IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper O

Difficulty Rating: 3.4333/1.5584

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The vectors $\mathbf{a}$ and $\mathbf{b}$ are such so that

$$|\mathbf{a}| = \sqrt{10}, \quad |\mathbf{b}| = 10 \quad \text{and} \quad \mathbf{a} \cdot \mathbf{b} = 30.$$

Find the value of $|\mathbf{a} \wedge \mathbf{b}|$. **(5)**

Question 2

Solve the following rational inequality.

$$\frac{4x-3}{2-x} < 1. \qquad \textbf{(5)}$$

Question 3

The curve with equation $y = f(x)$, passes through the point $(9,6)$ and satisfies

$$\frac{dy}{dx} = \frac{1}{1+\sqrt{x}}, \quad x \geq 0.$$

Use Euler's method, with a step of 0.25, to find, correct to 4 decimal places, the value of y at $x = 9.5$. **(6)**

Question 4

$$f(x) = \cos 2x.$$

a) Find the first three non zero terms in the Taylor expansion of $f(x)$, in powers of $\left(x - \frac{\pi}{4}\right)$. **(6)**

b) Use the first three terms of the expansion to show $\cos 2 \approx -0.416$. **(2)**

Question 5

With respect to a fixed origin O, four points have the following coordinates

$$A(-1,3,-1),\ B(1,2,-2),\ C(1,2,2) \quad \text{and} \quad D(k,k,k),$$

where k is a constant.

a) Determine the shortest distance between the straight lines AB and CD. **(6)**

b) Find, in terms of k, the volume of the tetrahedron $ABCD$. **(6)**

Question 6

Use L'Hospital's rule to find the value of the following limit

$$\lim_{x\to 0}\left[\frac{\tan x - x}{\sin 2x - \sin x - x}\right]. \qquad \textbf{(7)}$$

Question 7

The differential equation

$$\left(x^3+1\right)\frac{d^2y}{dx^2} - 3x^2\frac{dy}{dx} = 2 - 4x^3,$$

is to be solved subject to the boundary conditions $y = 0$, $\frac{dy}{dx} = 4$ at $x = 0$.

Use the substitution $u = \frac{dy}{dx} - 2x$, where u is a function of x, to show that the solution of the above differential equation is

$$y = x^4 + x^2 + 4x. \qquad \textbf{(12)}$$

Question 8

$$y = x^4 \cos x\,,\ x \in \mathbb{R}\,.$$

Use the Leibniz rule to find a simplified expression for $\dfrac{d^6y}{dx^6}$. **(8)**

Question 9

The general point $P\left(2t, \dfrac{2}{t}\right)$, $t \neq 0$, where t is a parameter, lies on the rectangular hyperbola, with Cartesian equation

$$xy = 4\,.$$

a) Find an equation of the normal to the hyperbola at the point P. **(3)**

The normal to the hyperbola at P meets the hyperbola again at the point Q.

The point M is the midpoint of PQ.

b) Find an equation of the locus of M, as t varies.
Give a simplified answer in the form $f(x, y) = 0$. **(9)**