IYGB GCE

Mathematics FP3

Advanced Level

Practice Paper M

Difficulty Rating: 3.3733/1.5228

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

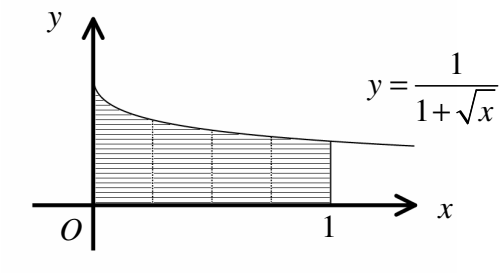
Question 1

The vectors $\mathbf{a}$ and $\mathbf{b}$, are not parallel.

Simplify fully the following expression

$$(2\mathbf{a}+\mathbf{b})\wedge(\mathbf{a}-2\mathbf{b}).$$ **(4)**

Question 2



The figure above shows part of the curve C with equation

$$y=\frac{1}{1+\sqrt{x}},\ x\geq 0.$$

It is required to estimate the area of the shaded region bounded by C, the coordinate axes and the straight line with equation $x=1$.

Use Simpson's rule with 4 equally spaced strips to estimate the area of this region, giving the answer correct to 3 decimal places. **(4)**

Question 3

Find, in terms of the positive constant k, the solution set of the following inequality.

$$\frac{x+k}{x+4k}>\frac{k}{x}.$$ **(7)**

Question 4

$$f(x) = x^2 \ln x,\ x > 0$$

a) Find the first three non zero terms in the Taylor expansion of $f(x)$, in powers of $(x-1)$. **(6)**

b) Use the first three terms of the expansion to show $\ln 1.1 \approx 0.095$. **(2)**

Question 5

The straight lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{j} + 3\mathbf{k})$$
$$\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{k})$$

where λ and μ are scalar parameters.

Show that l_1 and l_2 are skew and hence find the shortest distance between them. **(9)**

Question 6

$$y = x^3 \mathrm{e}^{2x},\ x \in \mathbb{R}.$$

Use the Leibniz rule to show that

$$\frac{d^k y}{dx^k} = \mathrm{e}^{2x} 2^{k-3} f(x,k),\ k \in \mathbb{N},$$

where $f(x,k)$ is a function to be found. **(7)**

Question 7

$$\frac{d^2y}{dx^2}-\left(1-6\mathrm{e}^{x}\right)\frac{dy}{dx}+10y\mathrm{e}^{2x}=5\mathrm{e}^{2x}\sin\left(2\mathrm{e}^{x}\right).$$

a) By using the substitution $x=\ln t$ or otherwise, show that the above differential equation can be transformed to

$$\frac{d^2y}{dt^2}+6\frac{dy}{dt}+10y=5\sin 2t\,. \qquad (\mathbf{7})$$

b) Hence find a general solution for the original differential equation. $(\mathbf{8})$

Question 8

The point $P\left(ap^2,2ap\right)$, where p is a parameter, lies on the parabola, with Cartesian equation

$$y^2=4ax\,,$$

where a is a positive constant.

The point F is the focus of the parabola and O represents the origin.

The straight line which passes through P and F meets the directrix of the parabola at the point Q, so that the area of the triangle OPQ is $\frac{15}{4}a^2$.

Show that one of the possible values of p is 3 and find in exact surd form the other 2 possible values. $(\mathbf{12})$

Question 9

The curve with equation $x = f(t)$, satisfies

$$\frac{d^2x}{dt^2} = -x, \qquad x(0) = 0, \qquad \frac{dx}{dt}(0) = 1.$$

Use Euler's method, with a step of 0.1, to find the approximate value of x at $t = 0.5$.

(9)