IYGB GCE

Core Mathematics C4

Advanced

Practice Paper C

Difficulty Rating: 3.2067 /1.4320

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The binomial expression $(1+x)^{\frac{1}{2}}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Determine the expansion of $(1+x)^{\frac{1}{2}}$ up and including the term in x^3. **(3)**

b) Use the expansion of part **(a)** to find the expansion of $\sqrt{4+2x}$, up and including the term in x^3. **(3)**

c) State the range of values of x for which the expansion of $\sqrt{4+2x}$ is valid. **(1)**

Question 2

A curve C has implicit equation

$$x^3 + 2xy = \mathrm{e}^y.$$

Show **clearly** that

$$\frac{dy}{dx} = \frac{x^3 + 2y}{x^3 + 2xy - 2x}. \qquad \textbf{(5)}$$

Question 3

The curve C has equation

$$y = x^{\frac{3}{2}}\sqrt{\ln x}\,,\ x > 0.$$

The region bounded by C, the x axis and the straight lines with equations $x = 1$ and $x = \mathrm{e}$ is rotated through $360°$ about the x axis.

Use integration by parts to show that the volume of the solid formed is

$$\frac{1}{16}\pi\left(3\mathrm{e}^4 + 1\right). \qquad \textbf{(6)}$$

Question 4

A curve C is given parametrically by the equations

$$x = t^2 - 8t + 12, \quad y = t - 4, \quad t \in \mathbb{R}.$$

a) Find the coordinates of the points where C crosses the coordinate axes. **(5)**

The point $P(-3,1)$ lies on C.

b) Show that the equation of the normal to C at the point P is

$$y + 2x + 5 = 0.$$ **(5)**

c) Show that a Cartesian equation of C is

$$y^2 = x + 4.$$ **(2)**

Question 5

The points A and B have coordinates $(11,15,4)$ and $(13,23,7)$, respectively.

a) Find a vector equation for the straight line l that passes through A and B. **(3)**

The point P lies on l, so that OP is perpendicular to l, where O is the origin.

b) Show, without verification, that the coordinates of P are $(7,-1,-2)$. **(5)**

c) Calculate the area of the triangle OAB. **(3)**

Question 6

Liquid dye is poured onto a large flat cloth and forms a circular stain, the area of which grows at a steady rate of $1.5\ \text{cm}^2\text{s}^{-1}$.

Calculate, correct to three significant figures, …

a) … the radius, in cm, of the stain after 4 seconds. $(\mathbf{3})$

b) … the rate, in $\text{cm}\,\text{s}^{-1}$, of increase of the radius of the stain after 4 seconds. $(\mathbf{4})$

Question 7

The mass, m grams, of a burning candle, t hours after it was lit up, satisfies the differential equation

$$\frac{dm}{dt} = -k\left(m - 10\right),$$

where k is a positive constant.

a) Solve the differential equation to show that

$$m = 10 + A\text{e}^{-kt},$$

where A is a non-zero constant. $(\mathbf{5})$

The initial mass of the candle was 120 grams, and 3 hours later its mass has halved.

b) Find the value of A and show further that

$$k = \frac{1}{3}\ln\left(\frac{11}{5}\right).$$ $(\mathbf{4})$

c) Calculate, correct to three significant figures, the mass of the candle after a further period of 3 hours has elapsed. $(\mathbf{2})$

Question 8

$$\frac{2u^2}{(u-1)(u+1)} \equiv A + \frac{B}{u+1} + \frac{C}{u-1}.$$

a) Find the value of A, B and C in the above identity. **(4)**

b) By using the substitution $u^2 = x+1$, or otherwise, find an exact value for

$$\int_3^8 \frac{\sqrt{x+1}}{x}\, dx. \qquad \textbf{(8)}$$

The table below shows some tabulated values for the equation $y = \frac{\sqrt{x+1}}{x}$, $3 \le x \le 8$.

x	3	4	5	6	7	8
y	0.6667	0.5590	0.4899		0.4041	0.3750

c) Complete the missing value in the table. **(1)**

d) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_3^8 \frac{\sqrt{x+1}}{x}\, dx. \qquad \textbf{(2)}$$

e) Calculate the difference between the exact value, found in part (b), and the trapezium rule estimate, found in part (d), and hence state whether the trapezium rule produces an overestimate or an underestimate. **(1)**