IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper W

Difficulty Rating: 3.9200/1.9231

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Write the polar equation

$$r = \cos\theta + \sin\theta,\ 0 \le \theta < 2\pi$$

in Cartesian form, and hence show that it represents a circle, further determining the coordinates of its centre and the size of its radius. **(5)**

Question 2

Given that $x > 0$ and $y > 0$, solve the simultaneous equations

$$\cosh(4x - 3y) = 1$$

$$y = \frac{1}{x}\mathrm{e}^{\operatorname{arsinh}\frac{4}{3}}.$$ **(6)**

Question 3

By using results for series expansions of standard functions, or otherwise, find the series expansion of $\ln\left(x^2 + 4x + 4\right)$ up and including the term in x^4. **(7)**

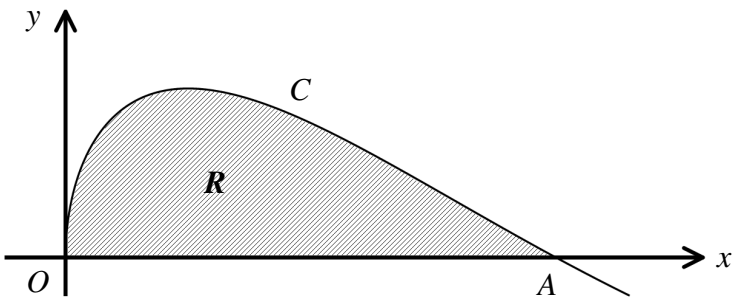
Question 4

The variables x and y satisfy the following coupled first order differential equations.

$$\frac{dx}{dt} = x - 2y \qquad \text{and} \qquad \frac{dy}{dt} = 5x - y.$$

Given further that $x = -1$, $y = 2$ at $t = 0$, solve the differential equations to obtain simplified expressions for x and y. **(12)**

Question 5



The figure above shows the curve C with parametric equations

$$x = t^2, \quad y = 4\sin 2t \ , \quad t \in \mathbb{R},\ t \geq 0 .$$

The curve crosses the x axis for the first time at the point A. The finite region R, shown shaded in the figure above, is bounded by C and the part of the x axis from the origin O to the point A. This region is revolved about the x axis to form a solid of revolution S.

a) Show that the volume of S is given by the integral

$$I = \pi \int_0^{\frac{\pi}{2}} 16t - 16\cos 4t \ dt . \qquad (\mathbf{4})$$

b) Hence find an exact value for the volume of S. $(\mathbf{6})$

Question 6

Find an exact value for

$$\int_{-1.5}^{1.5} 8x \arcsin\left(\frac{1}{3}x\right) \ dx . \qquad (\mathbf{9})$$

Question 7

a) Show that

$$\sin 7\theta \equiv 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta . \qquad (7)$$

b) By considering a suitable polynomial equation based on the result of part **(a)** show further

$$\operatorname{cosec}^2\left(\tfrac{1}{7}\pi\right) + \operatorname{cosec}^2\left(\tfrac{2}{7}\pi\right) + \operatorname{cosec}^2\left(\tfrac{3}{7}\pi\right) = 8 . \qquad (7)$$

Question 8

It is given that

$$f(r) = \frac{6r^4 + 6r^3 - ar^2 - ar + 1}{r(r+1)}, \quad r \in \mathbb{N},$$

where a is a non zero constant.

It is further given that

$$\sum_{r=1}^{n} f(r) = \frac{n^2(n+2)(2n+1)}{n+1} .$$

Determine the value of a. **(12)**