DIFFERENTIAL EQUATIONS

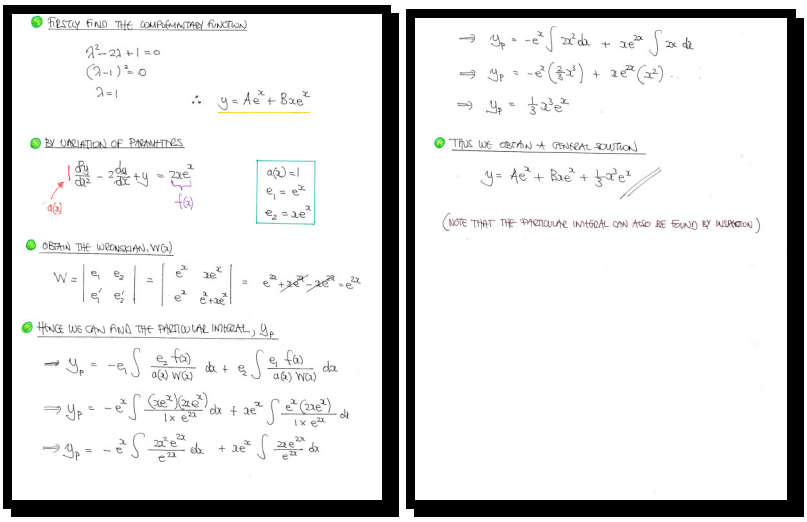
(by variation of parameters)

Question 1 (***)

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=2x\mathrm{e}^x.$$

$\boxed{}$, $\boxed{y=A\mathrm{e}^x+Bx\mathrm{e}^x+\frac{1}{3}x^3\mathrm{e}^x}$

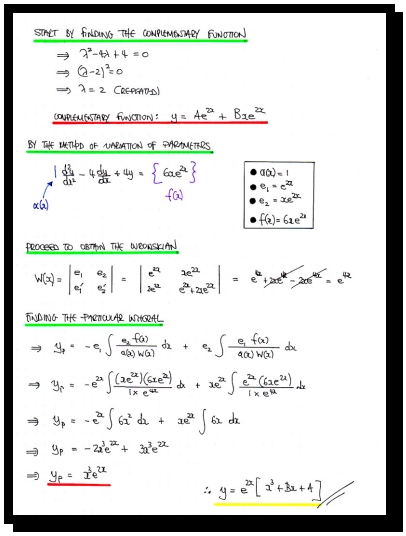


Question 2 (***)

Use the method of variation of parameters to find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=6xe^{2x}.$$

$\boxed{}$, $\boxed{y=Ae^{2x}+Bxe^{2x}+x^3e^{2x}}$

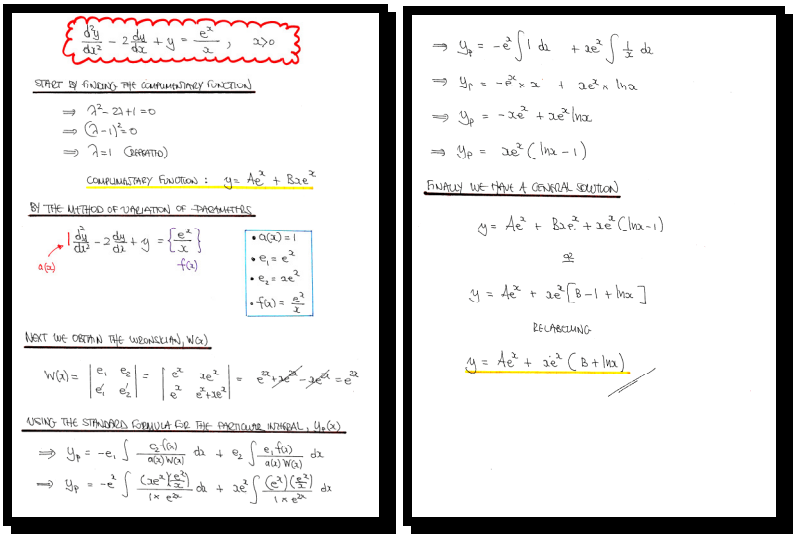


Question 3 (***)

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\frac{\mathrm{e}^x}{x},\ x>0.$$

$\boxed{}$, $\boxed{y=A\mathrm{e}^x+x\mathrm{e}^x\left(B+\ln x\right)}$

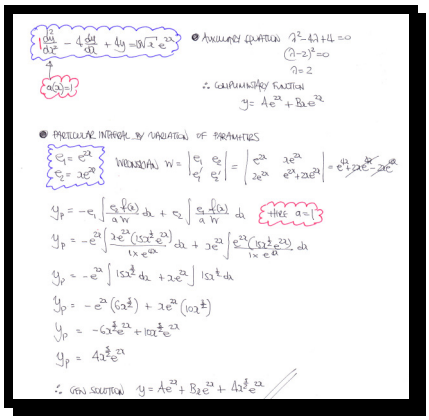


Question 4 (***)

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=15\sqrt{x}\,\mathrm{e}^{2x}\,.$$

$$\boxed{y=A\mathrm{e}^{2x}+Bx\mathrm{e}^{2x}+4x^{\frac{5}{2}}\mathrm{e}^{2x}}$$

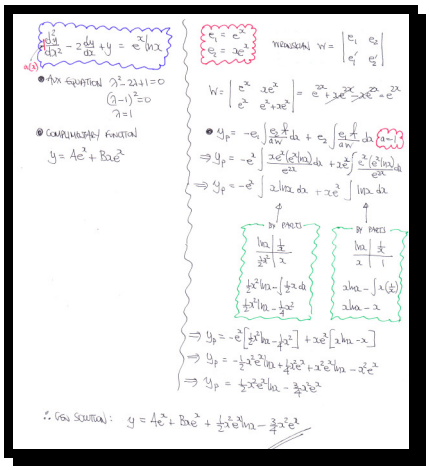


Question 5 (*+)**

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\mathrm{e}^x\ln x\,,\ \ x>0\,.$$

$$\boxed{y=A\mathrm{e}^{x}+Bx\mathrm{e}^{x}-\frac{3}{4}x^2\mathrm{e}^{x}+\frac{1}{2}x^2\mathrm{e}^{x}\ln x}$$

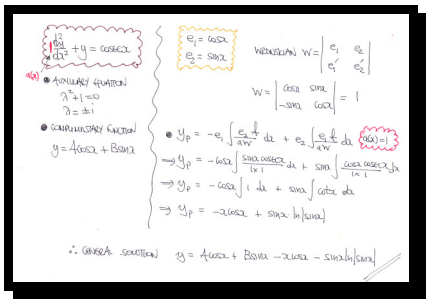


Question 6 (*+)**

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+y=\operatorname{cosec}x\,.$$

$$\boxed{y=A\cos x+B\sin x-x\cos x-\sin x\ln|\sin x|}$$

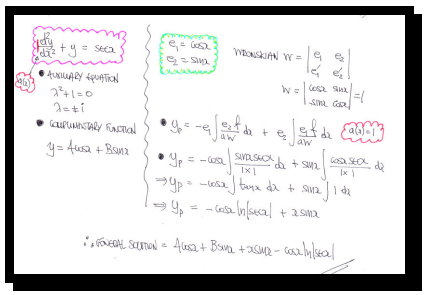


Question 7 (*+)**

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+y=\sec x\,.$$

$$\boxed{y=A\cos x+B\sin x+x\sin x-\cos x\ln|\sec x|}$$

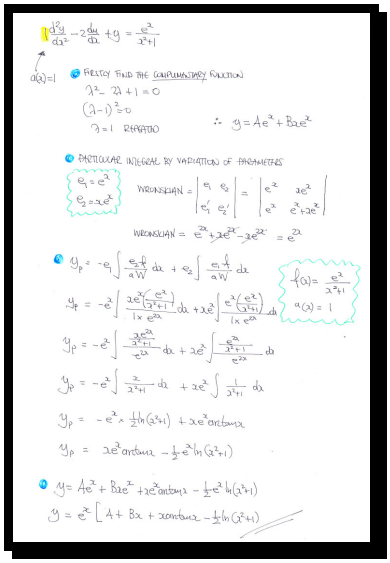


Question 8 (*+)**

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\frac{\mathrm{e}^x}{x^2+1}.$$

$$\boxed{y=\mathrm{e}^x\left[A+Bx+x\arctan x-\frac{1}{2}\ln\left(x^2+1\right)\right]}$$



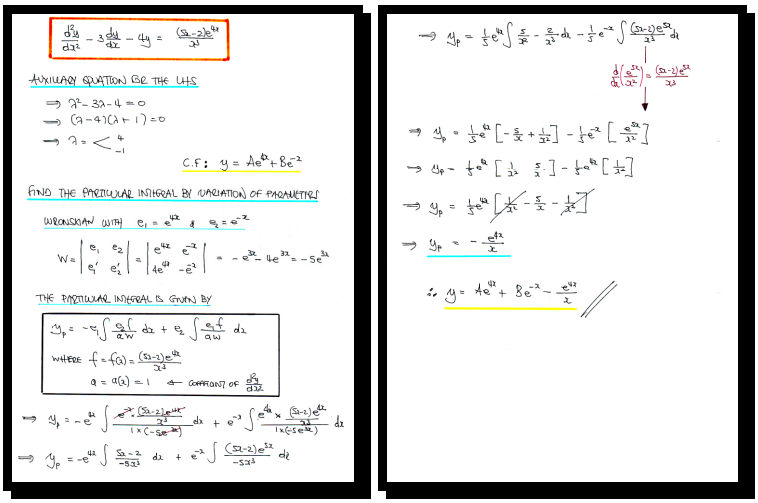
Question 9 (*+)**

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}-4y=\frac{(5x-2)\mathrm{e}^{4x}}{x^3}.$$

You may assume that $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{5x}}{x^2}\right)=\dfrac{(5x-2)\mathrm{e}^{5x}}{x^3}$.

$\square$, $\boxed{y=A\mathrm{e}^{4x}+B\mathrm{e}^{-x}-\dfrac{\mathrm{e}^{4x}}{x}}$

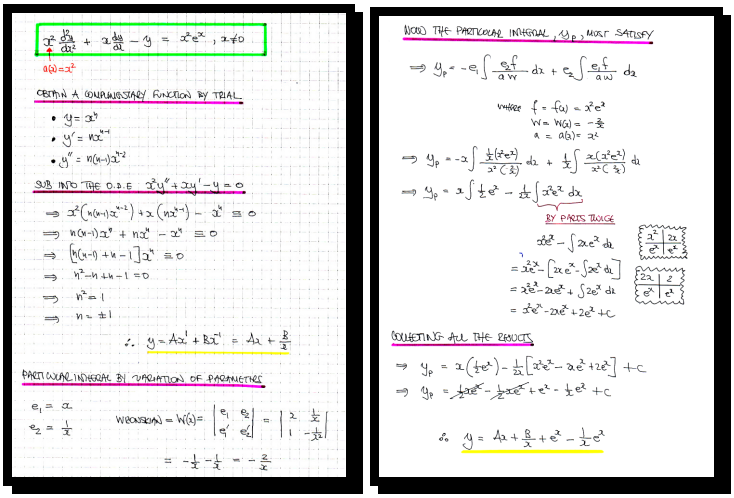


Question 10 (****)

Find the general solution of the following differential equation.

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2\,\mathrm{e}^x\,,\quad x\neq 0\,.$$

$\boxed{}$, $\boxed{y=Ax+\frac{B}{x}+\mathrm{e}^x-\frac{1}{x}\mathrm{e}^x}$

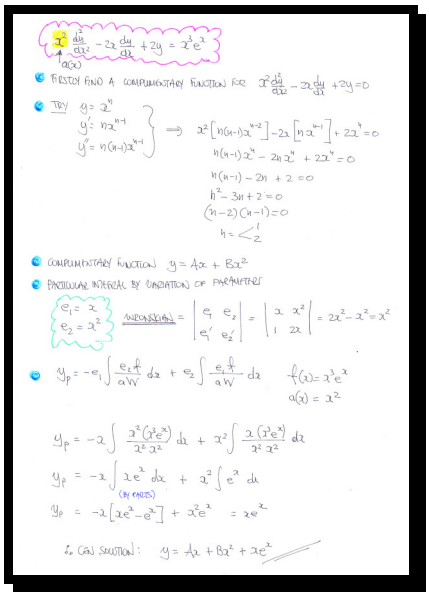


Question 11 (****)

Find the general solution of the following differential equation.

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^3\,\mathrm{e}^x\,,\quad x\neq 0\,.$$

$$\boxed{y=Ax+Bx^2+x\mathrm{e}^x}$$

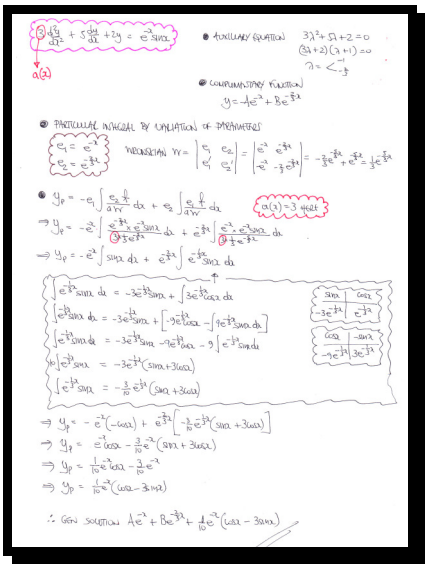


Question 12 (****)

Find the general solution of the following differential equation.

$$3\frac{d^2y}{dx^2}+5\frac{dy}{dx}+2y=\mathrm{e}^{-x}\sin x\,.$$

$$\boxed{y=A\mathrm{e}^{-x}+B\mathrm{e}^{x}+B\mathrm{e}^{-\frac{2}{3}x}+\frac{1}{10}\mathrm{e}^{-x}\left(\cos x-3\sin x\right)}$$



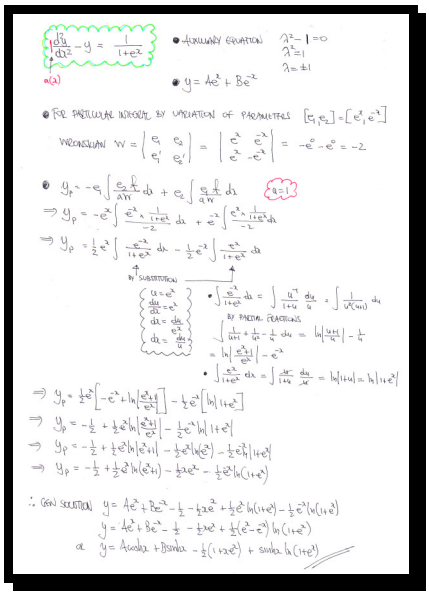
Question 13 (****)

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-y=\frac{1}{1+\mathrm{e}^x}\,.$$

$$\boxed{y=A\mathrm{e}^x+B\mathrm{e}^{-x}-\frac{1}{2}\left(1+x\mathrm{e}^x\right)+\frac{1}{2}\left(\mathrm{e}^x-\mathrm{e}^{-x}\right)\ln\left(1+\mathrm{e}^x\right)},$$

$$\boxed{y=A\cosh x+B\sinh x-\frac{1}{2}\left(1+x\mathrm{e}^x\right)+\sinh x\ln\left(1+\mathrm{e}^x\right)}$$



Question 14 (****)

Use variation of parameters to determine the specific solution of the following differential equation

$$x^2\frac{d^2y}{dx^2}-7x\frac{dy}{dx}+16y=16\ln x\,,$$

given further that $y=\frac{1}{2}$, $\frac{dy}{dx}=2$ at $x=1$.

$$\boxed{y=\frac{1}{2}+\left(1+x^4\right)\ln x}$$

