IYGB GCE

Mathematics FS1

Advanced Level

Practice Paper N

Difficulty Rating: 2.7000/1.2121

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A discrete random variable X has negative binomial distribution, with 6 successes required, each with probability of success 0.4.

Determine the value of …

a) … $\mathrm{E}(X)$. **(1)**

b) … $\mathrm{Var}(X)$. **(1)**

c) … $\mathrm{P}(X = 12)$. **(2)**

Question 2

The probability distribution of a discrete random variable X is given by

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	a	$\frac{1}{24}$

where a is a positive constant.

a) Explain why $a = 0$. **(1)**

b) Find the value of $\mathrm{E}(X)$. **(2)**

c) Calculate $\mathrm{Var}(X)$. **(3)**

Question 3

The discrete random variable X is modelled as being geometrically distributed with parameter 0.2.

a) State two conditions that must be satisfied by X, so that the geometric model is valid. **(2)**

b) Showing full workings, where appropriate, calculate the value of …

i. … $\mathrm{P}(X = 3)$. **(2)**

ii. … $\mathrm{P}(X > 8)$. **(2)**

iii. … $\mathrm{P}(5 \leq X < 13)$. **(3)**

Question 4

Bags of cement are filled by factory machinery. It is known from past records that the weights of these bags are Normally distributed with a standard deviation of 400 grams.

The bags of cement have an advertised weight of 50 kg.

At the start of each month a random sample of 25 bags are weighed, in order to test the functionality of the machinery.

The test is carried at the 5% level of significance, for

$$\mathrm{H}_0 : \mu = 50 \text{ kg} \qquad \text{versus} \qquad \mathrm{H}_1 : \mu \neq 50 \text{ kg}.$$

For this test…

a) … state the probability of a Type I error. **(1)**

b) … find the critical region, correct to the nearest gram. **(4)**

c) … determine the probability of a Type II error, if the mean weight has in fact changed to 50.1 kg. **(4)**

Question 5

The manager of a shop claims that the mean age of his customers is 33 years.

A sample of 64 customers is taken and this sample produces a mean of 35.6 years and a standard deviation of 8.2 years.

a) Stating your hypotheses and using a 1% level of significance, test whether or not the manager's claim is supported by the data. **(6)**

b) State two assumptions made in carrying this test, further explaining why this test is still valid even if the ages of the customers are not Normally distributed. **(2)**

Question 6

The discrete variable X is thought to have distribution $\mathrm{B}(5,0.2)$.

Some actual observations of X are summarized in the table below.

X	**Frequency**
0	15
1	36
2	17
3	10
4	1
5	1

Use a χ^2 test, at 5% level of significance, to investigate whether the above data can be modelled by $\mathrm{B}(5,0.2)$. **(10)**

Question 7

The discrete random variable X has distribution

$$X \sim \text{Po}(\lambda).$$

a) Derive **from first principles** $\text{G}_X(t)$, the probability generating function of X. **(5)**

b) Use $\text{G}_X(t)$ to show that the mean and variance of X are both λ. **(5)**

Another discrete random variable Y has distribution

$$Y \sim \text{Po}(\mu).$$

c) Use a method involving generating functions to show that

$$\text{E}(X+Y) = \lambda + \mu. \quad \textbf{(6)}$$

Question 8

Since his retirement, Fred goes fishing Monday to Friday, for 3 hours on each of these 5 days. The number of fish he catches every hour follows a Poisson distribution with mean 2.5.

a) Find the probability that Fred catches more than 9 fish on exactly 2 of the days, in a given 5 day fishing week. **(6)**

Fred buys a new type of bait and decides to test whether there is any difference to the rate at which he catches fish. He tries his new bait by going fishing on a Sunday and ends up catching 16 fish in 4 hours.

b) Carry out a significance test, at the 5% level, stating your hypotheses clearly. **(7)**