IYGB GCE

Mathematics FP4

Advanced Level

Practice Paper M

Difficulty Rating: 3.5733/1.6484

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The set S contains all the 2×2 matrices of the form $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$, where a and b are real numbers with $a^2 \neq b^2$.

Prove that S forms a group under matrix multiplication (composition), stating clearly any assumptions made. **(7)**

Question 2

Use Euclid's algorithm to find the Highest common factor of 560 and 1169. **(3)**

Question 3

A sequence of integers t_1, t_2, t_3, … is given by the recurrence relation

$$t_{n+1} = 3t_n + 2, \quad t_1 = 1, \quad n \in \mathbb{N}.$$

Prove by induction that its n^{th} term of the sequence is given by

$$t_n = 2\times 3^{n-1} - 1, \ n \in \mathbb{N}. \quad \textbf{(6)}$$

Question 4

A curve has equation

$$y = \ln(1+\cos x), \quad x \in \left[-\tfrac{1}{2}\pi, \tfrac{1}{2}\pi\right]$$

Show that the length this curve is $\ln\left(17+12\sqrt{2}\right)$ units. **(11)**

Question 5

The integral I_n is defined for $n \geq 0$ as

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx .$$

a) Show clearly that …

$$\ldots \; I_n = \left(\frac{\pi}{2}\right)^n - n(n-1) I_{n-2}, \; n \geq 2 . \qquad (\mathbf{5})$$

b) Hence find, in terms of π, exact expressions for …

i. $\ldots \int_0^{\frac{\pi}{2}} x^4 \cos x \, dx .$ $(\mathbf{4})$

ii. $\ldots \int_0^{\frac{\pi}{2}} x^5 \sin x \, dx .$ $(\mathbf{3})$

Question 6

The locus of a point, represented by the complex number z, satisfies the relationship

$$|z+1+\mathrm{i}| = |z-1+2\mathrm{i}| .$$

When this locus is transformed by the complex function

$$f(z) = kz + \mathrm{i}, \; k \in \mathbb{R},$$

the image of the locus traces the straight line with Cartesian equation

$$y = 2x - 8 .$$

Determine the value of k. $(\mathbf{11})$

Question 7

The 3×3 matrix $\mathbf{A}$ is given below.

$$\mathbf{A}=\begin{pmatrix}1 & -1 & 1\\ 3 & -3 & 1\\ 3 & -5 & 3\end{pmatrix}.$$

a) Given that $\mathbf{u}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ is an eigenvector of $\mathbf{A}$, find the corresponding eigenvalue. **(2)**

b) Given that $\lambda=-2$ is an eigenvalue of $\mathbf{A}$, find a corresponding eigenvector $\mathbf{v}$. **(3)**

The vector $\mathbf{w}$ is defined as $\mathbf{w}=\mathbf{u}+\mathbf{v}$.

c) Determine the vector $\mathbf{A}^7\mathbf{w}$. **(5)**

Question 8

$$\boxed{\text{B}},\ \boxed{\text{A}},\ \boxed{\text{N}},\ \boxed{\text{A}},\ \boxed{\text{N}},\ \boxed{\text{A}},\ \boxed{\text{S}}$$

The 7 letters shown above are written on separate pieces of card.

a) Find the number of arrangements which can be made if all 7 letters are used. **(2)**

b) Find the number of arrangements which can be made if all 7 letters are used and the three vowels are together. **(2)**

c) Find the number of arrangements which can be made if all 7 letters are used and the three vowels are together and the four consonants are together. **(3)**

d) Determine the number of ways in which 4 letters can be picked from the total of 7 letters. **(3)**

e) Calculate the number of arrangements of which 4 letters are used from the total of 7 letters. **(4)**