IYGB GCE

Core Mathematics C3

Advanced

Practice Paper X

Difficulty Rating: 4.02/2.0202

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve C has equation

$$y = \frac{x^2 - 6x + 12}{4x - 11},\ x \in \mathbb{R},\ x \neq \frac{11}{4}\ .$$

a) Find a simplified expression for $\frac{dy}{dx}$. **(4)**

b) Determine the range of values of x, for which y is decreasing. **(4)**

Question 2

Solve the following trigonometric equation

$$\sin 2\theta = \cot\theta, \qquad 0 \leq \theta \leq 180°. \qquad \textbf{(6)}$$

Question 3

The function f is defined by

$$f(x) = 1 + \sqrt{x},\ x \in \mathbb{R},\ x \geq 0.$$

a) Evaluate $ff(9)$. **(1)**

b) Find an expression for the inverse function, $f^{-1}(x)$. **(2)**

c) Sketch in the same diagram the graph of $f(x)$ and the graph of $f^{-1}(x)$, clearly marking the line of reflection between the two graphs. **(3)**

d) Show that $x = \frac{3 + \sqrt{5}}{2}$ is the only solution of the equation $f(x) = f^{-1}(x)$. **(4)**

Question 4

At the point P which lies on the curve with equation

$$x = \ln\left(y^3 - 4y\right),$$

the gradient is 2.

The point P is close to the point with coordinates $\left(\frac{11}{2}, \frac{13}{2}\right)$.

a) Show that the y coordinate of P is a solution of the equation

$$y = \frac{6y^2 - 8}{y^2 - 4}. \qquad \textbf{(6)}$$

b) By using an iterative formula based on the equation of part **(a)**, determine the coordinates of P correct to three decimal places. **(5)**

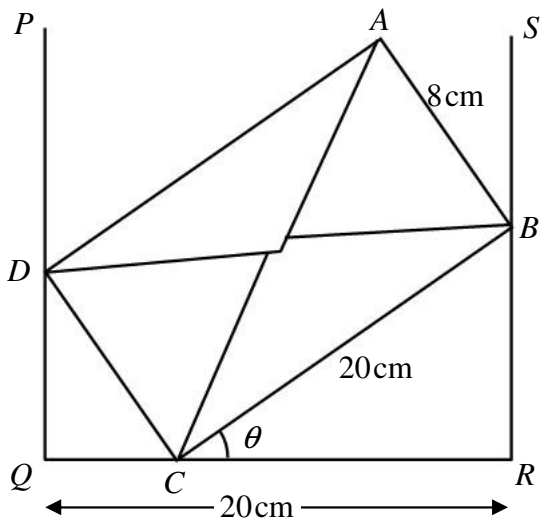
Question 5

A curve has equation

$$y = 4e^{2-x} - e^{4-2x}, \ x \in \mathbb{R}.$$

Use differentiation to find the exact coordinates of the stationary point of the curve, and further determine its nature. **(9)**

Question 6



The figure above shows the cross section of a letter inside a filling slot.

The letter $ABCD$ is modelled as a rectangle with $|AB| = 8\text{cm}$ and $|BC| = 20\text{cm}$.

The width of the filling slot QR is also 20 cm and the angle BCR is θ.

a) Show clearly that

$$5\cos\theta + 2\sin\theta = 5. \qquad (4)$$

b) Express $5\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, $R > 0$, $0 < a < 90°$. **(3)**

c) Hence, determine the value of θ. **(5)**

Question 7

The function f is defined as

$$f : x \mapsto \ln|4x-12|,\ x \in \mathbb{R},\ x \neq 3.$$

Consider the following sequence of transformations T_1, T_2 and T_3

$$\ln x \xrightarrow{T_1} \ln|x| \xrightarrow{T_2} \ln|x-12| \xrightarrow{T_3} \ln|4x-12|.$$

a) Describe geometrically the transformations T_1, T_2 and T_3 **(6)**

b) Hence sketch the graph of f.
Indicate clearly any intersections with the axes and the equation of its asymptote. **(3)**

Question 8

A curve has equation given by

$$y = x\sqrt{x+1},\ x \geq -1.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}. \quad \textbf{(5)}$$

The function f is defined as

$$f(x) = x\sqrt{x+1}\sin 2x,\ x \geq 1.$$

b) Show further that

$$f'\left(\frac{\pi}{2}\right) = -\pi\sqrt{\frac{\pi}{2}+1}\ . \quad \textbf{(5)}$$