IYGB GCE

Mathematics MMS

Advanced Level

Practice Paper V

Difficulty Rating: 4.3400/1.2048

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 16 questions in this question paper.
The total mark for this paper is 150.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

SECTION 1 - STATISTICS

Question 1

The number of hours worked in a given week by a group of 64 individuals is summarized in the table below.

Hours (nearest hour)	Frequency
1 – 10	5
11 – 20	16
21 – 25	14
26 – 30	17
31 – 40	10
41 – 59	2

a) Estimate, by linear interpolation, the value of the median. **(2)**

b) Estimate the mean and the standard deviation of these data. **(4)**

c) Establish, with justification, the skewness of the data. **(2)**

d) Determine the possibility whether the data contain any outliers. **(3)**

Question 2

In a histogram the commuting times of a group of individuals, correct to the nearest minute, are plotted on the x axis.

In this histogram the class $47-50$ has a frequency of 48 and is represented by a rectangle of base 6 cm and height 3.6 cm.

In the same histogram the class $51-55$ has a frequency of 30.

Determine the measurements, in cm, of the rectangle that represents the class $51-55$.

(3)

Question 3

At a college course 75% of the students are male and 25% are female.

It is further known that 60% of the male students own a bike and 40% of the female students own a bike.

A student is selected at random.

Given that the student selected owns a bike determine the probability that this students is female ...

a) ... by a two way table. **(4)**

b) ... by a tree diagram. **(4)**

c) ... by a Venn diagram. **(4)**

Question 4

Trains are arriving at the Villerpool train station **every hour** and these trains are either late or on time.

- If a train is on time the probability the next train is on time is 0.9.
- If a train is late the probability the next train is late is 0.6.

On a particular day the 07.00 train arrives on time.

a) Determine the probability that…

i. … the 10.00 train will arrive on time. **(4)**

ii. … **only one** out of the 08.00, 09.00 and 10.00 trains will arrive on time. **(4)**

b) Given that the 10.00 train arrived on time, find the probability that the 08.00 train was late. **(3)**

Question 5

An airline operates between London and New York.

The outward flight time, X hours, may be modelled by a Normal distribution with mean of $7\frac{1}{2}$ hours and a standard deviation of $\frac{2}{3}$.

a) Find the probability that one such flight will take more than $8\frac{1}{2}$ hours. **(3)**

The return flight time, Y hours, may also be modelled by a Normal distribution with mean μ hours and unknown standard deviation.

b) Given that $P(Y < \mu - 0.5) = 0.28$, determine $P(Y > \mu + 0.5 | Y > \mu - 0.5)$. **(3)**

Question 6

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X = x) = \begin{cases} k(4-x) & x = 0,1,2,3 \\ \frac{1}{2} & x = 4 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k = \frac{1}{20}$. **(2)**

Two independent observations of X are made, denoted by X_1 and X_2.

b) Find the probability distribution of Y, where $Y = X_1 + X_2$. **(3)**

c) Calculate $P(1.5 \le Y \le 4.5)$. **(1)**

Question 7

At "Stavros Restaurant" the owner is told by his chef that 25% of the customers order vegetarian food. The owner wants to check the validity of the chef's assertion so he checks a random sample of 20 orders, only to find 2 vegetarian orders.

a) Is there evidence, at the 10% level of significance, that the proportion of vegetarian orders is lower than 25%? **(6)**

At "Mavros Restaurant" the owner is told by his waiters that 25% of the customers order vegetarian food. The owner wants to check the validity of the waiters' belief so he checks a random sample of 100 orders.

b) Given that there are 31 vegetarian orders in the sample, use a distributional approximation, to test at the 5% level of significance, the belief of the waiters at "Mavros Restaurant". **(8)**

Question 8

The events A and B are such so that

$$\mathrm{P}(B|A)=\frac{3}{8}, \qquad \mathrm{P}(A|B)=\frac{4}{9}, \qquad \mathrm{P}(B|A')=\frac{15}{28}.$$

Determine the value of $\mathrm{P}(A)$. **(10)**

Question 9

It is an **actual fact** that "sleeping with your clothes and shoes on is strongly correlated with waking up with a headache".

Evidently the conclusion is that "sleeping with your clothes and shoes on causes a headache".

Discuss the validity of the above conclusion indicating how a strong correlation could be possible in the above scenario. **(2)**

SECTION 2 - MECHANICS

Question 10

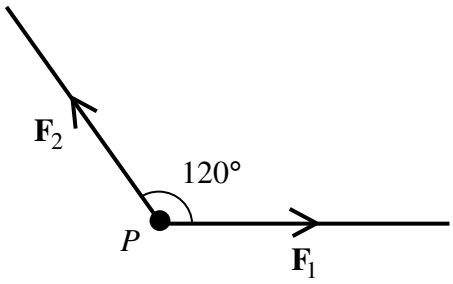
At time $t = 0$, relative to a fixed origin O, two particles, A and B, have position vectors $(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ m and $(-2\mathbf{i} + a\mathbf{j} + 6\mathbf{k})$ m, respectively, where a is a constant.

It is further given that A and B, are travelling with constant velocities of $(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})$ ms^{-1} and $(3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k})$ ms^{-1}, respectively.

The distance between A and B is least when $t = 4$s.

Determine the value of a. **(8)**

Question 11



The figure above shows two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, of magnitude 24 N and x N respectively, acting on a particle P.

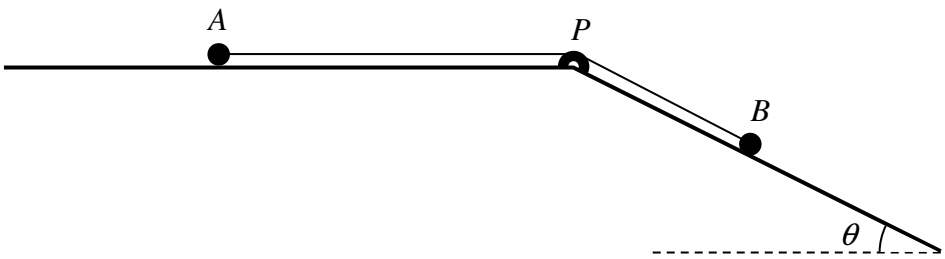
The angle between the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ is 120°.

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is the force $\mathbf{R}$, whose magnitude is $2x$ N.

a) Show clearly that $x = -4 + 4\sqrt{13}$. **(5)**

b) Calculate the value of $|\mathbf{F}_2 - \mathbf{F}_1|$, correct to three significant figures. **(5)**

Question 12



Two particles A and B have masses 2 kg and 3 kg, respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a rough horizontal table. The coefficient of friction between the particle A and the table is $\frac{1}{7}$.

The string lies along the table and passes over a small smooth pulley P which is fixed to the edge of the table. Particle B is at rest on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = 0.75$.

The coefficient of friction between the particle B and the plane is also $\frac{1}{7}$.

A constant force F, of magnitude 30 N, is applied to particle A, in the direction PA, while the string between the two particles is taut. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in the figure above.

a) Find the tension in the string while the system is in motion. **(7)**

The string suddenly breaks after 1.5 s.

b) Given that B never reaches P, determine the **total** distance that B travels **up** the plane. **(9)**

Question 13

A car travels along a straight horizontal road between two points, A and B, that are 960 m apart.

The car starts from rest at A and moves with constant acceleration of 2.5 ms^{-2} until it reaches a speed V ms^{-1}. It then travels at this constant speed before it decelerates uniformly at 1.25 ms^{-2}, coming to rest at B.

The car takes 60 seconds for the journey from A and B.

Determine the value of V. **(10)**

Question 14

A car moving on a straight road is modelled as a particle moving on the x axis, and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = \begin{cases} 4 - \frac{1}{2}t & 0 \le t \le 8 \\ 0 & t > 8 \end{cases}$$

The car starts from rest at the origin O.

a) Find a similar expression for the velocity of the car, as that of its acceleration. **(5)**

b) State the time it takes for the car to reach its maximum speed. **(1)**

c) Show that the displacement of P from O is given by

$$x = \begin{cases} 2t^2 - \frac{1}{12}t^3 & 0 \le t \le 8 \\ 16t - \frac{128}{3} & t > 8 \end{cases}$$ **(7)**

d) Calculate the time it takes the car to cover the first 1000 m. **(2)**

Question 15

A man is trying to push a box of weight W up the line of greatest slope of a rough plane inclined at an angle α to the horizontal, by applying a force parallel to a line of greatest slope.

The coefficient of friction between the man's feet and the plane is μ, $\mu > \tan\alpha$.

The coefficient of friction between the box and the plane is μ'.

The man and the box are both modelled as particles and air resistance is ignored.

Show, with detailed workings, that if the man is to succeed in pushing the box up this plane his weight must exceed

$$\left(\frac{\mu' + \tan\alpha}{\mu - \tan\alpha}\right)W . \qquad \textbf{(10)}$$

Question 16

A particle is projected from a point A, **down** the line of greatest slope of a smooth incline plane and moves along a straight line with constant acceleration of $7.84\ \text{ms}^{-2}$.

The particle experiences a constant normal reaction force of magnitude $29.4\ \text{N}$, throughout its motion.

a) Determine the mass of the particle. **(6)**

The particle reaches the point B, $2.5\ \text{s}$ after being projected and the point C, $2.5\ \text{s}$ after passing through B.

b) Given further that the distance BC is $100\ \text{m}$ greater than the distance AB, find the projection speed and the distance AB. **(8)**