IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper I

Difficulty Rating: 3.9900/1.3930

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

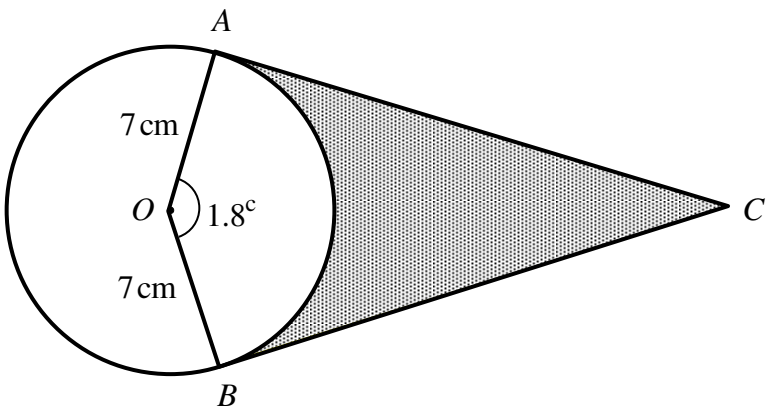
Question 1

A curve C has equation

$$y = \frac{x^2}{\ln x}, \quad x \in \mathbb{R}, \quad x > 0.$$

Find, in exact form, the equation of the tangent to C at the point where $x = \sqrt{e}$. **(7)**

Question 2



The figure above shows a circle with centre at O and radius 7 cm.

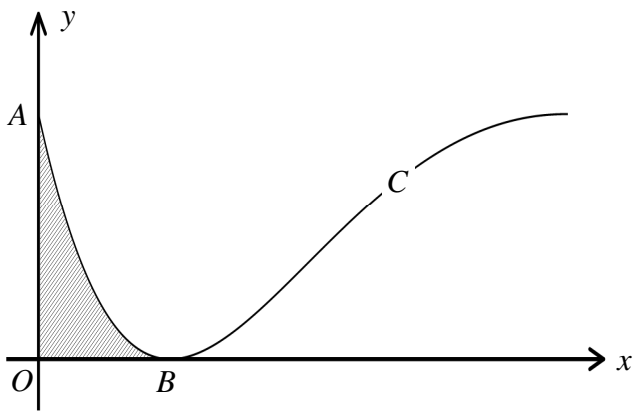
The points A and B lie on the circle so that the angle AOB is 1.8 radians.

The tangents to the circle at the points A and B meet at the point C.

The region shown shaded in the figure above, is enclosed by the two tangents AC and BC, and the circle.

Determine the area of this region. **(7)**

Question 3



The figure above shows the curve C, with parametric equations

$$x = t^2, \quad y = 1 + \cos t \ , \ 0 \le t \le 2\pi .$$

The curve meets the coordinate axes at the points A and B.

a) Show that the area of the shaded region bounded by C and the coordinate axes is given by the integral

$$\int_{t_1}^{t_2} 2t\left(1+\cos t\right)\ dt ,$$

where t_1 and t_2 are constants to be stated. **(4)**

b) Evaluate the above parametric integral to find an exact value for the area of the shaded region. **(4)**

Question 4

Prove by the method of **contradiction** that there are no integers a and b which satisfy the following equation.

$$a^2 - 8b = 7 \qquad \textbf{(6)}$$

Question 5

A function f is defined by

$$f(x) = 4 - \frac{1}{x-1}, \quad x \in \mathbb{R},\ x > 1.$$

a) Determine an expression for the inverse, $f^{-1}(x)$. **(4)**

b) Find the domain and range of $f^{-1}(x)$. **(2)**

Question 6

$$f(x) = \sin 2x, \quad x \in \mathbb{R}$$

$$g(x) = f\left(x + \frac{\pi}{4}\right) - f\left(x - \frac{\pi}{4}\right),\ x \in \mathbb{R}.$$

a) Show clearly that

$$g(x) = 2\cos 2x.$$ **(4)**

b) Express $g'(x)$ in terms of $f(x)$. **(2)**

Question 7

Three points, $A(4,4,1)$, $B(2,-2,0)$ and $C(6,3,7)$ are referred relative to a fixed origin O.

If A, B, C and the point D form the parallelogram $ABCD$, use vector algebra to find the coordinates of D and hence calculate the angle OCD. **(7)**

Question 8

$$f(x) = \sqrt{1-x}\,,\ -1 < x < 1\,.$$

a) Expand $f(x)$ up and including the term in x^3. **(4)**

b) Show clearly that

$$8 \times \sqrt{1 - \frac{1}{64}} = 3\sqrt{7}\,. \qquad \textbf{(2)}$$

c) By using the **first two** terms of the expansion obtained in part **(a)** and the result shown in part **(b)**, show further that

$$\sqrt{7} \approx \frac{127}{48}\,. \qquad \textbf{(3)}$$

Question 9

A curve C has implicit equation

$$ax^3 - 3xy + by^2 = 224\,,$$

where a and b are non zero constants.

The normal to the curve at the point $P(-2,6)$ has equation

$$15x - 13y + 108 = 0\,.$$

Determine the value of a and the value of b. **(12)**

Question 10

A pension broker gets paid £15 commission **per week** for every pension scheme he sells. Each week he sells a new pension scheme so that …

In the 1st week he gets paid £15 commission for the pension he just sold.

In the 2nd week he gets paid £30, £15 for the pension sold in the 1st week plus £15 for pension he sold in the 2nd week.

In the 3rd week he gets paid £45, £15 for the pension sold in the 1st week plus £15 for pension he sold in the 2nd week, plus £15 for the pension he sold in the 3rd week, and so on.

a) Find the commission he gets paid on the last week of the year. **(2)**

b) Find his annual earnings after one year in this job. **(2)**

His commission increases to £20 for new pension schemes sold during the 2nd year but decreases to £10 for the schemes he sold in the 1st year.

The broker continues to sell at the rate of one new pension scheme every week.

c) Find his annual earnings in the 2nd year. **(6)**

Question 11

By using the substitution $u = \sin 2x$, or otherwise, find an exact simplified value for the following trigonometric integral.

$$\int_0^{\frac{1}{4}\pi} \frac{1-\tan^2 x}{\sec^2 x + 2\tan x}\, dx\,. \qquad \textbf{(8)}$$

Question 12

A water tank has the shape of a hollow inverted hemisphere with a radius of 1 m.

It can be shown by calculus that when the depth of the water in the tank is h m, its volume, V m^3, is given by the formula

$$V = \frac{1}{3}\pi h^2(3-h).$$

Water is leaking from a hole at the bottom of the tank, in m^3 per hour, at a rate proportional to the volume of the water left in the tank at that time.

a) Show clearly that

$$\frac{dh}{dt} = -\frac{kh(3-h)}{3(2-h)},$$

where k is a positive constant. **(5)**

The water tank is initially full.

b) Solve the differential equation to show further that **(9)**

$$3h^2 - h^3 = 2e^{-kt}.$$