# IYGB

## Special Paper C

## Time: 3 hours 30 minutes

**Candidates may NOT use any calculator.**

**Information for Candidates**

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

**Scoring**

Total Score $=T$, Number of non attempted questions $=N$, Percentage score $=P$.

$P=\frac{1}{2}T+N$ (rounded up to the nearest integer)

Distinction $P\geq 70$, Merit $55\leq P\leq 69$, Pass $40\leq P\leq 54$

**Question 1**

The functions $f$ and $g$ are defined by

$$f(x) \equiv 3\sin x,\ x \in \mathbb{R},\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$g(x) \equiv 6 - 3x^2,\ x \in \mathbb{R}.$$

**a)** Find an expression for $f^{-1}g(x)$. **(2)**

**b)** Determine with justification the domain of $f^{-1}g(x)$. **(5)**

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**Question 2**

The sum of the first $k$ terms of a geometric progression is $180$.

It is further given that the sum of the first $k$ terms of this geometric progression is **twelve less** than its sum to infinity.

If the sum to infinity of the geometric progression is four times as large as its second term, use algebra to determine the value of $k$. **(10)**

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**Question 3**

The point $P$ lies on the straight line $L_1$, which is parallel to the vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and passes through the point with coordinates $(10,3,7)$, relative to an origin at $(0,0,0)$.

The point $Q$ lies on another straight line $L_2$, which is in the direction of the vector $4\mathbf{i} - \mathbf{j} + \mathbf{k}$ and passes through the point with coordinates $(9,1,0)$.

The straight line $L_3$ is perpendicular to both $L_1$ and $L_2$, and meets $L_1$ and $L_1$ at the points $P$ and $Q$, respectively.

Find the coordinates of $P$ and $Q$. **(10)**

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**Question 4**

Find the finite solution of the equation

$$\arctan\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\arctan x\,. \qquad \textbf{(10)}$$

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**Question 5**

$$I=\int_{1.5}^{2}\frac{(x-2)\left(2x^2-5x-1\right)}{(x-1)(x-3)}\,dx\,.$$

Use appropriate integrations techniques to show that

$$I=\frac{5}{4}-\ln k\,,$$

where $k$ is a positive integer. **(11)**

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**Question 6**

The island state of Trigland has declared an exclusive economic zone into the sea, which is within $6$ miles from every point of its coastline.

The island of Trigland is a rectilinear triangle of sides $13$, $14$ and $15$ miles.

Determine, in exact form, the total economic zone of Trigland, which consists of land and sea. **(7)**

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**Question 7**

$$f(x,y)\equiv\sqrt{\frac{x^4}{y^4}+\frac{y^4}{x^4}-2\left(\frac{x^3}{y^3}+\frac{y^3}{x^3}\right)+3\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)-4\left(\frac{x}{y}+\frac{y}{x}\right)+5}\,.$$

Simplify $f(x,y)$ in a form not involving square roots. **(8)**

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**Question 8**

The straight line $l$ has equation

$$(2+a)x+(2-a)y=2-5a,$$

where $a$ is a constant.

Show that $l$ passes through a fixed point $P$ for all values of $a$. **(8)**

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**Question 9**

A curve is defined in its largest real domain by the equation

$$y=\arccos\left[\frac{a\cos x+b}{a+b\cos x}\right],$$

where $a$ and $b$ are constants such $a>b>0$.

Show that $y$ increases with $x$ at a rate which lies between

$$\sqrt{\frac{a-b}{a+b}} \quad \text{and} \quad \sqrt{\frac{a+b}{a-b}}.$$

You may assume that $\frac{d}{dx}[\arccos x]=-\frac{1}{\sqrt{1-x^2}}$. **(10)**

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**Question 10**

By considering the solution of trigonometric equation

$$\sin(x-30)^\circ=\cos(x-45)^\circ,$$

find, in degrees, the exact value of $\arctan\left[\frac{1+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right]$. **(10)**

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**Question 11**

At time $t = 0$, one litre of a certain liquid chemical is added to a tank containing $20$ litres of water. The chemical reacts with the water forming a gas, and as a result of this reaction both the volumes of the water and the chemical are reduced.

At time $t$ minutes since the chemical reaction started, the respective volumes of the chemical and the water used in the reaction, are $(1-v)$ litres and $4(1-v)$ litres.

The rate at which the volume of the chemical in the tank reduces, is proportional to the product of the volume of the chemical and the volume of the water, still left in the tank.

Given that $2$ minutes after the reaction started the volume of the chemical remaining is $\frac{4}{19}$ of a litre, show that

$$2^t = \frac{v+4}{5v}. \qquad \textbf{(14)}$$

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**Question 12**

The sum to infinity $S$ of the convergent geometric series is given by

$$S = 1 + x + x^2 + x^3 + x^4 + \ldots, \quad |x| < 1,$$

By integrating the above equation between suitable limits, or otherwise, find

$$\sum_{r=1}^{\infty} \left[ \frac{1}{r \times 2^r} \right].$$

***You may assume that integration and summation commute.*** **(8)**

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**Question 13**

The distinct points $A$, $B$ and $C$ lie on the curve with equation

$$xy = p^2,$$

where $p$ is a positive constant.

Given that $ABC$ is a right angle, show that the tangent to the curve at $B$, is perpendicular to $AC$. **(10)**

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**Question 14**

Sketch the graph of

$$y = x|x-1| - x|x+4|, \quad x \in \mathbb{R}.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of any cusps of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*] **(10)**

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**Question 15**

$$I = \int_{-\frac{5}{2}}^{\frac{7}{2}} \frac{4x+1}{\sqrt{35+4x-4x^2}}\, dx.$$

By writing $35+4x-4x^2$ in completed the square form, followed by a suitable trigonometric substitution, show that

$$I = \frac{3}{2}\pi. \qquad \textbf{(10)}$$

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**Question 16**

Solve the following simultaneous equations

$$(91-2x)^3 = 216xy^2 \qquad \text{and} \qquad (37-2y)^3 = 216x^2y,$$

where $x \in \mathbb{R}$, $y \in \mathbb{R}$. **(12)**

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**Question 17**

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] \;=\; u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

By showing a detailed method prove that

$$\prod_{k=1}^{n}\left[\frac{2k-1}{2k+2}\right] \;=\; \binom{2n+1}{n}\frac{1}{4^n(2n+1)}. \qquad \textbf{(12)}$$

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**Question 18**

A spherical cap of depth $a$ is removed from a sphere of radius $na$, where $n$ is a positive constant, such that $n > \frac{1}{2}$. The volume of the spherical cap is less than half the volume of the sphere.

The remainder of the sphere is moulded to a right circular cone whose base is equal to that of the circular plane face of the spherical cap removed.

Given that the height of the cone is $ma$, where $m$ is a positive constant, show that

$$m = (n+p)(2n+q),$$

where $p$ and $q$ are integers to be found. **(13)**

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**Question 19**

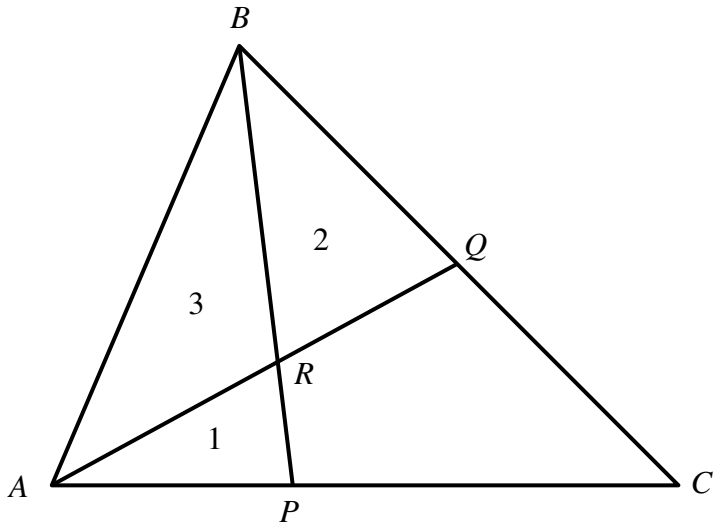
Solve the following trigonometric equation.

$$2\cot 2x = \sec x \sec\left(\frac{2\pi}{15}\right) + 2\tan\left(\frac{2\pi}{15}\right), \quad 0 \le x < 2\pi .$$

Give the answers in the form $k\pi$, where $k$ is rational. **(12)**

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**Question 20**




The figure above shows a triangle $ABC$.

The point $P$ lies on $AC$ and the point $Q$ lies on $BC$.

The point $R$ is the intersection of $BP$ and $AQ$.

Given that the respective areas of the triangles $APR$, $BQR$ and $ABR$ are 1, 2 and 3 square units, determine the exact area of the quadrilateral $CPRQ$. **(8)**

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