IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper M

Difficulty Rating: 3.3533/1.5113

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Find, in terms of k, the inverse of the following 2×2 matrix.

$$\mathbf{M}=\begin{pmatrix} k & k+1 \\ k+1 & k+2 \end{pmatrix}.$$

Verify your answer by multiplication. **(6)**

Question 2

$$w=\frac{-9+3\mathrm{i}}{1-2\mathrm{i}}.$$

Find the modulus and the argument of the complex number w. **(5)**

Question 3

The roots of the quadratic equation

$$2x^2-8x+9=0$$

are denoted, in the usual notation, as α and β.

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^2-1 \quad \text{and} \quad \beta^2-1. \qquad \textbf{(7)}$$

Question 4

The 3×3 matrix $\mathbf{R}$ is defined by

$$\mathbf{R}=\begin{pmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

The image of the straight line L, when transformed by $\mathbf{R}$, is the straight line with Cartesian equation

$$\frac{x+2}{3}=\frac{y-1}{2}=\frac{z-1}{4}.$$

Find a Cartesian equation for L. **(8)**

Question 5

The point P lies on the curve with equation

$$y=x^2,\quad x\geq 0.$$

The straight line L_1 is parallel to the x axis and passes through P. The finite region R_1 is bounded by the curve, L_1 and the y axis.

The straight line L_2 is parallel to the y axis and passes through P. The finite region R_2 is bounded by the curve, L_2 and the x axis.

When R_1 is fully revolved about the y axis the volume of the solid formed is equal to the volume of the solid formed when R_2 is fully revolved about the x axis.

Determine the x coordinate of P. **(8)**

Question 6

Given that z and w are complex numbers prove that

$$|z+w|^2 - |z-\overline{w}|^2 = 4 \operatorname{Re} z \operatorname{Re} w,$$

where $\overline{w}$ denotes the complex conjugate of w. **(8)**

Question 7

Relative to a fixed origin O, the following points are given.

$$A(7,2,6), \quad B(9,10,4) \quad \text{and} \quad C(-3,-2,-2).$$

a) Determine a vector, with integer components, which is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$, and hence deduce a Cartesian equation of the plane Π, which passes through A, B and C.

*You may **NOT** use the vector (cross) product for this part.* **(7)**

The straight line l is perpendicular to Π and passes through the point $P(11,3,-4)$.

The point Q is the intersection of l and Π.

b) Find the coordinates of Q. **(5)**

c) Calculate the distance PQ. **(2)**

Question 8

Prove by induction that for all natural numbers n,

$$4^{n+1} + 5^{2n-1}$$

is divisible by 21. **(7)**

Question 9

$$f(n)=\sum_{r=1}^{n}\left[r^3-r\right],\qquad n\in\mathbb{N}.$$

a) Use standard summation results to find a fully factorized expression for $f(n)$.

(4)

b) Hence solve the equation

$$\sum_{r=5}^{10}\left[r^3-r+6k\right]\;-\;\sum_{r=1}^{12}\left[r^2+k^2\right]\;=\;70 \qquad \textbf{(8)}$$