D OPERATOR

Summary of D Operator

Basic Definitions

- $D\{\ \} \equiv \dfrac{d}{dx}(\ )$,

  or depending on the independent variable

  $$D\{\ \} \equiv \frac{d}{dt}(\ ),\ D\{\ \} \equiv \frac{d}{dy}(\ ),\ D\{\ \} \equiv \frac{d}{du}(\ ) \text{ etc}$$

- $D^2\{\ \} \equiv \dfrac{d^2}{dx^2}(\ ),\ D^3 \equiv \dfrac{d^3}{dx^3}\{\ \},\ D^4\{\ \} \equiv \dfrac{d^4}{dx^4}(\ )$ etc

- $\left(aD^2+bD+c\right)\{f(x)\} \equiv aD^2\{f(x)\}+bD\{f(x)\}+c\{f(x)\}$

- $\left(\dfrac{1}{aD^2+bD+c}\right)\{f(x)\} \equiv \left(aD^2+bD+c\right)^{-1}\{f(x)\}$

- $\dfrac{1}{D}\{\ \} \equiv \int(\ )\,dx$

D Operations on Simple Functions

- $\left[f(D)\right]\left\{\mathrm{e}^{kx}\right\} \equiv \mathrm{e}^{kx}\left[f(k)\right]$

- $\left[f(D)\right]\left\{\mathrm{e}^{kx}V(x)\right\} \equiv \mathrm{e}^{kx}\left[f(D+k)\right]\{V(x)\}$

- $\left[f\left(D^2\right)\right]\{\sin kx\} \equiv \left[f\left(-k^2\right)\right]\sin kx$

- $\left[f\left(D^2\right)\right]\{\cos kx\} \equiv \left[f\left(-k^2\right)\right]\cos kx$

GENERAL PRACTICE

Question 1

Simplify each of the following expressions.

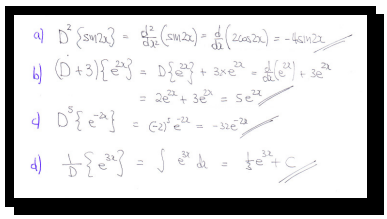
a) $D^2\{\sin 2x\}$.

b) $(D+3)\{\mathrm{e}^{2x}\}$.

c) $D^5\{\mathrm{e}^{-2x}\}$.

d) $\dfrac{1}{D}\{\mathrm{e}^{3x}\}$.

$\boxed{-4\sin 2x}$, $\boxed{5\mathrm{e}^{2x}}$, $\boxed{-32\mathrm{e}^{-2x}}$, $\boxed{\frac{1}{3}\mathrm{e}^{3x}+C}$



Question 2

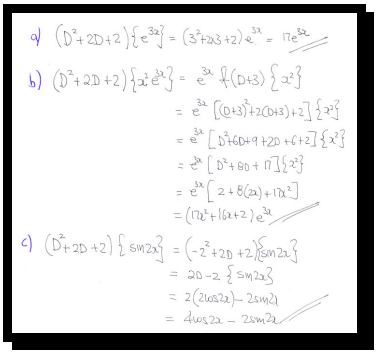
Simplify each of the following expressions.

a) $\left(D^2+2D+2\right)\left\{\mathrm{e}^{3x}\right\}$.

b) $\left(D^2+2D+2\right)\left\{x^2\,\mathrm{e}^{3x}\right\}$.

c) $\left(D^2+2D+2\right)\{\sin 2x\}$.

$\boxed{17\,\mathrm{e}^{3x}}$, $\boxed{\left(17x^2+16x+2\right)\mathrm{e}^{3x}}$, $\boxed{4\cos 2x-2\sin 2x}$



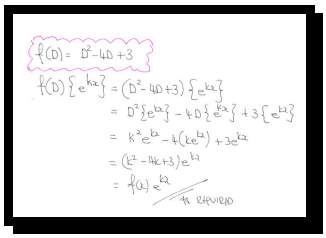
Question 3

$$f(D)=D^2-4D+3.$$

Show directly, by the definition of the D operator as $D\equiv\frac{d}{dx}(\ )$, that

$$f(D)\left\{\mathrm{e}^{kx}\right\}=f(k)\mathrm{e}^{kx}.$$

proof



Question 4

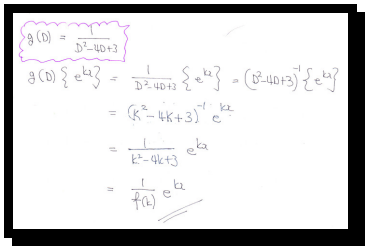
$$g(D)=\frac{1}{D^2-4D+3}.$$

Show directly, by the definition of the D operator as $D\equiv\frac{d}{dx}(\ )$, that

$$f(D)\left\{\mathrm{e}^{kx}\right\}=\frac{1}{f(k)}\mathrm{e}^{kx},$$

where k is a constant.

proof



Question 5

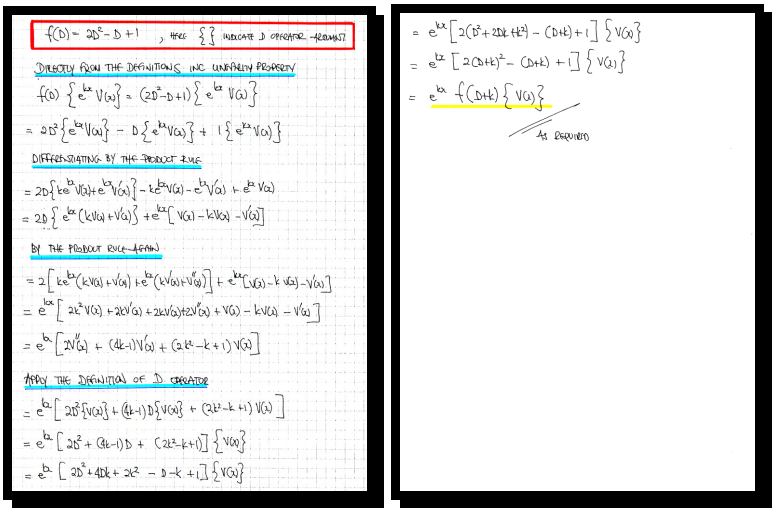
$$f(D)=2D^2-D+1.$$

Show directly, by the definition of the D operator as $D\equiv\dfrac{d}{dx}(\ \ )$, that

$$f(D)\left\{e^{kx}V(x)\right\}=e^{kx}f(D+k)\left\{V(x)\right\},$$

where $V(x)$ is a smooth function of x and k is a constant.

, proof



Question 6

Simplify each of the following expressions.

a) $\dfrac{1}{D^2+4D+3}\left\{30\mathrm{e}^{-2x}\right\}$.

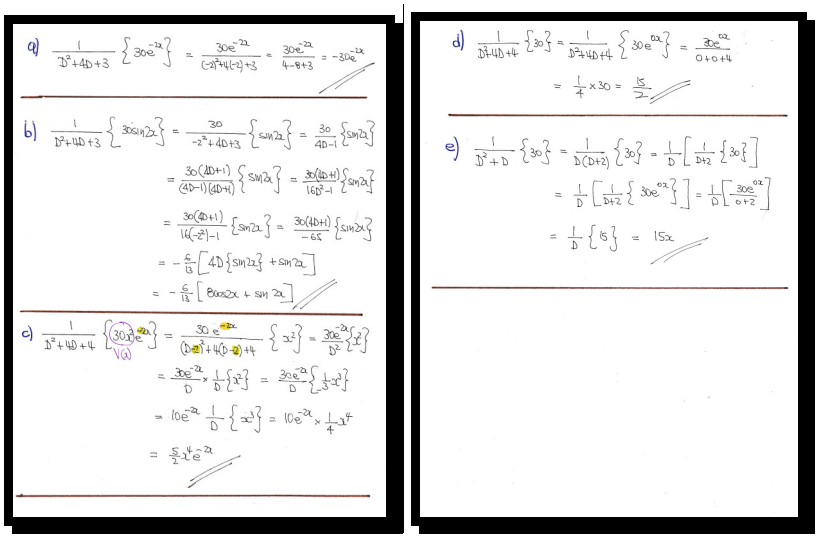
b) $\dfrac{1}{D^2+4D+3}\left\{30\sin 2x\right\}$.

c) $\dfrac{1}{D^2+4D+4}\left\{30x^2\,\mathrm{e}^{-2x}\right\}$.

d) $\dfrac{1}{D^2+4D+4}\left\{30\right\}$.

e) $\dfrac{1}{D^2+2D}\left\{30\right\}$.

$\boxed{}$, $\boxed{-30\mathrm{e}^{-2x}}$, $\boxed{-\dfrac{6}{13}\left[8\cos 2x+\sin 2x\right]}$, $\boxed{\dfrac{5}{2}x^4\,\mathrm{e}^{-2x}}$, $\boxed{\dfrac{15}{2}}$, $\boxed{15x}$



Question 7

Simplify each of the following expressions.

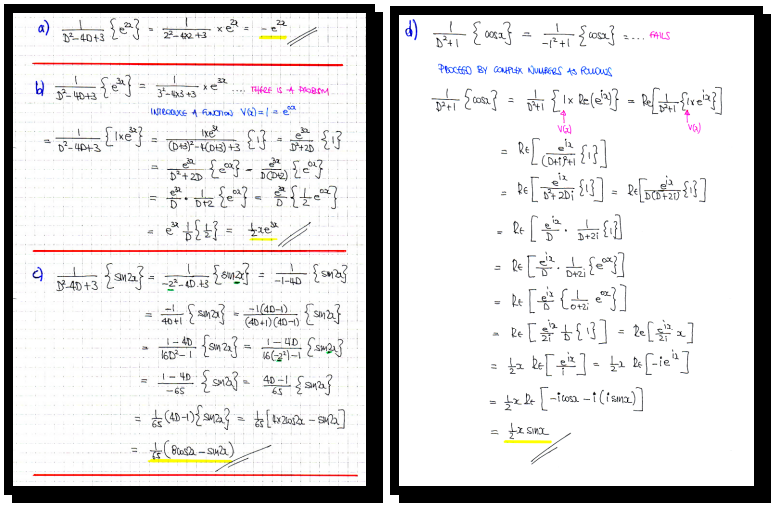
a) $\dfrac{1}{D^2-4D+3}\left\{\mathrm{e}^{2x}\right\}$.

b) $\dfrac{1}{D^2-4D+3}\left\{\mathrm{e}^{3x}\right\}$.

c) $\dfrac{1}{D^2-4D+3}\left\{\sin 2x\right\}$.

d) $\dfrac{1}{D^2+1}\left\{\cos x\right\}$.

$\boxed{}$, $\boxed{-\mathrm{e}^{2x}}$, $\boxed{\frac{1}{2}x\mathrm{e}^{3x}}$, $\boxed{\frac{1}{65}\left[8\cos 2x-\sin 2x\right]}$, $\boxed{\frac{1}{2}x\sin x}$



Question 8

Simplify each of the following expressions.

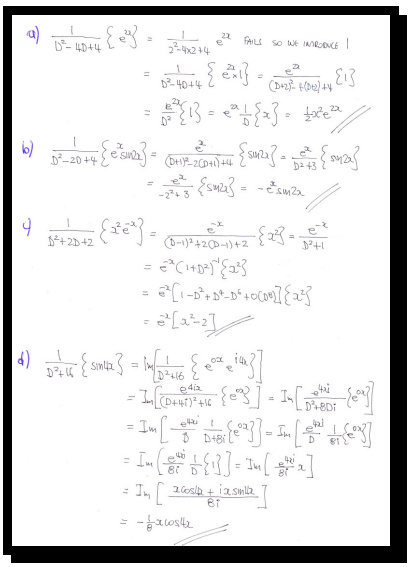
a) $\dfrac{1}{D^2-4D+4}\left\{\mathrm{e}^{2x}\right\}$.

b) $\dfrac{1}{D^2-4D+4}\left\{\mathrm{e}^{x}\sin 2x\right\}$.

c) $\dfrac{1}{D^2+2D+2}\left\{x^2\mathrm{e}^{-x}\right\}$.

d) $\dfrac{1}{D^2+16}\left\{\sin 4x\right\}$.

$\boxed{\frac{1}{2}x^2\mathrm{e}^{2x}}$, $\boxed{-\mathrm{e}^{x}\sin 2x}$, $\boxed{\mathrm{e}^{-x}\left(x^2-2\right)}$, $\boxed{-\frac{1}{8}x\cos 4x}$



Question 9

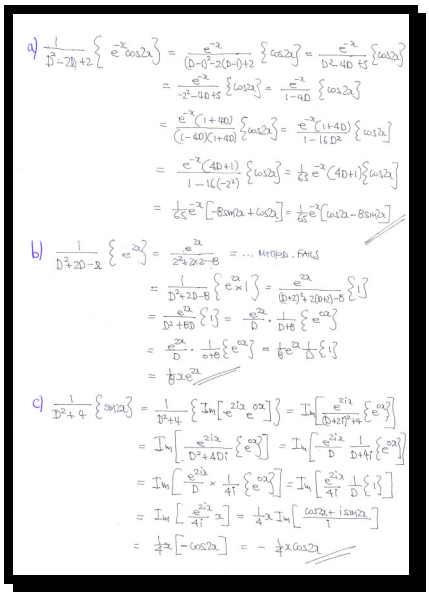
Simplify each of the following expressions.

a) $\dfrac{1}{D^2-2D+2}\left\{\mathrm{e}^{-x}\cos 2x\right\}$.

b) $\dfrac{1}{D^2+2D-8}\left\{\mathrm{e}^{2x}\right\}$.

c) $\dfrac{1}{D^2+4}\left\{\sin 2x\right\}$.

$\boxed{\dfrac{1}{65}\mathrm{e}^{-x}\left[\cos 2x-8\sin 2x\right]}$, $\boxed{\dfrac{1}{8}x\mathrm{e}^{2x}}$, $\boxed{-\dfrac{1}{4}x\cos 2x}$



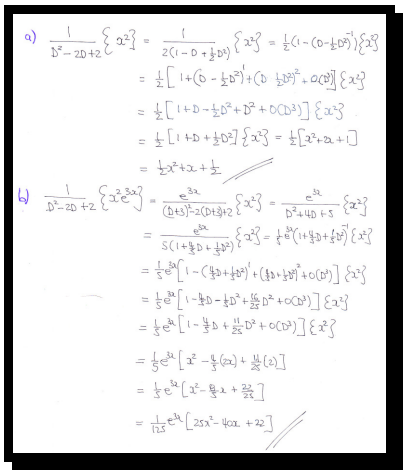
Question 10

Simplify each of the following expressions.

a) $\dfrac{1}{D^2-2D+2}\left\{x^2\right\}$.

b) $\dfrac{1}{D^2-2D+2}\left\{x^2\,\mathrm{e}^{3x}\right\}$.

$\boxed{\dfrac{1}{2}x^2+x+\dfrac{1}{2}}$, $\boxed{\dfrac{1}{125}\mathrm{e}^{3x}\left[25x^2-40x+22\right]}$



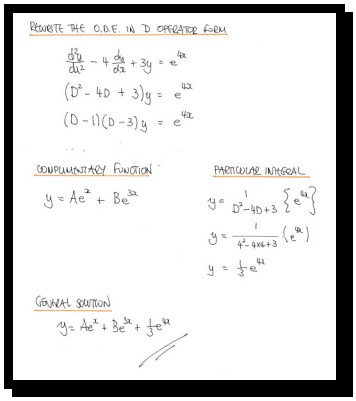
O.D.E.s APPLICATIONS

Question 1

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=\mathrm{e}^{4x}.$$

, $\boxed{y=A\mathrm{e}^{3x}+B\mathrm{e}^{x}+\frac{1}{3}\mathrm{e}^{4x}}$

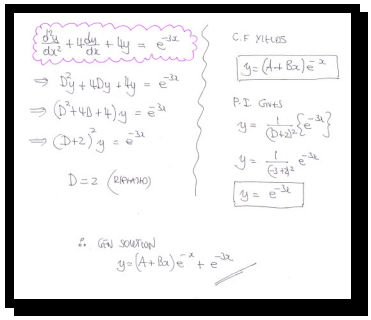


Question 2

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=\mathrm{e}^{-3x}.$$

$\boxed{y=\left(A+Bx\right)\mathrm{e}^{-x}+\mathrm{e}^{-3x}}$

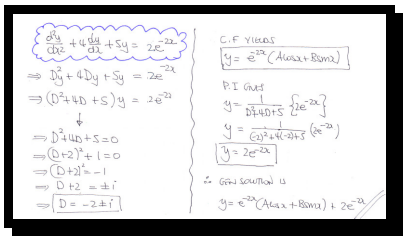


Question 3

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=2\mathrm{e}^{-2x}.$$

$$\boxed{y=\mathrm{e}^{-2x}\left[A\cos x+B\sin x\right]+2\mathrm{e}^{-2x}}$$

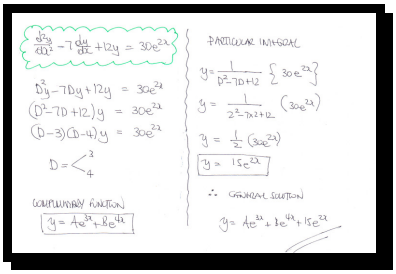


Question 4

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-7\frac{dy}{dx}+12y=30\mathrm{e}^{2x}.$$

$$\boxed{y=A\mathrm{e}^{3x}+B\mathrm{e}^{4x}+15\mathrm{e}^{2x}}$$

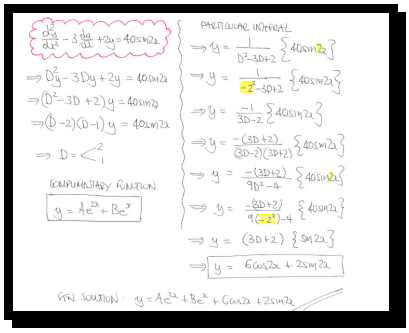


Question 5

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=40\sin 2x\,.$$

$$\boxed{y=A\mathrm{e}^{2x}+B\mathrm{e}^{x}+6\cos 2x+\sin 2x}$$

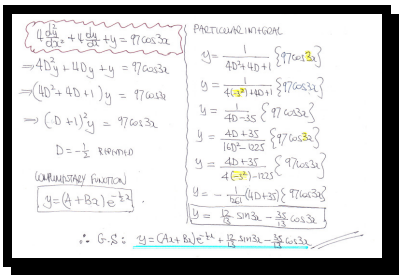


Question 6

Use D operator techniques, to find the general solution of the following differential equation

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=97\cos 3x\,.$$

$$\boxed{y=(Ax+B)\mathrm{e}^{-\frac{1}{2}x}+\frac{12}{13}\sin 3x-\frac{35}{13}\cos 3x}$$

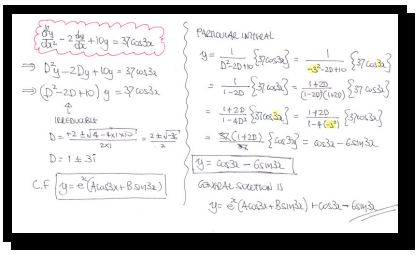


Question 7

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+10y=37\cos 3x\,.$$

$$\boxed{y=\mathrm{e}^{x}\left(A\cos 3x+B\sin 3x\right)+\cos 3x-6\sin 3x}$$

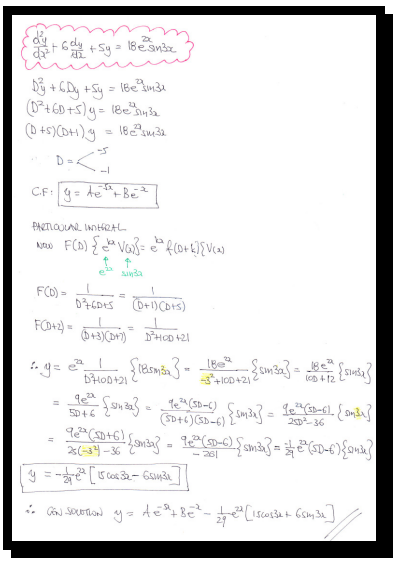


Question 8

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+5y=18\mathrm{e}^{2x}\sin 3x\,.$$

$$\boxed{y=A\mathrm{e}^{-5x}+B\mathrm{e}^{-x}-\frac{1}{29}\mathrm{e}^{x}\left[15\cos 3x+6\sin 3x\right]}$$

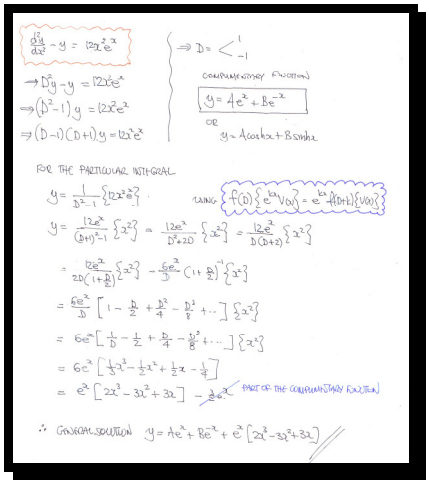


Question 9

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-y=12x^2\mathrm{e}^x.$$

$$\boxed{y=A\mathrm{e}^x+B\mathrm{e}^{-x}+\mathrm{e}^x\left[2x^3-3x^2+3x\right]}$$

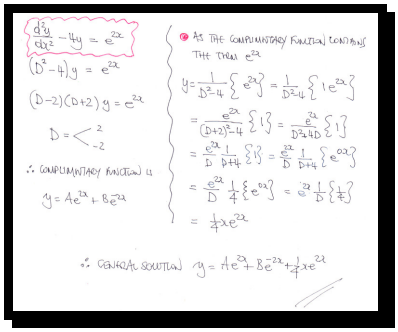


Question 10

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-4y=\mathrm{e}^{2x}.$$

$$\boxed{y=A\mathrm{e}^{2x}+B\mathrm{e}^{-2x}+\frac{1}{4}x\mathrm{e}^{2x}}$$

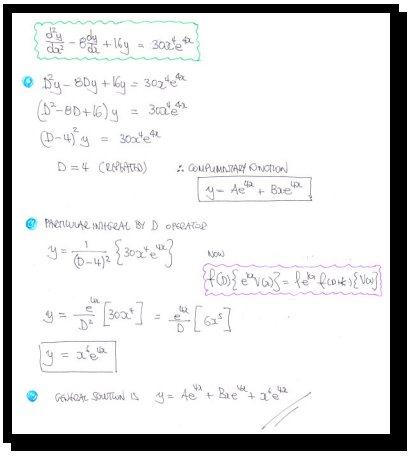


Question 11

Use D operator techniques, to find the general solution of the following differential equation

$$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=30x^4\,\mathrm{e}^{4x}.$$

$$\boxed{y=A\mathrm{e}^{4x}+Bx\mathrm{e}^{4x}+x^6\,\mathrm{e}^{4x}}$$



Question 12

Find the general solution of the following differential equation.

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=10\mathrm{e}^{-2x},$$

a) ... by using D-operator techniques only.

b) ... by using the substitution $Y=(D+2)y$, in a method involving D-operator techniques only.

$\boxed{}$, $\boxed{y=A\mathrm{e}^{3x}+B\mathrm{e}^{-2x}-2x\mathrm{e}^{-2x}}$

