IYGB GCE

Mathematics FS1

Advanced Level

Practice Paper P

Difficulty Rating: 2.8000/1.2500

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The water from a lake is tested and is found to contain on average three bacteria per litre of water. A sample of 250 ml is collected from the lake.

a) Determine the probability that the 250 ml water sample will contain …

i. … exactly two bacteria. **(2)**

ii. … at least two bacteria. **(2)**

A larger sample of two litres of water is collected from the lake.

b) Find the probability that this larger sample will contain less than ten but no less than six bacteria. **(3)**

Question 2

Mini-cakes having a mean weight of 145 grams and standard deviation 9 grams are packed in boxes of 12.

A box of these mini-cakes is selected at random.

If the weights of these mini-cakes are Normally distributed, determine the probability that the mean weight of the mini-cakes in the box will be greater than 150 grams. **(5)**

Question 3

On a certain day, postman Mat has 200 letters to deliver. In general, 2% of the letters that postman Mat delivers, are delivered to the wrong address.

a) Determine the probability that postman Mat delivers 6 letters to the wrong address that day. **(2)**

b) Use a distributional approximation, to find the probability that postman Mat delivers more than 8 letters to the wrong address that day. **(4)**

Question 4

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} k(2-x)^2 & x=-2,-1,0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the value of the constant k. **(2)**

b) Find the value of …

i. … $\mathrm{E}(X)$. **(2)**

ii. … $\mathrm{E}(X^2)$. **(2)**

c) Determine …

i. … $\mathrm{E}(1-15X)$. **(2)**

ii. … $\mathrm{Var}(1-15X)$. **(3)**

Question 5

During Saturday afternoons, customers are known to walk into a certain clothes shop at the rate of 8 every 10 minutes.

The new shop manager wants to find if that rate has changed since he took over.

a) Find the critical region, at the 5% level of significance, to investigate whether the rate of 8 customers walking into the shop every 10 minutes has changed.

State your hypotheses clearly. **(7)**

b) State the actual significance level for a test using the critical region of part **(a)**. **(1)**

During a Saturday afternoon, 14 customers walked into the shop in a random 10 minute interval.

c) Complete the test. **(2)**

Question 6

A bank manager investigates the number of customers served by his staff.

He records the number of customers being served in 100 consecutive five minute time intervals and his data is summarized in the table below.

Number of Customers	Frequency
0	5
1	38
2	32
3	17
4	7
5	1
6 or more	0

The manager further asserts that his data can be modelled by a Poisson distribution.

Use a χ^2 test, at 1% level of significance, to investigate the validity of the manager's assertion. **(12)**

Question 7

A coin is biased so that the probability of obtaining "heads" in any toss is p, $p \neq \frac{1}{2}$.

The coin is tossed repeatedly until a "head" is obtained.

The probability of obtaining "heads" after an even number of tosses is $\frac{2}{5}$.

Determine the value of p. **(9)**

Question 8

The discrete random variable X has probability mass function

$$\mathrm{P}(X = x) = \frac{x^2}{k}, \quad x = 1, 2, 3, 4, 5$$

where k is a positive constant.

The probability generating function of X is $\mathrm{G}_X(t)$.

a) Find an expression for $\mathrm{G}_X(t)$, in terms of k.

Give the answer in simplified sigma $[\Sigma]$, notation. **(3)**

b) Find $\frac{d^2}{dt^2}\left[\mathrm{G}_X(t)\right]$, in terms of k.

Give the answer in simplified sigma $[\Sigma]$, notation. **(5)**

c) State the value of k. **(1)**

d) Use $\mathrm{G}_X(t)$ and its derivatives to calculate the value of $\mathrm{Var}(X)$.

No credit will be given if the value of $\mathrm{Var}(X)$ *is obtained from calculations based on the probability mass function of* X. **(6)**