INTEGRAL EQUATIONS

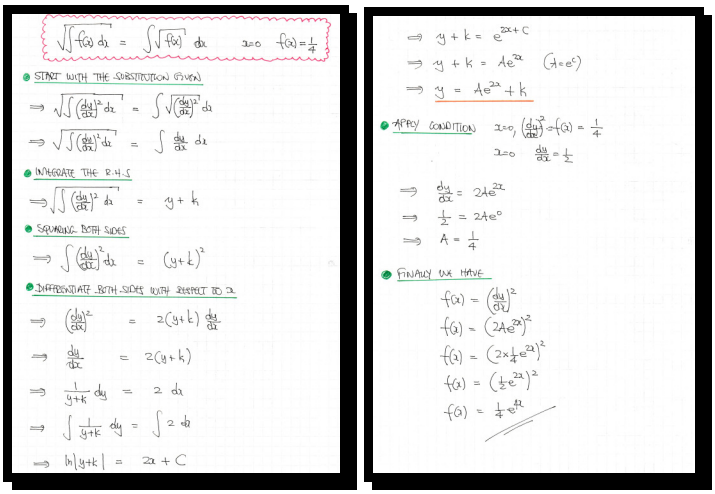
Question 1

The non zero function $f(x)$ satisfies the integral equation

$$\sqrt{\int f(x)\,dx} = \int \sqrt{f(x)}\,dx\,, \qquad f(0)=\frac{1}{4}.$$

Use the substitution $f(x)=\left(\frac{dy}{dx}\right)^2$, to find a simplified expression for $f(x)$.

$\square$, $\boxed{f(x)=\frac{1}{4}e^{4x}}$



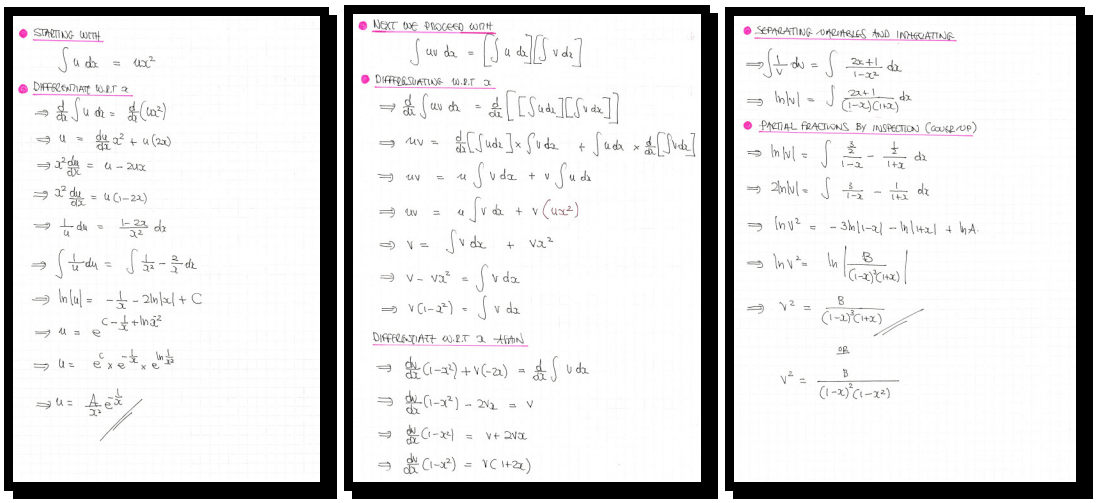
Question 2

The non zero functions $u(x)$ and $v(x)$ satisfy the integral equations

$$\int u(x)\,dx = x^2u(x) \qquad \text{and} \qquad \int u(x)v(x)\,dx = \left[\int u(x)\,dx\right]\left[\int v(x)\,dx\right].$$

Determine, in terms of an arbitrary constant, a simplified expression for $u(x)$ and a similar expression for $\left[v(x)\right]^2$.

$$\square, \quad \boxed{u(x)=\frac{Ae^{-\frac{1}{x}}}{x^2}}, \quad \boxed{\left[v(x)\right]^2=\frac{B}{(1-x)^2\left(1-x^2\right)}}$$



Question 3

The function f satisfies the following relationship.

$$f(x)=\int_1^x \left[f(t)\right]^2\,dt\,, \quad f(2)=\frac{1}{2}\,.$$

Determine the value of $f\left(\frac{1}{2}\right)$.

$\square$, $\boxed{f\left(\frac{1}{2}\right)=\frac{2}{7}}$

