IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper U

Difficulty Rating: 4.0333/2.0339

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve with equation $y = f(x)$, satisfies

$$\frac{d^2y}{dx^2} = 1 + x\sin y,$$

subject to the boundary conditions $y = 1$, $\frac{dy}{dx} = 2$, at $x = 1$.

Use the approximations

$$\left(\frac{d^2y}{dx^2}\right)_{r+1} \approx \frac{y_{r+2} - 2y_{r+1} + y_r}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx}\right)_{r+1} \approx \frac{y_{r+2} - y_r}{2h},$$

to determine, correct to 4 decimal places, the value of y at $x = 1.1$.

Use $h = 0.05$ throughout this question. **(9)**

Question 2

Find the value of the following limit

$$\lim_{x \to 0}\left[\frac{e^{5x} - 5x - 1}{\sin 4x \sin 3x}\right]. \quad \textbf{(7)}$$

Question 3

Sketch the graph of the curve with equation

$$y(y-2) = 4x + 3.$$

The sketch must include the coordinates of any intersections with the axes and the coordinates of the point where the tangent to the curve is parallel to the y axis. **(7)**

Question 4

It is given that the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ satisfy

$$\mathbf{b} \wedge \mathbf{c} = 2\mathbf{i} \qquad \text{and} \qquad \mathbf{a} \wedge \mathbf{c} = \mu\mathbf{j},$$

where μ is a scalar constant.

It is further given that the vector expression defined as

$$(\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}) \wedge (\mathbf{a} + 2\mathbf{b} + k\mathbf{c}),$$

where k is a scalar constant, is parallel to the vector $\mathbf{i} - \mathbf{j}$.

Determine the condition that μ and k must satisfy. **(8)**

Question 5

$$I = \int_0^8 2^x \, dx$$

a) Use Simpson's rule with 8 equally spaced intervals to verify that

$$I \approx \frac{1105}{3}.$$

[The answer must be supported with detailed calculations.] **(4)**

b) Find the exact value of I, by writing $2^x = \mathrm{e}^{x \ln 2}$. **(3)**

c) Hence show that

$$\ln 2 \approx \frac{9}{13}.$$ **(2)**

Question 6

Use the substitution $w = \dfrac{dy}{dx}$ to solve the following differential equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = (1-x)^2,\ |x| < 1$$

subject to the boundary conditions $y = -4.5$ and $\dfrac{dy}{dx} = -1$ at $x = 0$.

Give the answer in the form $y = \alpha(x-3)^2 + \beta\ln(x+1)$, where α and β are constants to be found. **(12)**

Question 7

Use the substitution $t = \tan\left(\frac{1}{2}x\right)$ to find the exact value for the integral

$$\int_0^{\frac{1}{2}\pi} \frac{2}{1+\sin x+2\cos x}\ dx$$

All relevant results used in this evaluation must be carefully derived. **(12)**

Question 8

An ellipse has a focus at $(5,-3)$ and directrix with equation $y = 2x - 7$.

Given that the eccentricity of the ellipse is $\dfrac{\sqrt{5}}{10}$, find the coordinates of the points of intersection of the ellipse with the straight line with equation $y = -3$. **(11)**
