Jordan-Gauss Elimination

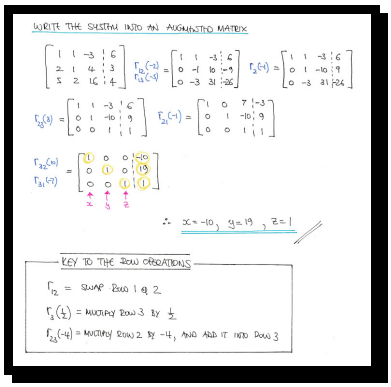
Unique Solutions

Question 1

Solve the following simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\begin{pmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix}$$

V , [ ] , $x = -10, \quad y = 19, \quad z = 1$

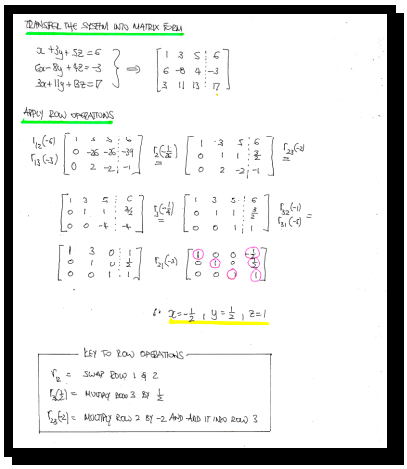


Question 2

$$\begin{aligned} x\ +3y\ +5z &= 6 \\ 6x\ -8y\ +4z &= -3 \\ 3x+11y+13z &= 17 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

V, $\square$, $\boxed{x=-\frac{1}{2},\ y=\frac{1}{2},\ z=1}$

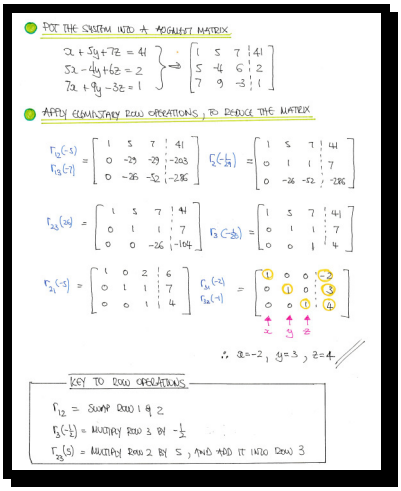


Question 3

$$\begin{aligned} x\ +5y+7z&=41\\ 5x-4y+6z&=\ \ 2\\ 7x+9y-3z&=\ \ 1 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$\boxed{}$, $\boxed{x=-2,\ \ y=3,\ \ z=4}$

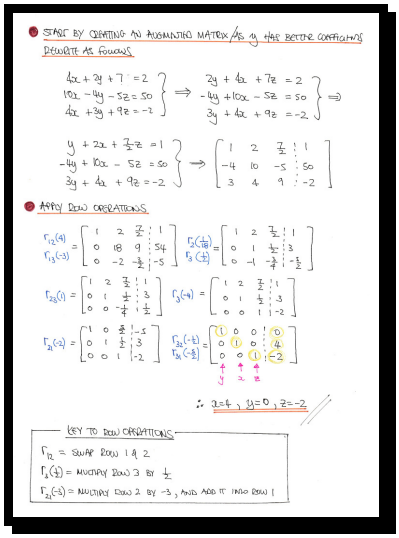


Question 4

$$\begin{aligned} 4x &+2y+7z= 2 \\ 10x &-4y-5z= 50 \\ 4x &+3y+9z=-2 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$\boxed{}$, $\boxed{x=4,\ y=0,\ z=-2}$

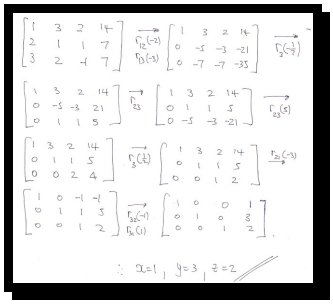


Question 5

$$\begin{aligned} x+3y+2z&=14\\ 2x+y+z&=7\\ 3x+2y-z&=7 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\boxed{x=1,\ y=3,\ z=2}$$

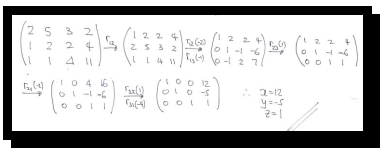


Question 6

$$\begin{aligned} 2x+5y+3z&=2\\ x+2y+2z&=4\\ x+y+4z&=11 \end{aligned}$$

Solve the above simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\boxed{x=12,\ y=-5,\ z=1}$$

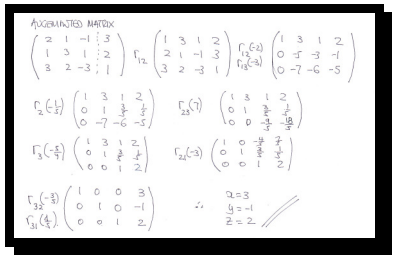


Question 7

$$\begin{aligned} 2x + y - z &= 3 \\ x + 3y + z &= 2 \\ 3x + 2y - 3z &= 1 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\boxed{x = 3,\ y = -1,\ z = 2}$$

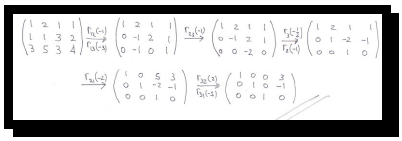


Question 8

Solve the following simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$\boxed{x = 3,\ y = -1,\ z = 0}$$

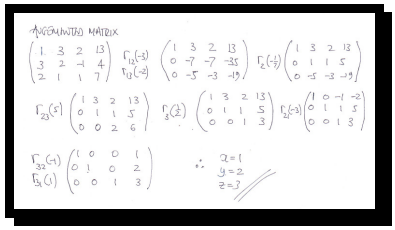


Question 9

$$\begin{aligned} x+3y+2z&=13\\ 3x+2y-z&=4\\ 2x+y+z&=7 \end{aligned}$$

Solve the above system of simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\boxed{x=1,\ y=2,\ z=3}$$

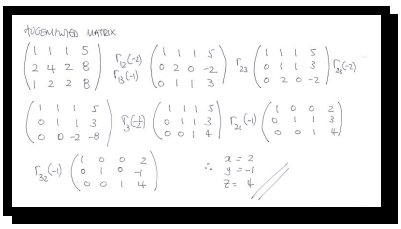


Question 10

Solve the following simultaneous equations by manipulating their augmented matrix into reduced row echelon form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 5 \\ 8 \\ 8 \end{pmatrix}$$

$$\boxed{x=2,\ y=-1,\ z=4}$$

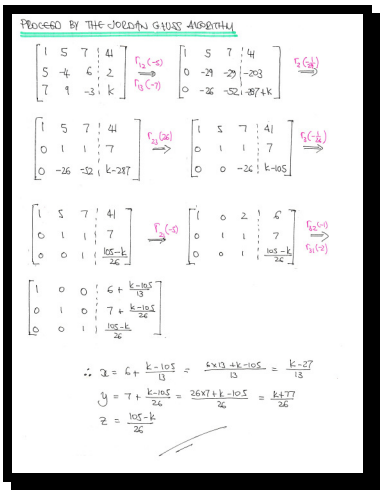


Question 11

$$\begin{aligned} x+5y+7z &= 41 \\ 5x-4y+6z &= 2 \\ 7x+9y-3z &= k \end{aligned}$$

Use the Jordan Gauss algorithm to determine the solution of the above system of simultaneous equations, giving the answers in terms of the constant k.

$\square$, $\boxed{x=\dfrac{k-27}{13},\quad y=\dfrac{k+77}{26},\quad z=\dfrac{105-k}{26}}$



Non-Unique Solutions

Question 1

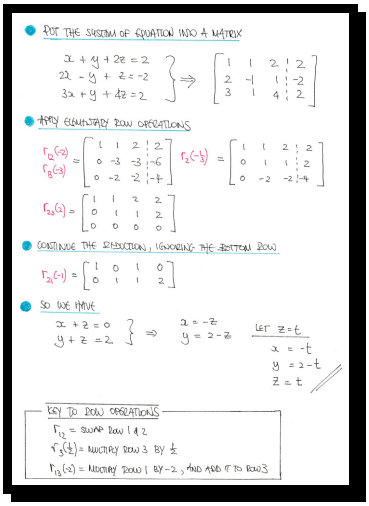
$$\begin{aligned} x+y+2z &= 2 \\ 2x-y+z &= -2 \\ 3x+y+4z &= 2 \end{aligned}$$

Show, by reducing the augmented matrix of the above system of equations into row echelon form, that the solution can be written as

$$x=-t\,, \quad y=2-t\,, \quad z=t$$

where t is a scalar parameter.

[ ] , [proof]



Question 2

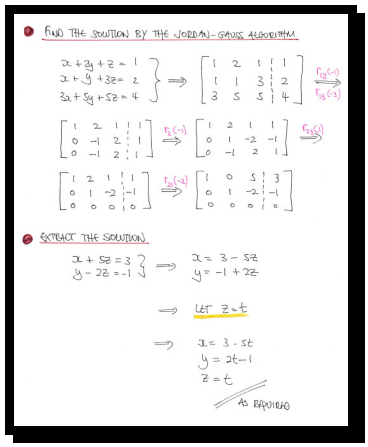
$$\begin{aligned} x+2y+\ z&=1 \\ x+\ y+3z&=2 \\ 3x+5y+5z&=4 \end{aligned}$$

Show that the solution of the above simultaneous equations is

$$x=3-5t\,,\quad y=2t-1\,,\quad z=t$$

where t is a parameter.

V, [ ], proof



Question 3

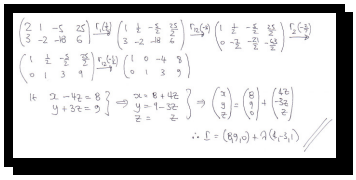
$$\begin{aligned} 3x-2y-18z&=6\\ 2x+y-5z&=25 \end{aligned}$$

Show, by reducing the above system of equations into row echelon form, that the solution can be written as

$$\mathbf{r}=8\mathbf{i}+9\mathbf{j}+\lambda\left(4\mathbf{i}-3\mathbf{j}+\mathbf{k}\right),$$

where λ is a scalar parameter.

proof



Question 4

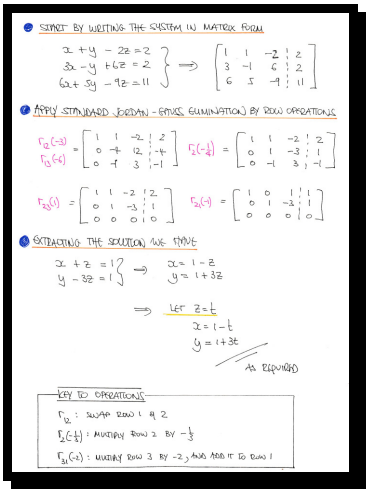
$$\begin{aligned} x+y-2z&=2\\ 3x-y+6z&=2\\ 6x+5y-9z&=11 \end{aligned}$$

Show, by reducing the above equation system into row echelon form, that the consistent solution of the system can be written as

$$x=1-t\,,\qquad y=3t+1\,,\qquad z=t$$

where t is a scalar parameter.

[ ], [proof]



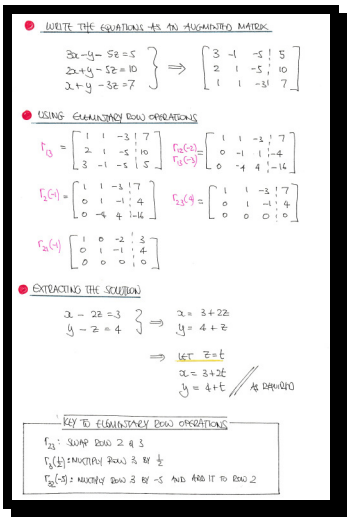
Question 5

$$\begin{aligned} 3x-y-5z&=5\\ 2x+y-5z&=10\\ x+y-3z&=7 \end{aligned}$$

Show, by reducing the above system into row echelon form, that the consistent solution of the system can be written as

$$x=2t+3\,,\quad y=t+4\,,\quad z=t\,.$$

V , [ ] , proof



Question 6

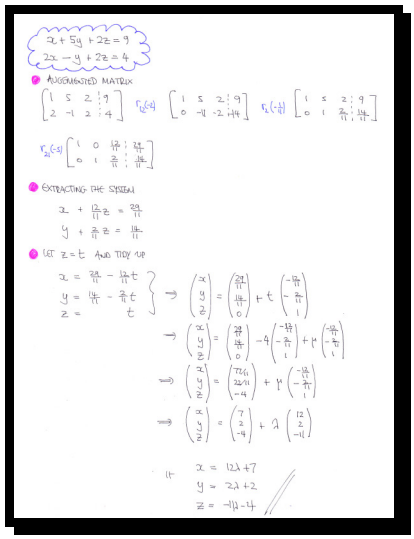
$$\begin{aligned} x+5y+2z&=9\\ 2x-y+2z&=4 \end{aligned}$$

Show, by reducing the above system of equations into row echelon form, that the solution can be written as

$$x=A\lambda+B\,,\ y=C\lambda+D\,,\ x=E\lambda+F$$

where A, B, C, D, E and F are integers, and λ is a scalar parameter.

V, $\boxed{x=12\lambda+7}$, $\boxed{y=2\lambda+2}$, $\boxed{z=-11\lambda-4}$

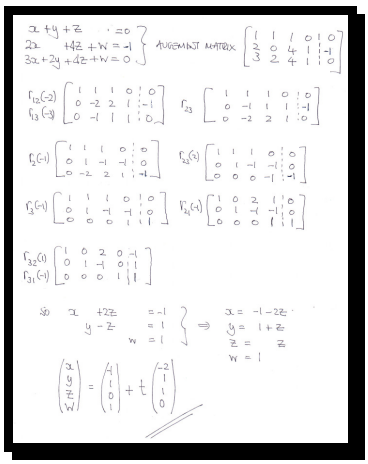


Question 7

$$\begin{aligned} x+y+z&=0\\ 2x+4z+w&=-1\\ 3x+2y+4z+w&=0 \end{aligned}$$

Find a general solution of the above system of simultaneous equations.

V, $\begin{pmatrix} x\\ y\\ z\\ w \end{pmatrix} = \begin{pmatrix} -1\\ 1\\ 0\\ 1 \end{pmatrix} + t\begin{pmatrix} -2\\ 1\\ 1\\ 0 \end{pmatrix}$



Cramer's Rule

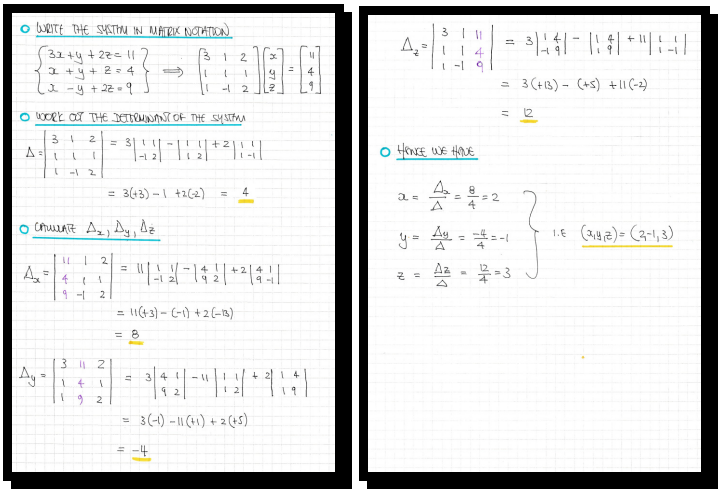
Question 1

Use Cramer's rule to solve the following system of simultaneous equations.

$$\begin{aligned} 3x + y + 2z &= 11 \\ x + y + z &= 4 \\ x - y + 2z &= 9 \end{aligned}$$

No credit will be given for using alternative solution methods.

$\boxed{}$, $\boxed{x=2}$, $\boxed{y=-1}$, $\boxed{z=3}$



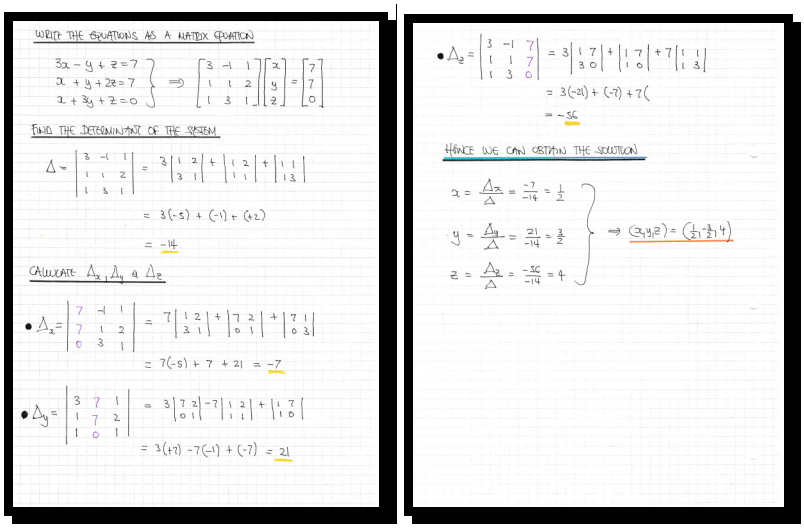
Question 2

Use Cramer's rule to solve the following system of simultaneous equations.

$$\begin{aligned} 3x - y + z &= 7 \\ x + y + 2z &= 7 \\ x + 3y + z &= 0 \end{aligned}$$

No credit will be given for using alternative solution methods.

$\boxed{}$, $\boxed{x=\frac{1}{2}}$, $\boxed{y=-\frac{3}{2}}$, $\boxed{z=4}$



Question 3

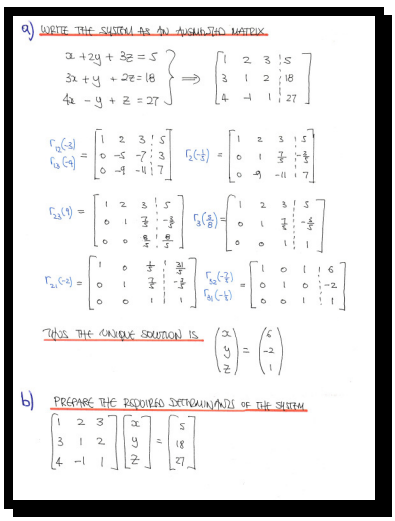
$$\begin{aligned} x+2y+3z&=5\\ 3x+y+2z&=18\\ 4x-y+z&=27 \end{aligned}$$

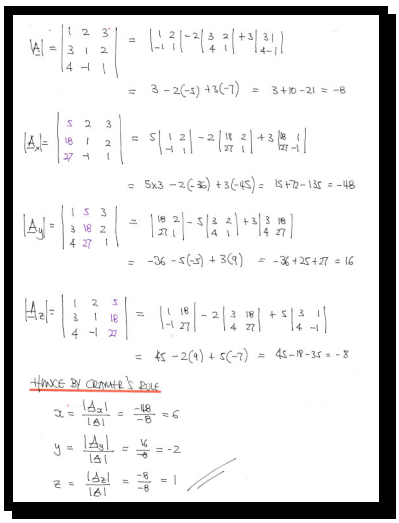
Solve the above system of the simultaneous equations ...

a) ... by manipulating their augmented matrix into reduced row echelon form.

b) ... by using Cramer's rule.

$\boxed{}$, $\boxed{x=6,\ y=-2,\ z=1}$



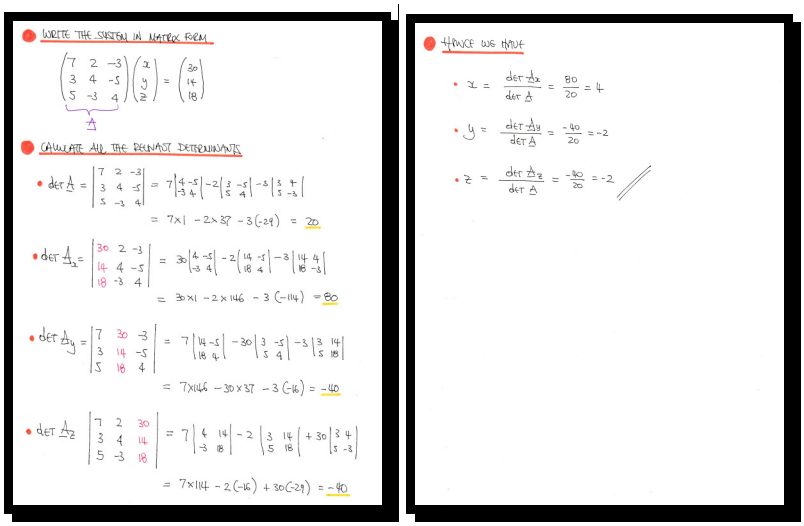


Question 4

$$\begin{aligned} 7x &+2y &-3z &= 30 \\ 3x &+4y &-5z &= 14 \\ 5x &-3y &+4z &= 18 \end{aligned}$$

Solve the above system of the simultaneous equations by using Cramer's rule.

$\boxed{}$, $\boxed{x=4,\ y=-2,\ z=-2}$

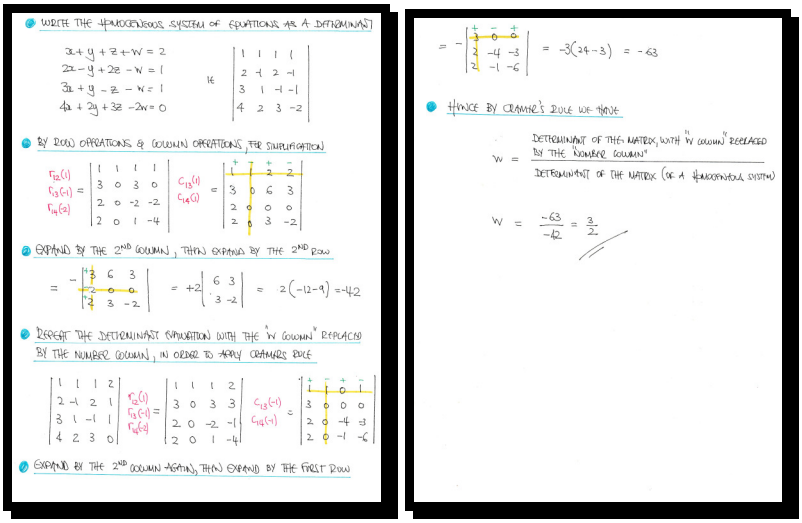


Question 5

$$\begin{aligned} x+\ y+\ z+\ w&=2\\ 2x-\ y+2z-\ w&=1\\ 3x+\ y-\ z-\ w&=1\\ 4x+2y+3z-2w&=0 \end{aligned}$$

Use Cramer's rule to find the value of w in the above system of the simultaneous equations

$$\boxed{}\ ,\quad \boxed{w=\frac{3}{2}}$$



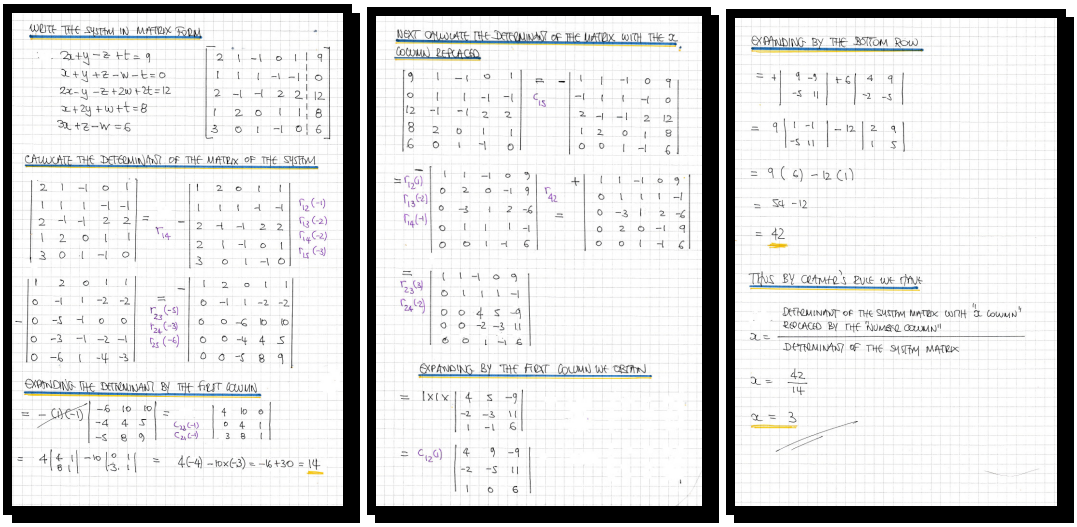
Question 6

Use Cramer's rule to find the value of x in the following system of simultaneous equations.

$$\begin{aligned} 2x + y - z + t &= 9 \\ x + y + z - w - t &= 0 \\ 2x - y - z + 2w + 2t &= 12 \\ x + 2y + w + t &= 8 \\ 3x + z - w &= 6 \end{aligned}$$

No credit will be given for using alternative solution methods.

$\boxed{}$, $\boxed{x=3}$



Matrix Inverse

Question 1

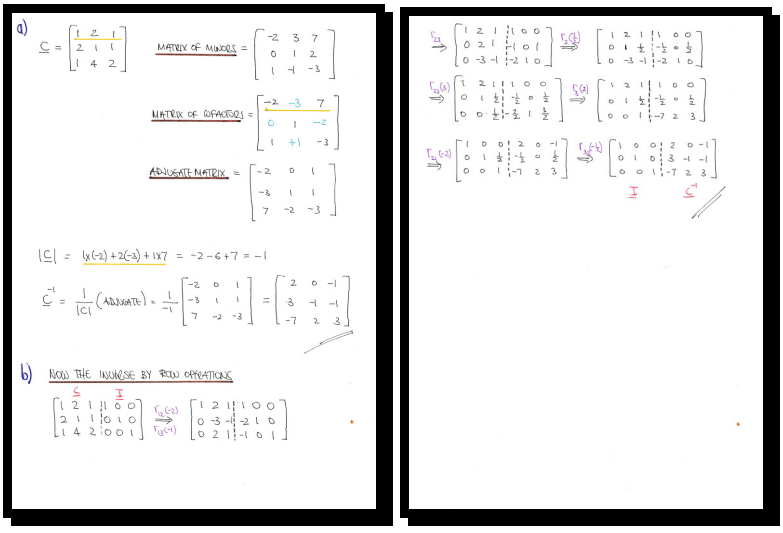
The 3×3 matrix $\mathbf{C}$ is given below.

$$\mathbf{C}=\begin{pmatrix}1&2&1\\2&1&1\\1&4&2\end{pmatrix}.$$

a) Use the standard method for finding the inverse of a 3×3 matrix, to determine the elements of $\mathbf{C}^{-1}$.

b) Verify the answer of part **(a)** by obtaining the elements of $\mathbf{C}^{-1}$, by using a method involving elementary row operations.

V, , $\mathbf{C}^{-1}=\begin{pmatrix}2&0&-1\\3&-1&-1\\-7&2&3\end{pmatrix}$



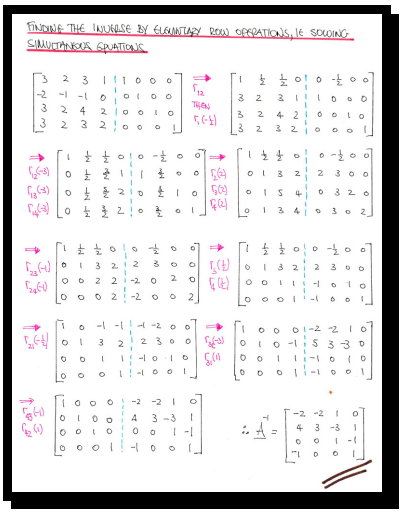
Question 2

The 4×4 matrix $\mathbf{A}$ is given below.

$$\mathbf{A}=\begin{pmatrix} 3 & 2 & 3 & 1 \\ -2 & -1 & -1 & 0 \\ 3 & 2 & 4 & 2 \\ 3 & 2 & 3 & 2 \end{pmatrix}.$$

Find $\mathbf{A}^{-1}$, by using a method involving elementary row operations.

$$\boxed{},\quad \boxed{\mathbf{A}=\begin{pmatrix} -2 & -2 & 1 & 0 \\ 4 & 3 & -3 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}}$$



Various

Question 1

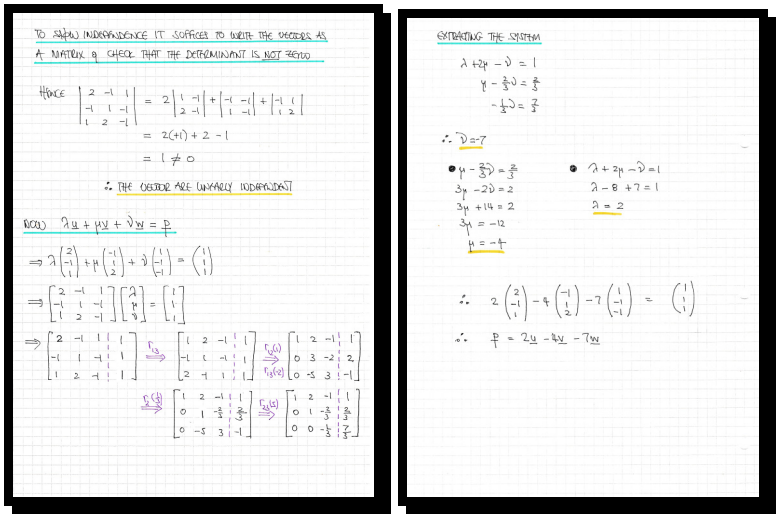
The following four vectors are given.

$$\mathbf{u}=\begin{bmatrix}2\\-1\\1\end{bmatrix},\quad \mathbf{v}=\begin{bmatrix}-1\\1\\2\end{bmatrix},\quad \mathbf{w}=\begin{bmatrix}1\\-1\\-1\end{bmatrix},\quad \mathbf{p}=\begin{bmatrix}1\\1\\1\end{bmatrix}.$$

a) Show that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

b) Express $\mathbf{p}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

$\boxed{}$, $\boxed{\mathbf{p}=2\mathbf{u}-4\mathbf{v}-7\mathbf{w}}$



Question 2

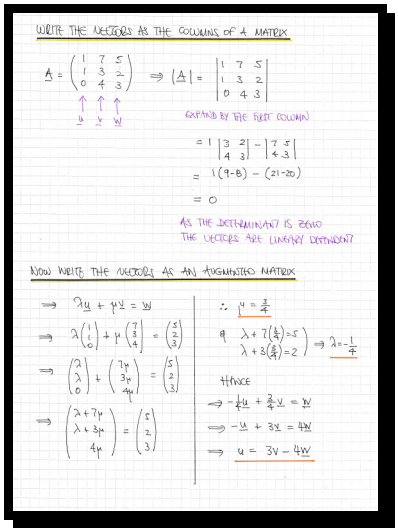
The following three vectors are given.

$$\mathbf{u}=\begin{bmatrix}1\\1\\0\end{bmatrix},\qquad \mathbf{v}=\begin{bmatrix}7\\3\\4\end{bmatrix},\qquad \mathbf{w}=\begin{bmatrix}5\\2\\3\end{bmatrix}.$$

a) Show that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent.

b) Find a linear relationship, with integer coefficients, between $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

$\boxed{}$, $\boxed{\mathbf{u}=3\mathbf{v}-4\mathbf{w}}$



Question 3

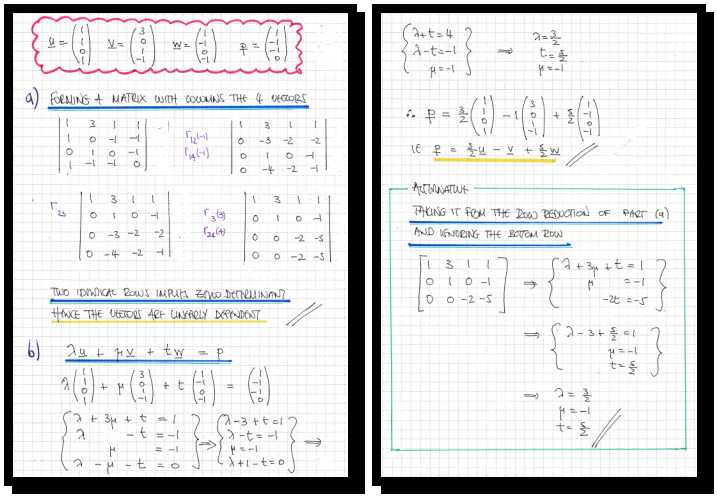
The following four vectors are given.

$$\mathbf{u}=\begin{bmatrix}1\\1\\0\\1\end{bmatrix},\quad \mathbf{v}=\begin{bmatrix}3\\0\\1\\-1\end{bmatrix},\quad \mathbf{w}=\begin{bmatrix}1\\-1\\0\\-1\end{bmatrix},\quad \mathbf{p}=\begin{bmatrix}1\\-1\\-1\\0\end{bmatrix}.$$

a) Show that these four vectors are linearly dependent.

b) Express $\mathbf{p}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

$\boxed{}$, $\boxed{\mathbf{p}=\frac{3}{2}\mathbf{u}-\mathbf{v}+\frac{5}{2}\mathbf{w}}$



Created by T. Madas