IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper N

Difficulty Rating: 3.6600/1.7094

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

a) Simplify $\dfrac{1}{r(r+1)} - \dfrac{1}{(r+1)(r+2)}$ into a single fraction. **(2)**

b) Hence show that

$$\sum_{r=1}^{20} \left[\frac{1}{r(r+1)(r+2)} \right] = \frac{115}{462}. \quad \textbf{(5)}$$

Question 2

$$f(x) \equiv \mathrm{e}^{-2x} \cos 4x, \ x \in \mathbb{R}.$$

Find the Maclaurin expansion of $f(x)$ up and including the term in x^4. **(6)**

Question 3

$$f(y) \equiv \frac{4y}{y^4 - 1}, \ y \in \mathbb{R}, \ |y| \neq 1.$$

a) Express $f(y)$ into three partial fractions. **(5)**

b) Hence evaluate the improper integral

$$\int_2^{\infty} f(y) \ dy,$$

showing clearly the limiting processes used. **(5)**

c) Find, in exact form, the mean value of f, in the interval $\{y \in \mathbb{R} : 2 \leq y \leq 4\}$. **(3)**

Question 4

$$z = \mathrm{e}^{\mathrm{i}\theta},\ -\pi < \theta \le \pi .$$

a) Show that …

i. … $z^n + \dfrac{1}{z^n} = 2\cos n\theta .$

ii. ... $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta .$ **(3)**

b) Hence show further that

$$\cos^4\theta \sin^2\theta = \frac{1}{16} + \frac{1}{32}\cos 2\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 6\theta . \quad \textbf{(6)}$$

Question 5

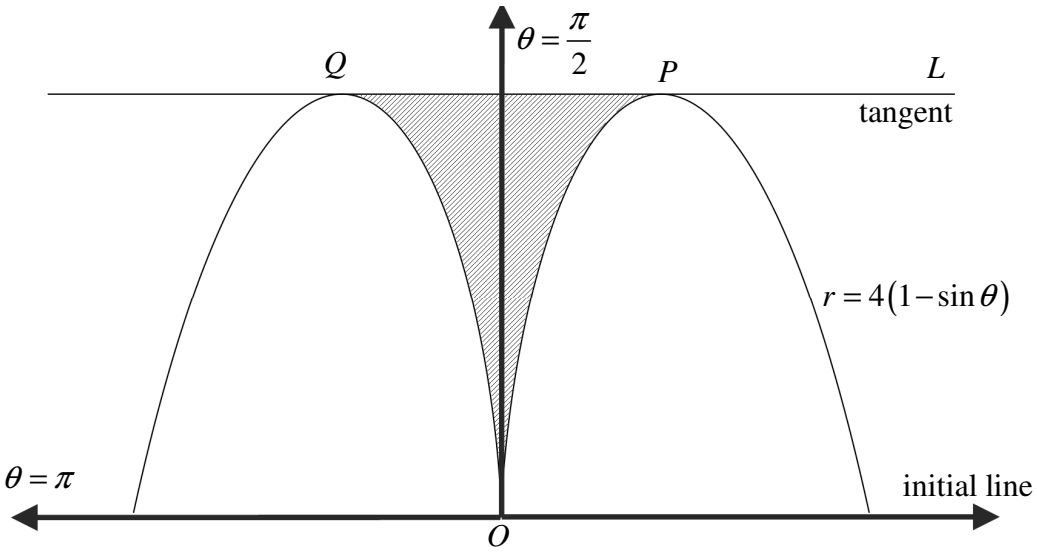
The function f is given by

$$f(x) \equiv \mathrm{e}^{2x+2}\left(\mathrm{e}^{2x} - 4\right), \quad x \in \mathbb{R} .$$

Show that

$$f\left[\ln\left(2\cosh\tfrac{1}{2}\right)\right] = \left(\mathrm{e}^2 - 1\right)^2 . \quad \textbf{(7)}$$

Question 6



The figure above shows the graph of the curve with polar equation

$$r = 4(1-\sin\theta),\ 0 \le \theta \le \pi .$$

The straight line L is a tangent to the curve parallel to the initial line, touching the curve at the points P and Q.

a) Find the polar coordinates of P and the polar coordinates of Q. **(5)**

b) Show that the area of the shaded region is exactly

$$15\sqrt{3} - 8\pi .$$ **(8)**

Question 7

$$z^3 = \left(1+\mathrm{i}\sqrt{3}\right)^8 (1-\mathrm{i})^5,\ z \in \mathbb{C} .$$

Determine the three roots of the above equation.

Give the answers in the form $k\sqrt{2}\,\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \le \pi$, $k \in \mathbb{Z}$. **(7)**

Question 8

It is given that the variables $x = f(t)$ and $y = g(t)$ satisfy the following coupled first order differential equations.

$$\frac{dx}{dt} = x + \frac{2}{3}y \qquad \text{and} \qquad \frac{dy}{dt} = 3y - \frac{3}{2}x\,.$$

Given further that $x = 1$, $y = 3$ at $t = 0$, solve the differential equations to obtain simplified expressions for $f(t)$ and $g(t)$. **(13)**