IYGB

Special Paper N

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $=T$, Number of non attempted questions $=N$, Percentage score $=P$.

$P=\frac{1}{2}T+N$ (rounded up to the nearest integer)

Distinction $P\geq 70$, Merit $55\leq P\leq 69$, Pass $40\leq P\leq 54$

Question 1

$$2^{m+1}+2^{m}=3^{n+2}-3^{n}.$$

Given that m and n are positive integers, find the value of m and the value of n. **(6)**

Question 2

The point A lies on the straight line L with equation

$$y=2x+3.$$

The point B has coordinates $(4,1)$ and the point C is the reflection of A about L.

Determine the possible coordinates of A, given that AB is perpendicular to AC. **(10)**

Question 3

A water tank is full of water.

The tank has 3 outlet pipes, each having a constant drainage rate, when the water is allowed to flow out of the tank.

Let A, B and C be labels for each of the three outlet pipes.

If only A and B are turned on, it takes 12 hours to drain the tank.

If only A and C are both turned on, it takes 15 hours to drain the tank.

If only B and C are both turned on, it takes 20 hours to drain the tank.

a) Find how long does each outlet pipe on its own take to drain a full tank. **(8)**

b) Determine the time it takes to drain a full tank, if all three outlet pipes are turned on. **(1)**

Question 4

It is given that

$$a^2 + b^2 = c^2, \quad a \in \mathbb{N}, \; b \in \mathbb{N}.$$

Show that a and b cannot both be odd. **(6)**

Question 5

A curve has equation

$$y = 2kx^{\frac{3}{2}} - \frac{25}{16}\ln kx, \quad x \in \mathbb{R}, \quad x > 0.$$

where k is a positive constant.

The point A lies on the curve, where $x = \frac{1}{k}$.

Given that the normal to the curve at A passes through the origin O, determine an equation of the normal to the curve at A. **(12)**

Question 6

The sum of the first k terms of an arithmetic progression is 110.

The sum of the first $2k$ terms of the same arithmetic progression is 946.

Given further that $k \neq 1$, determine the first term and the common difference of the arithmetic progression. **(10)**

Question 7

Solve the following quadratic in x, giving the answers in terms of k.

$$k^2x^2 - \left(k^3 + k + 1\right)x + k^2 + k = 0, \quad k \neq 0. \qquad \textbf{(11)}$$

Question 8

Solve the following trigonometric equation, for $0 \le x < 2\pi$.

$$2\cos x \sin^2 x - 2\cos^2 x \sin x + \cos^2 x - 4\sin^2 x + 3\cos x \sin x + 2\sin x - 2\cos x = 0. \quad \textbf{(11)}$$

Question 9

Use polynomial division to find the exact value of

$$\int_0^1 \frac{x^4(1-x)^4}{x^2+1}\, dx. \quad \textbf{(7)}$$

You may assume that

$$\int \frac{1}{1+x^2}\, dx = \arctan x + \text{constant}.$$

Question 10

On a clearly labelled set of axes, draw a detailed sketch of the graph of

$$y = (\arcsin x)^2 \arccos x, \ -1 \le x \le 1. \quad \textbf{(9)}$$

You may assume that

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}.$$

Question 11

Find in exact simplified form an exact expression for the sum of the first n terms of the following series

$$1 + 11 + 111 + 1111 + 11111 + \ldots \quad \textbf{(10)}$$

Question 12

Solve the following differential equation

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2,$$

given further $y = 0$, $\frac{dy}{dx} = 2$ at $x = 0$.

Give the answer in the form $x = f(y)$. **(12)**

Question 13

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] \;=\; u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

By showing a detailed method prove that if n is even

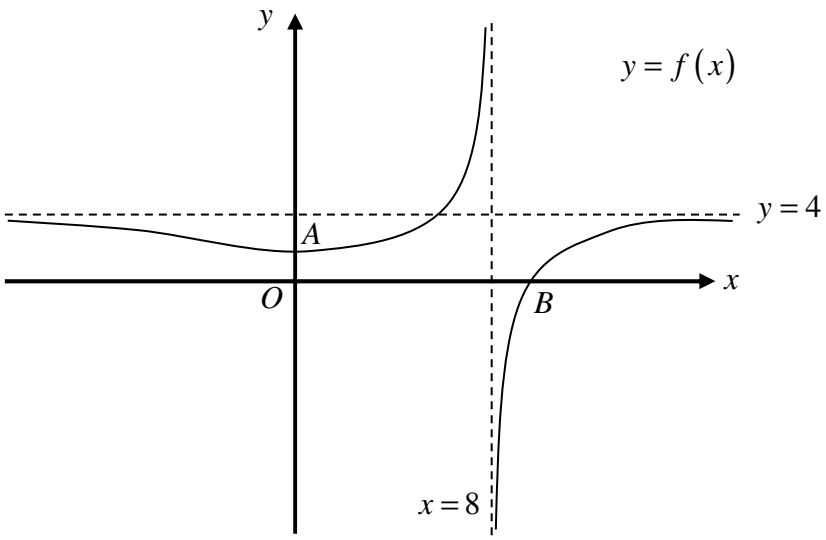
$$\prod_{r=1}^{n}\left[\cos\frac{2\pi}{n} + \cos\frac{2\pi}{n}\cot\frac{(2r-1)\pi}{n}\right] = 1. \qquad \textbf{(10)}$$

Question 14

Determine the range of values of x that satisfy the inequality

$$\left|\frac{x+3}{x}\right| \geq \left|\frac{x}{2-x}\right|. \qquad \textbf{(11)}$$

Question 15



The figure above shows the curve with equation $y = f(x)$.

The curve has a local minimum at $A(0,2)$ and crosses the x axis at $B(9,0)$.

The straight lines with equations $y = 4$ and $x = 8$ are asymptotes to the curve.

Sketch on separate set of axes the graph of …

a) … $y = \dfrac{1}{f(x)}$. **(4)**

b) … $y = f'(x)$. **(3)**

c) … $y^2 = f(x)$. **(3)**

In each case, give if possible, the equations of any asymptotes, the coordinates of any stationary points and the coordinates of any points where the curve meets the coordinate axes.

Question 16

The function f, defined for all real numbers, satisfies the following relationship

$$f(x)+4f(-x)\equiv 1+x^2\int_{-1}^{1} f(u)\ du\,.$$

Determine as an exact fraction the value of

$$\int_{-1}^{1} f(x)\ dx\,. \qquad (\mathbf{9})$$

Question 17

Show that if n is an integer, then

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos(2n\theta)\cos\theta\ d\theta \;=\; \frac{2(-1)^n}{1-(2n)^2}\,. \qquad (\mathbf{10})$$

Question 18

$$x^3+px+q=0\,,\ x\in\mathbb{R}$$

The cubic equation above is given in terms of the real constants, p and q.

Use differentiation to determine the conditions that p and q must satisfy so that the above equation has …

- … one real root.
- … three real roots, of which one is repeated.
- … three distinct real roots. $(\mathbf{12})$

Question 19

Use a fully detailed method to show that

$$\arctan\left[\sqrt{6}+\sqrt{3}-\sqrt{2}-2\right]=37.5°\,. \qquad \textbf{(12)}$$

Question 20

In the acute triangle ABC the following information is given.

- The point P lies on AB so that $AP:PB = 1:2$.
- The point Q lies on AC so that $AQ:QC = 1:3$.
- The point D is the intersection of CP and BQ.

The straight line through A and D is extended so that it meets BC at the point R.

Determine the ratio $BR:RC$. **(13)**
