IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper E

Difficulty Rating: 3.885/1.3238

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Differentiate each of the following expressions with respect to x.

a) $y = (2x + \ln x)^3$. **(3)**

b) $y = \dfrac{x^2}{3x - 1}$. **(3)**

c) $y = \sin^4 3x$. **(3)**

Question 2

a) Use the trapezium rule with 5 equally spaced ordinates to estimate the value of the following integral.

$$\int_0^{\frac{1}{3}\pi} e^{\tan^2 x} \, dx.$$ **(4)**

b) Use the answer of part **(a)** to estimate the value of

$$\int_0^{\frac{1}{3}\pi} e^{\sec^2 x} \, dx.$$ **(3)**

c) Discuss briefly whether the estimates of the previous parts of the question are likely to be accurate, stating further whether they are overestimates or underestimates to the true values of these integrals. **(3)**

Question 3

Prove by contradiction that if p and q are positive integers, then

$$\frac{p}{q} + \frac{q}{p} \geq 2.$$ **(5)**

Question 4

The points $A(t,3,2)$ and $B(5,2,2t)$, where t is a scalar constant, are referred relative to a fixed origin O.

Given that $\left|\overrightarrow{AB}\right| = \sqrt{21}$, find the possible values of t. **(7)**

Question 5

An oil company is drilling for oil.

It costs £5000 to drill for the first 10 metres into the ground.

For the next 10 metres it costs an extra £1200 compared with the first 10 metres, thus it costs £6200. Each successive 10 metres drilled into the ground costs an extra £1200, compared with the cost of drilling the previous 10 metres.

a) Find the cost of drilling 200 metres into the ground. **(4)**

The company has a budget of £15,000,000.

b) Determine the maximum depth, in metres, that can be reached on this budget. **(5)**

Question 6

The volume of the water, V m^3, in a container satisfies

$$V = x^3 \mathrm{e}^{-x^2},$$

where x m is the depth of the water in the container.

Find the rate of increase of the volume of the water in the container when its depth is 0.5 m and is rising at the rate of 0.01 m s^{-1}. **(6)**

Question 7

The functions f and g are defined as

$$f(x) = \frac{1}{3}(x + 2a),\ x \in \mathbb{R},$$

$$g(x) = |2x - a|,\ x \in \mathbb{R},$$

where a is a positive constant.

a) Sketch in the same set of axes the graph of $f(x)$ and the graph of $g(x)$. **(4)**

The sketch must include the coordinates of any points where these graphs meet the coordinate axes.

b) Find, in terms of a, the coordinates of the points of intersection between the graphs of $f(x)$ and $g(x)$. **(5)**

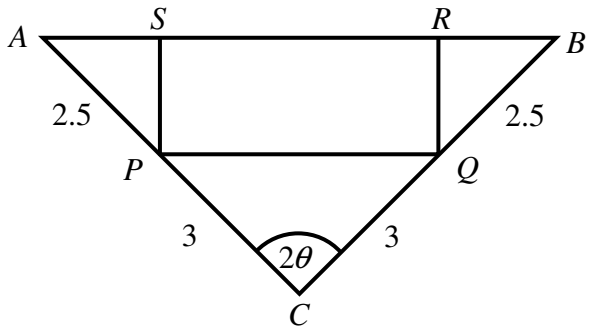
Question 8

$$\frac{dy}{dx}\sec x = y^2 - y$$

Solve the differential equation above, subject to the boundary condition, $y = \frac{1}{2}$ at $x = 0$, to show that

$$y = \frac{1}{1 + e^{\sin x}}.$$ **(10)**

Question 9



The figure above shows an isosceles triangle ABC where the angle $ACB = 2\theta$.

A rectangle $PQRS$ is drawn inside ABC, so that S and R lie on AB, P lies on AC and Q lies on BC.

It is further given that $|AP| = |BQ| = 2.5$ and $|PC| = |QC| = 3$.

a) Show clearly that the perimeter of $PQRS$ is

$$5\cos\theta + 12\sin\theta.$$ **(3)**

b) Express $5\cos\theta + 12\sin\theta$ in the form $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$. **(3)**

c) Find the value of θ, given that the perimeter of $PQRS$ is 10. **(4)**

Question 10

It is given that for some constants A and B

$$6\sin x \equiv A(\cos x + \sin x) + B(\cos x - \sin x).$$

a) Find the value of A and the value of B. **(3)**

b) Hence find

$$\int \frac{6\sin x}{\cos x + \sin x}\, dx.$$ **(5)**

Question 11

The function $f(x)$ is defined

$$f(x) = x^2(x+2),\ x \in \mathbb{R},\ x > 0.$$

a) Show that $f(x)$ is invertible. **(3)**

b) Solve the equation

$$f(x) = f^{-1}(x). \quad \textbf{(6)}$$

Question 12

A curve lies entirely above the x axis and has parametric equations

$$x = 2t^5, \qquad y = \frac{1}{1+2t^{\frac{5}{2}}}, \qquad t \geq 0.$$

The finite region R is bounded by the curve, the x axis, the y axis and the straight line with equation $x = 2$.

Use integration in parametric to find the exact area of R. **(8)**
