IYGB GCE

Mathematics MMS

Advanced Level

Practice Paper J

Difficulty Rating: 3.5133/0.8043

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 16 questions in this question paper.
The total mark for this paper is 150.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

SECTION 1 - STATISTICS

Question 1

The table below shows the maximum daytime temperature, in °C, at a certain city centre, and the amount of a certain pollutant in mg per litre.

Maximum Temperature	10	12	14	16	18	20	22	24
Amount of Pollutant	513	475	525	530	516	520	507	521

a) Find, using a statistical calculator, the value of the product moment correlation coefficient for the above data. **(1)**

b) State, with justification, the value of the product moment correlation coefficient, if the maximum daily temperatures were to be measured in degrees Fahrenheit. **(2)**

c) Test, at the 10% level of significance, whether there is evidence of positive correlation in these bivariate data. **(4)**

Question 2

In a histogram the heights, h cm, of primary school pupils are plotted on the x axis.

In this histogram the class $120 \leq h < 130$ has a frequency of 72 and is represented by a rectangle of base 4.2 cm and height 9 cm.

In the same histogram a different class is represented by a rectangle of base 2.1 cm and height 8 cm.

Determine the frequency of this class. **(3)**

Question 3

A company decides to give their 23 employees a skills test in order to decide if any of these employees need to be retrained.

The maximum possible score in this test is 50 and the results are summarised in an ordered stem and leaf diagram.

0	5
1	9,9
2	1,6,8
3	3,4,5,7
4	2,3,4,4,8,9,9
5	0,0,0,0,0,0

where $\overline{2\|9} = 29$.

a) Find the median score of the test. **(1)**

b) Determine the interquartile range of the scores. **(2)**

The company decides to retrain any employee whose score is less than the **lower quartile minus the interquartile range**.

c) Show clearly that only one employee will undergo retraining. **(2)**

d) Draw a suitably labelled box plot for this data, clearly indicating any outliers, as found in part **(c)**. **(3)**

e) Determine with justification the skewness of the scores. **(2)**

Question 4

The events A and B are such so that

$$\mathrm{P}(A) = \mathrm{P}(B), \quad \mathrm{P}(A' \cap B') = \frac{17}{24}, \quad \mathrm{P}(A' \cup B') = \frac{19}{24}.$$

Find the value of $\mathrm{P}(A \cap B)$. **(6)**

Question 5

Bag X contains 3 one pound coins and 2 two pound coins.

Bag Y contains 1 one pound coin and 3 two pound coins.

A statistical experiment consists of

- a coin being picked **at random** from bag X and placed into bag Y.
- then a coin being picked **at random** from bag Y and placed back into bag X.

Find the probability …

a) … that a one pound coin will be picked on both occasions in this experiment. **(2)**

b) … that at the end of the experiment both bags contain £7. **(2)**

A third coin is picked **at random** from bag X at the end of the experiment.

c) Determine the probability that it will be a two pound coin. **(3)**

Question 6

The probability distribution of the discrete random variable X is given by

x	2	3	4
P($X = x$)	$0.4 - a$	$2a$	$0.6 - a$

where a is a constant.

a) State the range of the possible values of a. **(2)**

Two independent observations of X, denoted by X_1 and X_2, are considered.

b) Determine, in terms of a, a simplified expression for $\mathrm{P}(X_1 + X_2 = 6)$. **(5)**

Question 7

The times, in minutes, taken by Year 6 students to complete the SATS Science test are assumed to be Normally distributed with mean of 48 and standard deviation of 5.

8% of the students finished the exam in less than t minutes.

a) Find the value of t, correct to the nearest minute. **(4)**

b) Determine the probability that a randomly chosen student took more than 57 minutes to complete the test. **(4)**

20 students that sat the SATS Science test are selected at random.

c) Calculate, correct to 3 significant figures, the probability that more than 2 of these 20 students took more than 57 minutes to complete the test. **(4)**

It is claimed that the "top set students" take less time to complete the exam as the mean finishing time of a random sample of 6 "top set students" was 44 minutes.

d) Test this claim at the 1% level of significance. **(6)**

Question 8

A discrete random variable X has distribution

$$X \sim \mathrm{B}\left(6, \frac{1}{3}\right).$$

a) By showing a full method, find $\mathrm{P}(X = 2)$. **(3)**

Two independent observations of X, denoted by X_1 and X_2 are considered.

b) Determine the probability that the sum of these two observations will be less than 2. **(4)**

Eight independent observations of X are selected at random.

c) Determine the probability that half of these observations will be a 2. **(3)**

Question 9

a) Define the word population, when used in a statistical context. **(1)**

b) Describe briefly what do you understand by the word census, when used in a statistical context. **(2)**

c) Give four different reasons when you might use a census. **(4)**

SECTION 2 – MECHANICS

Question 10

The points A, B and C lie in that order on a road, where the distance $AB = 476$ m and the distance $BC = 855$ m.

The car passes through A with speed 24 ms^{-1} decelerating uniformly until it passes through B with speed 10 ms^{-1}.

a) Find the deceleration of the car as it travels from A to B. **(2)**

b) Calculate the time it took the car to travel from A to B. **(2)**

As the car passes through B it begins to accelerate uniformly until it passes through C, 45 s after passing through B.

c) Find the acceleration of the car as it travels from B to C. **(2)**

d) Determine the speed of the car as it passes through C. **(2)**

e) Find the **average** speed for the journey from A to C. **(2)**

Question 11

The acceleration $\mathbf{a}$ ms^{-2} of a particle P of mass 0.2 kg, t s after a given instant is given by

$$\mathbf{a} = (2t - 4)\mathbf{i} + 3\mathbf{j},\ t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing along the positive x axis and along the positive y axis, respectively.

a) Find the magnitude of the resultant force acting on P, when $t = 4$. **(4)**

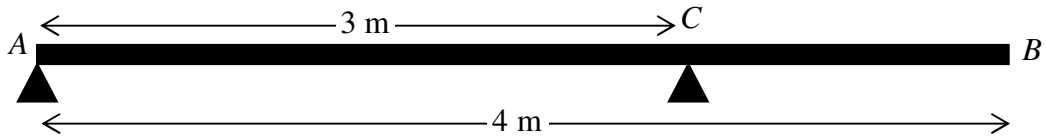
It is further given that when $t = 0$, P is at the point A with position vector $(-18\mathbf{i} - 24\mathbf{j})$ m and has velocity $(3\mathbf{i} - 9\mathbf{j})$ ms^{-1}.

b) Find the value of t when the particle is at rest. **(5)**

c) Show that when $t = 6$, P is on the y axis and state its distance from A. **(6)**

d) Determine the value of t when the particle is on the x axis. **(3)**

Question 12



A plank of wood AB has length 4 m and mass 40 kg. The plank is smoothly supported at A and at C, where $AC = 3$ m, as shown in the figure above.

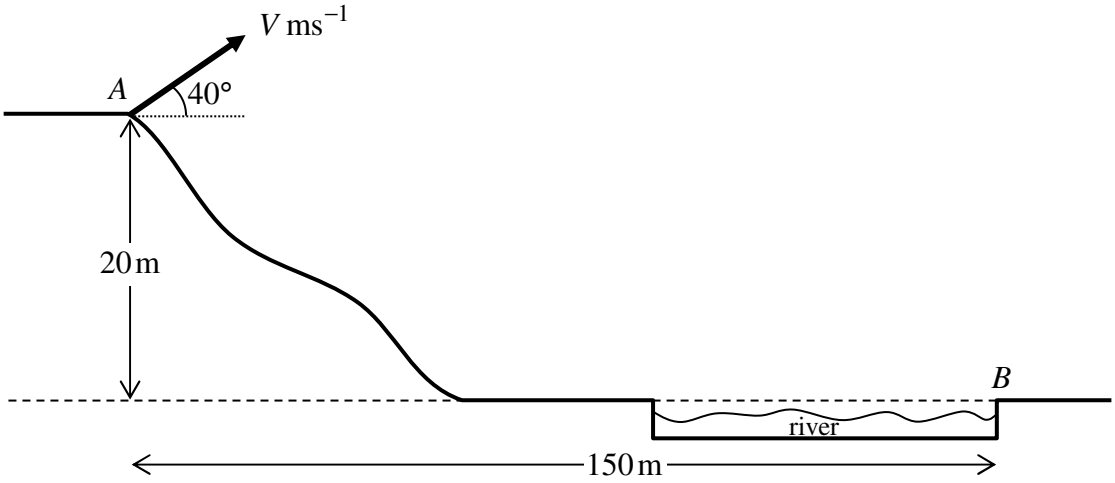
A man of mass 80 kg stands on the plank at a distance d m from A.

The plank, with the man standing on it, remains in equilibrium with AB horizontal, and the reactions on the plank at A and at C equal.

The plank is modelled as a uniform rod and the man as a particle.

Determine the value of d. **(5)**

Question 13

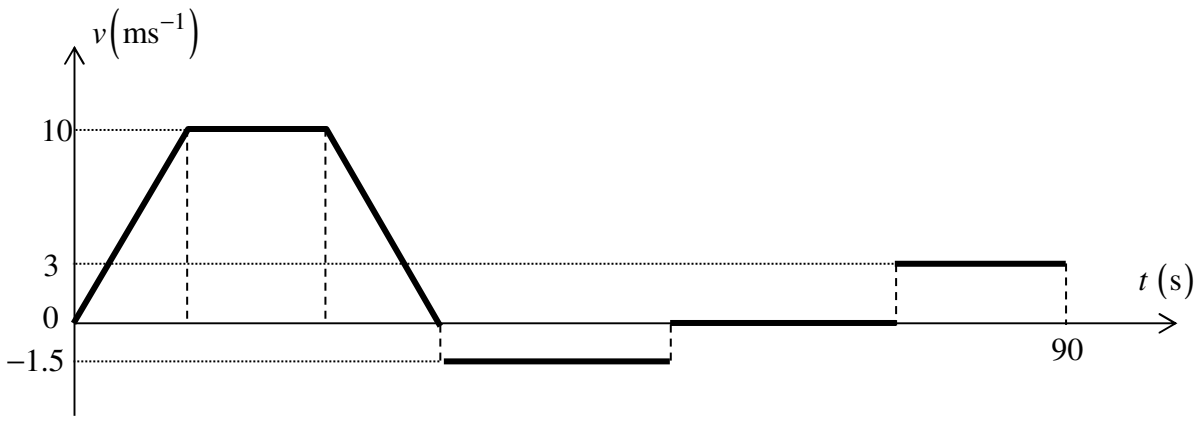


A golf ball is struck from a point A with a speed V ms^{-1} at an angle of elevation $40°$, so it can clear a river. The ball is modelled as a particle moving freely under the action of its weight. In order to clear the river the ball must land further than a point B on the opposite river bank. The point B lies a vertical distance of 20 m below the level of A and a horizontal distance of 150 m from A, as shown in the figure above.

a) Find, to two decimal places, the minimum value of V. **(10)**

b) Given instead that $V = 50$, determine the magnitude and direction of the velocity of the ball 5 s after it was struck. **(6)**

Question 14



The figure above shows the velocity time graph (t, v) of Andy riding a bike.

Andy starts cycling from rest with constant acceleration of $1.25\ \text{ms}^{-2}$ until he reaches a speed $10\ \text{ms}^{-1}$ which he maintains for $18\ \text{s}$.

Andy hears his friend Maya calling him so he decelerates to rest in $4\ \text{s}$.

He leaves his bike where he stopped and walks back at constant speed of $1.5\ \text{ms}^{-1}$ to meet Maya, reaching her after a further period of $10\ \text{s}$.

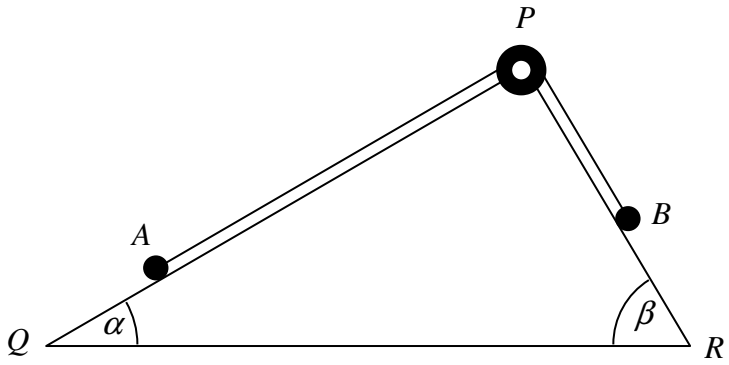
The two children talk to each other for $T\ \text{s}$. Andy then runs back to his bike at constant speed of $3\ \text{ms}^{-1}$. It takes a total time of $90\ \text{s}$ since Andy started his ride.

a) Find the time for which Andy is accelerating. **(2)**

b) Find the value of Andy's deceleration. **(2)**

c) Determine the value of T. **(2)**

Question 15



Two particles A and B have masses M and m, respectively.

The particles are attached to the ends of a light inextensible string.

The string passes over a small smooth pulley P which is fixed at the top of the cross section of a triangular prism QPR, where $\measuredangle PQR = \arctan\frac{1}{2}$ and $\measuredangle PRQ = \arctan 2$.

The string lies in the vertical plane which contains the pulley and lines of greatest slope of the inclined planes, PR and PQ, as shown in the figure above.

The system is in equilibrium, with the string taut, A on PQ and B on PR.

If the equilibrium is limiting with A about to slip down PQ, show that

$$M = m(2+\mu),$$

where μ is the coefficient of friction between B and PR.

You may assume that QP is smooth. **(11)**

Question 16

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship S is sailing with constant velocity.

At 12.00 hours, S is passing through the point with position vector $(18\mathbf{i} - 5\mathbf{j})$ km and half an hour later S is passing through the point with position vector $(16\mathbf{i} - 2\mathbf{j})$ km.

a) Calculate the speed of S. **(2)**

The position vector of S, $\mathbf{s}$ km, t hours after noon, is given by

$$\mathbf{s} = f(t)\mathbf{i} + g(t)\mathbf{j}.$$

b) Determine an expression for $f(t)$ and an expression for $g(t)$. **(2)**

At 14.00, to an observer on S, a lighthouse L appears due north of S.

At 15.30, to an observer on S, the same lighthouse L appears north-east of S.

c) Find the position vector of L. **(5)**