IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper M

Difficulty Rating: 3.5333/1.6210

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=12\left(x+\mathrm{e}^x\right). \qquad (7)$$

Question 2

Find the exact value of the following integral.

$$\int_{\mathrm{e}}^{\infty}\frac{1-\ln x}{x^2}\,dx. \qquad (6)$$

Question 3

$$f(r)\equiv r^2(r+1)^2-(r-1)^2r^2,\ r\in\mathbb{N}.$$

a) Simplify $f(r)$ as far as possible. **(1)**

b) Use the method of differences to show that

$$\sum_{r=1}^{20}r^3 = 44100. \qquad (6)$$

Question 4

By considering the binomial expansion of $(\cos\theta+\mathrm{i}\sin\theta)^4$ show that

$$\tan 4\theta\equiv\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}. \qquad (6)$$

Question 5

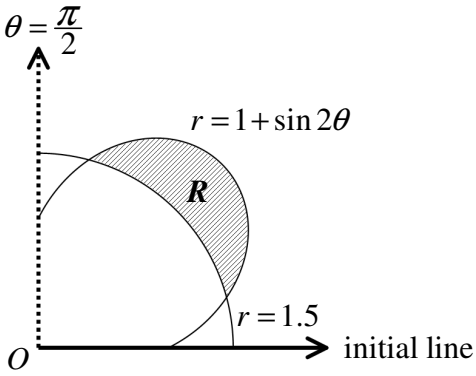
$$y = e^{2x} \sin 3x .$$

a) Use standard results to find the series expansion of y, up and including the term in x^4. **(5)**

b) Hence find an approximate value for

$$\int_0^{0.1} e^{2x} \sin 3x \ dx . \quad \textbf{(3)}$$

Question 6



The diagram above shows the curves with polar equations

$$r = 1 + \sin 2\theta , \ 0 \le \theta \le \tfrac{1}{2}\pi ,$$

$$r = 1.5 , \ 0 \le \theta \le \tfrac{1}{2}\pi .$$

a) Find the polar coordinates of the points of intersection between the two curves. **(3)**

The finite region R, is bounded by the two curves and is shown shaded in the figure.

b) Show that the area of R is

$$\tfrac{1}{16}\left(9\sqrt{3} - 2\pi\right). \quad \textbf{(8)}$$

Question 7

$$5\cosh x + 3\sinh x = 12$$

Express the left side of the above equation in the form $R\cosh(x+\alpha)$, where R and α are positive constants, and use it to show that

$$x = \ln\left(A \pm \sqrt{B}\right),$$

where A and B are constants to be found. **(11)**

Question 8

$$y = \arctan x, \quad x \in \mathbb{R}.$$

a) By writing the above equation in the form $x = g(y)$, show that

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}. \qquad \textbf{(4)}$$

The function f is defined as

$$f(x) = \arctan\sqrt{x}, \quad x \in \mathbb{R}, \; x \geq 0.$$

b) Show further that

$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(3x+1)(x+1)^{-2}. \qquad \textbf{(4)}$$

Question 9

$$\frac{dy}{dx}+\frac{y}{x}=\frac{5}{\left(x^2+2\right)\left(4x^2+3\right)},\ x>0.$$

Given that $y=\frac{1}{2}\ln\frac{7}{6}$ at $x=1$, show that the solution of the above differential equation can be written as

$$y=\frac{1}{2x}\ln\left(\frac{4x^2+3}{2x^2+4}\right). \qquad \textbf{(11)}$$