IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper V

Difficulty Rating: 4.0133/2.0134

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=0\,. \qquad (4)$$

Question 2

$$z^5=\mathrm{i}\,,\quad z\in\mathbb{C}\,.$$

a) Solve the equation, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, $r>0, -\pi<\theta\le\pi$. (5)

b) Plot the roots of the equation as points in an Argand diagram. (1)

Question 3

a) Sketch the graph of $y=\operatorname{arsech}x$, defined for $0<x\le 1$. (3)

b) Show clearly that

$$\frac{dy}{dx}=-\frac{1}{x\sqrt{1-x^2}}\,. \qquad (4)$$

c) Hence evaluate

$$\int_{\frac{1}{2}}^{1}\operatorname{arsech}x\;\;dx\,.$$

Give the answer in the form $\lambda\left[2\pi-3\ln\left(2+\sqrt{3}\right)\right]$, where λ is a rational number to be found. (8)

Question 4

By showing formally all the limiting processes evaluate the following integral

$$\int_0^{\frac{1}{4}\pi} \frac{1}{x} - \frac{\sin 2x}{1-\cos 2x}\, dx \ .$$

Give the answer in the form $\ln\left[\frac{\pi\sqrt{2}}{n}\right]$, where n is a positive integer to be found. **(8)**

Question 5

Consider the following infinite convergent series.

$$\frac{3}{1\times 2} - \frac{5}{2\times 3} + \frac{7}{3\times 4} - \frac{9}{4\times 5} + \frac{11}{5\times 6} - \ldots.$$

a) Use the method of differences, to find the sum of this series. **(8)**

b) Verify the answer of part **(a)** by using a method based on the Maclaurin expansion of $\ln(1+x)$. **(8)**

Question 6

The following polar equations are given.

$$r_1 = \cos\theta, \quad 0 \le \theta \le \pi \ .$$

$$r_2 = \frac{1}{\cos\theta - \sin\theta}, \quad -\frac{1}{4}\pi \le \theta \le \frac{5}{4}\pi \ .$$

Find, in exact simplified form, the area of the **smaller** of the two finite regions, bounded by r_1 and r_2. **(8)**

Question 7

Use appropriate integration techniques to find an exact simplified value for

$$\int_0^{\frac{1}{4}\pi} \frac{10}{2-\tan x}\, dx\,. \qquad \textbf{(10)}$$

Question 8

$$f(x) = 2\arcsin\sqrt{x} - \arcsin(2x-1), \quad 0 \le x \le 1\,.$$

By considering $f'(x)$ sketch the graph of $f(x)$. **(8)**
