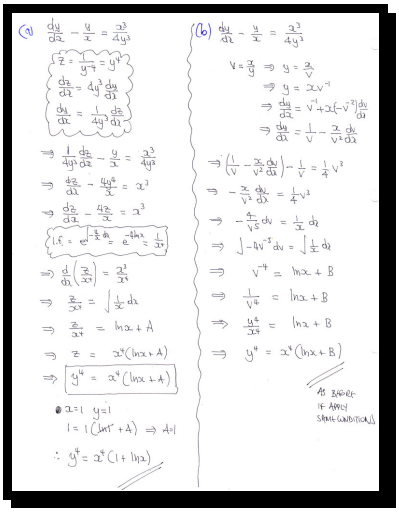
subject to the condition $y=1$ at $x=1$.

a) Find an equation of C by using the substitution $z=y^4$.

b) Find an equation of C by using the substitution $v=\frac{x}{y}$.

Give the answer in the form $y^4=f(x)$.

$$\boxed{y^4=x^4(1+\ln x)}$$

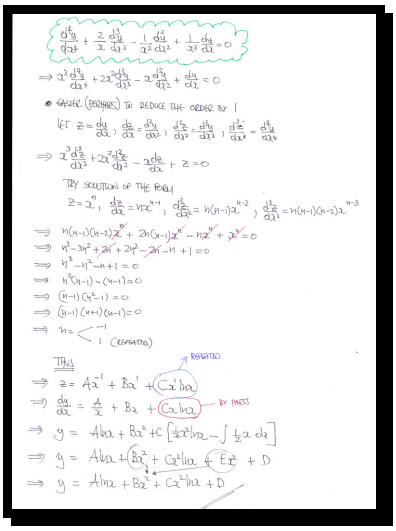


Question 92 (****)

Find the general solution of the following differential equation

$$\frac{d^4y}{dx^4}+\frac{2}{x}\frac{d^3y}{dx^3}-\frac{1}{x^2}\frac{d^2y}{dx^2}+\frac{1}{x^3}\frac{dy}{dx}=0.$$

$$\boxed{y=A\ln x+Bx^2+Cx^2\ln x+D}$$



Question 93 (****)

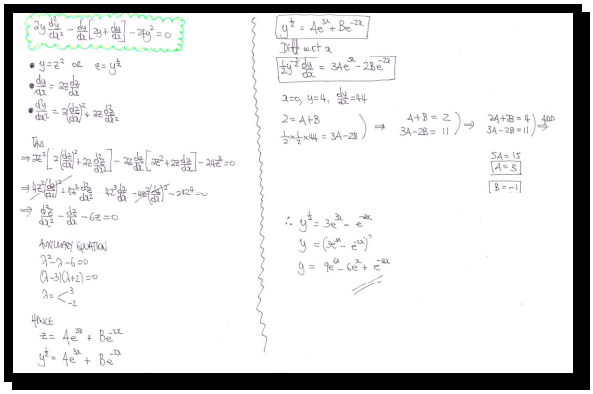
Use the substitution $z=\sqrt{y}$, where $y=f(x)$, to solve the differential equation

$$\frac{d^2y}{dx^2}+\frac{1}{y}\left(\frac{dy}{dx}\right)^2-5\frac{dy}{dx}+2y=0,$$

subject to the boundary conditions $y=4, \frac{dy}{dx}=44$ at $x=0$.

Give the answer in the form $y=f(x)$.

$$\boxed{y=9\mathrm{e}^{6x}-6\mathrm{e}^{x}+\mathrm{e}^{-4x}}$$

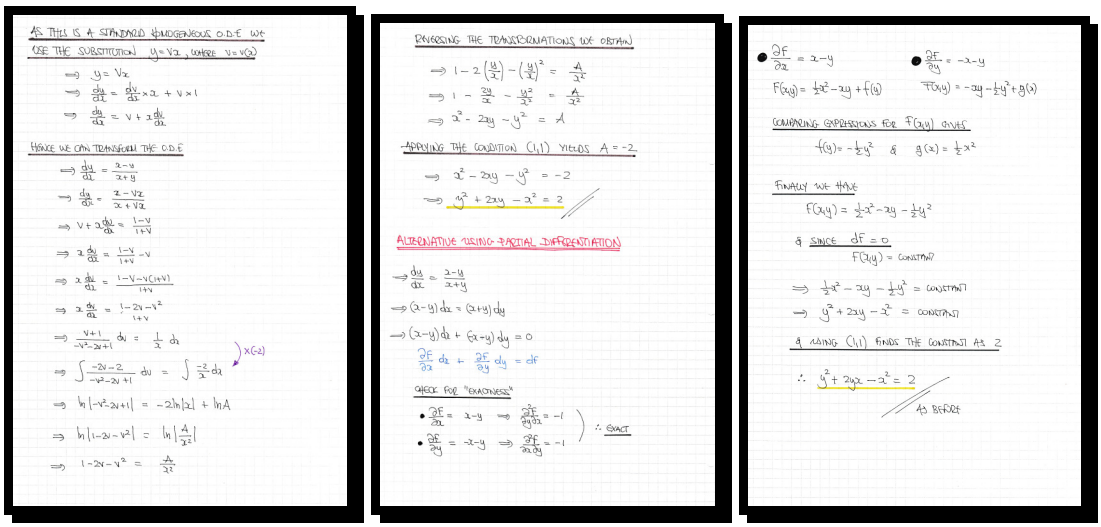


Question 94 (****)

Solve the differential equation

$$\frac{dy}{dx}=\frac{x-y}{x+y},\ y(1)=1.$$

$\boxed{}$, $\boxed{y^2+2xy-x^2=2}$



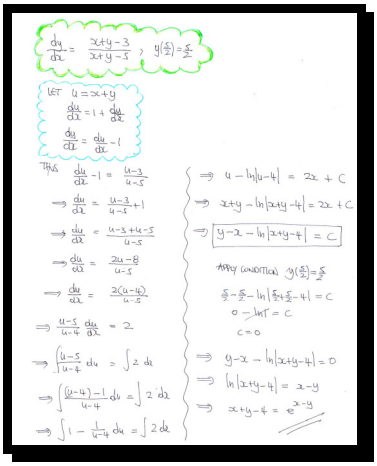
Question 95 (****)

Solve the differential equation

$$\frac{dy}{dx}=\frac{x+y-3}{x+y-5},$$

subject to the condition $y=\frac{5}{2}$ at $x=\frac{5}{2}$.

$$\boxed{x+y-4=\mathrm{e}^{x-y}}$$



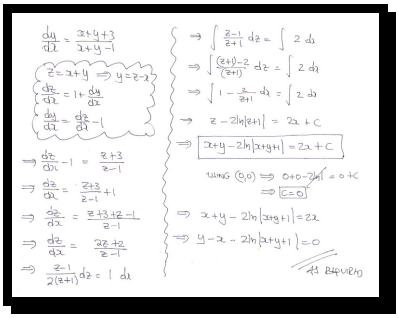
Question 96 (****)

$$\frac{dy}{dx}=\frac{x+y+3}{x+y-1},\ y(0)=0.$$

Show that the solution of the above differential equation is

$$y-x-2\ln(x+y+1)=0.$$

proof



Question 97 (****)

$$2x\frac{d^2y}{dx^2}+\left(1-3x^{\frac{1}{2}}\right)\frac{dy}{dx}+y=0.$$

The above differential equation is to be solved by a substitution.

a) Given that $y=f\left(x\right)$ and $t=x^{\frac{1}{2}}$, show clearly that …

i. … $\dfrac{dy}{dx}=\dfrac{1}{2t}\dfrac{dy}{dt}$.

ii. … $\dfrac{d^2y}{dx^2}=\dfrac{1}{4t^2}\dfrac{d^2y}{dt^2}-\dfrac{1}{4t^3}\dfrac{dy}{dt}$.

b) Hence show further that the differential equation

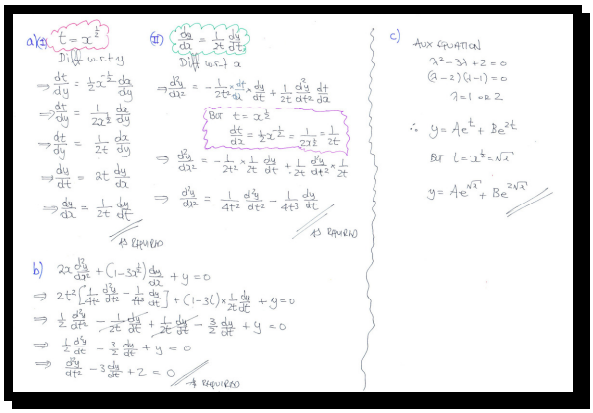
$$2x\frac{d^2y}{dx^2}+\left(1-3x^{\frac{1}{2}}\right)\frac{dy}{dx}+y=0,$$

can be transformed to the differential equation

$$\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=0.$$

c) Find a general solution of the **original** differential equation, giving the answer in the form $y=f\left(x\right)$.

$$\boxed{y=A\mathrm{e}^{\sqrt{x}}+B\mathrm{e}^{2\sqrt{x}}}$$



Question 98 (****)

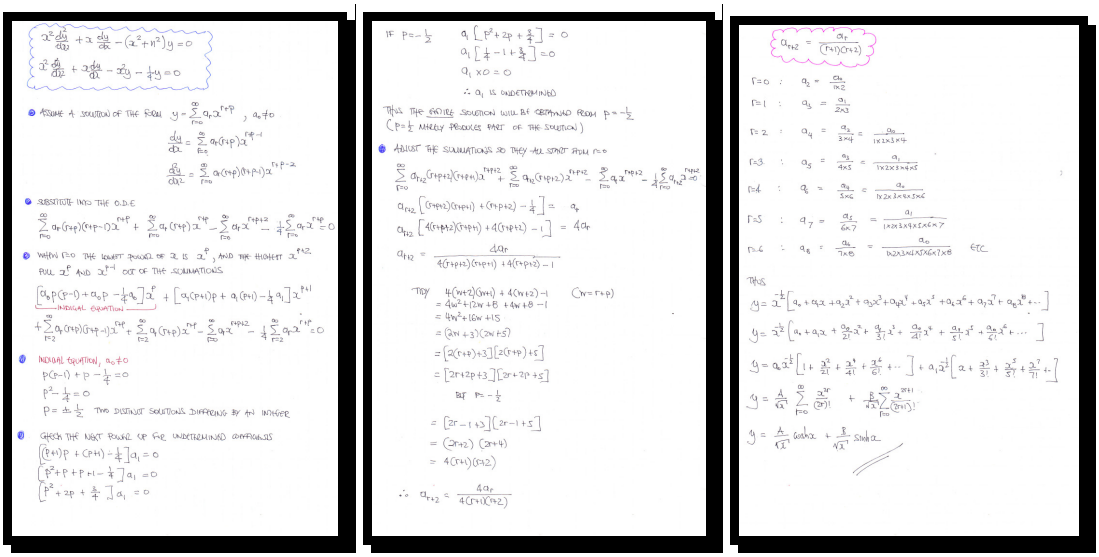
$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-\left(x^2+n^2\right)y=0\,.$$

The above differential equation is known as modified Bessel's Equation.

Use the Frobenius method to show that the general solution of this differential equation, for $n=\frac{1}{2}$, is

$$y=x^{-\frac{1}{2}}\left[A\cosh x+B\sinh x\right].$$

proof



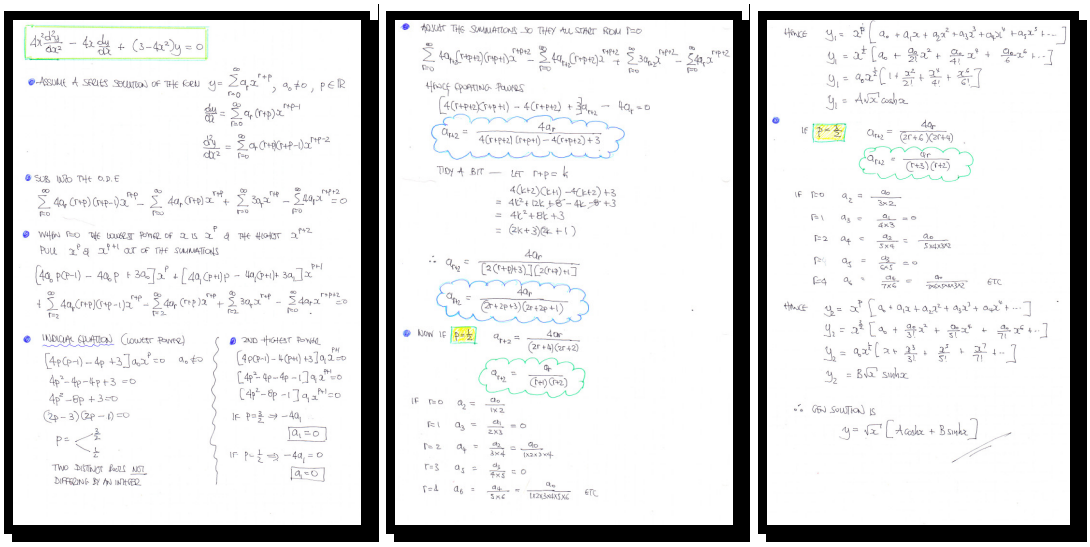
Question 99 (****)

Use the Frobenius method to find a general solution, as an infinite series, for the following differential equation

$$4x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(3-4x^2\right)y=0\,.$$

Give the final answer in terms of elementary functions.

$$\boxed{y=\sqrt{x}\left(A\cosh x+B\sinh x\right)}$$



Question 100 (****)

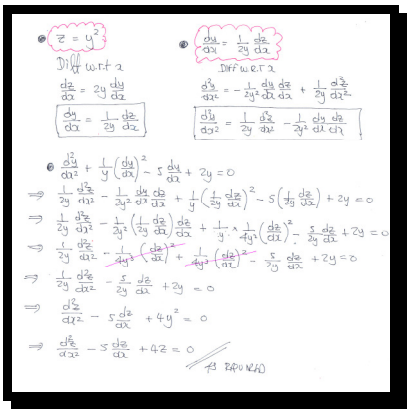
Show clearly that the substitution $z = y^2$, where $y = f(x)$, transforms the differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 2y = 0,$$

into the differential equation

$$\frac{d^2z}{dx^2} - 5\frac{dz}{dx} + 4z = 0$$

proof



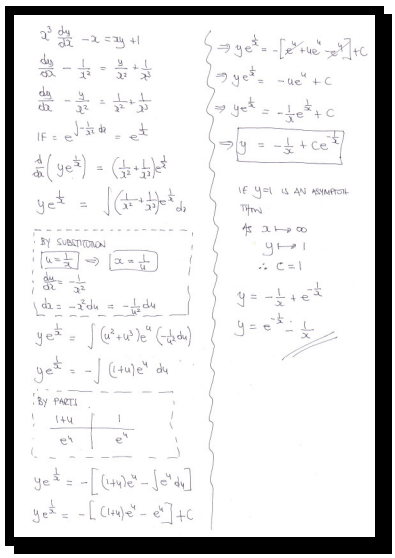
Question 101 (****)

The curve with equation $y = f(x)$ has the line $y = 1$ as an asymptote and satisfies the differential equation

$$x^3 \frac{dy}{dx} - x = xy + 1, \ x \neq 0 .$$

Solve the above differential equation, giving the solution in the form $y = f(x)$.

$$\boxed{y = \mathrm{e}^{-\frac{1}{x}} - \frac{1}{x}}$$



Question 102 (****)

Given that if $x=t^{\frac{1}{2}}$, where $y=f\left(x\right)$, show clearly that

a) $\dfrac{dy}{dx}=2t^{\frac{1}{2}}\dfrac{dy}{dt}$.

b) $\dfrac{d^2y}{dx^2}=4t\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}$.

The following differential equation is to be solved

$$x\frac{d^2y}{dx^2}-\left(8x^2+1\right)\frac{dy}{dx}+12x^3y=12x^5,$$

subject to the boundary conditions $y=\dfrac{10}{3}$, $\dfrac{d^2y}{dx^2}=10$ at $x=0$.

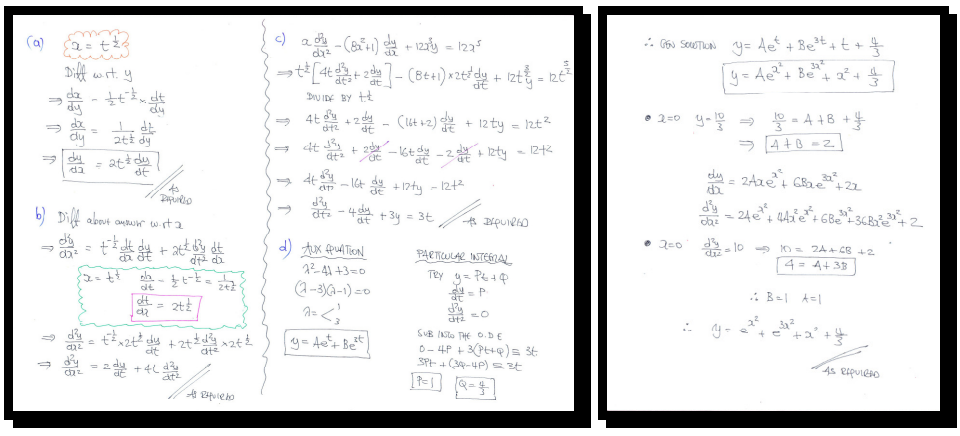
c) Show further that the substitution $x=t^{\frac{1}{2}}$, where $y=f\left(x\right)$, transforms the above differential equation into the differential equation

$$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=3t.$$

d) Show that a solution of the **original** differential equation is

$$y=\mathrm{e}^{3x^2}+\mathrm{e}^{x^2}+x^2+\frac{4}{3}.$$

proof



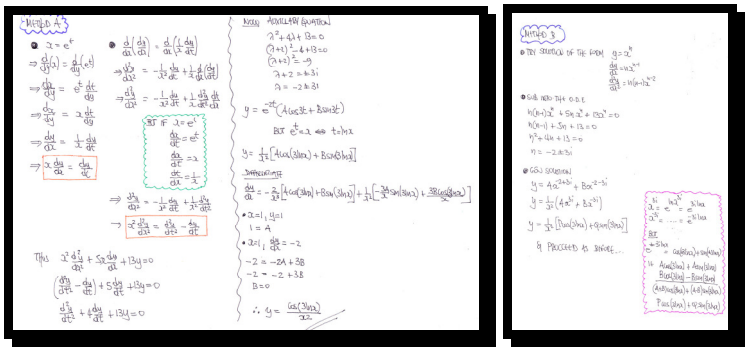
Question 103 (****)

The curve with equation $y = f(x)$ satisfies

$$x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = 0, \ x > 0 .$$

By using the substitution $x = \mathrm{e}^t$, or otherwise, determine an equation for $y = f(x)$, given further that $y = 1$ and $\frac{dy}{dx} = -2$ at $x = 1$.

$$\boxed{y = \frac{\cos(3\ln x)}{x^2}}$$



Question 104 (****)

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}\cot x+2y\operatorname{cosec}^2 x=2\cos x-2\cos^3 x\,.$$

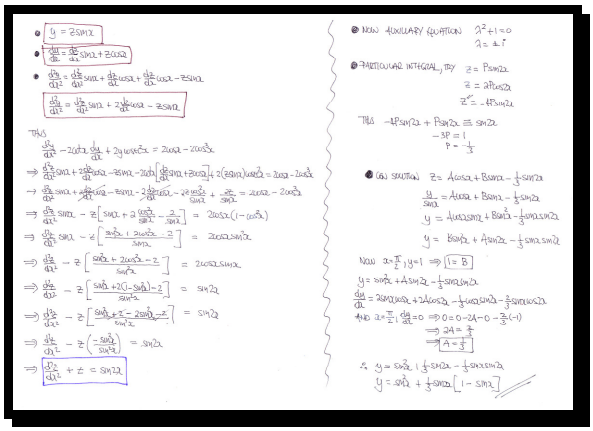
Use the substitution $y=z\sin x$, where z is a function of x, to solve the above differential equation subject to the boundary conditions $y=1$, $\dfrac{dy}{dx}=0$ at $x=\dfrac{\pi}{2}$.

Give the answer in the form

$$y=a\sin^2 x+b(1-\sin x)\sin 2x\,,$$

where a and b are constants to be found.

$$\boxed{a=1}\,,\quad \boxed{b=\frac{1}{3}}$$

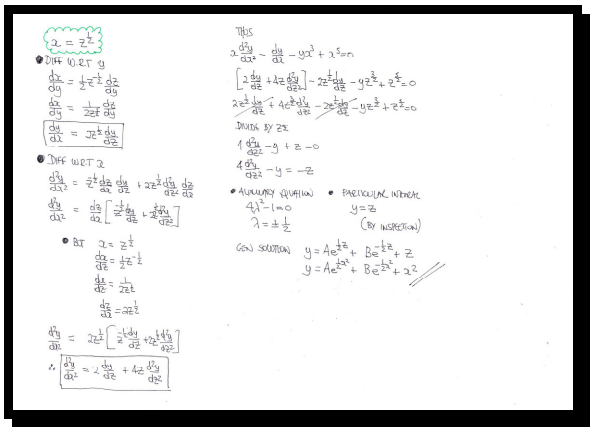


Question 105 (****)

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-x^3y+x^5=0\,.$$

Use the substitution $x=z^{\frac{1}{2}}$, where $y=f\left(x\right)$, to find a general solution of the above differential equation.

$$\boxed{y=A\mathrm{e}^{\frac{1}{2}x^2}+B\mathrm{e}^{-\frac{1}{2}x^2}+x^2}$$



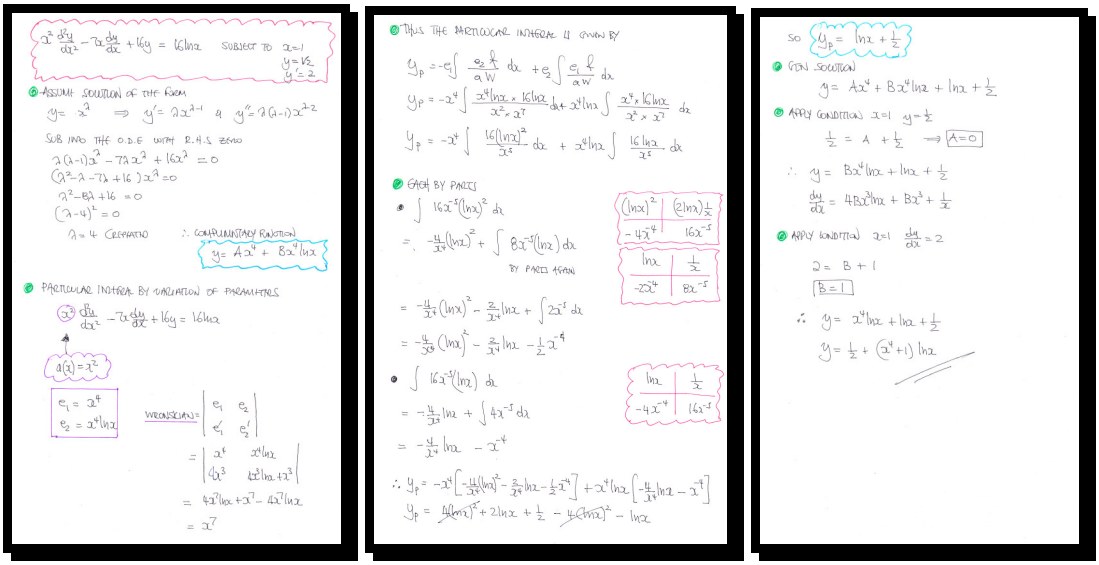
Question 106 (****)

Use variation of parameters to determine the specific solution of the following differential equation

$$x^2\frac{d^2y}{dx^2}-7x\frac{dy}{dx}+16y=16\ln x\,,$$

given further that $y=\frac{1}{2}$, $\frac{dy}{dx}=2$ at $x=1$.

$$\boxed{y=\frac{1}{2}+\left(1+x^4\right)\ln x}$$



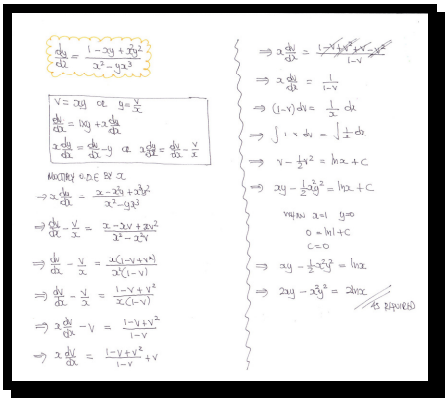
Question 107 (**+)**

Solve the differential equation

$$\frac{dy}{dx}=\frac{1-xy+x^2y^2}{x^2-yx^3},\ x>0,$$

subject to the condition $y(1)=0$.

$$\boxed{2xy-x^2y^2=2\ln x}$$



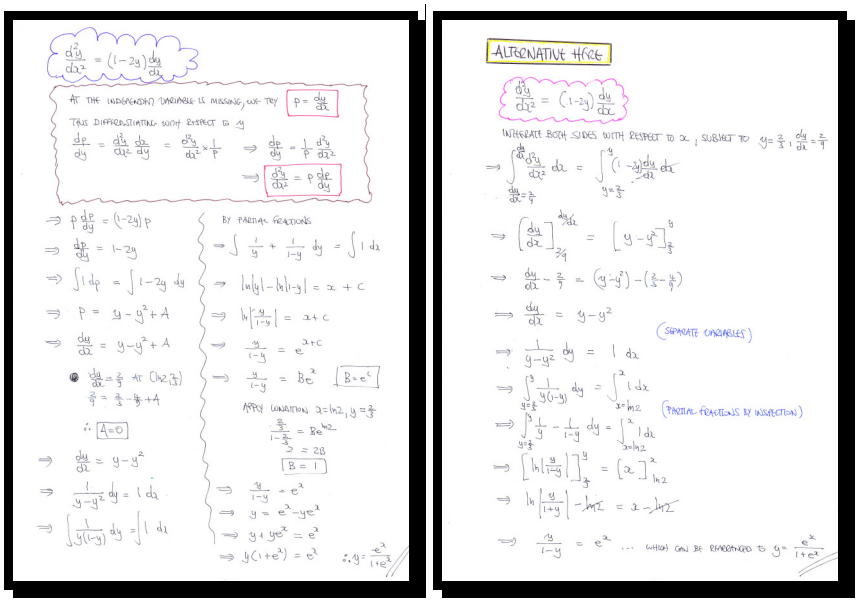
Question 108 **(****+)**

The curve C, has gradient $\frac{2}{9}$ at the point with coordinates $\left(\ln 2, \frac{2}{3}\right)$, and satisfies the differential relationship

$$\frac{d^2y}{dx^2} = (1-2y)\frac{dy}{dx}, \quad y < \frac{1}{2}.$$

Find an equation for C, giving the answer in the form $y = f(x)$.

$$\boxed{y = \frac{e^x}{1+e^x} = \frac{1}{e^x + e^{-x}} = \frac{1}{2}\operatorname{sech} x}$$



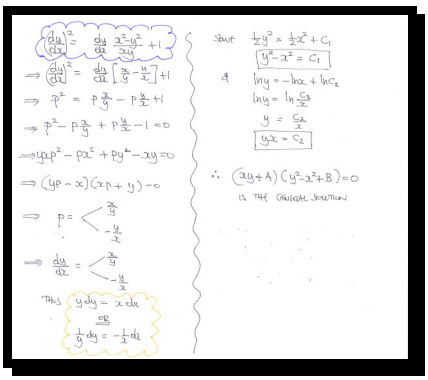
Question 109 (**+)**

By writing $\dfrac{dy}{dx}=p$ and seeking a suitable factorization find a general solution for the non linear differential equation

$$\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}\left(\frac{x^2-y^2}{xy}\right)+1.$$

Give the solution in the form $F(x,y)G(x,y)=0$.

$$\boxed{(xy+A)\left(x^2-y^2+B\right)=0}$$

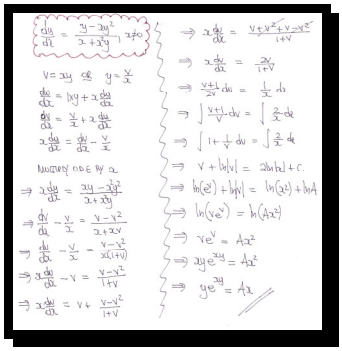


Question 110 (**+)**

Find a general solution for the differential equation

$$\frac{dy}{dx}=\frac{y-xy^2}{x+yx^2},\ x\neq 0\,.$$

$$\boxed{y\,\mathrm{e}^{xy}=Cx}$$



Question 111 **(****+)**

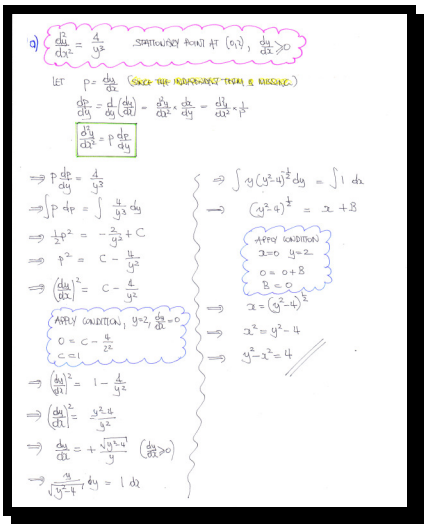
The curve C, has a stationary point at $(0,2)$ and satisfies the differential relationship

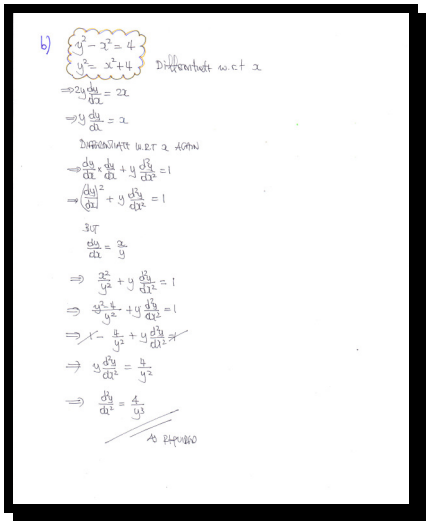
$$\frac{d^2y}{dx^2}=\frac{4}{y^3},\ y\neq 0.$$

a) Given further that $\frac{dy}{dx}\geq 0$ along C, determine a simplified expression for the Cartesian equation of C.

b) Verify by differentiation the answer to part **(a)**.

$$\boxed{y^2-x^2=4}$$





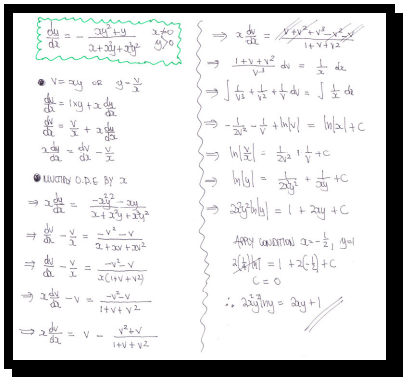
Question 112 (**+)**

Solve the differential equation

$$\frac{dy}{dx}=-\frac{xy^2+y}{x+yx^2+x^3y^2},\ x\neq 0,\ y>0,$$

subject to the condition $y\left(\frac{1}{2}\right)=1$.

$$\boxed{2x^2y^2\ln y=2xy+1}$$



Question 113 **(****+)**

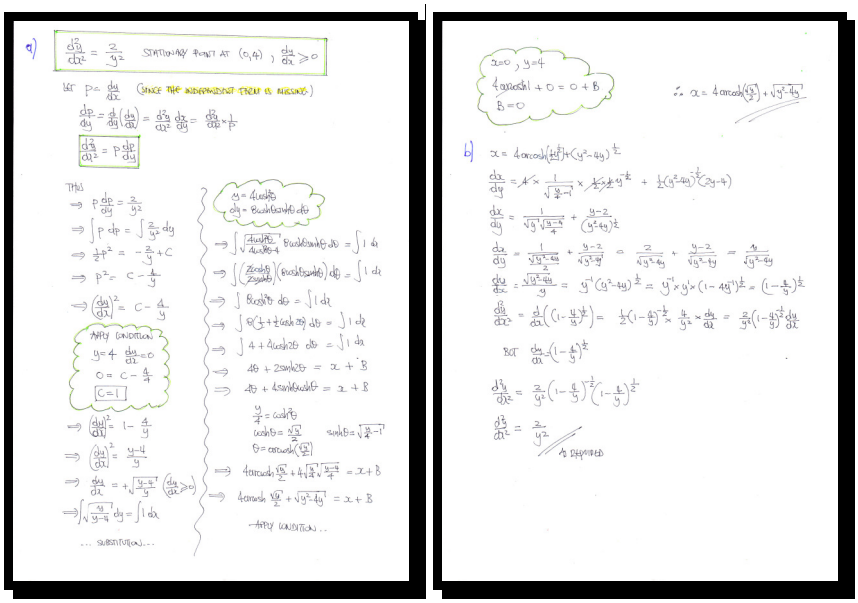
The curve C, has a stationary point at $(0,4)$ and satisfies the differential equation

$$\frac{d^2y}{dx^2}=\frac{2}{y^2},\ y\neq 0.$$

a) Given further that $\frac{dy}{dx}\geq 0$ along C, determine a simplified expression for the Cartesian equation of C, giving the answer in the form $x=f(y)$.

b) Verify by differentiation the answer to part **(a)**.

$$\boxed{x=4\operatorname{arcosh}\left(\frac{1}{2}\sqrt{y}\right)+\sqrt{y^2-4y}}$$



Question 114 **(****+)**

The curve C with Cartesian equation $f(x,y)=0$, satisfies the differential equation

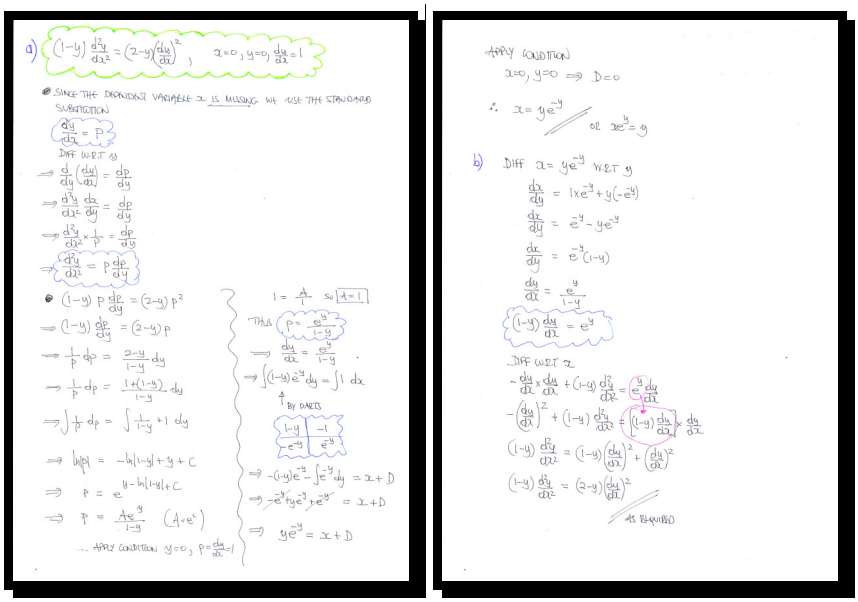
$$(1-y)\,y''=(2-y)(y')^2.$$

It is further given that $y(0)=0$ and $y'(0)=1$

a) Determine a simplified expression for the Cartesian equation of C.

b) Verify by differentiation the answer to part **(a)**.

$$\boxed{x=ye^{-y}}$$



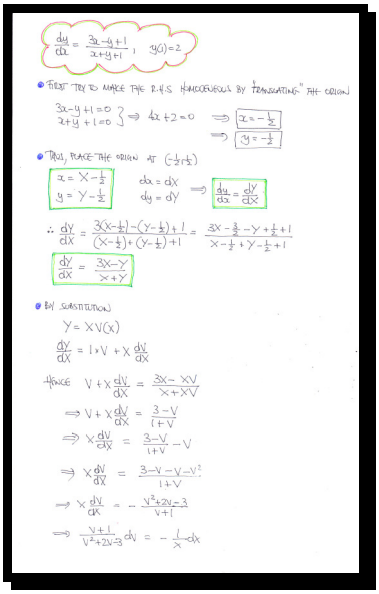
Question 115 (**+)**

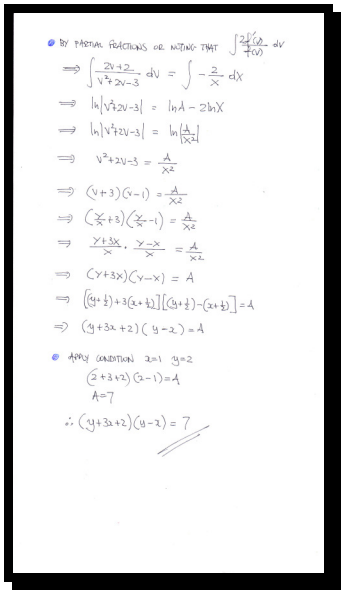
$$\frac{dy}{dx}=\frac{3x-y+1}{x+y+1},\ y(1)=2.$$

Solve the differential equation to show that

$$(y-x)(y+3x+2)=7.$$

proof





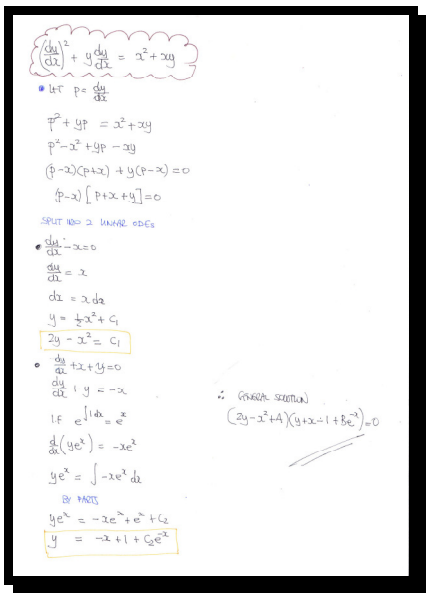
Question 116 (**+)**

By writing $\dfrac{dy}{dx}=p$ and seeking a suitable factorization find a general solution for the non linear differential equation

$$\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=x^2+xy\,.$$

Give the solution in the form $F\left(x,y\right)G\left(x,y\right)=0$.

$$\boxed{\left(2y-x^2+A\right)\left(x+y-1+B\,\mathrm{e}^{-x}\right)=0}$$



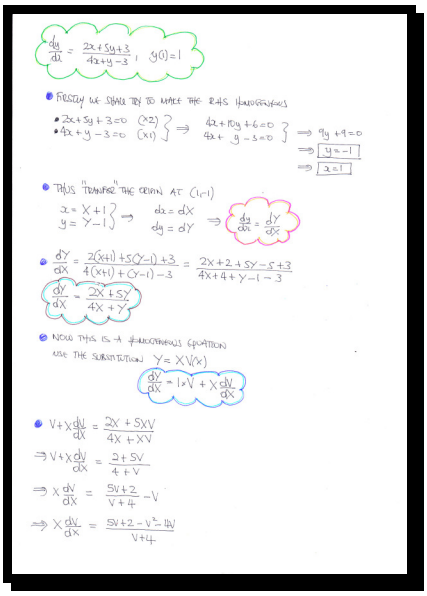
Question 117 (**+)**

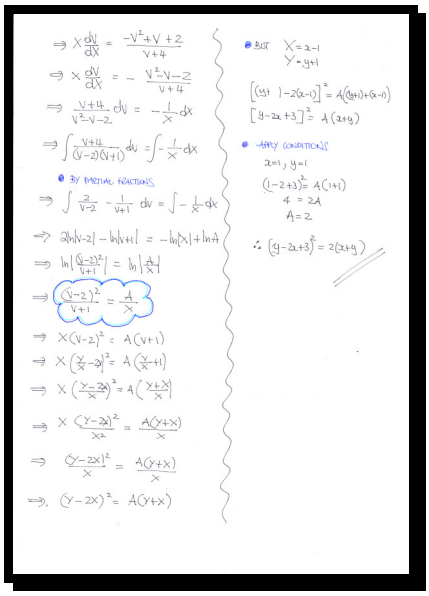
$$\frac{dy}{dx}=\frac{2x+5y+3}{4x+y-3},\ y(1)=1.$$

Solve the differential equation to show that

$$(y-2x+3)^2=2(x+y).$$

proof





Question 118 (*****)

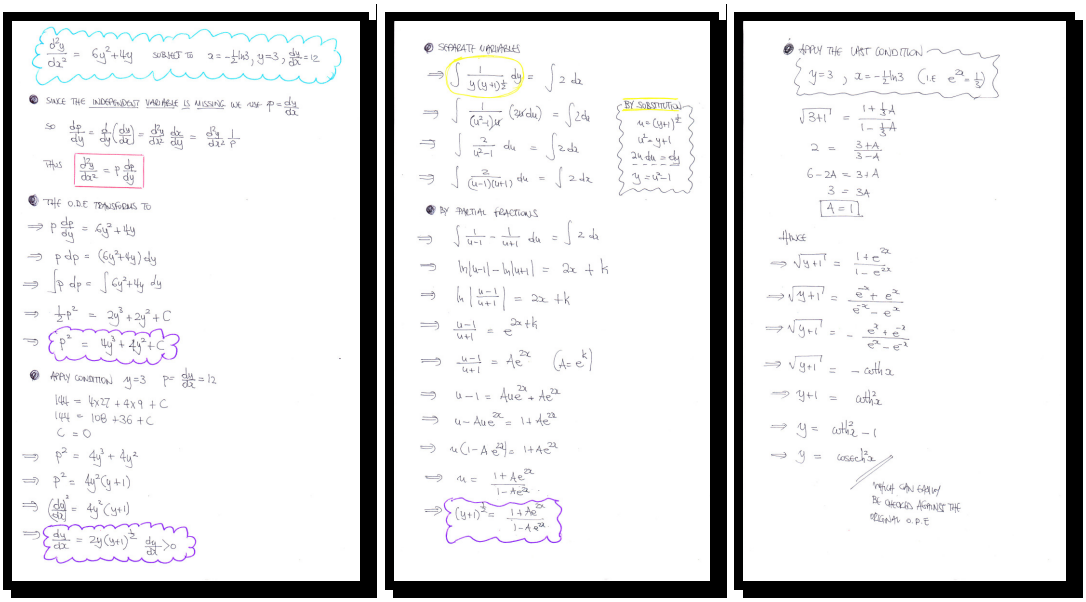
The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} = 6y^2 + 4y\,, \quad \frac{dy}{dx} \geq 0\,.$$

If $y = 3$, $\dfrac{dy}{dx} = 12$ at $x = -\dfrac{1}{2}\ln 3$, solve the differential equation to show that

$$y = \text{cosech}^2 x\,.$$

proof



Question 119 (*****)

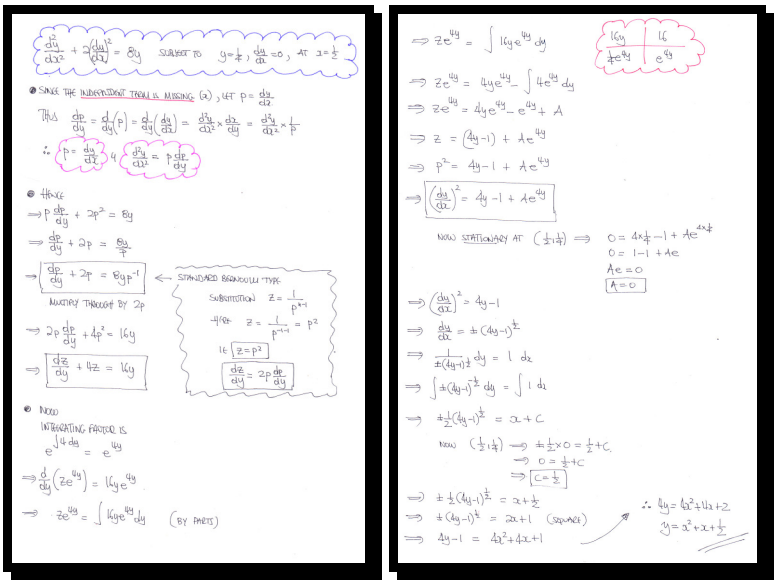
The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 8y\,.$$

Given further that the curve has a stationary point at $\left(\frac{1}{2}, \frac{1}{4}\right)$, solve the differential equation to show that

$$y = x^2 + x + \frac{1}{2}\,.$$

proof



Question 120 (*****)

The curve C, has gradient 1 at the origin and satisfies the differential relationship

$$\frac{d^2y}{dx^2}\sqrt{1-2y}=\frac{dy}{dx}(3y-2), \quad y<\frac{1}{2}.$$

Find an equation for C, giving the answer in the form $y=f(x)$.

$$\boxed{y=\frac{\sin x}{1+\sin x}=(\sec x-\tan x)\tan x}$$



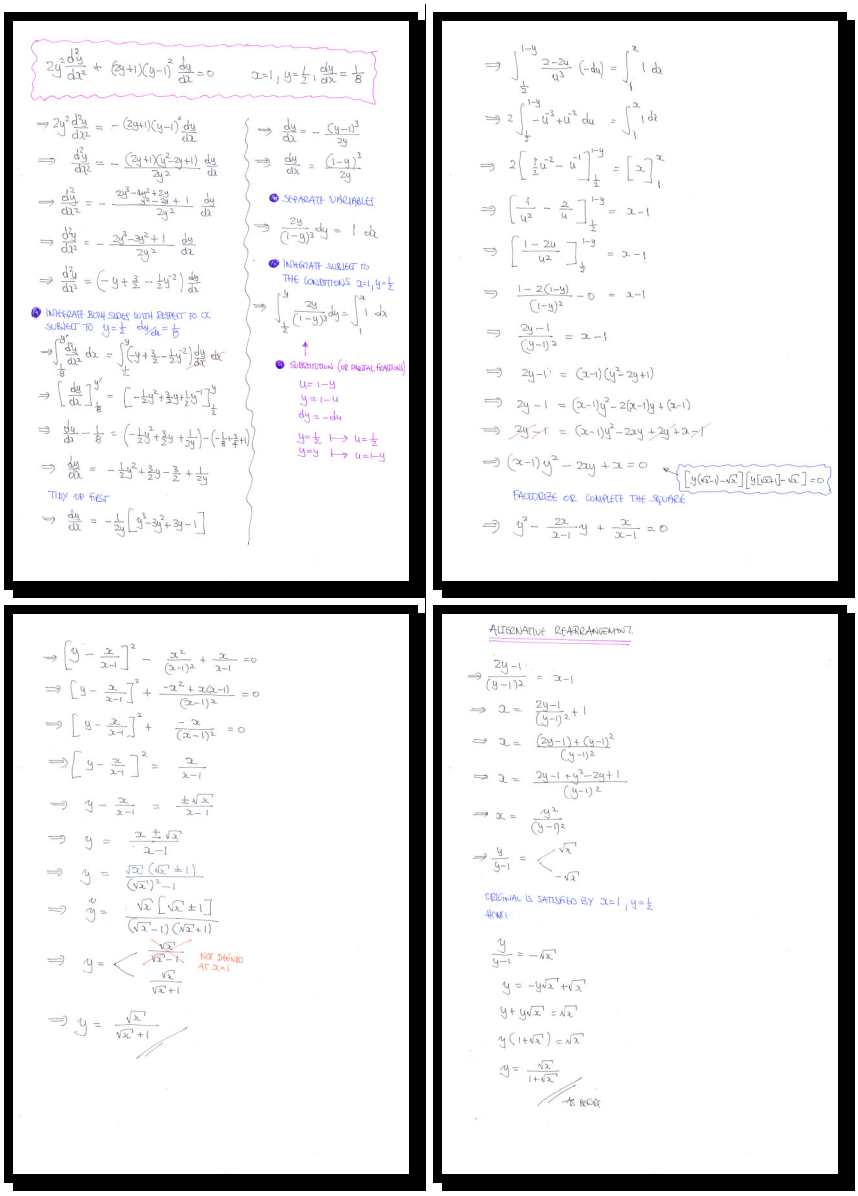
Question 121 **(*****)**

The curve C, has gradient $\frac{1}{8}$ at the point with coordinates $\left(1,\frac{1}{2}\right)$ and further satisfies the differential relationship

$$2y^2\frac{d^2y}{dx^2}+(2y+1)(y-1)^2\frac{dy}{dx}=0, \qquad y\neq 0.$$

Find an equation for C, giving the answer in the form $y=f(x)$.

$$\boxed{y=\frac{\sqrt{x}}{1+\sqrt{x}}}$$

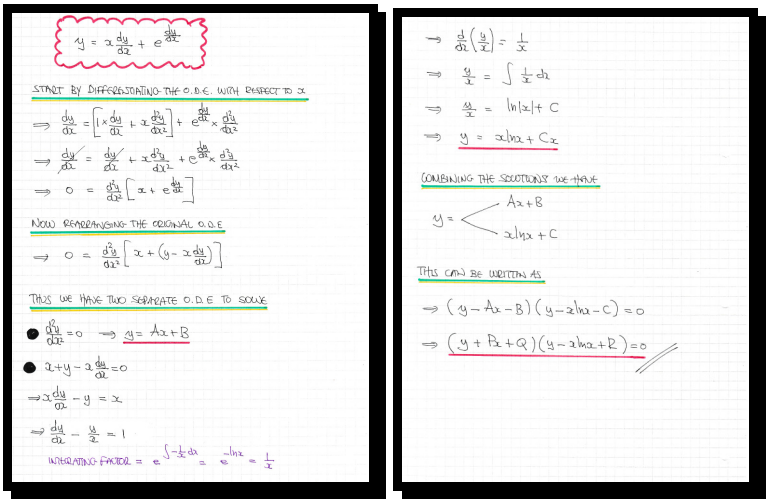


Question 122 (*****)

Find a general solution of the following differential equation.

$$y = x\frac{dy}{dx} + \mathrm{e}^{\frac{dy}{dx}}.$$

$\boxed{}$, $\boxed{(y + Ax + B)(y - x\ln x + Cx) = 0}$



Created by T. Madas