DIAGONALIZING

CONICS & QUADRICS

CONICS

Question 1

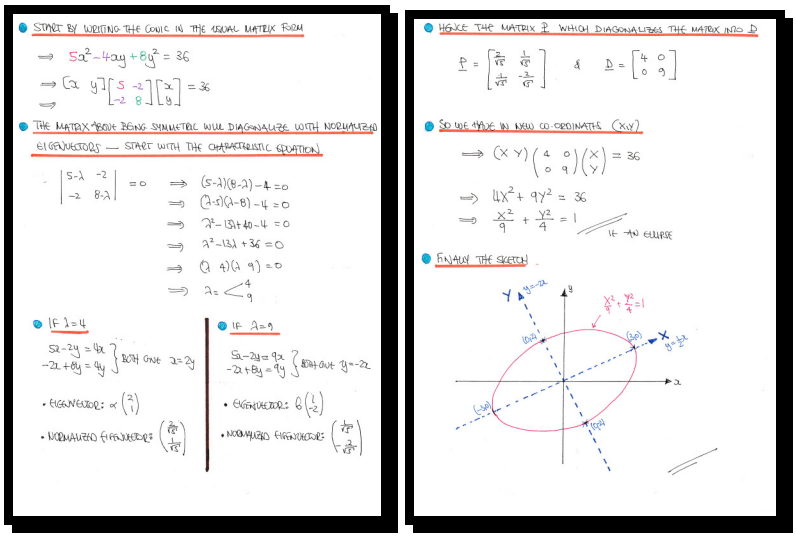
$$5x^2-4xy+8y^2=36\,.$$

The conic with the above Cartesian equation is rotated out of its standard position.

By suitably removing the xy term, give a Cartesian equation for the conic in the rotated system.

You are expected to sketch and name the conic.

$\square$, $\boxed{\text{ellipse}:\dfrac{X^2}{4}+\dfrac{Y^2}{9}=1}$



Question 2

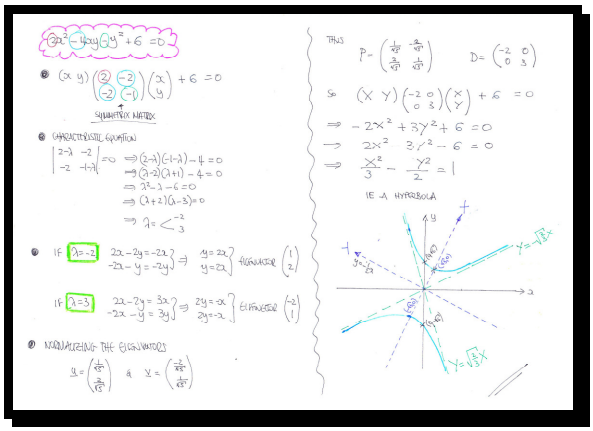
$$2x^2 - 4xy - y^2 + 6 = 0 .$$

The conic with the above Cartesian equation is rotated out of its standard position.

By suitably removing the xy term, give a Cartesian equation for the conic in the rotated system.

You are expected to sketch and name the conic.

$$\text{hyperbola}: \frac{X^2}{3} - \frac{Y^2}{2} = 1$$



Question 3

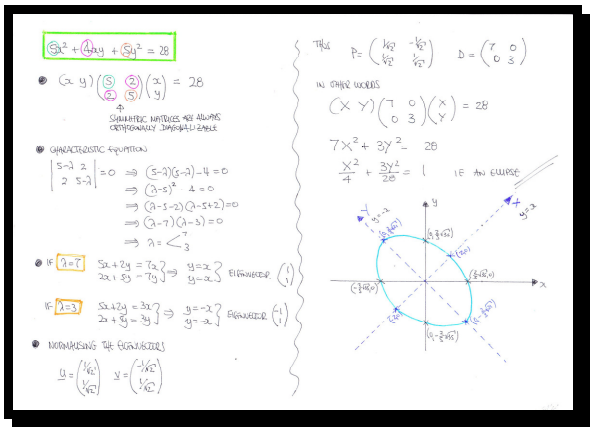
$$5x^2+4xy+5y^2=28\,.$$

The conic with the above Cartesian equation is rotated out of its standard position.

By suitably removing the xy term, give a Cartesian equation for the conic in the rotated system.

You are expected to sketch and name the conic.

$$\boxed{\text{ellipse}:\ \frac{X^2}{4}+\frac{3Y^2}{28}=1}$$



Question 4

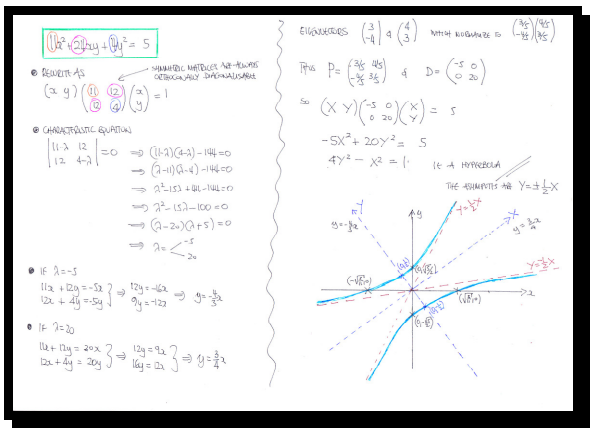
$$11x^2+24xy+4y^2=5\,.$$

The conic with the above Cartesian equation is rotated out of its standard position.

By suitably removing the xy term, give a Cartesian equation for the conic in the rotated system.

You are expected to sketch and name the conic.

$$\boxed{\text{hyperbola}:\ 4Y^2-X^2=1}$$



Question 5

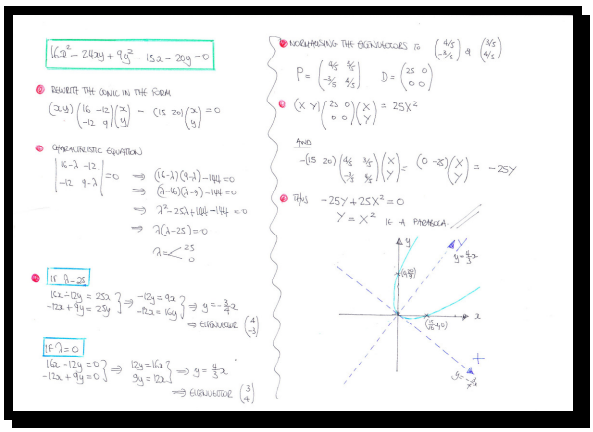
$$16x^2 - 24xy + 9y^2 - 15x - 20y = 0 .$$

The conic with the above Cartesian equation is rotated out of its standard position.

By suitably removing the xy term, give a Cartesian equation of the conic in the rotated system.

You are expected to sketch and name the conic.

$$\boxed{\text{parabola} : \ Y = X^2}$$



Question 6

$$9x^2-4xy+6y^2-10x-20y=100\,.$$

The conic with the above Cartesian equation has been rotated and translated out of its standard position.

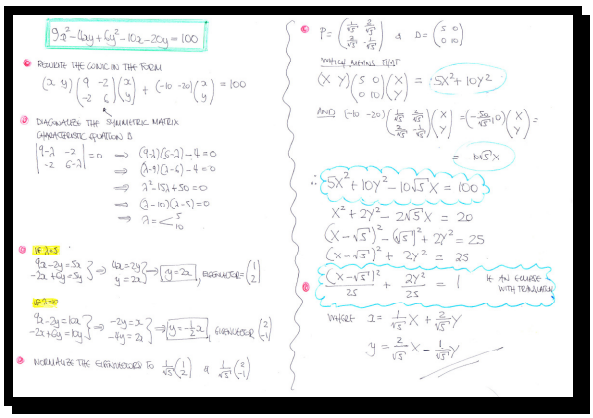
Show that a Cartesian equation in a suitable coordinate system is

$$\frac{(X-k)^2}{25}+\frac{2Y^2}{25}=1\,,$$

where k is an exact constant to be found.

You are not expected to sketch the conic.

$$\boxed{k=\sqrt{5}}$$



Question 7

$$x^2+2xy+y^2+8x+y=0\,.$$

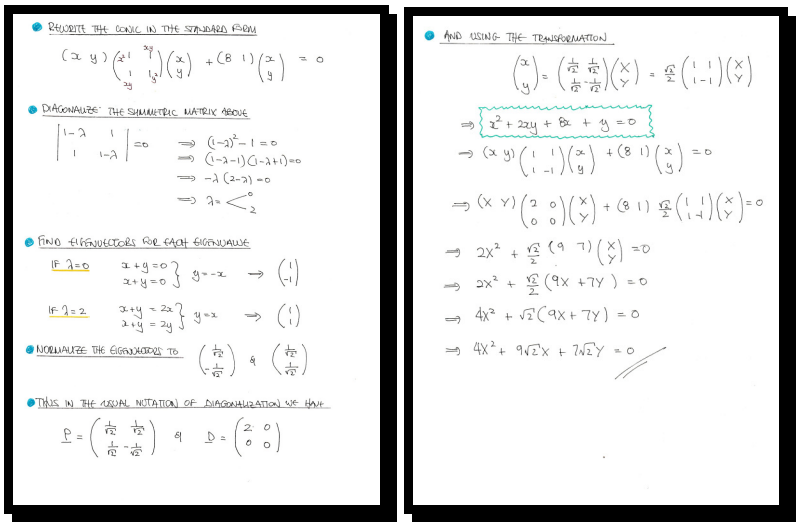
The conic with the above Cartesian equation has been rotated and translated out of its standard position.

Show that a Cartesian equation of the conic in a suitable coordinate system is

$$4X^2+9\sqrt{2}\,X+7\sqrt{2}\,Y=0\,.$$

You are not expected to sketch the conic.

[ ], [proof]

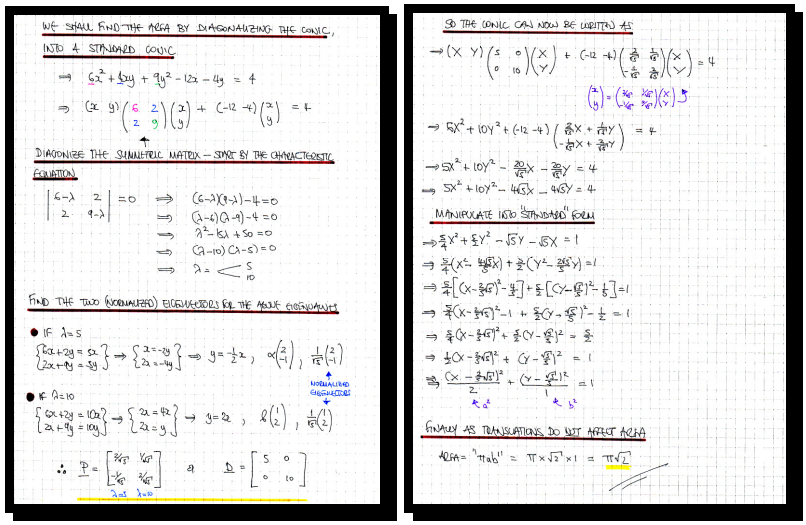


Question 8

Find, showing a detailed method, the area enclosed by the ellipse with the following Cartesian equation.

$$6x^2+4xy+9y^2-12x-4y=4\,.$$

$\boxed{}$, $\boxed{\text{area}=\pi\sqrt{2}}$



Question 9

$$3x^2-8xy+12y^2-30x-64y=90\,.$$

The conic with the above Cartesian equation has been rotated and translated out of its standard position.

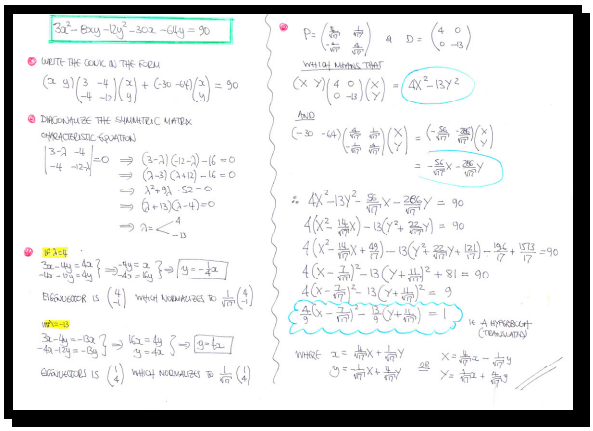
Show that a Cartesian equation of the conic in a suitable coordinate system is

$$\frac{4}{9}(X-a)^2-\frac{13}{9}(Y+b)^2=1,$$

where a and b are exact constants to be found.

You are not expected to sketch the conic.

$$\boxed{a=\frac{7}{\sqrt{17}},\quad b=\frac{11}{\sqrt{17}}}$$



QUADRICS

Question 1

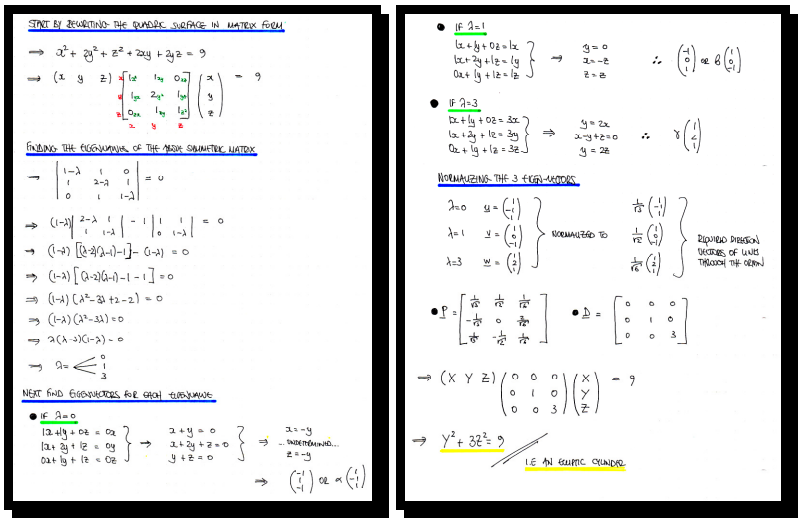
$$x^2+2y^2+z^2+2xy+2yz=9\,.$$

The quadric surface, with the above Cartesian equation, is rotated out of its standard position.

By suitably removing the cross terms, give the Cartesian equation of the quadric surface in the rotated system.

You are expected to name the conic and give the direction of any axes of symmetry in the original coordinate system.

$\square$, elliptic cylinder : $Y^2+3Z^2=9$ or $\dfrac{Y^2}{9}+\dfrac{Z^2}{3}=1$



Question 2

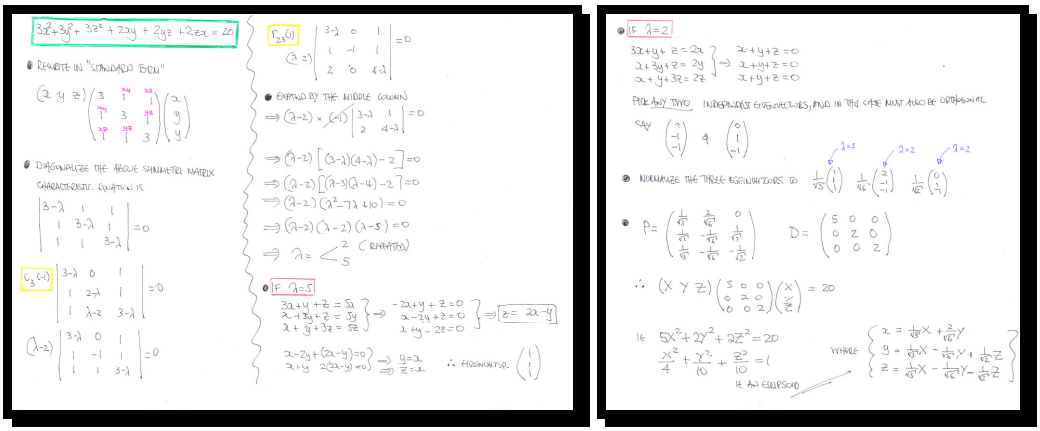
$$3x^2+3y^2+3z^2+2xy+2yz+2xz=20\,.$$

The quadric surface, with the above Cartesian equation, is rotated out of its standard position.

By suitably removing the cross terms, give the Cartesian equation of the quadric surface in the rotated system.

You are expected to name the conic and give the direction of any axes of symmetry in the original coordinate system.

$$\text{ellipsoid}: 5X^2+2Y^2+2Z^2=20 \quad \text{or} \quad \frac{x^2}{4}+\frac{Y^2}{10}+\frac{Z^2}{10}=1$$



Question 3

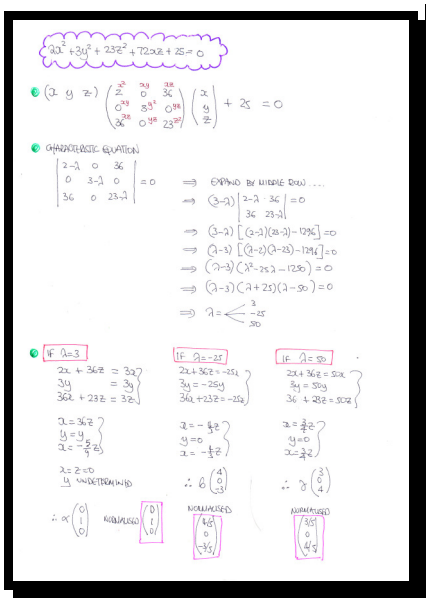
$$2x^2+3y^2+23z^2+72xz+25=0\,.$$

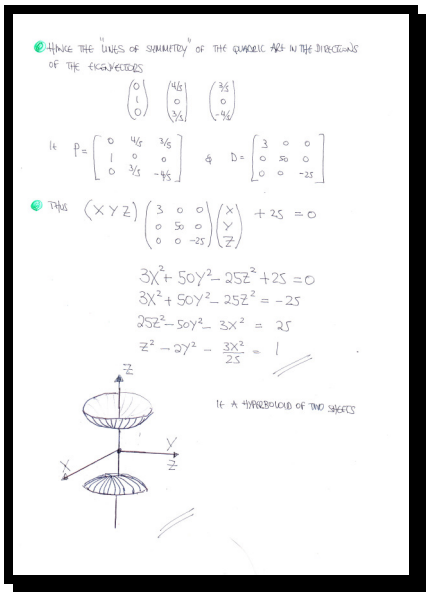
The quadric surface, with the above Cartesian equation, is rotated out of its standard position.

By suitably removing the cross terms, give the Cartesian equation of the quadric surface in the rotated system.

You are expected to name the conic and give the direction of any axes of symmetry in the original coordinate system.

$$\boxed{\text{hyperboloid of 2 sheets}: Z^2-2Y^2-\frac{3X^2}{25}=1}$$





Question 4

$$x^2+y^2+z^2+xy+yz+xz=2\,.$$

The quadric surface, with the above Cartesian equation, is rotated out of its standard position.

By suitably removing the cross terms, give the Cartesian equation of the quadric surface in the rotated system.

You are expected to name the conic and give the direction of any axes of symmetry in the original coordinate system.

$\boxed{}$, $\boxed{\text{ellipsoid}: \dfrac{X^2}{1}+\dfrac{Y^2}{4}+\dfrac{Z^2}{4}=1}$

