IYGB GCE

Mathematics FP4

Advanced Level

Practice Paper N

Difficulty Rating: 3.2333/1.45458

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

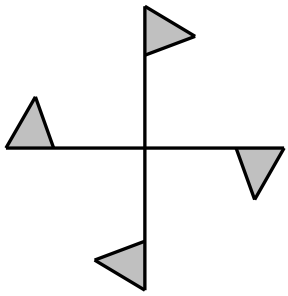
This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



a) Construct a table for the symmetry group G of the above shape. **(5)**

b) Construct a table for the group $H = \{3,6,9,12\}$ under multiplication modulo 15 and hence show that G and H are isomorphic. **(4)**

Question 2

By finding a suitable Cartesian locus in the complex z plane, shade the region R that satisfies the inequality

$$|z-3| \leq |z+3\mathrm{i}|. \quad \textbf{(6)}$$

Question 3

The part of the curve with equation

$$y = x^3, \quad 0 \leq x \leq 1$$

is rotated through 2π radians about the x axis.

Show that the area surface generated is

$$\frac{\pi}{27}\left[10\sqrt{10}-1\right]. \quad \textbf{(6)}$$

Question 4

There are 8 boys and 7 girls in the student council of a school.

A committee of 8 people is to be selected from the members of this council to organize a sports day.

a) Find the number of different ways in which the committee can be selected if all the members are available. **(2)**

b) Determine the number of different ways in which the committee can be selected if the committee is to have more girls than boys. **(4)**

Question 5

The integral I_n is defined for $n \geq 0$ as

$$I_n = \int_0^1 x^n (1-x)^{\frac{3}{2}} \, dx, \; n \in \mathbb{N}.$$

Show that

$$I_n = \left(\frac{2n}{2n+5} \right) I_{n-1}, \; n \geq 1,$$

and use it to find as an exact fraction the value of I_3. **(10)**

Question 6

When a, $a \in \mathbb{N}$, is divided by b, $b \in \mathbb{N}$, the quotient is 20 and the remainder is 17.

a) Find the remainder when a is divided by 5. **(4)**

Suppose that when a positive integer is divided by 8 the remainder is 6, and when the same positive integer is divided by 18 the remainder is 3.

b) Determine whether such positive integer exists. **(4)**

Question 7

The 2×2 matrix $\mathbf{M}$ satisfies $\mathbf{M}=\mathbf{PDP}^{-1}$ where

$$\mathbf{P}=\begin{pmatrix}-1 & 4\\ 3 & 1\end{pmatrix} \quad \text{and} \quad \mathbf{D}=\begin{pmatrix}1 & 0\\ 0 & 27\end{pmatrix}.$$

a) Determine the elements of $\mathbf{M}$. **(3)**

b) State the eigenvalues, and the corresponding eigenvectors of $\mathbf{M}$. **(2)**

c) Find an equation of the straight line of invariant points under the transformation described by $\mathbf{M}$. **(2)**

It is further given that

$$\mathbf{M}^n=\frac{1}{13}\begin{pmatrix}4\times 3^{3n+1}+1 & 4\times 3^{3n}-4\\ 3^{3n+1}-3 & 3^{3n}+12\end{pmatrix}.$$

d) Deduce that $3^{3n+2}+4$ is divisible by 13, for all positive integers n. **(4)**

Question 8

$$f(z)\equiv(z+2\mathrm{i})^2,\ z\in\mathbb{C}.$$

The complex function f maps points, of the form $x+\mathrm{i}y$, from the z plane onto points, of the form $u+\mathrm{i}v$, in the w plane.

The straight line L lies in the z plane and has Cartesian equation

$$y=x-1.$$

Find an equation of the image of L in the w plane, giving the answer in the form

$$v=g(u), \quad \textbf{(5)}$$

where g, is a real function to be found. **(4)**

Question 9

A sequence of numbers is generated by the recurrence relation

$$u_n = 5u_{n-1} - 4u_{n-2} - 12n + 31, \quad n \in \mathbb{N}, \ n \geq 2$$

with $u_0 = 7$, $u_1 = 9$.

Determine a simplified expression for the n^{th} term of this sequence. **(10)**