IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper W

Difficulty Rating: 3.8467/1.8576

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = x + y + y^2, \quad y(0.9) = 3.75, \quad y(1) = 4$$

Using, in the standard notation, the approximation

$$\left(\frac{dy}{dx}\right)_r \approx \frac{y_{r+1} - y_{r-1}}{2h},$$

with $h = 0.1$, the value of y at $x = 0.8$ was estimated to be k.

Determine the value of k. **(7)**

Question 2

a) Use Simpson's rule with 5 equally spaced ordinates to estimate the value of

$$\int_0^{\frac{1}{3}\pi} e^{\sec^2 x}\, dx. \quad \textbf{(4)}$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_0^{\frac{1}{3}\pi} e^{\tan^2 x}\, dx. \quad \textbf{(3)}$$

c) Explain whether the estimates of the previous parts of the question are likely to be accurate. **(1)**

Question 3

$$y = x^4 \mathrm{e}^{3x}, \ x \in \mathbb{R}.$$

Use the Leibniz rule to show that

$$\frac{d^k y}{dx^k} = \mathrm{e}^{3x} 3^{k-4} f(x,k), \ k \in \mathbb{N},$$

where $f(x,k)$ is a function to be found. **(8)**

Question 4

Sketch the parabola with equation

$$y^2 - 4y - 2x = 2.$$

The sketch must include the …

- … coordinates of points of intersection with the coordinate axes.
- … coordinates of the vertex of the parabola.
- … coordinates of the focus of the parabola.
- … equation of the directrix of the parabola. **(8)**

Question 5

$$x\frac{d^2 y}{dx^2} - \frac{dy}{dx} - x^3 y + x^5 = 0.$$

Use the substitution $x = z^{\frac{1}{2}}$, where $y = f(x)$, to find a general solution of the above differential equation. **(12)**

Question 6

$$y = \tan^2 x .$$

a) Show that

$$\frac{d^4 y}{dx^4} = 120\sec^6 x - 120\sec^4 x + 16\sec^2 x . \qquad \textbf{(6)}$$

b) Determine the first 5 terms in the Taylor expansion of $\tan^2 x$, in ascending powers of $\left(x - \frac{\pi}{3} \right)$. **(4)**

Question 7

Show with detailed workings that

$$\lim_{x \to \infty} \left[\left(1 + \frac{a}{x} \right)^{bx} \right] = e^{ab} . \qquad \textbf{(10)}$$

Question 8

An irregular pyramid with a triangular base ABC has vertex at the point V.

The equation of the straight line VC is

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The plane face ABV has equation $2x - 3y - z = 1$.

If the point D lies on the plane face VBC and has position vector $\frac{10}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + 5\mathbf{k}$, show that the equation of the line VB can be written as

$$\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}),$$

where μ is a scalar parameter. **(12)**